Scattering amplitudes from the amplituhedron NMHV volume forms

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Based on 1512.04954 with Livia Ferro, Tomasz Łukowski, Matteo Parisi

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Scattering amplitudes . . .

Scattering amplitudes are central objects in QFT. Interesting

- as an intermediate step to compute observables;
- as a means to gain insight into the formal structure of a specific model.

How are they traditionally computed?

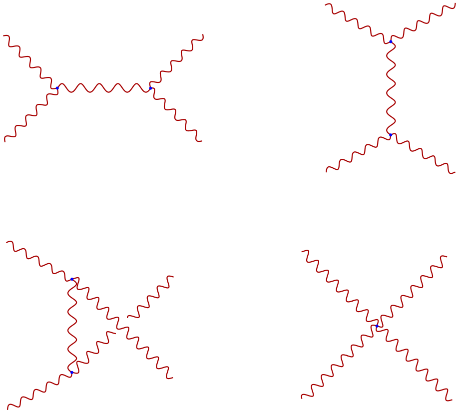
1. Stare at Lagrangian and extract the Feynman rules;
2. draw every possible Feynman diagram contributing to the process of interest;
3. evaluate each one of those and add up the results.

Straightforward enough. What could possibly go wrong?

. . . are complicated?

Consider tree-level gluon amplitudes in QCD.

$2g \to 2g$: 4 diagrams



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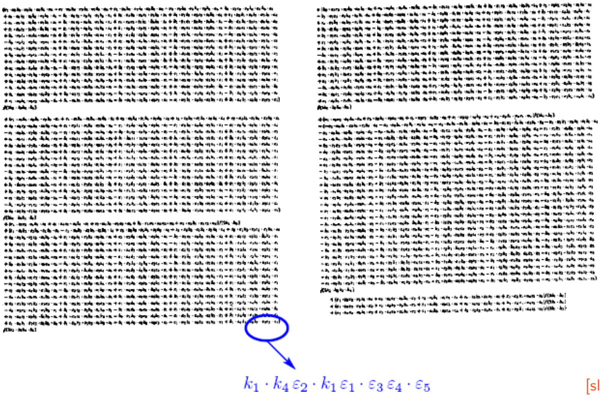
Consider tree-level gluon amplitudes in QCD.

$2g \to 3g$: 25 diagrams



[slide by Z. Bern]

. . . are simpler than expected!

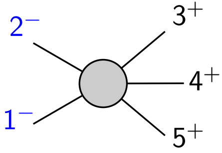
Consider tree-level gluon amplitudes in QCD.

$2g \to 3g$: 10 colour-ordered diagrams

$$A_5^{\text{tree}}(1^{\pm}, 2^{\pm}, 3^{\pm}, 4^{\pm}, 5^{\pm}) = 0$$
$$A_5^{\text{tree}}(1^{\mp}, 2^{\pm}, 3^{\pm}, 4^{\pm}, 5^{\pm}) = 0$$

$$A_5^{\text{tree}}(1^{-}, 2^{-}, 3^{+}, 4^{+}, 5^{+}) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

$$\langle ij\rangle = \epsilon_{\alpha\beta}\lambda_i^{\alpha}\lambda_j^{\beta} = \begin{vmatrix} \lambda_i^1 & \lambda_j^1 \\ \lambda_i^2 & \lambda_j^2 \end{vmatrix}$$



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$$A_n^{\text{MHV}}(1^{+},\ldots,i^{-},\ldots,j^{-},\ldots,n^{+}) = \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

MHV = maximally helicity-violating

[Parke, Taylor]

The simplest quantum field theory

[Arkani-Hamed, Cachazo, Kaplan]

Most symmetric theory in 4D is *planar* $\mathcal{N}=4$ super Yang-Mills.

