Supersymmetric solutions of SU(2)-FI gauged $\mathcal{N} = 2$, $d = 4$ supergravity

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T. Ortín, C. Santoli, arXiv:1609.08694

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- SU(2) Fayet-Iliopoulos gauging;
- solutions from classification: the $\overline{\mathbb{CP}}^3$ model;
- solutions from dimensional reduction.

$\mathcal{N} = 2$, $d = 4$ supergravity: field content

- supergravity multiplet $\mapsto$ $\left(g_{\mu\nu}, \Psi_{\mu}{}^{\alpha}, A^{0}{}_{\mu}\right)$
- n_V vector multiplets $\mapsto$ $\left(A^{i}{}_{\mu}, \chi^{\mathrm{i}\alpha}, Z^{i}\right)$
- n_H hypermultiplets $\mapsto$ $\left(\xi^{A}, q^{u}\right)$

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- n_H hypermultiplets $\mapsto$ $\left(\xi^A, q^u\right)$

Classical solutions $\rightarrow$ bosonic fields only

$$g_{\mu\nu}\ ,\ A^\Lambda{}_\mu\ ,\ Z^i\ ,\ q^u$$

with $\Lambda = 0, \ldots, n_V$, $i = 1, \ldots, n_V$, $u = 1, \ldots, 4n_H$

$\mathcal{N}=2$, $d=4$ supergravity: field content

- Z^i parametrize a special Kähler manifold, $\mathcal{M}_V$, base of a symplectic bundle with sections

$$\mathcal{V}^M = \begin{pmatrix} \mathcal{L}^\Lambda \\ \mathcal{M}_\Lambda \end{pmatrix} = \mathrm{e}^{\mathcal{K}/2} \begin{pmatrix} \mathcal{X}^\Lambda \\ \mathcal{X}_\Lambda \end{pmatrix} ,$$

where $\mathcal{X}^\Lambda = \mathcal{X}^\Lambda \left(Z^i\right)$ and $\mathcal{K}$ is the Kähler potential, defining the geometry.

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  where $\mathcal{X}^\Lambda = \mathcal{X}^\Lambda \left(Z^i\right)$ and $\mathcal{K}$ is the Kähler potential, defining the geometry.

- q^u parametrize a quaternionic Kähler manifold, $\mathcal{M}_H$.

$\mathcal{N} = 2$, $d = 4$ supergravity: the ungauged theory

$$e^{-1}\mathscr{L} = R + 2\mathcal{G}_{ij*}\partial_\mu Z^i \partial^\mu Z^{*j^*} + 2\mathsf{H}_{uv}\partial_\mu q^u \partial^\mu q^v$$

$$+ 2\mathfrak{Im}\,\mathcal{N}_{\Lambda\Sigma}F^{\Lambda\mu\nu}F^{\Sigma}{}_{\mu\nu} - 2\mathfrak{Re}\,\mathcal{N}_{\Lambda\Sigma}F^{\Lambda\mu\nu}\star F^{\Sigma}{}_{\mu\nu}$$

where $\mathcal{G}_{ij*} \mapsto$ metric of special Kähler,

$\mathsf{H}_{uv} \mapsto$ metric of quaternionic Kähler,

$\mathcal{M}_\Lambda = \mathcal{N}_{\Lambda\Sigma}\mathcal{L}^\Sigma$, $\mathcal{D}_{i*}\mathcal{M}^*_\Lambda = \mathcal{N}_{\Lambda\Sigma}\mathcal{D}_{i*}\mathcal{L}^{*\Sigma}$

$\Rightarrow$ geometry determines the action.

