Intersecting surface defects and 2d Conformal Field Theory

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[1610.03501] Gomis, Le Floch, YP, Peelaers
[1612.xxxxx] YP, Peelaers

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Oviedo

Outline

- Introduction: class-$\mathcal{S}$, CFT, partition functions, AGT
- Surface defects and their intersection
  - Construction
  - Two simplest intersecting defect systems
  - Partition functions, correlators, dualities
  - Higgsing
- Summary, conjectures
- Open problems

Introduction

class-$\mathcal{S}$ theories

Liouville/Toda

AGT

surface defect (class-$\mathcal{S}$ construction)

AGT with surface defect

Motivations

- QFTs are well studied on smooth spaces ($S^n, S^n \times S^1$, ...), spaces with boundaries (D^n, $\mathbb{R}^k, \ldots$)
- Explore QFTs on intersecting spaces, e.g., $\mathbb{R}^2_{x_1,x_2=0} \cup \mathbb{R}^2_{x_3,x_4=0} \subset \mathbb{R}^4$, $\mathbb{R}^2_{x_1,x_2=0} \cap \mathbb{R}^2_{x_3,x_4=0} = (0,0,0,0)$
- Enrich the family of surface defects in four dimensions
- Generalize AGT correspondence to include intersecting surface defects
- Explore new dualities

Class $\mathcal{S}$ of type A_{n_f} [Gaiotto]

- 4d $\mathcal{N} = 2$ theories on M^4
- Labeled by punctured Riemann surfaces $\Sigma_{g,n} \Rightarrow \mathcal{T}_{g,n}$ on M^4
- M^4 unrelated to Σ
- Some of them in weak coupling regime $\Rightarrow$ quiver gauge theories

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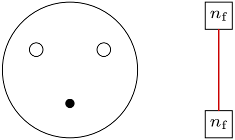
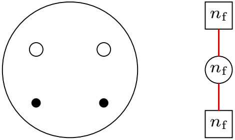
- Examples: consider Riemann spheres



n_f^2 Free hypers



$SU(n_\mathrm{f})$ SQCD

Partition functions Z^M

- For QFTs $\mathcal{T}$ on space M who have Lagrangians
- Defined formally as path integral

$$Z^M(\mathcal{T}) \equiv \int \mathcal{D}\,[\text{fields}]\, e^{-S_M[\text{fields}]}$$

- For some theories of class-$\mathcal{S}$, simplified to ordinary integrals/sums

$$Z^M(\mathcal{T}) = \sum\!\!\!\!\!\!\int Z_{\text{cl}}(\Phi) Z_{1-\text{loop}}(\Phi) Z_{\text{instanton}}(\Phi)$$

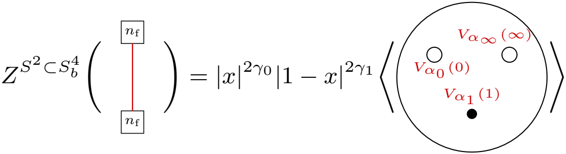
- Examples will be shown later

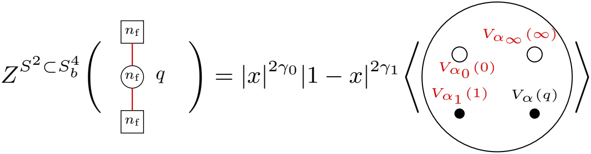
Liouville/Toda CFT [Teschner, '95; Zamolodchikov, Zamolodchikov, '96;]

- Liouville theory: 2d CFT on $\Sigma_{g,n}$; Toda, the generalized version
- Depend on a param b ($\leftrightarrow$ central charge)
- Liouville $\leftrightarrow W_2 \sim Virasoro$, Toda $\leftrightarrow W_{n_\mathrm{f}}$
- Vertex operators $V_\alpha(x)$:
  - Location: x
  - Momentum: α
- Special ones: degenerate vertex op. $V_{\alpha_{\mathrm{deg}}}(x)$
  - Pick $\mathcal{R}$: irrep of $\mathfrak{su}(n_\mathrm{f})$
  - **deg.** momentum $\alpha_{\mathrm{deg}} \propto \Omega_{\mathcal{R}}$
- Insert $V_\alpha(x)$ at the punctures
- Correlation functions $\langle V_{\alpha_0}(0) V_{\beta_1}(x_1) \ldots V_{\beta_n}(x_n) V_{\alpha_1}(1) V_{\alpha_\infty}(\infty)\rangle$

AGT relation [Alday, Gaiotto, Tachikawa]

- S_b^4-partition functions (of $\mathcal{T}_{g,n}$) $=$ Liouville/Toda correlators (on $\Sigma_{g,n}$);





Surface defect (class-$\mathcal{S}$ construction) [Gomis, Le Floch; Gadde, Gukov; Gaiotto, Kim, ...]

