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Deformed symmetries in noncommutative and multifractional spacetimes

G. Calcagni and M. Ronco, arXiv:1608.01667 [hep-th]

Oviedo V Postgraduate Meeting On Theoretical Physics
November 17th, 2016

SAPIENZA UNIVERSITÀ DI ROMA

IEM

CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

OBJECTIVES AND MOTIVATIONS

Quantum Gravity

Top-down (discrete?) approaches:

- String theory;
- Loop quantum gravity;
- Group field theory;
- Causal dynamical triangulations;
- Causal sets;
- Spin foams;
- …………

More ambitious but no phenomenology!

Bottom-up (continuous?) approaches:

- Asymptotic safety;
- Horava-Lifshitz gravity;
- Non-local gravity theories;
- Non-commutative geometry;
- Multi-fractional theories
- ………….

Less ambitious with phenomenology!

Any links???

OBJECTIVES AND MOTIVATIONS

Waiting for data
(Lorentz violations, CMB anisotropies, gravitational waves,...)

→ *Look for shared points!!*

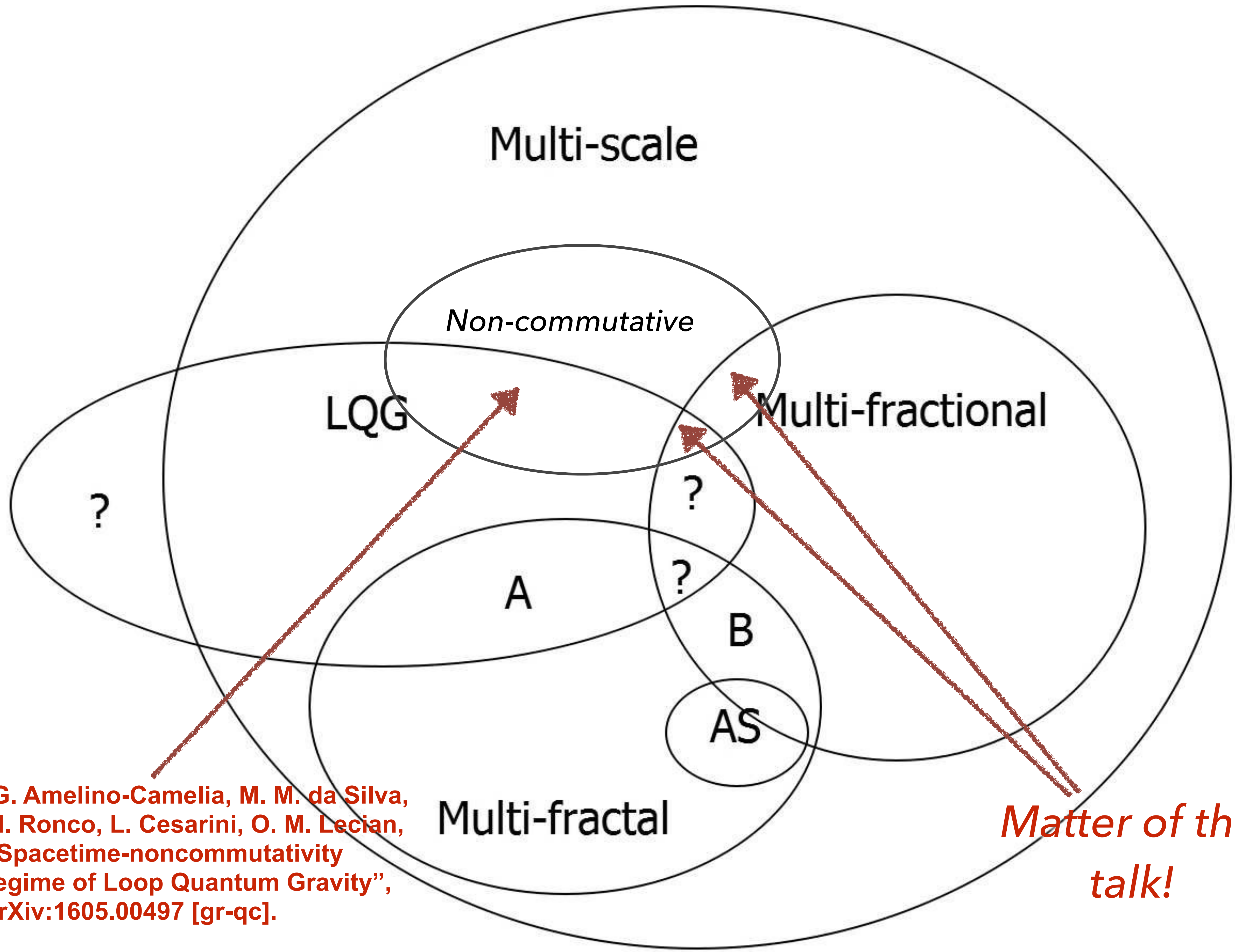
We compare:
Multi fractional theories
Noncommutative geometries