$\mathcal{N}=2$, $d=4$ supergravity: possible gaugings

Gauging of the isometries of the special Kähler manifold

$$e^{-1}\mathscr{L} = R + 2\mathcal{G}_{ij^*}\mathfrak{D}_\mu Z^i \mathfrak{D}^\mu Z^{*j^*} + 2\mathsf{H}_{uv}\partial_\mu q^u \partial^\mu q^v$$

$$+ 2\Im\mathfrak{m}\,\mathcal{N}_{\Lambda\Sigma} F^{\Lambda\mu\nu} F^{\Sigma}{}_{\mu\nu} - 2\Re\mathfrak{e}\,\mathcal{N}_{\Lambda\Sigma} F^{\Lambda\mu\nu} \star F^{\Sigma}{}_{\mu\nu}$$

$$+\frac{1}{4} g^2 \left(\Im\mathfrak{m}\,\mathcal{N}\right)^{-1|\Lambda\Sigma} \mathcal{P}_\Lambda \mathcal{P}_\Sigma$$

where $\mathfrak{D}_\mu Z^i = \partial_\mu Z^i + g A^{\Lambda}{}_\mu k_\Lambda{}^i\,,$

$$F^\Lambda = dA^\Lambda + \tfrac{1}{2} g f_{\Sigma\Gamma}{}^{\Lambda} A^\Sigma \wedge A^\Gamma\,,$$

$$k_\Lambda{}^i = i\partial^i \mathcal{P}_\Lambda\,.$$

$\mathcal{N} = 2,\ d = 4$ supergravity: possible gaugings

Gauging of the isometries of the quaternionic Kähler manifold

$$\begin{aligned} e^{-1}\mathscr{L} = {} & R + 2\mathcal{G}_{ij^*}\mathfrak{D}_\mu Z^i \mathfrak{D}^\mu Z^{*j^*} + 2\mathsf{H}_{uv}\mathfrak{D}_\mu q^u \mathfrak{D}^\mu q^v \\ & + 2\,\mathfrak{Im}\,\mathcal{N}_{\Lambda\Sigma} F^{\Lambda\mu\nu} F^{\Sigma}{}_{\mu\nu} - 2\,\mathfrak{Re}\,\mathcal{N}_{\Lambda\Sigma} F^{\Lambda\mu\nu} \star F^{\Sigma}{}_{\mu\nu} \\ & + \frac{1}{4} g^2 \left(\mathfrak{Im}\,\mathcal{N}\right)^{-1|\Lambda\Sigma} \mathcal{P}_\Lambda \mathcal{P}_\Sigma - 2g^2 \mathsf{H}_{uv} \mathsf{k}_\Lambda{}^u \mathsf{k}_\Sigma{}^v \mathcal{L}^\Lambda \mathcal{L}^\Sigma \\ & - \frac{1}{2} g^2 \left(\mathcal{G}^{ij^*} \mathcal{D}_i \mathcal{L}^\Lambda \mathcal{D}_{j^*} \mathcal{L}^{*\Sigma} - 3\mathcal{L}^{*\Lambda}\mathcal{L}^\Sigma\right) \mathsf{P}_\Lambda{}^x \mathsf{P}_\Sigma{}^x \end{aligned}$$

where $\mathfrak{D}_\mu q^u = \partial_\mu q^u + g A^\Lambda{}_\mu \mathsf{k}_\Lambda{}^u$

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&+ \frac{1}{4}g^2\,(\Im\mathfrak{m}\,\mathcal{N})^{-1|\Lambda\Sigma}\,\mathcal{P}_\Lambda\mathcal{P}_\Sigma - 2g^2\mathsf{H}_{uv}\mathsf{k}_\Lambda{}^u\mathsf{k}_\Sigma{}^v\mathcal{L}^\Lambda\mathcal{L}^\Sigma \\
&- \frac{1}{2}g^2\left(\mathcal{G}^{ij^*}\mathcal{D}_i\mathcal{L}^\Lambda\mathcal{D}_{j^*}\mathcal{L}^{*\Sigma} - 3\mathcal{L}^{*\Lambda}\mathcal{L}^\Sigma\right)\mathsf{P}_\Lambda{}^x\mathsf{P}_\Sigma{}^x
\end{aligned}
$$

where $\mathfrak{D}_\mu q^u = \partial_\mu q^u + g A^\Lambda{}_\mu \mathsf{k}_\Lambda{}^u$

$\Rightarrow$ we need $A^\Lambda{}_\mu$ transforming in the adjoint of the gauge group.

They come in multiplets with Z^i $\Rightarrow$ the gauge group must be a subgroup of the isometry group of the special Kähler manifold.

