Near-horizon extreme Kerr magnetospheres

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Outline

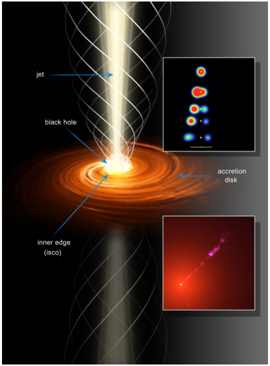
Physical setup

Active galactic nuclei (AGN)
are the brightest regions at the center of a galaxy.

Spinning supermassive black holes
are believed to be hosted at the center of the AGN.

AGN is a wonderful playground
of high-energy physics phenomena in strong gravity regime, yet to be fully understood. Among them:

1. matter accretion onto the black hole;
2. collimated jets;
3. magnetospheres with different field lines topologies: radial, vertical, parabolic, hyperbolic.



Theoretical setup

Some important facts to know:

- a rotating black hole immersed in an external magnetic field induces an electric field with Lorentz invariant $\tilde{F}_{\mu\nu} F^{\mu\nu} \neq 0$ [Wald ('74)];
- a pair-production mechanism operates to produce a plasma-filled magnetosphere until $\tilde{F}_{\mu\nu} F^{\mu\nu} = 0$;
- the magnetosphere is force-free. It means that the plasma rest-mass density is negligible with respect to the electromagnetic energy density;
- force-free magnetosphere extracts electromagnetically energy and angular momentum from the rotating black hole [Blandford, Znajek ('77)].

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Outline

Derivation of FFE equations (1)

Let $g_{\mu\nu}$ be the background spacetime metric and A_μ be the gauge potential. The Maxwell field is $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$. It obeys Maxwell's equations:

$$\nabla_{[\sigma} F_{\mu\nu]} = 0, \qquad \nabla_\nu F^{\mu\nu} = j^\mu$$

with j^μ being the electric current density.

The full energy-momentum tensor is

$$T^{\mu\nu} = T^{\mu\nu}_{em} + T^{\mu\nu}_{matter}$$

Assumption 1: we neglect any backreaction to the spacetime geometry.

The energy-momentum conservation

$$0 = \nabla_\nu T^{\mu\nu}_{em} + \nabla_\nu T^{\mu\nu}_{matter} = -F^{\mu\nu} j_\nu + \nabla_\nu T^{\mu\nu}_{matter},$$

governs the transfer of energy and momentum between the electromagnetic field and the matter content.

Derivation of FFE equations (2)

Assumption 2: the exchange of energy and momentum from the EM field and the matter is negligible.

Then, energy-momentum conservation implies that

$$\boxed{F_{\mu\nu} j^\nu = 0,}$$

the Lorentz force density is zero.

Thus, FFE equations are

$$\boxed{\nabla_{[\sigma} F_{\mu\nu]} = 0, \qquad \nabla_\nu F^{\mu\nu} = j^\mu, \qquad F_{\mu\nu} j^\nu = 0,}$$

or, eliminating j^μ,

$$\nabla_{[\sigma} F_{\mu\nu]} = 0, \qquad F_{\mu\nu} \nabla_\sigma F^{\nu\sigma} = 0.$$

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Some properties

FFE equations:

$$\nabla_{[\sigma} F_{\mu\nu]} = 0, \qquad \nabla_\nu F^{\mu\nu} = j^\mu, \qquad F_{\mu\nu} j^\nu = 0,$$

or, in differential form,

$$\boxed{dF = 0, \qquad d \star F = \star J, \qquad J \wedge \star F = 0,}$$

1. Any vacuum Maxwell solution ($j^\mu = 0$) is trivially force-free;
2. Assume $j^\mu \neq 0$.
   Because $F_{\mu\nu} j^\nu = 0$, then $F_{[\mu\nu} F_{\sigma\rho]} j^\rho = 0 \Rightarrow F_{[\mu\nu} F_{\sigma\rho]} = 0$ ($F \wedge F = 0$)
   In other words, force-free fields are degenerate: $F_{\mu\nu} = \alpha_\mu \beta_\nu - \alpha_\nu \beta_\mu$;
3. FFE is nonlinear $\Rightarrow$ no general superposition principle.
   A sufficient condition for linear superposition of two solutions F_1 and F_2 is to have collinear currents $J_1 \propto J_2$, up to an arbitrary function.