Dimensionality changes with scale!

Coordinates do not commute!

1. M. Arzano, G. Calcagni, D. Oriti, M. Scalisi, Phys.Rev. D84 (2011) 125002, [arXiv:1107.5308 [hep-th]]
2. G. Calcagni and M. Ronco, [arXiv:1608.01667 [hep-th]]

MULTI-SCALE LANDSCAPE



G. Amelino-Camelia, M. M. da Silva, M. Ronco, L. Cesarini, O. M. Lecian, "Spacetime-noncommutativity regime of Loop Quantum Gravity", arXiv:1605.00497 [gr-qc].

Matter of this talk!

MULTIFRACTIONAL: BASIC NOTIONS

Main characteristic:
the spacetime dimension changes with the probed scale!

Main ingredient: non-trivial integration measure

$$d^D q(x) = dq^0(x^0)\, dq^1(x^1) \cdots dq^{D-1}(x^{D-1}) = d^D x\, v_0(x^0) \cdots v_{D-1}(x^{D-1})$$

- *geometric coordinates:* $q^\mu(x^\mu)$
- *fractional coordinates:* x^μ
- *measure weights:* $v_\mu(x^\mu)$

Most general measure:

$$q^\mu = x^\mu + \sum_{n=1}^{\infty} \frac{l_n}{\alpha_n} \left|\frac{x^\mu}{l_n}\right|_n^{\alpha} \left[1 + A_n \cos\left(\log\left(\omega_n \left|\frac{x^\mu}{l_\infty}\right|\right)\right) + B_n \sin\left(\log\left(\omega_n \left|\frac{x^\mu}{l_\infty}\right|\right)\right)\right]$$

G. Calcagni, arXiv:1609.02776 [gr-qc]

Coarse-grained polinomial measure (continuous regime)

Logarithmic oscillations (discrete regime)

$$q(\exp(2\pi/\omega)x) \simeq \exp(2\pi/\omega)q(x)$$

MULTIFRACTIONAL: BASIC NOTIONS

Four existing multi fractional theories

- *ordinary derivatives*
- *fractional derivatives*
- *weighted derivatives*
- *q derivatives*

We consider only these last two!

weighted derivatives:

- *Free theory lagrangian is invariant under standard Poincare' transformations*

$$\partial_\mu \to v_\mu^{-1/2}\partial_\mu(v_\mu^{1/2})$$

q derivatives:

- *Free theory lagrangian is invariant under non-linear q-Poincare' transformations*

$$\partial_\mu \to \partial/\partial q^\mu(x^\mu)$$

simplified model:

$$q^\mu(x^\mu) = x^\mu + \frac{l_*}{\alpha}\left|\frac{x^\mu}{l_*}\right|^\alpha$$

$$d_H := \frac{d\ln\mathcal{V}(l)}{d\ln l} \Rightarrow d_H^{UV} = D\alpha$$

$$d_S := -2\frac{d\ln\mathcal{P}(\tau)}{d\ln\tau} \Rightarrow d_S^{UV} = D\alpha$$

G. Calcagni, "ABC of multi-fractal spacetimes and fractional sea turtles", Eur. Phys. J. C76 no. 8 (2016) 459, [arXiv:1602.01470 [hep-th]]

NONCOMMUTATIVE: BASIC NOTIONS

Main characteristic:
quantum spacetime picture!