- Maximal susy: spectrum is organized in a single supermultiplet with 2 gluons ($g^{\pm}$), 8 gluinos ($\psi^{\pm}$), 6 scalars (φ), all *massless*.

$$\Omega = g^{+} + \eta^{A}\psi_{A} + \frac{1}{2}\eta^{A}\eta^{B}\varphi_{AB} + \\ + \frac{1}{3!}\eta^{A}\eta^{B}\eta^{C}\epsilon_{ABCD}\,\bar{\psi}^{D} + \frac{1}{4!}\eta^{A}\eta^{B}\eta^{C}\eta^{D}\epsilon_{ABCD}\,g^{-}$$

The simplest quantum field theory

[Arkani-Hamed, Cachazo, Kaplan]

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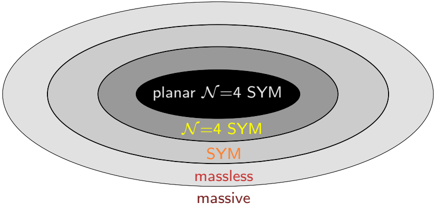
- (Ordinary + Dual) superconformal symmetries give rise to an infinite-dimensional Yangian algebra $Y\big(\mathfrak{psu}(2,2|4)\big)$

  [Drummond, Henn, Plefka]

- At weak coupling : more constrained, easier to compute
  At strong coupling : amenable to AdS/CFT techniques



[picture by L. Dixon]

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$\mathcal{N}=4$ SYM is a supersymmetric version of QCD:

- Tree-level gluon amplitudes coincide
- One-loop gluon amplitudes satisfy

$$\mathcal{A}_n^{\text{QCD}} = \mathcal{A}_n^{\mathcal{N}=4} - 4\mathcal{A}_n^{\mathcal{N}=1} + \mathcal{A}_n^{\text{scalar}}$$

The power of momentum twistors

"Masslessness" of the spectrum + conformal symmetry $\longrightarrow$ introduce momentum supertwistors for describing the kinematics.

Instead of	four-momenta p^μ	$\mu = 0, 1, 2, 3$ and
	Grassmann-odd η^{A}	$\mathsf{A} = 1, 2, 3, 4$
use	mom. supertwistors $\mathcal{Z}^{\mathcal{A}}$	$\mathcal{A} = \begin{pmatrix} \alpha, \dot{\alpha} = 0, 1, \dot{0}, \dot{1} \\ \mathsf{A} = 1, 2, 3, 4 \end{pmatrix}$

The geometry of momentum twistor superspace $\mathbb{CP}^{3|4}$ ensures masslessness of momenta and momentum conservation.

Generating function for *every* tree-level $\mathcal{N}=4$ SYM amplitude

$$\mathcal{L}_{n,k} = \frac{1}{\mathrm{GL}(k)} \int \frac{\mathrm{d}^{k\times n} c_{\alpha i}}{(1\cdots k)(2\cdots k+1)\cdots(n\cdots k-1)}\, \delta^{4|4}(C\cdot\mathcal{Z})$$

[(Arkani-Hamed, Cachazo, Cheung, Kaplan) (Mason, Skinner)]

Towards the amplituhedron

Two remarkable results inspired the amplituhedron:

$$\mathcal{L}_{n,k} = \frac{1}{\mathrm{GL}(k)} \int \frac{\mathrm{d}^{k\times n} c_{\alpha i}}{(1\cdots k)(2\cdots k+1)\cdots(n\cdots k-1)}\, \delta^{4|4}(C\cdot \mathcal{Z})$$

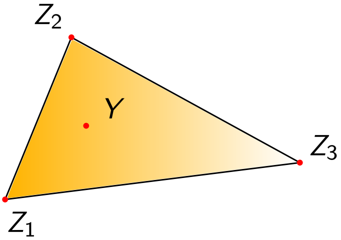
One-to-one correspondence between residues of $\mathcal{L}_{n,k}$ and cells of the positive Grassmannian $G_+(k,n)$.

[Arkani-Hamed, Bourjaily, Cachazo, Goncharov, Postnikov, Trnka]

NMHV tree-level amplitudes can be thought of as volumes of polytopes in twistor space.

[Hodges]