- Insert degenerate puncture(s)/vertex operator(s);
- Labeled by a representation $\mathcal{R}$ of $\mathfrak{su}(n_\mathrm{f})$ with highest weight $\Omega_\mathcal{R}$;

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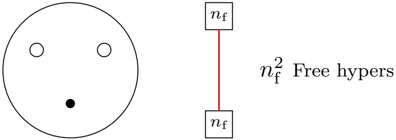

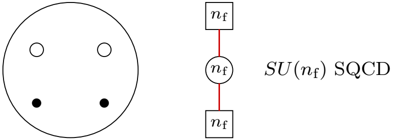

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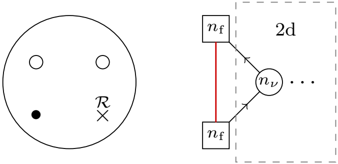
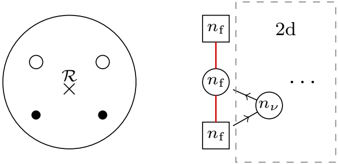
- Examples (after), **$\mathcal{R}$ determines 2d quiver** (inside dashed box)



n_{f}^2 Free hypers + defect



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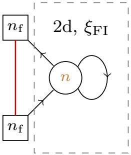
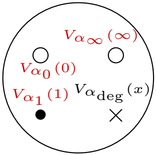
- Example: $\alpha_{\text{deg}} = -b\Omega_{\text{symm}^n \square} = -nbh_1$, $x \propto e^{-2\pi\xi_{\text{FI}}}$,

$Z^{S^2\subset S_b^4}\Big($



$\Big) = |x|^{2\gamma_0}|1-x|^{2\gamma_1}\Big\langle$



$\Big\rangle$

Intersecting surface defects

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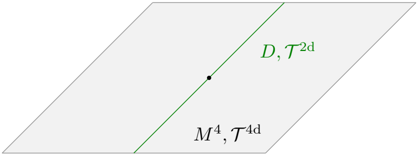

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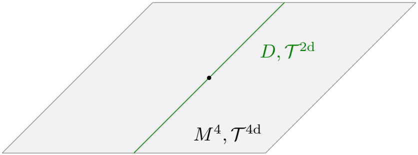
- 

- Physical quantity: the partition function of the 4d-2d coupled system

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  - $P \equiv D_{\mathrm{L}} \cap D_{\mathrm{R}} \subset M^4$, 0d $\mathcal{N} = 2$ theory $\mathcal{T}^{0\mathrm{d}}$ (0d $\mathcal{N} = 2$ Fermi or chiral).

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  - $\mathcal{T}^{4\text{d}}$, $\mathcal{T}_{\text{L,R}}^{2\text{d}}$ and $\mathcal{T}^{0\text{d}}$ couple supersymmetrically.

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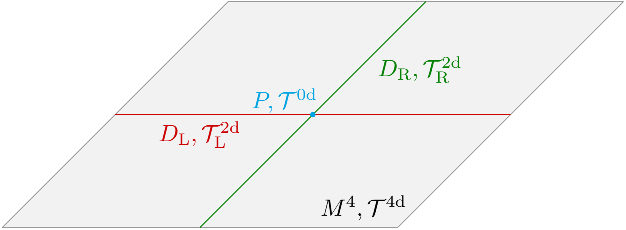

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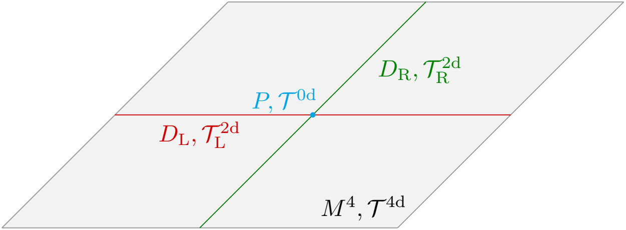

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- Examples ($\mathbb{R}^4 \supset \mathbb{R}^2_{x_3,x_4} \supset \{0\}$):

$$\mathcal{A}^{\mathrm{4d}\ \mathcal{N}=2} \to \mathcal{A}^{\mathcal{N}=(2,2)}_{x_3,x_4} \oplus \Phi^{\mathcal{N}=(2,2)}_{x_3,x_4}, \qquad \mathcal{Q}^{\mathcal{N}=2} \to \Phi^{\mathcal{N}=(2,2)}_{x_3,x_4} \oplus \tilde{\Phi}^{\mathcal{N}=(2,2)}_{x_3,x_4}$$

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- Higher-dim vector multiplet gauge lower-dim global symmetries
- Bulk and defect fields superpotential $\Rightarrow$ relations between $\mathcal{R}$-charges and masses.