Main ingredient: coordinates do not commute

$$[X_\mu, X_\nu] = i\Theta_{\mu\nu}(X^\alpha)$$

useful tool: Weyl maps

$$F(X^\mu) =: \Omega[f(x^\mu)]$$

$$[x^\mu, x^\nu] = 0 \quad \forall \mu, \nu$$

to make it a one-to-one correspondence need a star product:

non-invertible relation: many ways of quantising!

$$f(x^\mu) \star g(x^\nu) := \Omega^{-1}[F(X^\mu)G(X^\nu)]$$

NONCOMMUTATIVE: BASIC NOTIONS

Canonical noncommutative spacetime:

$$[X_\mu, X_\nu] = i\theta_{\mu\nu}$$

star product: $$e^{ip_\mu x^\mu} \star e^{ik_\nu x^\nu} = e^{i(p_\mu + k_\mu)x^\mu} e^{-\frac{i}{2}p^\mu \theta_{\mu\nu} k^\nu}$$

kappa-Minkowski noncommutative spacetime:

$$[X_\mu, X_\nu] = -i\lambda\delta_{\mu 0} X_\nu + i\lambda\delta_{\nu 0} X_\mu$$

coordinates close a Lie algebra!

star product: $$e^{ip_\mu x^\mu} \star e^{ik_\nu x^\nu} = e^{i(p_i + k_i e^{-\lambda p_0})x^i + (p_0 + k_0)x^0}$$

transform under (non-linear) kappa-Poincare' transformations:

$$[P_\mu, P_\nu] = 0, \quad [J_i, P_0] = 0, \quad [J_i, P_j] = i\epsilon_{ijk} P_k,$$
$$[J_i, J_j] = i\epsilon_{ijk} J_k, \quad [J_i, N_j] = -i\epsilon_{ijk} N_k,$$
$$[N_i, P_0] = iP_i \quad [N_i, N_j] = -i\epsilon_{ijk} N_k,$$
$$[N_i, P_j] = i\delta_{ij}\left(\frac{1 - e^{-2\lambda P_0}}{2\lambda} + \frac{\lambda}{2}\vec{P}^2\right) - i\lambda P_i P_j$$

PREVIOUS RESULTS

M. Arzano, G. Calcagni, D. Oriti, M. Scalisi, Phys.Rev. D84 (2011) 125002, [arXiv:1107.5308 [hep-th]]

start from canonical noncommutativity:

$$[Q, Q_0] = i\lambda^2 \Rightarrow \int d^D q v_c(q) f(q)$$

Cyclicity preserving measure!

map it into kappa-Minkowski:

$$[X, X_0] = i\lambda X \Rightarrow \begin{cases} Q = \lambda \ln\left(\frac{X}{\lambda}\right) \\ Q_0 = X_0 \end{cases}$$

$$\int d^D q v_c(q) f(q) = \int d^D x v_c[q(x)] \left|\frac{dq}{dx}\right| f[q(x)] \Rightarrow v_c(x) = v_c[q(x)] \left|\frac{dq}{dx}\right|$$

correspondence:

$$v_c(x) = \frac{1}{|x|}$$

Coincides with multi fractional measure in the boundary-effect regime with nonfractional time!!

$$x/l_\infty \sim 1, \quad \alpha_0 \equiv 1$$

GENERALISING THE CORRESPONDENCE

G. Calcagni and M. Ronco,"Deformed symmetries in noncommutative and multifractional spacetimes", arXiv:1608.01667 [hep-th]]

$$v_c(x) = \frac{1}{|x|}$$

→ *Major drawback of previous derivation: cyclicity-invariant measure breaks kappa-Poincare' symmetries!*

A. Agostini, G. Amelino-Camelia, M. Arzano, F. D'Andrea, Int.J.Mod.Phys. A21, 3133 (2006).

to overcome this obstacle work with Heisenberg algebras:

$$[X, X_0] = i\lambda X, \quad [X, K] = i, \quad [X_0, K_0] = -i, \quad [X, K_0] = 0, \quad [X_0, K] = i\lambda K,$$

$$[Q, Q_0] = i\lambda^2, \quad [Q, P] = i, \quad [Q_0, P_0] = -i, \quad [Q, P_0] = [Q_0, P] = 0.$$

establish a map:

$$Q = \int dX\, v(X), \quad Q_0 = X_0, \quad P = \frac{1}{v(X)} K, \quad P_0 = K_0$$

Multifractional relation between geometric and fractional coordinates!