Positive means inside



Triangle in $\mathbb{RP}^2$

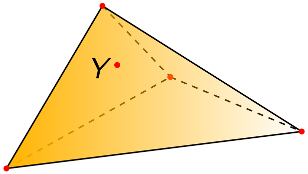
Interior of a triangle

$$Y^A = c_1 Z_1^A + c_2 Z_2^A + c_3 Z_3^A \qquad , \qquad c_1, c_2, c_3 > 0$$

Points inside are described by the positive triple

$$(c_1 \; c_2 \; c_3)/\mathrm{GL}(1)$$

Positive means inside



Simplex in $\mathbb{RP}^{n-1}$

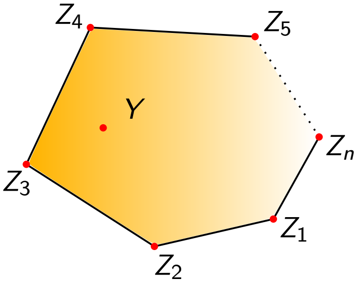
Interior of a simplex

$$Y^A = \sum_i c_i Z_i^A \quad , \quad c_i > 0$$

Points inside are described by the positive n-tuple

$$(c_1 \; c_2 \; \ldots \; c_n)/\mathrm{GL}(1) \quad , \quad \text{a point in } G_+(1,n) \; .$$

Positive also means convex



Polygon in $\mathbb{RP}^m$

Interior of a n-gon with vertices $Z_1, \ldots, Z_n$ is only canonically defined if

$$Z = \begin{pmatrix} Z_1^1 & Z_2^1 & \ldots & Z_n^1 \\ \vdots & \vdots & & \vdots \\ Z_1^{1+m} & Z_2^{1+m} & \ldots & Z_n^{1+m} \end{pmatrix} \in M_+(1+m, n)$$

Tree-level amplituhedron

Interior of an n-polyhedron in $\mathbb{RP}^m$

$$\mathfrak{A}^{\text{tree}}_{n,1;m}[Z] = \left\{ Y^A = \sum_i c_i Z_i^A \,,\; \begin{matrix} C = (c_1 \;\ldots\; c_n) \in G_+(1,n) \\ Z = (Z_1 \;\ldots\; Z_n) \in M_+(1+m,n) \end{matrix} \right\}$$

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Generalize this picture
to account for N^{k}MHV amplitudes

Tree-level amplituhedron

$$\mathfrak{A}^{\text{tree}}_{n,k;m}[Z] = \left\{ Y \in G(k,k+m) \ : \ Y = C\cdot Z \ , \ \begin{array}{l} C \in G_+(k,n) \\ Z \in M_+(k+m,n) \end{array} \right\}$$

[Arkani-Hamed, Trnka]

The volume form

Volume form

Top-dimensional differential form $\tilde{\Omega}^{(m)}_{n,k}$ defined on $\mathfrak{A}^{\text{tree}}_{n,k;m}$ with only logarithmic singularities on its boundaries.

top-dimensional : $Y \in G(k, k+m) \longrightarrow \tilde{\Omega}^{(m)}_{n,k}$ is an mk-form

log-singularity : approaching any boundary, $\tilde{\Omega}^{(m)}_{n,k} \sim \dfrac{\mathrm{d}\alpha}{\alpha}$

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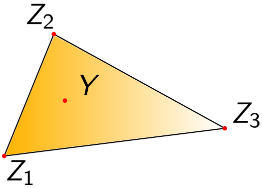

If $Y = \alpha_1 Z_1 + \alpha_2 Z_2 + Z_3$,

$$\tilde{\Omega}^{(2)}_{3,1} = \frac{\mathrm{d}\alpha_1}{\alpha_1} \wedge \frac{\mathrm{d}\alpha_2}{\alpha_2} = \frac{1}{2}\,\frac{\langle 123\rangle^2\,\langle Y \mathrm{d}^2 Y\rangle}{\langle Y12\rangle\langle Y23\rangle\langle Y31\rangle}$$

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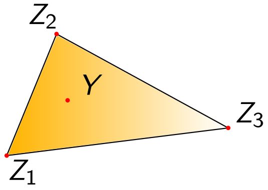

If $Y = \alpha_1 Z_1 + \alpha_2 Z_2 + Z_3$,

$$\Omega^{(2)}_{3,1} = \underbrace{\frac{1}{2}\frac{\langle 123\rangle^2}{\langle Y12\rangle\langle Y23\rangle\langle Y31\rangle}}_{\text{Area of (dual) triangle}} \equiv [1\,2\,3]$$

Get to the amplitude

Morally ...

$$\frac{\mathcal{A}_{n,k}}{\mathcal{A}_{n,0}} = \int_{\mathfrak{A}^{\text{tree}}_{n,k;m}} \tilde{\Omega}^{(m)}_{n,k}(Y, Z)$$

"Scattering amplitudes are volumes of (dual) amplituhedra"