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$$+\frac{1}{4}g^2\left(\Im\mathfrak{m}\,\mathcal{N}\right)^{-1|\Lambda\Sigma}\mathcal{P}_\Lambda\mathcal{P}_\Sigma - 2g^2\mathrm{H}_{uv}\mathrm{k}_\Lambda{}^u\mathrm{k}_\Sigma{}^v\mathcal{L}^\Lambda\mathcal{L}^\Sigma$$

$$-\frac{1}{2}g^2\left(\mathcal{G}^{ij^*}\mathcal{D}_i\mathcal{L}^\Lambda\mathcal{D}_{j^*}\mathcal{L}^{*\Sigma} - 3\mathcal{L}^{*\Lambda}\mathcal{L}^\Sigma\right)\mathsf{P}_\Lambda{}^x\mathsf{P}_\Sigma{}^x$$

where $\varkappa\mathsf{K}^x{}_{uv}\mathsf{k}_\Lambda{}^v = \partial_u\mathsf{P}_\Lambda{}^x + \epsilon^{xyz}\mathsf{A}^y{}_u\mathsf{P}_\Lambda{}^z$

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& - \frac{1}{2} g^2 \left(\mathcal{G}^{ij^*} \mathcal{D}_i \mathcal{L}^\Lambda \mathcal{D}_{j^*} \mathcal{L}^{*\Sigma} - 3\mathcal{L}^{*\Lambda}\mathcal{L}^\Sigma\right) \mathsf{P}_\Lambda{}^x \mathsf{P}_\Sigma{}^x
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⇒ if no hypermultiplets are present, $\mathsf{P}_\Lambda{}^x$ can still be constants

⇒ Fayet-Iliopoulos terms, satisfying $\epsilon^{xyz} \mathsf{P}_\Lambda{}^y \mathsf{P}_\Sigma{}^z = f_{\Lambda\Sigma}{}^\Gamma \mathsf{P}_\Gamma{}^x$.

↙ U(1) ↘ SU(2)

$\mathcal{N} = 2$, $d = 4$ supergravity: $n_H = 0$

Possible gaugings if $n_H = 0$:

- special Kähler isometries $\Rightarrow$ SEYM, include non-Abelian fields, known solutions;
- Fayet-Iliopoulos terms
  - U(1)-FI, admit AdS vacua, widely studied;
  - SU(2)-FI, admit AdS vacua and include non-Abelian fields, no known solutions.

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\begin{aligned}
V = & -\frac{1}{4} g^2 \left(\Im\mathfrak{m}\,\mathcal{N}\right)^{-1|\Lambda\Sigma} \mathcal{P}_\Lambda \mathcal{P}_\Sigma \\
& + \frac{1}{2} g^2 \left(\mathcal{G}^{ij^*} \mathcal{D}_i \mathcal{L}^\Lambda \mathcal{D}_{j*} \mathcal{L}^{*\Sigma} - 3\mathcal{L}^{*\Lambda} \mathcal{L}^\Sigma\right)
\end{aligned}
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where $\mathsf{P}_\Lambda{}^x = -\delta^x_\Lambda$.

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$$V = -\frac{1}{4}g^2 \left(\Im\mathfrak{m}\,\mathcal{N}\right)^{-1|\Lambda\Sigma} \mathcal{P}_\Lambda \mathcal{P}_\Sigma + \frac{1}{2}g^2 \left(\mathcal{G}^{ij^*}\mathcal{D}_i\mathcal{L}^\Lambda \mathcal{D}_{j^*}\mathcal{L}^{*\Sigma} - 3\mathcal{L}^{*\Lambda}\mathcal{L}^\Sigma\right)$$

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There are no maximally supersymmetric vacua.

$\mathcal{N} = 2,\ d = 4$ supergravity: supersymmetric solutions

Aim $\mapsto$ finding new supersymmetic solutions

- solve the equations provided by the general classification[1] of timelike supersymmetric (at least $\frac{1}{8}$-BPS) solutions;
- dimensionally reduce known solutions, since $d = 4, 5, 6$ supergravities with 8 supercharges are related[2].

[1] P. Meessen, T. Ortín, Nucl.Phys. B863 (2012) (arXiv:1204.0493)

[2] P. A. Cano, T. Ortín, C. Santoli, arXiv:1607.02095

Solutions from classification

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Recipe from the classification:

- Introduce X s.t. $\frac{\mathcal{V}^M}{X} = \mathcal{R}^M + i\mathcal{I}^M$.