Nonfractional time!

$$[X_0, K_0] = [Q_0, P_0] = -i,$$

$$[X, K] = [Q, P] = i.$$

GENERALISING THE CORRESPONDENCE

require compatibility between map and Heisenberg algebras:

$$0 = [P, Q_0] = [\frac{1}{v(X)} K, X_0] = \frac{1}{v(X)}[K, X_0] + [\frac{1}{v(X)}, X_0]K$$

$$= \frac{1}{v(X)}(-i\lambda K) - \frac{v'(X)}{v^2(X)}[X, X_0]K = -\frac{i\lambda}{v(X)}\left[1 + \frac{v'(X)}{v(X)}X\right]K$$

$$-\int \frac{dX}{X} = \int \frac{dv}{v} \quad \Rightarrow \quad v(X) = \frac{\lambda}{|X|}$$

Same
result but without using cyclicity
arguments!
kappa-Poincare' symmetries are
safe!

Missing?
Comparison of symmetries!!

SYMMETRY COMPARISON

q-theory is invariant under q-Poincare' transformations:

$$q^\mu(x'^\mu) = \Lambda^\mu{}_\nu q^\nu(x^\nu) + a^\mu \quad \Rightarrow \quad [\mathcal{N}, P] = iP_0\,, \quad [\mathcal{N}, P_0] = iP\,, \quad [P_0, P] = 0$$

with:

$$\mathcal{N} = i\left(Q\frac{\partial}{\partial Q_0} - Q_0\frac{\partial}{\partial Q}\right)\,, \quad P_0 = i\frac{\partial}{\partial Q_0}, \quad P = -i\frac{\partial}{\partial Q}$$

These transformations are linear in q but highly nonlinear in x!

$$Q(X) = X + \frac{l_*}{\alpha}\left|\frac{X}{l_*}\right|^\alpha,$$

$$P(K) = \frac{K}{1+\alpha^{-1}|l_* K|^{1-\alpha}}$$

$$[\mathcal{N}, K_0] = iP(K), \quad [K, K_0] = 0,$$

$$[\mathcal{N}, K] = \frac{iK_0K^2}{P^2(K)v(1/K)}$$

G. Calcagni, JCAP 1312 (2013) 041, [arXiv:1307.6382 [hep-th]]

Nonlinear symmetry algebras can be a sign of noncommutativity!

Check it!!

SYMMETRY COMPARISON

discover if coordinates do not commute by imposing Jacobi identities!

$$[\mathcal{N}, Q_0] = iQ, \quad [\mathcal{N}, Q] = iQ_0 \quad \Rightarrow \quad [\mathcal{N}, X_0] = iQ(X), \quad [\mathcal{N}, X] = iX_0 v^{-1}(X)$$

$$\begin{aligned} 0 &= [[\mathcal{N}, X], X_0] + [[X_0, \mathcal{N}], X] + [[X, X_0], \mathcal{N}] \\ &= iX_0[v^{-1}(X), X_0] + [[X, X_0], \mathcal{N}] \\ &= -iX_0[X, X_0]\frac{v'}{v^2} + [[X, X_0], \mathcal{N}] \end{aligned}$$

two possibilities:

$$[X, X_0] = if(X) \quad \vee \quad [X, X_0] = 0$$

Nonconclusive proof!

Substituting in the above equation:

$$[X, X_0] = i\frac{\lambda^2}{v(x)} \quad \Rightarrow \quad [X, X_0] = i\lambda X, \quad v(X) = \frac{\lambda}{X}$$

MULTIFRACTIONAL FROM NONCOMMUTATIVE

multifractional mass Casimir is standard in p momenta but highly deformed in k!

kappa-Poincare' mass Casimir:

$$\mathcal{C} = -\left(\frac{2}{\lambda}\sinh\frac{\lambda K_0}{2}\right)^2 + e^{\lambda K_0} K^2 \quad \Rightarrow \quad P_0(K_0), P(K)??$$

from the on-shellness for the massless case:

$$e^{-\lambda K_0}\mathcal{C} = 0 \quad \Rightarrow \quad P_0 = \frac{2}{\lambda} e^{-\frac{\lambda K_0}{2}} \sinh\frac{\lambda K_0}{2}\,, \quad P = K \;\rightarrow\; \mathcal{C} = -P_0^2 + P^2$$

since we know:

$$P^\mu(K^\mu) = \frac{1}{q^\mu(1/K^\mu)} \;\rightarrow\; Q = X\,, \quad Q_0 = \frac{\lambda}{1 - e^{-\lambda/X_0}}$$

geometric coordinates from kappa-Poincare' mass Casimir

MULTIFRACTIONAL FROM NONCOMMUTATIVE

2-ball volume:

$$V = \int_{ball} dQ^0 dQ^1 \quad \Rightarrow \quad d_H = 1 - \frac{\lambda/R}{1-e^{\lambda/R}}$$