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NHEK metric and properties

NHEK spacetime describes the region near the horizon of the extreme Kerr. It can be derived from extreme Kerr ($a = M$) metric, performing the scaling

$$T \to \frac{\lambda}{2M}t, \quad R \to \frac{r-M}{\lambda M}, \quad \Phi \to \phi - \frac{t}{2M};$$

In Poincaré coordinates, NHEK metric reads

$$ds^2 = 2M^2\Gamma(\theta)\left[-R^2dT^2 + \frac{dR^2}{R^2} + d\theta^2 + \gamma^2(\theta)(d\Phi + RdT)^2\right],$$

Its main properties are:

1. it has an enhanced isometry group $SL(2,\mathbb{R}) \times U(1)$ generated by:

$$Q_0 = \partial_\Phi, \quad H_+ = \sqrt{2}\partial_T, \quad H_0 = T\partial_T - R\partial_R,$$
$$H_- = \sqrt{2}\left[\frac{1}{2}\left(T^2 + \frac{1}{R^2}\right)\partial_T - TR\partial_R - \frac{1}{R}\partial_\Phi\right],$$

obeying $[H_0, H_\pm] = \mp H_\pm, \quad [H_+, H_-] = 2H_0, \quad [Q_0, H_\pm] = 0 = [Q_0, H_0].$

2. it has no globally timelike Killing vectors. ∂_T is timelike for $\gamma^2(\theta) < 1$ and becomes null at the velocity of light surface $\gamma^2(\theta) = 1$.

Outline

Defining the problem

We want to find solutions to FFE equations around NHEK spacetime,

$$\boxed{dF = 0, \qquad d \star F = \star J, \qquad J \wedge \star F = 0,}$$

further obeying the highest-weight (HW) conditions:

$$\boxed{\mathcal{L}_{H_+} F = 0, \quad \mathcal{L}_{H_0} F = hF, \quad \mathcal{L}_{Q_0} F = iqF,}$$

where $h \in \mathbb{C}$ is the weight of F, while $q \in \mathbb{Z}$ is the $U(1)$-charge.

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Solving FFE around NHEK

1. Define real $SL(2,\mathbb{R})$ covariant basis for:
   1-form μ^i, such that $\mathcal{L}_{H_+}\mu^i = 0 = \mathcal{L}_{H_0}\mu^i$, and
   2-form w^j, such that $\mathcal{L}_{H_+}w^j = 0$, $\mathcal{L}_{H_0}w^j = w^j$;
2. Consider A, F and J in the HW representation and expand them:

   $$A_{(h,q)} = \Phi_{(h,q)} a_i(\theta)\mu^i, \quad F_{(h,q)} = \Phi_{(h-1,q)} f_i(\theta) w^i, \quad J_{(h,q)} = \Phi_{(h,q)} j_i(\theta)\mu^i,$$

   where $H_+\Phi_{(h,q)} = 0 = \partial_\theta \Phi_{(h,q)}$, $H_0\Phi_{(h,q)} = h\Phi_{(h,q)}$, $Q_0\Phi_{(h,q)} = iq\Phi_{(h,q)}$.
   Maxwell's equations constraint the functions f_i and j_i in terms of a_i.
3. Fix the gauge $a_4 = 0$, $\forall h$.
4. Rewrite the force-free condition $J \wedge \star F = 0$ to get three nonlinear ODEs in terms of a_1, a_2, a_3.
5. Classify solutions according to their HW representation labeled by (h, q).

Outline

Potentially physical solutions (definition)

Thus far, we have a list of complex (and therefore) unphysical solutions.
A potentially physical solution must be

1. real;
2. magnetically dominated or null, i.e., we demand that the Lorentz scalar invariant $\star(F \wedge \star F) = -\frac{1}{2}F^{\mu\nu}F_{\mu\nu} \leq 0$;
3. such that the energy and angular momentum flux densities to be finite

$$\dot{\mathcal{E}} \equiv \sqrt{-\gamma}\, T^{\mu}{}_{\nu} n_{\mu} (\partial_T)^{\nu} \propto E(\theta) R^{2-2h},$$
$$\dot{\mathcal{J}} \equiv \sqrt{-\gamma}\, T^{\mu}{}_{\nu} n_{\mu} (\partial_\Phi)^{\nu} \propto J(\theta) R^{1-2h},$$

   (with n^{μ} the unit normal and γ induced metric on constant R surface)

   3.1. either at the spatial boundary of the NHEK spacetime, or
   3.2. with respect to an asymptotically flat observer.

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Potentially physical solutions (finite energy and angular momentum, 1st class)

A potentially physical solution must be

3. such that the energy and angular momentum flux densities to be finite

$$\dot{\mathcal{E}} \equiv \sqrt{-\gamma}\, T^{\mu}{}_{\nu} n_{\mu} (\partial_T)^{\nu} \propto E(\theta) R^{2-2h},$$
$$\dot{\mathcal{J}} \equiv \sqrt{-\gamma}\, T^{\mu}{}_{\nu} n_{\mu} (\partial_\Phi)^{\nu} \propto J(\theta) R^{1-2h},$$

at the spatial boundary $R \to \infty$ of the NHEK spacetime implies

$$\boxed{\mathrm{Re}(h) > 1.}$$

Such class of solutions might be useful to discuss holography in near-horizon geometries and we call them near-horizon solutions.