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- Metric: $ds^2 = e^{2U}\left(dt + \hat{\omega}\right)^2 - e^{-2U}\gamma_{\underline{mn}}dx^m dx^n$,

$$\text{where } e^{2U} = 2\left|X\right|^2$$

$$\gamma_{\underline{mn}} = V^x{}_{\underline{m}} V^y{}_{\underline{n}} \delta_{xy} \, .$$

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- Choose a model $\rightarrow$ solve $\mathcal{R}^M = \mathcal{R}^M\left(\mathcal{I}^M\right)$.
- Solve the coupled system for $V^x{}_{\underline{m}} \, , \hat{\omega} \, , A^\Lambda{}_{\underline{m}} \, , \mathcal{I}^M$.

Solutions from classification: the $\overline{\mathbb{CP}}^3$ model

Simplest model admitting an $\mathsf{SU}(2)$ gauging: $\overline{\mathbb{CP}}^3$

- 3 vector multiplets;
- the scalars Z^i parametrize a $\frac{\mathsf{U}(1,3)}{\mathsf{U}(1)\times\mathsf{U}(3)}$ coset space;

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- $e^{-\mathcal{K}} = 1 - Z^i Z^i \Rightarrow 0 \leqslant \sum_i \left|Z^i\right|^2 < 1\,;$
- evaluate all the geometrical quantities for the Lagrangian:

$$\mathcal{G}_{ij*}\,,\ \mathcal{V}\,,\ \mathcal{D}_i\mathcal{V}\,,\ \mathcal{N}_{\Lambda\Sigma}\,,\ (\mathcal{N})^{-1|\Lambda\Sigma}\ ;$$

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- $\mathcal{R}_\Lambda = \frac{1}{2}\eta_{\Lambda\Sigma}\mathcal{I}^\Sigma\,,\ \mathcal{R}^\Lambda = -2\eta^{\Lambda\Sigma}\mathcal{I}_\Sigma\,, \quad \eta_{\Lambda\Sigma} = (+\,-\,-\,-)\ .$

Solutions from classification: the $\overline{\mathbb{CP}}^3$ model

Gauging: SU(2) subgroup of the isometries of the Kähler manifold.

- Acts in the adjoint on $\mathcal{X}^i$ and Z^i, leaves $\mathcal{X}^0$ invariant

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Gauging: $\mathrm{SU}(2)$ subgroup of the isometries of the Kähler manifold.

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$$\Rightarrow \quad k_\Lambda{}^x = \delta_\Lambda^y \, \epsilon_y{}^x{}_z Z^z \,,$$

$$\mathcal{P}_\Lambda = \delta_\Lambda^x \, i \, e^{\mathcal{K}} \, \epsilon_{xyz} Z^y Z^{*z} \,,$$

$$\mathfrak{D}_\mu Z^x = \partial_\mu Z^x - g\epsilon^x{}_{yz} A^y{}_\mu Z^z \,,$$

$$F^0{}_{\mu\nu} = 2\partial_{[\mu} A^0{}_{\nu]} \,,$$

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- Explicit construction of the potential $\Rightarrow$ can be negative
  we found no minima, no AdS_4 vacua.

Solutions from classification: the $\overline{\mathbb{CP}}^3$ model

Assumption: $\mathcal{I}_\Lambda = \mathcal{R}^\Lambda = 0$

- Scalars: $Z^i = \frac{\mathcal{I}^i}{\mathcal{I}^0}$.
- Metric: $ds^2 = e^{2U}\left(dt + \hat{\omega}\right)^2 - e^{-2U}\gamma_{\underline{mn}}dx^m dx^n$,

$$\text{where } e^{2U} = 2\left|X\right|^2 = 2\left(\left(\mathcal{I}^0\right)^2 - \mathcal{I}^i\mathcal{I}^i\right)^{-1}$$

$$d\hat{\omega} = 0 \Rightarrow \text{set } \hat{\omega} = 0.$$

- Vectors: $A^\Lambda{}_t = 0$.