The physics, i.e. the kinematics of scattering particles, is encoded in Z^A variables, bosonized version of momentum supertwistors $\mathcal{Z}^{\mathcal{A}}$:

$$Z_i^A = \begin{pmatrix} \lambda_i^\alpha \\ \tilde{\mu}_i^{\dot{\alpha}} \\ \phi_1 \cdot \chi_i \\ \vdots \\ \phi_k \cdot \chi_i \end{pmatrix}, \quad \text{with} \quad \begin{array}{ll} (\lambda_i^\alpha, \tilde{\mu}_i^{\dot{\alpha}}) & \text{are bosonic d.o.f. of } \mathcal{Z}^{\mathcal{A}} \\ \chi_i^A & \text{are fermionic d.o.f. of } \mathcal{Z}^{\mathcal{A}} \\ \phi_\alpha^A & \text{are } \textit{auxiliary} \text{ fermionic d.o.f.} \end{array}$$

Get to the amplitude

Precisely ...

$$\frac{\mathcal{A}_{n,k}}{\mathcal{A}_{n,0}} = \int d^{m\cdot k}\phi \;\; \Omega^{(m)}_{n,k}(Y^*, Z)$$

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$(\lambda_i^\alpha, \tilde{\mu}_i^{\dot\alpha})$	are bosonic d.o.f. of $\mathcal{Z}^{\mathcal{A}}$
χ_i^A	are fermionic d.o.f. of $\mathcal{Z}^{\mathcal{A}}$
ϕ_α^A	are *auxiliary* fermionic d.o.f.

1. Scattering amplitudes in planar $\mathcal{N} = 4$ super Yang-Mills
   - Heavy computations for simple amplitudes
   - $\mathcal{N} = 4$ SYM
2. Introduction to the tree-level Amplituhedron
   - Positive geometry
   - Amplitudes as volumes
3. **NMHV volume forms from symmetry**
   - Capelli differential equations
   - The $k = 1$ solution
   - Examples of NMHV volume forms
4. Conclusions

Covariance and scaling properties

[Ferro, Łukowski, AO, Parisi]

Integral representation of the volume

$$\Omega^{(m)}_{n,k}(Y,Z) = \int_\gamma \frac{\mathrm{d}^{k\times n} c_{\alpha i}}{(1\cdots k)\cdots(n\cdots k-1)} \prod_{\alpha=1}^{k} \delta^{k+m}(Y^A_\alpha - c_{\alpha i} Z^A_i)$$

Look for symmetry properties: obvious ones are

- ⋆ $\mathrm{GL}(k+m)$-covariance

$$\Omega^{(m)}_{n,k}(Y\cdot g, Z\cdot g) = \frac{1}{(\det g)^k}\,\Omega^{(m)}_{n,k}(Y,Z)$$

- ⋆ $\mathrm{GL}_+(k)\otimes\mathrm{GL}_+(1)\otimes\cdots\otimes\mathrm{GL}_+(1)$-scaling

$$\Omega^{(m)}_{n,k}(h\cdot Y, \lambda\cdot Z) = \frac{1}{(\det h)^{k+m}}\,\Omega^{(m)}_{n,k}(Y,Z)$$

The Capelli differential equations

[Ferro, Łukowski, AO, Parisi]

New observation:

Capelli equations

$$\det\left(\frac{\partial}{\partial W_{a_\mu}^{A_\nu}}\right)_{\substack{1\leq\nu\leq k+1\\1\leq\mu\leq k+1}} \Omega_{n,k}^{(m)}(Y,Z)=0 \qquad , \qquad W_a^A=(Y_\alpha^A, Z_i^A)$$

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Example: $m=2, k=1, n=4$

$$\det{}_{2\times 2}\begin{pmatrix}\partial_{Y^1} & \partial_{Z_1^1} & \partial_{Z_2^1} & \partial_{Z_3^1} & \partial_{Z_4^1}\\ \partial_{Y^2} & \partial_{Z_1^2} & \partial_{Z_2^2} & \partial_{Z_3^2} & \partial_{Z_4^2}\\ \partial_{Y^3} & \partial_{Z_1^3} & \partial_{Z_2^3} & \partial_{Z_3^3} & \partial_{Z_4^3}\end{pmatrix}\Omega_{4,1}^{(2)}=0$$