Example: the quivers

- $M^4 = S^4_b$, $D_{\rm L} = S^2_{\rm L}$, $D_{\rm R} = S^2_{\rm R}$, $\{{\rm N}, {\rm S}\} = S^2_{\rm L} \cap S^2_{\rm R}$

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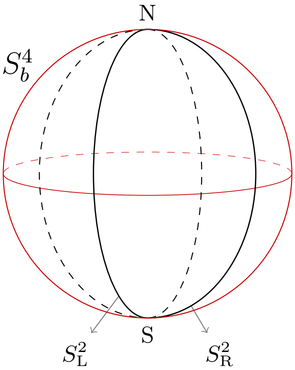

$$S_b^4: \quad \ell^{-2}(x_1^2+x_2^2)+\tilde\ell^{-2}(x_3^2+x_4^2)+r^{-2}x_5^2=1$$

$$S_{\rm L}^2: \quad \ell^{-2}(x_1^2+x_2^2)+ \qquad +r^{-2}x_5^2=1$$

$$S_{\rm R}^2: \quad +\tilde\ell^{-2}(x_3^2+x_4^2)+r^{-2}x_5^2=1$$

$$\boxed{b^2 = \ell/\tilde\ell}$$

Example: the quivers

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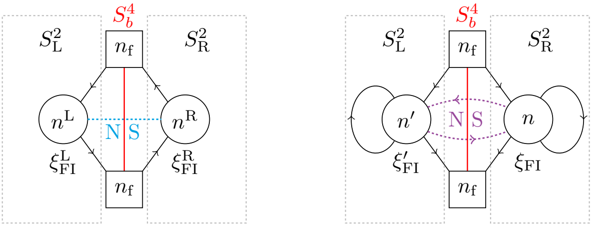
- 4d-2d-0d coupled system: Fermi-type and Chiral-type quivers



4d free hypers	2d chirals	0d Fermis	0d chirals
on S^4_b	on S^2_L, S^2_R respectively	on N, S	on N, S

Example: partition function

- The full partition function reads

$$Z^{S^2_{\rm L}\cap S^2_{\rm R}\subset S^4_b} = Z^{S^4_b}_{\rm freehyper} \sum_{B^{\rm L,R}} \int d\sigma^{\rm L,R} \prod_{\alpha={\rm L,R}} Z^{S^2_\alpha}_{\rm SQCD(A)}(\sigma^\alpha, B^\alpha)$$

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- the 0d Fermi/chiral contribution at N- and S-poles

$$Z^{\mathrm{N/S}}_{\mathrm{F/C}}(\sigma, B) = \begin{cases} \prod_{a=1}^{n^{\mathrm{R}}} \prod_{b=1}^{n^{\mathrm{L}}} \Delta^{\mathrm{N/S}}_{ab} \\ \prod_{a=1}^{n^{\mathrm{R}}} \prod_{b=1}^{n^{\mathrm{L}}} \prod_{\pm} (\Delta^{\mathrm{N/S}}_{ab} \pm (b^2 + b^{-2})/2)^{-1} \end{cases}$$

where $\Delta^{\mathrm{N/S}}_{ab} \equiv b^{-1}(i\sigma^{\mathrm{L}}_a \pm B^{\mathrm{L}}_a/2) - b(i\sigma^{\mathrm{R}}_b \pm B^{\mathrm{R}}_b/2)$.

Example: compare with Liouville [Fateev, et.al, '09; Zamolodchikov, Zamolodchikov;]

- Liouville CFT on $\Sigma_{g=0} = S^2$; parameter b
  - $\mathfrak{su}(2)$ fund. $\square (= \mathrm{symm}^1 \square = \wedge^1 \square)$, highest weight $\Omega_\square = 1/2$.