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  $d\hat{\omega} = 0 \Rightarrow$ set $\hat{\omega} = 0$.
- Vectors: $A^\Lambda{}_t = 0$.
- To determine $\hat{V}^x$, $\mathcal{I}^\Lambda$, $A^\Lambda{}_x$, solve:

$$d\hat{V}^x - g\epsilon^x{}_{yz}\hat{A}^y \wedge \hat{V}^z + \frac{1}{\sqrt{2}}g\mathcal{I}^y\hat{V}^y \wedge \hat{V}^x = 0\,,$$

$$F^0{}_{xy} = -\frac{1}{\sqrt{2}}\epsilon_{xyz}\left(\partial_z\mathcal{I}^0 + \frac{1}{\sqrt{2}}g\mathcal{I}^0\mathcal{I}^z\right)\,,$$

$$F^z{}_{xy} = -\frac{1}{\sqrt{2}}\epsilon_{xyw}\left(\mathfrak{D}_w\mathcal{I}^z + \frac{1}{\sqrt{2}}g\left(e^{-2U}\delta^{zw} + \mathcal{I}^w\mathcal{I}^z\right)\right)\,.$$

Solutions from classification: solution 1

Hedgehog Ansatz, radial symmetry

$$\mathcal{I}^0 = \mathcal{I}^0(r)\,, \quad \mathcal{I}^x = \sqrt{2}x^x f(r)\,, \quad V^x{}_{\underline{m}} = \delta^x_{\underline{m}} V(r)$$

$$A^x{}_{\underline{m}} = \epsilon^x{}_{mn} x^n h(r)\,, \quad A^0{}_{\underline{m}} = \text{generalised Dirac monopole}\,.$$

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- No solutions with $\mathcal{I}^x = 0$.
- Solution 1: $AdS_2 \times S^2$, depends on 2 parameters
  - $ds^2 = \frac{\rho^2}{R_1^2} d\tau^2 - \frac{R_1^2}{\rho^2} d\rho^2 - R_2^2 d\Omega_2^2\,;$
  - $\mathcal{I}^0 \propto \rho^{-1}\,, \quad Z^i \propto x^i \rho^j\,;$
  - $A^x{}_{\underline{m}} \propto \epsilon^x_{mn} x^n \rho^{2j}\,;$

  where $R_1\,, R_2\,, j$ are functions of the parameters.

Solutions from classification: solution 2

Domain-wall Ansatz, x^1 dependence

$$\mathcal{I}^\Lambda = \mathcal{I}^\Lambda(x^1)\,, \quad V^x{}_{\underline{m}} = \delta^x_{\underline{m}} V(x^1)\,, \quad A^x{}_{\underline{m}} = 0\,.$$

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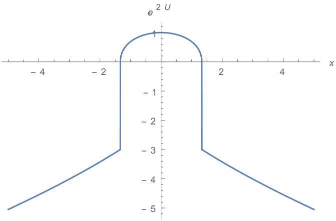
- Solution 2:
  - depends on 3 parameters, only some values give physical solutions.
  - $\mathcal{I}^0\,,\mathcal{I}^1 \neq 0\,, \quad \mathcal{I}^2\,,\mathcal{I}^3 = 0\,.$
  - Example of e^{2U} for certain values of the parameters:



Solutions from classification: solution 3

Domain-wall Ansatz, x^1 dependence

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Solutions from classification: solution 3

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- Solution 3: $\mathbb{R} \times \mathbb{H}^3$, depends on the parameter b
  - $ds^2 = \frac{2}{b^2} dt^2 - \frac{b^2}{2g^2} \frac{dx^m dx^m}{(x^1)^2}\,;$
  - $\mathcal{I}^0 = \frac{\sqrt{2}}{b}\,, \quad \mathcal{I}^1\,, \mathcal{I}^2\,, \mathcal{I}^3 = 0\,;$
  - $A^0{}_{2,3} = \text{const.}\,, \quad A^3{}_2 = -A^2{}_3 = \left(g x^1\right)^{-1}\,.$

Solutions from dimensional reduction

Dimensional reduction

$6d:\qquad \mathcal{N}=(1,0)\qquad \tilde{g}_{\tilde{\mu}\tilde{\nu}}\qquad \tilde{B}_{\tilde{\mu}\tilde{\nu}}\qquad \tilde{A}^{A}{}_{\tilde{\mu}}\qquad \tilde{\varphi}$