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Example: $m=2, k=1, n=4$

$$\begin{vmatrix}\partial_{Y^A} & \partial_{Z_i^A}\\ \partial_{Y^B} & \partial_{Z_i^B}\end{vmatrix}\Omega_{4,1}^{(2)}(Y,Z)=0 \quad , \quad \begin{vmatrix}\partial_{Z_i^A} & \partial_{Z_j^A}\\ \partial_{Z_i^B} & \partial_{Z_j^B}\end{vmatrix}\Omega_{4,1}^{(2)}(Y,Z)=0$$

for all values of $A, B=1,2,3$

All $k=1$ volume forms

[following Gel'fand, Graev, Retakh]

Master formula

$$\Omega^{(m)}_{n,1}(Y,Z)=\int_0^{+\infty}\left(\prod_{A=2}^{1+m}\mathrm{d}s_A\right)\frac{m!}{(s\cdot Y)^{1+m}}\prod_{i=m+2}^{n}\theta(s\cdot Z_i)$$

to be compared with

$$\Omega^{(m)}_{n,1}(Y,Z)=\int_\gamma\frac{\mathrm{d}c_1\ldots\mathrm{d}c_n}{c_1\cdots c_n}\,\delta^{1+m}(Y^A-c_iZ_i^A)$$

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- New integral lives in the *dual* Grassmannian $G(1,1+m)$. Fixed number of integration variables $s_2,\ldots,s_{1+m}$.
- integration domain $\mathcal{D}^{(m)}_n$ is shaped by θ-functions.
- Integrand $(s\cdot Y)^{-(1+m)}$ is free of singularities.

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[following Gel'fand, Graev, Retakh]

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to be compared with $\Omega^{(m)}_{n,1}(Y,Z) = \int_\gamma \frac{\mathrm{d}c_1 \ldots \mathrm{d}c_n}{c_1\cdots c_n}\, \delta^{1+m}(Y^A - c_i Z_i^A)$

Known results are

$$\Omega^{(2)}_{n,1} = \sum_i [1\, i\, i+1] \quad , \quad \Omega^{(4)}_{n,1} = \sum_{i<j} [1\, i\, i+1\, j\, j+1]$$

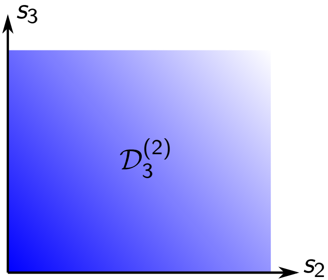
$$[1\, i\, i+1] = \frac{\langle 1\, i\, i+1\rangle^2}{\langle Y\, 1\, i\rangle\langle Y\, i\, i+1\rangle\langle Y\, i+1\, 1\rangle} \quad , \quad [1\, i\, i+1\, j\, j+1] = \frac{\langle 1\, i\, i+1\, j\, j+1\rangle^4}{\langle Y\, 1\, i\, i+1\, j\rangle\cdots\langle Y\, j+1\, 1\, i\, i+1\rangle}$$

Two-dimensional examples

Toy model: $m = 2$, useful for visualization purposes.

Three-point volume form

$$\Omega^{(2)}_{3,1} = 2\int_0^{+\infty} \frac{\mathrm{d}s_2\,\mathrm{d}s_3}{(s\cdot Y)^3} = [1\,2\,3]$$

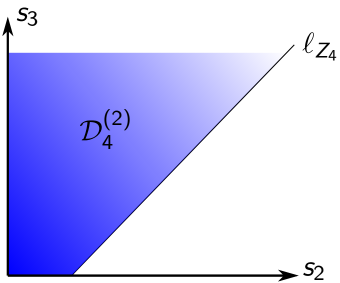


Two-dimensional examples

Toy model: $m = 2$, useful for visualization purposes.

