The holographic Weyl semi-metal

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Based on C. Copetti, J.F.P., K. Landsteiner, to appear soon.

Outline

1. Weyl semi-metals.
2. Holographic set-up.
3. Computation of the axial conductivity.
4. Results and interpretation.
5. Conclusions.

Weyl semi-metals

- New state of matter in 3D.
- Conduction and valence bands touch in pointlike singularities.
- Quasiparticle excitations: left and right-handed Weyl fermions.
- Broken T symmetry allows WF to sit at different points.
- Broken inversion allows WF to sit at different energies.
- At strong coupling, semiclassical understanding breaks down.

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AdS/CFT !

Effective description and the axial anomaly

Two points with definite chirality and linear dispersion relation:
Dirac system with mass M and time-reversal breaking term b.

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - eA\!\!\!/ + M + \vec{b}\cdot\vec{\gamma}\gamma_5\right)\psi.$$

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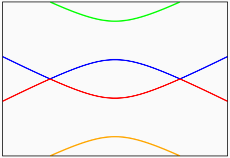
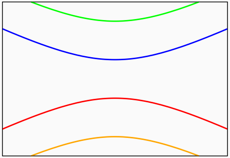


$|b| > |M|$



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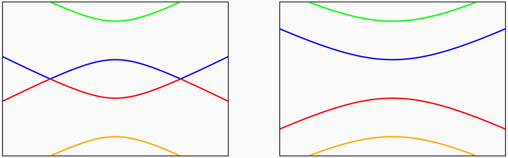


$|b| > |M|$ $\qquad$ $|b| < |M|$

The axial anomaly

$$\partial_\mu J_5^\mu = \frac{1}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + 2M\bar{\psi}\gamma_5\psi$$

implies the anomalous Hall effect

$$\vec{J} = \frac{1}{2\pi^2}\vec{b}_{\mathrm{eff}} \times \vec{E}, \text{ with } \vec{b}_{eff} = \sqrt{b^2 - M^2}\,\hat{e}_z.$$

What are we looking for? Phenomena we want to reproduce

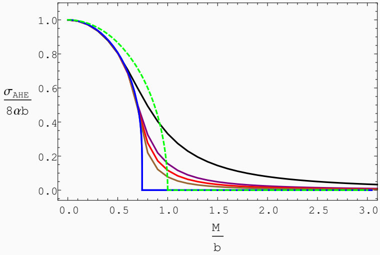
Hall conductivity that shows two phases:

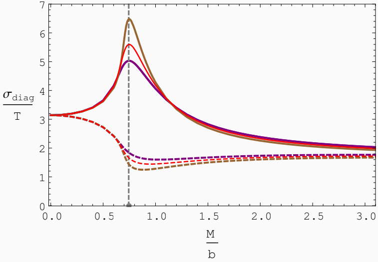
- M/b is the physical parameter governing the phase structure.
- One of the phases has finite conductivity.
- The other one has zero conductivity.
- $T \neq 0$ turns quantum phase transition into a crossover.

Longitudinal and transverse conductivity:

- The phase transition is between a topological and a trivial SM.
- The conductivities are smaller in the trivial phase but not zero.
- Some but not all of the degrees of freedom are gapped out.

Previous results of the model [1]





$T = 0$, $T = 0.03$ b, $T = 0.04$ b, $T = 0.05$ b, $T = 0.1$ b.

[1] K. Landsteiner, Y. Liu and Y. W. Sun, *"Quantum phase transition between a topological and a trivial semimetal from holography,"* Phys. Rev. Lett. **116** (2016) no.8, 081602 [arXiv:1511.05505 [hep-th]].

Holographic model

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R+\frac{12}{L^2}\right)-\frac{1}{4}F^2-\frac{1}{4}F_5^2\right.$$
$$\left.+\frac{\alpha}{3}A\wedge\left(F_5\wedge F_5+\mathbf{3}F\wedge F\right)-(D\Phi)^2-V(\Phi)\right]$$

U(1) electromagnetic and axial symmetries: V_μ and A_μ.

Mass deformation: non-normalizable mode of Φ.

- Scalar with mexican-hat potential: $m^2|\Phi|^2+\frac{\lambda}{2}|\Phi|^4$.
- Only charged with respect to the axial vector: $D=\partial_\mu - iqA_\mu$.