Dimensional reduction

$6d$: $\mathcal{N} = (1,0)$ $\tilde{g}_{\tilde{\mu}\tilde{\nu}}$ $\tilde{B}_{\tilde{\mu}\tilde{\nu}}$ $\tilde{A}^{A}{}_{\tilde{\mu}}$ $\tilde{\varphi}$

$5d$: $\begin{array}{rcl} n_V & = & 5 \\ C_{0rs} & = & \frac{1}{3!}\eta_{rs} \end{array}$ $\hat{g}_{\hat{\mu}\hat{\nu}}$ $\hat{A}^{I}{}_{\hat{\mu}} = (\hat{A}^{0,1,2}_{\hat{\mu}}, \hat{A}^{A}{}_{\hat{\mu}})$ $\hat{\phi}^{r}$

Dimensional reduction

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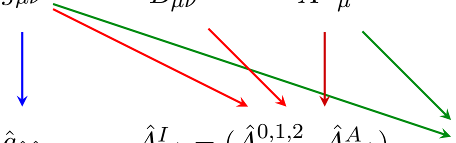

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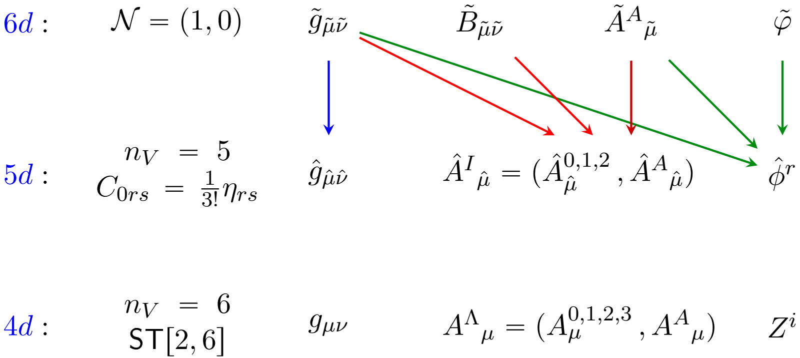
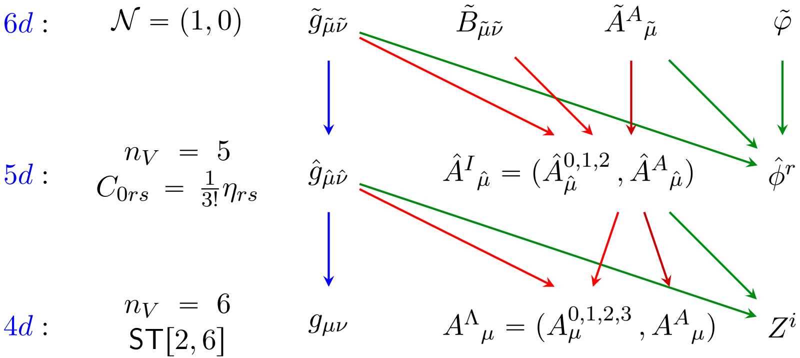
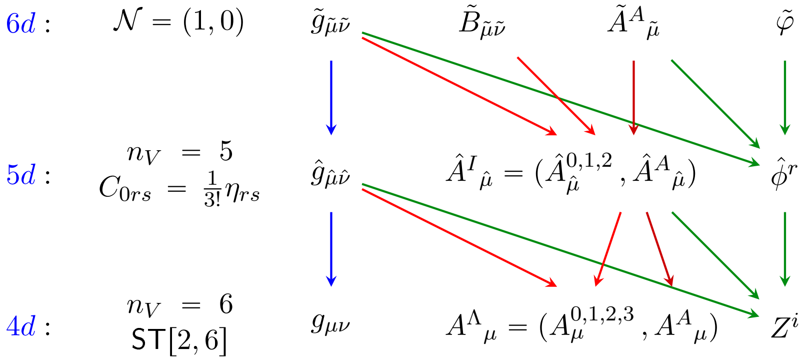