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- Consider 3 generic vertex op., 1 degenerate vertex, $\alpha_{\mathrm{deg}} = -b/2 - b^{-1}/2$

$$\left\langle V_{\alpha_0}(0) V_{-b/2-b^{-1}/2}(x,\bar{x}) V_{\alpha_1}(1) V_{\alpha_\infty}(\infty) \right\rangle$$

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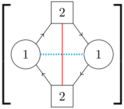
$$\overset{\text{5th}}{=} \frac{1}{A_{\text{F}}(x,\bar{x})} Z^{S^2_{\text{L}} \cup S^2_{\text{R}} \subset S^4_b}$$



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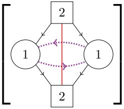

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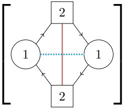
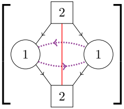
$$\overset{\text{5th}}{=} \frac{1}{A_{\text{F}}(x,\bar{x})} Z^{S^2_{\text{L}} \cup S^2_{\text{R}} \subset S^4_b}$$ 

$$\overset{\text{5th}}{=} \frac{1}{A_{\text{C}}(x,\bar{x})} Z^{S^2_{\text{L}} \cup S^2_{\text{R}} \subset S^4_b}$$ 

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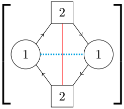

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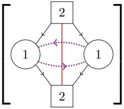


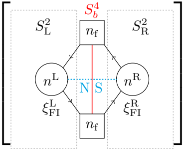
- $n_{\text{f}} = 2$, $\xi_{\text{FI}} = \xi'_{\text{FI}} = \xi^{\text{R}}_{\text{FI}} = -\xi^{\text{L}}_{\text{FI}} \equiv \xi$.
- Left-right mass relations $b^{-1}m_i = bm'_i + \frac{i}{2}(b^2 - b^{-2})$, *etc*: **superpotential**

CFT	$\mathcal{R}, \mathcal{R}' = \square$	x	$\alpha' s$
Quiver	$n = n' = \ldots = 1$	$e^{-2\pi\xi}$	masses

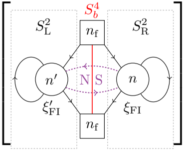
Seiberg-like duality

- Hint from previous slide: quivers of F-type and C-type have equal **partition function**

$$\frac{1}{A_{\mathrm{F}}(x,\bar{x})} Z \left[\right.$$



$$\left. \right] \overset{\text{2nd}}{=} \frac{1}{A_{\mathrm{C}}(x,\bar{x})} Z \left[\right.$$



$$\left. \right]$$

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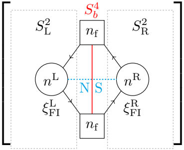
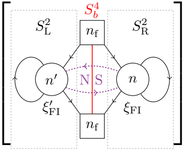
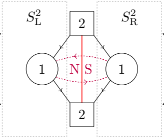


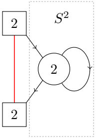


- Parameters relations
  - $n = n^{\mathrm{R}}$, $n' = n_{\mathrm{f}} - n^{\mathrm{L}}$
  - $\xi_{\mathrm{FI}} = \xi'_{\mathrm{FI}} = \xi^{\mathrm{R}}_{\mathrm{FI}} = -\xi^{\mathrm{L}}_{\mathrm{FI}} \equiv \xi$
  - Mass relations $m_j + i/2 = m^{\mathrm{R}}_j$, $m'_j + i/2 = \tilde{m}^{\mathrm{L}}_j$, *etc.*

Symmetry enhancement

- Round sphere limit $b \to 1$.
- $\alpha_{\rm deg} = -b/2 - b^{-1}/2 \to -b \sim$ highest weight of $\mathcal{R} \equiv \mathrm{symm}^2\,\Box_{\mathfrak{su}(2)}$.
- By [Gomis, Le Floch], intersecting defect $\xrightarrow{b\to 0}$ single defect on one S^2:

$Z^{S^2_{\rm L} \cup S^2_{\rm R} \subset S^4_b}\Big($ 

$\Big) \xrightarrow{b \to 1} Z^{S^2 \subset S^4}\Big($ 

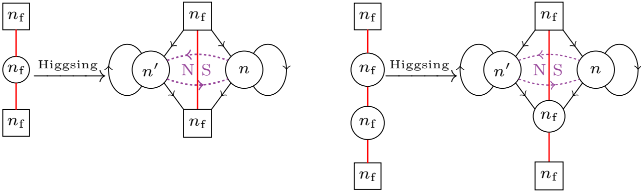
$\Big)$

Higgsing

- Procedure of constructing surface defects [Gaiotto, Razamat, Rastelli; Gaiotto, Kim;].
- $\mathcal{T}_{\text{UV}}^{\mathcal{N}=2} \xrightarrow{\text{Higgsing}} \mathcal{T}_{\text{IR}}^{\mathcal{N}=2} + $ surface defects
- C-type Quiver from Higgsing, I.P.F. with surface defects from Higgsing
- Factorization [Pan, Peelaers, to appear]: $Y \to Y^{\text{L}}, Y^{\text{R}}$,