Anomaly: Chern-Simons term with the **right** coefficients (1,3).

Consistent currents and holographic dictionary

Consistent electromagnetic and axial currents:

$$\mathcal{J}^{\mu} = \lim_{r\to\infty} \sqrt{-g}\,[F^{\mu r} + 4\alpha\epsilon^{r\mu\nu\rho\sigma} A_{\nu} F_{\rho\sigma}]$$

$$\mathcal{J}_5^{\mu} = \lim_{r\to\infty} \sqrt{-g}\left[F_5^{\mu r} + \frac{4\alpha}{3}\epsilon^{r\mu\nu\rho\sigma} A_{\nu} F_{\rho\sigma}\right]$$

Electromagnetic current is conserved and axial one is anomalous.

Boundary conditions fix duality:

$$\lim_{r\to\infty} r\phi = M, \qquad \lim_{r\to\infty} A_z = b.$$

We expect AdS asymptotics and anisotropy, so background is:

$$ds^2 = u(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{u} + hdz^2.$$

Axial conductivity

We want to compute the axial conductivity.

It serves as a check of our model and anomaly induced transport.

In holography, conductivities obtained with Kubo formulae:

$$\sigma^5_{mn} = \lim_{\omega \to 0} \frac{1}{i\omega} \langle \mathcal{J}^5_m \mathcal{J}^5_n \rangle (\omega, \vec{k} = 0)$$

Fluctuations on dual fields with infalling boundary cond. in the IR.

Backreaction couples the axial vector with the metric.

Operator mixing important for retarded Green's function [2].

[2] M. Kaminski, K. Landsteiner, J. Mas, J. P. Shock and J. Tarrio, *"Holographic Operator Mixing and Quasinormal Modes on the Brane"*, JHEP **1002** (2010) 021 doi:10.1007/JHEP02(2010)021 [arXiv:0911.3610 [hep-th]].

Anomaly is inducing transport

Axial conductivity requires computation of $\langle \mathcal{J}^5_m \mathcal{J}^5_n \rangle$.

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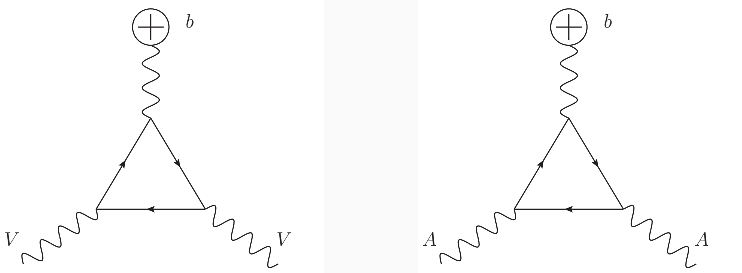
The leading contribution to this 2-point functions are:

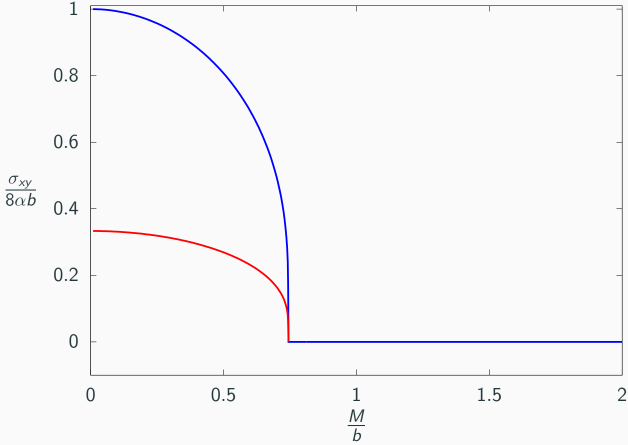


Same triangle diagram we have for the well-known chiral anomaly.

[1] K. Landsteiner, Y. Liu and Y. W. Sun, *"Quantum phase transition between a topological and a trivial semimetal from holography,"* Phys. Rev. Lett. **116** (2016) no.8, 081602 [arXiv:1511.05505 [hep-th]].

Results

The axial conductivity should be $\mathbf{1/3}$ of the vector conductivity.