Potentially physical solutions (finite energy and angular momentum, 2nd class)

A potentially physical solution must be

3. such that the energy and angular momentum flux densities to be finite

$$\dot{\mathcal{E}} \equiv \sqrt{-\gamma}\, T^{\mu}{}_{\nu} n_{\mu} (\partial_T)^{\nu} \propto E(\theta) R^{2-2h},$$
$$\dot{\mathcal{J}} \equiv \sqrt{-\gamma}\, T^{\mu}{}_{\nu} n_{\mu} (\partial_\Phi)^{\nu} \propto J(\theta) R^{1-2h},$$

with respect to an asymptotically flat observer.
An asymptotically flat observer measures

$$\mathcal{E}'{}_{out} - \Omega_{ext} \mathcal{J}'{}_{out} \sim \lambda^{2-2h} \dot{\mathcal{E}}, \quad \mathcal{J}'{}_{out} \sim \lambda^{1-2h} \dot{\mathcal{J}}$$

Here, the prime means derivative wrt the asymptotically observer's time.
Hence, solutions which admit finite and nonvanishing fluxes are those with [see also 1602.01833]

$$\boxed{\frac{1}{2} \leq \mathrm{Re}(h) \leq 1, \quad \text{and} \quad \left(\mathrm{Re}(h) - \frac{1}{2}\right) J(\theta) = 0.}$$

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Outline

Maximally symmetric solution and Meissner-like effect

Invariance under the full isometry group $SL(2,\mathbb{R}) \times U(1)$ implies

$$A = A_0(\theta)(RdT + d\Phi).$$

Force-free condition $J \wedge \star F = 0$ implies $A_0' O[A_0] = 0$.

The solution corresponding to $A_0 = const$ is electrically dominated.
The other one with $O[A_0] = 0$ is a solution to vacuum electrodynamics:

$$O[A_0] = 0 \Rightarrow A_0(\theta) = Q_e \cos\left[\theta_0 + 2\arctan(\cos(\theta))\right], \quad Q_e : \text{electric charge}$$

For Kerr black hole $Q_e = 0$ and therefore there is no electromagnetic field close to the horizon region at extremality.
This is the so-called Meissner-like effect for black holes [see also Bicak, Janis (1985) and 1602.01833].

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$(h = 1, q = 0)$ regular solution

This solution is the only one that we found to be regular at the future horizon:

$$A_{(h=1,q=0)} = a(\theta) d \left(T - \frac{1}{R} \right) ,$$

where $a(\theta)$ is a function of the polar coordinate, obeying the Znajek's boundary condition at the horizon:

$$a(\theta) = 2M \left[(\Omega - \Omega_H) \partial_\theta \Psi \right]_{r=r_H} ,$$

with Ω is the angular velocity of the magnetic lines and Ψ is the magnetic flux.

Moreover, by a different analysis in [1602.01833], this solution was shown to be the universal near-horizon limit for force-free plasma around extreme Kerr spacetime and might be astrophysically relevant.

Summary

- we solved FFE around NHEK;
- we refined and extended the list of formal solutions;
- we introduced physical criteria to select potentially physical solutions;
- we realised that not all the NHEK spacetime is physical due to the presence of the velocity of light surface: physical regions are those close to the north and south poles.

One of the main left questions is how to glue these near-horizon solutions to asymptotically flat spacetime.

Thank you for your attention

Potentially physical solutions (reality condition)

A potentially physical solution must be

1. real

Since $A^*_{(h,q)} = \Phi_{(h^*,-q)} a_i^* \mu^i$:

1.1. if $h \in \mathbb{R}$, $q = 0$, and a_i is real then the solution is real;

1.2. if J and J^* are collinear, then one can linearly superpose the two solutions F and F^* to get the real solution;

1.3. otherwise, no general superposition principle and one might attempt to construct real solutions in different ways.

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Potentially physical solutions (magnetically dominated or null)

A potentially physical solution must be

2. magnetically dominated or null.

We demand that the Lorentz scalar invariant $\star(F \wedge \star F) \leq 0$.

Physically, for electrically dominated solutions there exists a local inertial frame where the magnetic field is zero. This, in turn, means that drift velocity of charged particles is superluminal.
Mathematically, FFE equations with $\star(F \wedge \star F) > 0$ are not deterministic (not hyperbolic).