$d_H^{IR} \simeq 2$

$d_H^{UV} \simeq 1$

return probability:

$$P(\sigma) = \int d^D P \, \exp[-Q^0(\sigma)\, P_\mu P^\mu] \propto [Q^0(\sigma)]^{-D/2}$$

$$\Rightarrow \quad d_S = D\lambda/[(e^{\lambda/\sigma} - 1)\sigma]$$

$d_S^{IR} \simeq 2$

$d_S^{UV} \simeq 0$

Two problems:

1. Resulting measure is not of multifractional type;
2. Dimensional flow does not coincide with that of kappa-Minkowski.

M. Arzano and T. Trze´sniewski, Phys. Rev. D 89, 124024 (2014) [arXiv:1404.4762].

NONCOMMUTATIVE FROM MULTIFRACTIONAL

Is it possible to read off the noncommutative (star) product from the q-theory action?

$$S_q = -\frac{1}{2}\int d^2q \left(\partial_{q^\mu}\phi\partial^{q^\mu}\phi + m^2\phi^2 + \frac{2\sigma}{n!}\phi^n\right)$$

$$= \frac{1}{2}\int dq^0 dq^1 \left[(\partial_{q^0}\phi)^2 - (\partial_{q^1}\phi)^2 - m^2\phi^2 - \frac{2\sigma}{n!}\phi^n\right]$$

$$= \frac{1}{2}\int dx^0 dx^1 \left[\frac{v_1}{v_0}(\partial_0\phi)^2 - \frac{v_0}{v_1}(\partial_1\phi)^2 - v_0 v_1 m^2\phi^2\right)$$

$$\neq$$

$$S_\star = -\frac{1}{2}\int d^2x \left(\partial_\mu\phi\star\partial^\mu\phi + m^2\phi\star\phi + \frac{2\sigma}{n!}\phi\star\phi\star\ldots\star\phi\right)$$

True also for the case with weighted derivatives!

NONCOMMUTATIVE FROM MULTIFRACTIONAL

$$e^{ik_\mu x^\mu} \star e^{i\widetilde{k}_\nu x^\nu} := \exp\left[i(k_\mu + \widetilde{k}_\mu)x^\mu + i\frac{\ell_*}{\alpha}(k_1 + \widetilde{k}_1)\left|\frac{x_1}{\ell_*}\right|^\alpha - i\left(\frac{k_1}{|\ell_* k_1|^{\alpha-1}} + \frac{\widetilde{k}_1}{|\ell_* \widetilde{k}_1|^{\alpha-1}}\right)\frac{x_1}{\alpha}\right]$$

try to define a star product from the multifractional nonlinear composition of momenta!

G. Calcagni, JCAP 1312 (2013) 041, [arXiv:1307.6382 [hep-th]]

using the BCH lemma

$$e^{ik_\mu x^\mu} \star e^{i\widetilde{k}_\nu x^\nu} := \Omega^{-1}\left(e^{ik_\mu X^\mu} e^{i\widetilde{k}_\nu X^\nu}\right) \simeq \Omega^{-1}\left(e^{i(k_\mu + \widetilde{k}_\mu)X^\mu - \frac{k_\mu \widetilde{k}_\nu}{2}[X^\mu, X^\nu]}\right)$$