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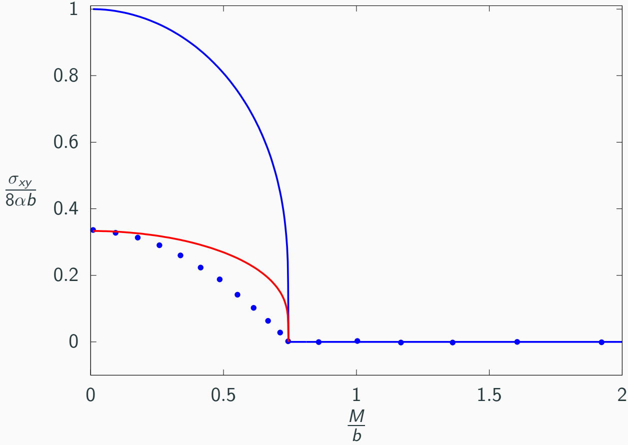

Wait, what? Where is the 1/3?

Interpretation

External axial legs get renormalized by interaction with scalar.

Renormalization for axial field, i.e. $A^{\mathrm{IR}} = \sqrt{Z_A} A$, but not for V.

We know from the 60's that the triangle can't get renormalized.

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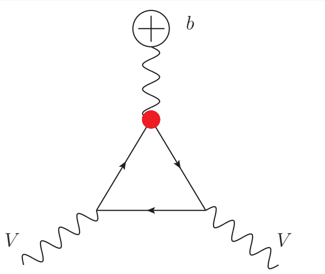
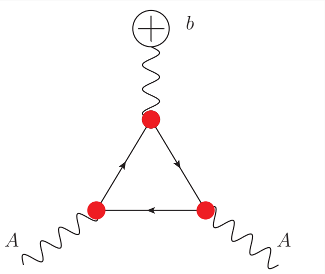
In the vector diagram (a) there was only one such external leg.

Now we have three in the axial diagram (b).



(a)



(b)

It fits!

Besides the 1/3 factor, there is a screening of the gauge coupling.

$$\frac{\sigma^5_{xy}}{\sigma_{xy}} = \frac{1}{3} Z_A = \frac{1}{3}\left(\frac{A_z(r_\mathrm{H})}{A_z(r_\mathrm{B})}\right)^2 = \frac{1}{3}\left(\frac{A_z(r_\mathrm{H})}{b}\right)^2$$

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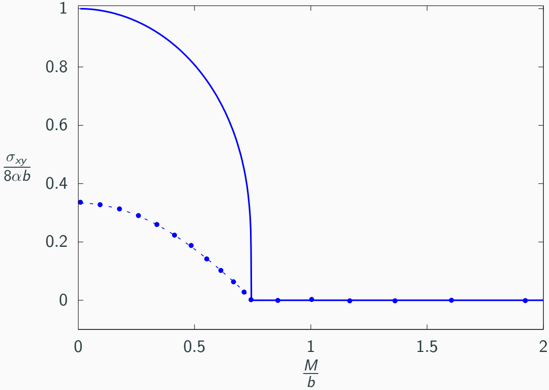
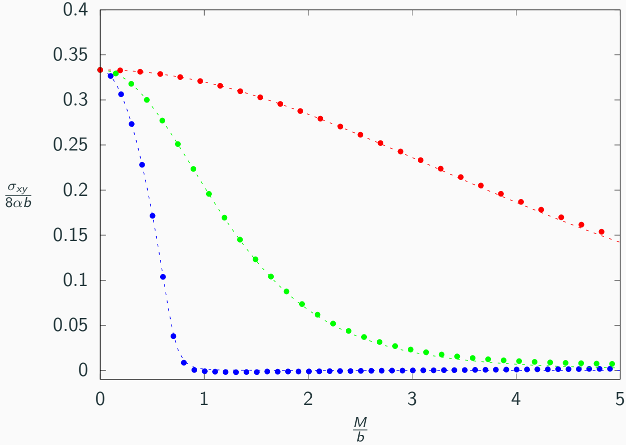


Also for finite temperature



The result also holds for $T \neq 0$. T = 0.05 b, T = 0.5 b, T = 2 b.

Conclusions

- The holographic model seems to be a good effective theory.
- We confirm the major role the anomaly has in transport.
- But we also see how important a non-trivial RG flow can be.
- It is a solid result that still holds at finite temperature.
  This is a good hint of the topological origin of the effect.