$$Z_{\text{inst}}(Y) \sim Z_{\text{vortex}}(Y^{\text{L}}) Z_{\text{vortex}}(Y^{\text{R}}) Z_{\text{0dchiral}}(Y^{\text{L}}, Y^{\text{R}})$$

- Example:



Summary/conjectures

The conjecture: ingredients

- Players
  - 4d-2d-0d coupled quiver gauge theories: F- and C-type
  - Degenerate Liouville/Toda correlators

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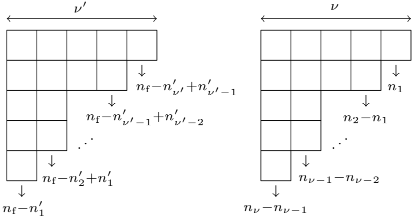
- Other input: $\xi \leftrightarrow x$, $m, \tilde{m} \leftrightarrow \alpha_0, \alpha_\infty$
  - ξ: 2d FI parameter;
  - $m, \tilde{m}$: 2d masses;
  - x: position of the deg. vertex operator;
  - α_0, α_∞ momenta of other vertex operators.

The conjecture: inputs $\mathcal{R}$, $\mathcal{R}'$

- $\mathcal{R}$, $\mathcal{R}'$ determines 2d part of the F-type quivers:



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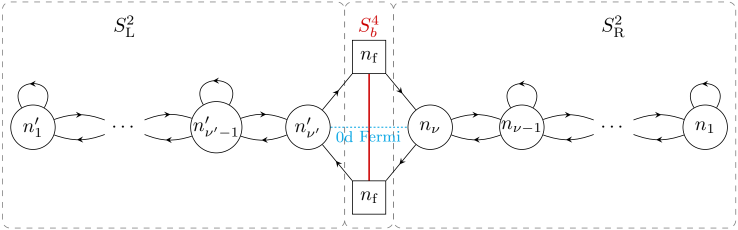

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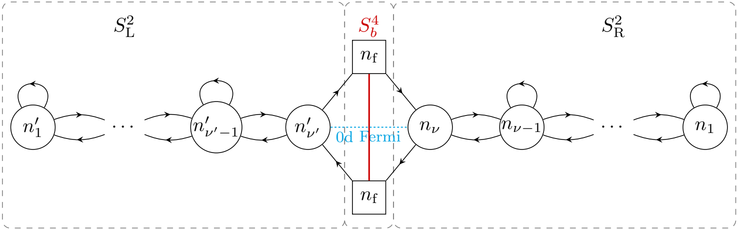

- $\mathcal{R}$, $\mathcal{R}'$ determines the degenerate momentum: $\alpha_{\mathrm{deg}} = -b\Omega_{\mathcal{R}} - b^{-1}\Omega_{\mathcal{R}'}$:

$$\left\langle \hat{V}_{\alpha_0}(0)\hat{V}_{\alpha_{\mathrm{deg}}}(x,\bar{x})\hat{V}_{\alpha_1}(1)\hat{V}_{\alpha_\infty}(\infty)\right\rangle$$

The conjectures

- Full $(S^4 \supset S^2_{\rm L} \cup S^2_{\rm R})$-partition function of the F-type quiver gauge theory $=$ Liouvlle/Toda degenerate correlator with $\hat{V}_{-b\Omega_{\mathcal{R}} - b^{-1}\Omega_{\mathcal{R}'}}$
- A special set of F-type quivers are Seiberg-dual to C-type quivers:

where $n^{\rm L} + n' = n_{\rm f}$, $n = n^{\rm R}$.

Open problems

Open problems

- F-type quiver from Higgsing?
- Seiberg-like, hopping dualities when $\mathcal{T}^{4\text{d}}$ is interacting?
- Brane realization of the Seiberg-like duality (C-type $\leftrightarrow$ F-type)?
- Generalize to intersecting Levi-type defects, and W_ρ correlators?
- What is the low energy effective theories for intersecting GLSM's? NLSM's on **intersecting worldsheets**?

And ...

Thank you for your attention!