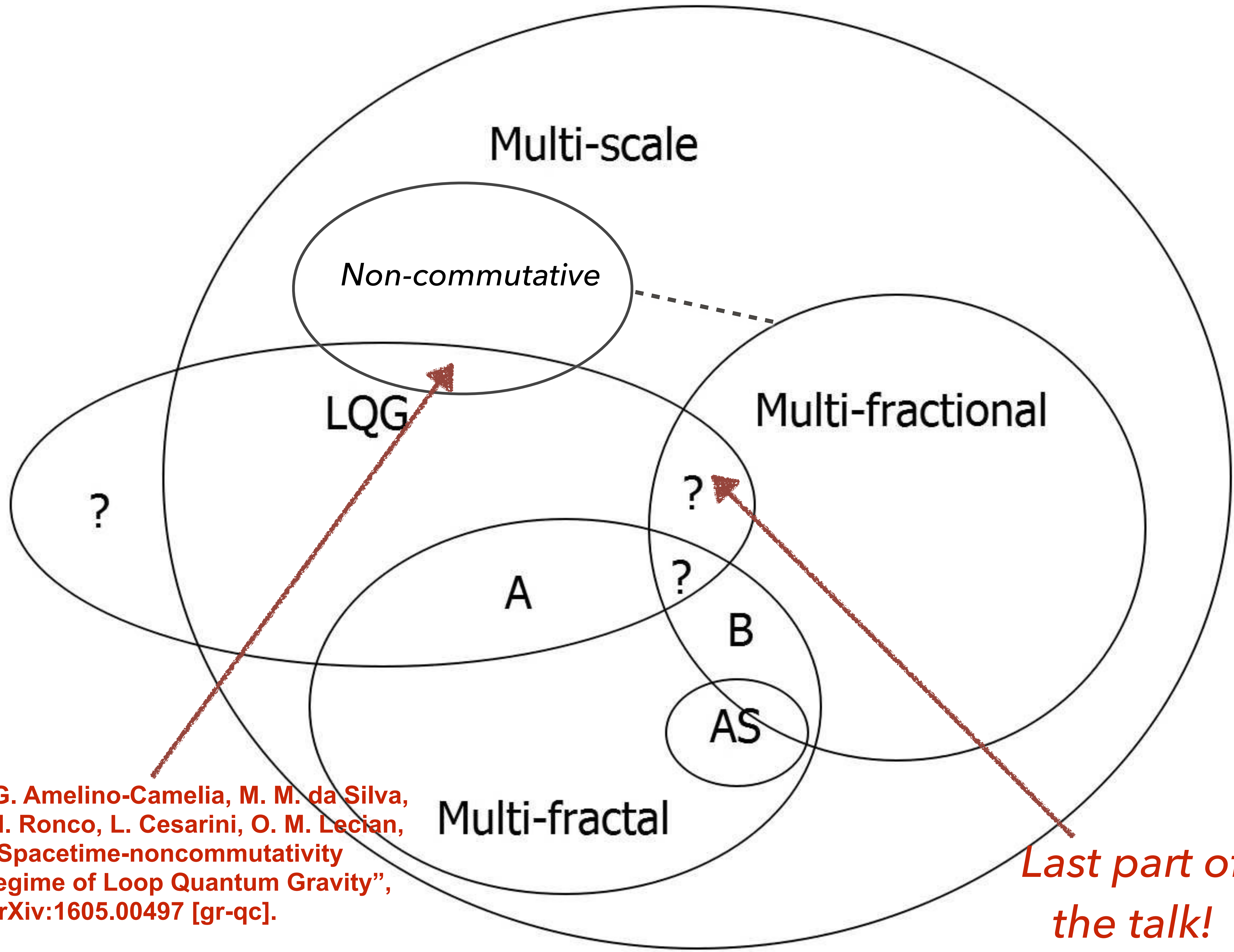
$$= \Omega^{-1}\left(e^{i(k_\mu + \widetilde{k}_\mu)X^\mu + \frac{k_0 \widetilde{k}_1 - k_1 \widetilde{k}_0}{2}[X^1, X^0]}\right)$$

$$[X^1, X^0] = \frac{2i}{k_0 \widetilde{k}_1 - k_1 \widetilde{k}_0}\left[\frac{l_*}{\alpha}(k_1 + \widetilde{k}_1)\left|\frac{X^1}{l_*}\right|^\alpha - \left(\frac{k_1}{|l_* k_1|^{\alpha-1}} + \frac{\widetilde{k}_1}{|l_* \widetilde{k}_1|^{\alpha-1}}\right)\frac{X^1}{\alpha}\right]$$

ill defined!!

(cause of problem: multifractional measures are factorizable)

MULTI-SCALE LANDSCAPE



G. Amelino-Camelia, M. M. da Silva, M. Ronco, L. Cesarini, O. M. Lecian, “Spacetime-noncommutativity regime of Loop Quantum Gravity”, arXiv:1605.00497 [gr-qc].