Four-point volume form

$$\Omega^{(2)}_{4,1} = 2\int_0^{+\infty} ds_2\, ds_3\, \frac{\theta(s\cdot Z_4)}{(s\cdot Y)^3} = [1\,2\,3] + [1\,3\,4]$$

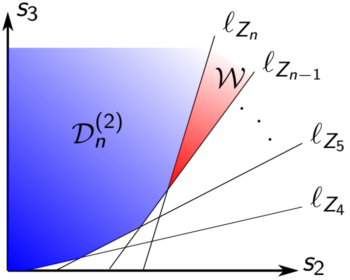


Two-dimensional examples

Toy model: $m = 2$, useful for visualization purposes.

n-point volume form

$$\Omega^{(2)}_{n,1} = 2 \int_0^{+\infty} \mathrm{d}s_2\, \mathrm{d}s_3 \, \frac{\prod_{i=4}^{n} \theta(s \cdot Z_i)}{(s \cdot Y)^3} = \sum_{i=2}^{n-1} [1\, i\, i+1]$$

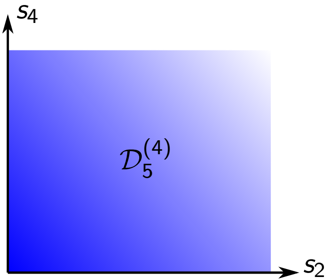


Physical examples

Realistic case: $m = 4$, hard to visualize.

Five-point volume form

$$\Omega^{(4)}_{5,1} = 4! \int_0^{+\infty} \frac{ds_2 \ldots ds_5}{(s \cdot Y)^5} = [1\,2\,3\,4\,5]$$

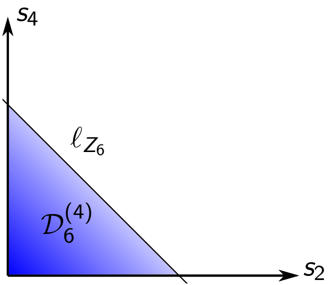


Physical examples

Realistic case: $m = 4$, hard to visualize.

Six-point volume form

$$\Omega^{(4)}_{6,1} = 4! \int_0^{+\infty} ds_2 \ldots ds_5 \, \frac{\theta(s \cdot Z_6)}{(s \cdot Y)^5}$$

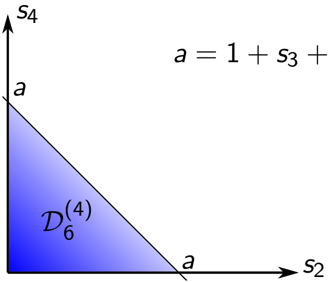


Physical examples

Realistic case: $m = 4$, hard to visualize.

Six-point volume form

$$\Omega_{6,1}^{(4)} = 4! \int_0^{+\infty} \mathrm{d}s_3\, \mathrm{d}s_5 \int_0^{a} \mathrm{d}s_2 \int_0^{a-s_2} \mathrm{d}s_4\, (s \cdot Y)^{-5}$$



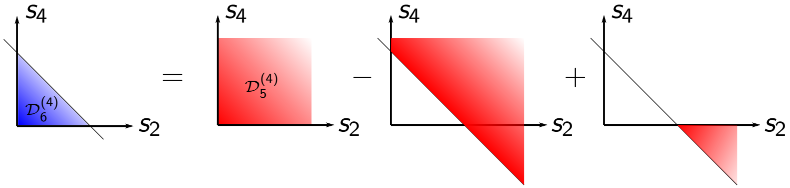
$a = 1 + s_3 + s_5$

Physical examples

Realistic case: $m = 4$, hard to visualize.

Six-point volume form

$$\Omega_{6,1}^{(4)} = [1\,2\,3\,4\,5] + [1\,2\,3\,5\,6] + [1\,3\,4\,5\,6]$$



Conclusions and outlook

Summarizing,

- The amplituhedron construction allows to think of scattering amplitudes in planar $\mathcal{N} = 4$ SYM as volumes of "polytopes".
- Volume forms corresponding to tree-level NMHV amplitudes are fully constrained by symmetry $\longrightarrow$ Capelli equations.
- Our master formula explicitely computes the "volume" of a region in a *dual* Grassmannian.

Conclusions and outlook

Summarizing,

- The amplituhedron construction allows to think of scattering amplitudes in planar $\mathcal{N} = 4$ SYM as volumes of "polytopes".
- Volume forms corresponding to tree-level NMHV amplitudes are fully constrained by symmetry $\longrightarrow$ Capelli equations.
- Our master formula explicitely computes the "volume" of a region in a *dual* Grassmannian.

What needs to be done?

⋆ Understand whether the Capelli equations hint at a realization of Yangian symmetry in the amplituhedron framework.

⋆ Use this knowledge to move beyond NMHV volume forms.

Thank you!

[picture by A. Gilmore]