Last part of the talk!

MULTIFRACTIONAL GRAVITY

G. Calcagni, "Multi-scale gravity and cosmology ", JCAP 1312 (2013) 041, [arXiv:1307.6382 [hep-th]]

q-theory:

$$^q\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}\left(\frac{1}{v_\mu}\partial_\mu g_{\nu\sigma} + \frac{1}{v_\nu}\partial_\nu g_{\mu\sigma} - \frac{1}{v_\sigma}\partial_\sigma g_{\mu\nu}\right)$$

$$^q R^\rho_{\mu\sigma\nu} = \frac{1}{v_\sigma}\partial_\sigma\, {}^q\Gamma^\rho_{\mu\nu} - \frac{1}{v_\nu}\partial_\nu\, {}^q\Gamma^\rho_{\mu\sigma} + {}^q\Gamma^\tau_{\mu\nu}\, {}^q\Gamma^\rho_{\sigma\tau} - {}^q\Gamma^\tau_{\mu\sigma}\, {}^q\Gamma^\rho_{\nu\tau}$$

$$^q S = \frac{1}{2\kappa^2}\int d^D x v(x)\sqrt{-g}({}^q R - 2\Lambda) + S_m$$

weighted-theory:

$$S_g = \frac{1}{2\kappa^2}\int d^D x e^{\Phi/\beta}\sqrt{-g}\left[R - \Omega\partial_\mu\Phi\partial^\mu\Phi - U(v)\right]$$

$$\Omega := \frac{9\omega}{4\beta^2} e^{\frac{2}{\beta}\Phi} + (D-1)\left(\frac{1}{2\beta_*} - \frac{1}{\beta}\right), \qquad \Phi(x) = \log v(x)$$

MULTIFRACTIONAL HYPERSURFACE-DEFORMATION ALGEBRA

M. Bojowald, G. M. Paily, Phys.Rev. D86 (2012) 104018, [arXiv:1112.1899 [gr-qc]]

(effective) loop quantum gravity:

$$\{H^Q[N], H^Q[M]\} = D\left[\beta(h^{ij}, \pi^{ij}) h^{jk}(N\partial_j M - M\partial_j N)\right]$$

q-theory:

$$\{D^q[M^k], D^q[N^j]\} = D^q\left[\frac{1}{v_j(x^j)}(M^j\partial_j N^k - N^j\partial_j M^k)\right],$$

$$\{D^q[N^k], H^q[M]\} = H^q\left[\frac{1}{v_j(x^j)}N^j\partial_j M\right],$$

$$\{H^q[N], H^q[M]\} = D^q\left[\frac{h^{jk}}{v_j(x^j)}(N\partial_j M - M\partial_j N)\right] \longrightarrow \beta = \frac{1}{v_i(x^i)}$$

weighted-theory:

$$H[N] = H_0[N] + H_\phi[N] = \int d^3x\, N(H_0 + \sqrt{h}\mathcal{H}_\phi)$$

$$\{H[N], H[M]\} = \{H_0[N], H_0[M]\} + \int d^3x\, N(x) \int d^3y\, M(y) \times \{\mathcal{H}_0(x), \sqrt{h}\}\mathcal{H}_\phi(y) + \int d^3x\, N(x) \int d^3y\, M(y) \times \mathcal{H}_\phi(x)\{\sqrt{h}, \mathcal{H}_0(y)\} = D\left[h^{jk}(N\partial_j M - M\partial_j N)\right] \longrightarrow \beta = 1$$

CONCLUSIONS

Achievements:

- Comparison between multifractional and noncommutative spacetimes;
- No definite duality nor correspondence found;
- Multifractional are not noncommutative;
- Non commutative are not multifractional;
- Similarity in the integration measure;
- Similarity in the symmetries;
- Canonical noncommutative multi fractional is dual to kappa-Minkowski;
- Algebra of gravitational constraints: deformed in the q-theory, standard in the weighted theory.

Outlook:

- Study multifractional with non-factorizable measure;
- Extend the analysis to the case with fractional derivatives;
- Compare dimensional flow in multifractional and noncommutative.

MUCHAS GRACIAS POR VUESTRA ATENCIÓN!