Dimensional reduction

$6d$:	$\mathcal{N} = (1,0)$	$\tilde{g}_{\tilde{\mu}\tilde{\nu}}$	$\tilde{B}_{\tilde{\mu}\tilde{\nu}}$ $\quad$ $\tilde{A}^{A}{}_{\tilde{\mu}}$	$\tilde{\varphi}$
$5d$:	$n_V = 5$ $C_{0rs} = \frac{1}{3!}\eta_{rs}$	$\hat{g}_{\hat{\mu}\hat{\nu}}$	$\hat{A}^{I}{}_{\hat{\mu}} = (\hat{A}^{0,1,2}_{\hat{\mu}}\,, \hat{A}^{A}{}_{\hat{\mu}})$	$\hat{\phi}^{r}$
$4d$:	$n_V = 6$ $\mathsf{ST}[2,6]$	$g_{\mu\nu}$	$A^{\Lambda}{}_{\mu} = (A^{0,1,2,3}_{\mu}\,, A^{A}{}_{\mu})$	Z^{i}

Dimensional reduction





$6d$:
$\mathcal{N} = (1,0)$
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$5d$:
$n_V = 5$
$C_{0rs} = \frac{1}{3!}\eta_{rs}$
$\hat{g}_{\hat{\mu}\hat{\nu}}$
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Z^i

Dimensional reduction

- SU(2)-FI gauging in all the theories related by dimensional reduction;
- known solutions[3] of the $\mathcal{N} = (1, 0)\,, d = 6\,,$ SU(2)-FI gauged theory with 2 isometries $\mapsto$ can be reduced to solutions of:
  - $\mathcal{N} = 2\,, d = 5$ with $n_V = 5$ and $C_{0rs} = \frac{1}{3!}\eta_{rs}$;
  - $\mathcal{N} = 2\,, d = 4$ with $n_V = 6\,,$ ST$[2, 6]$ model. The scalars parametrize a $\frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)} \times \frac{\mathrm{SO}(2,5)}{\mathrm{SO}(2)\times\mathrm{SO}(5)}$ coset space;
- dictionary relating the fields;
- the coupling constants are related by $g_6 = \sqrt{12} g_5 = -\frac{1}{\sqrt{2}} g_4\,.$

[3] M. Cariglia, O. A. P. Mac Conamhna, Class. Quant. Grav. 21 (2004) (arXiv:hep-th/0402055)

Dimensional reduction: solution 1

$6d$: $\mathbb{M}_3 \times S^3$, $\tilde{\varphi} =$ const. , $\tilde{A}^A \propto \sigma^A$ (meron)

↓

$5d$: $\mathbb{M}_2 \times S^3$, constant scalars , $\hat{A}^A \propto \sigma^A$ (meron)

↓

$4d$: $\mathbb{R} \times S^3$, constant scalars , $A^A \propto \sigma^A$ (meron)

Dimensional reduction: solution 2

$$6d: \; d\tilde{s}^2 = f(r)(dt^2 - dz^2) - f(r)^{-1}(dr^2 + a^2 r^2 d\Omega_3^2) \;\; \text{(black string)}$$

$$\tilde{\varphi} \neq 0\,, \qquad \tilde{A}^A \propto \sigma^A \;\; \text{(meron)}\,, \qquad \tilde{H} \text{ electric and magnetic.}$$

- Horizon at $r = 0$;
- if $r \to 0 \Rightarrow \mathrm{AdS}_3 \times \mathrm{S}^3$;
- for certain parameters, $\mathrm{AdS}_3 \times \mathrm{S}^3$ is a solution.

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Isometries:

- z
- ϕ in $d\Omega_3^2 = \frac{1}{4}\left((d\phi + \cos\theta d\psi)^2 + d\theta^2 + \sin^2\theta d\psi\right)^2$

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$\tilde{\varphi} \neq 0\,,\qquad \tilde{A}^A \propto \sigma^A$ (meron) , $\qquad \tilde{H}$ electric and magnetic.

$\downarrow$ along z

$5d$: singular at $r = 0$, asymptotically $\mapsto$ no known vacuum.

$\downarrow$ along ϕ

$4d$: same problems.

Dimensional reduction: solution 2

$6d$: $d\tilde{s}^2 = f(r)(dt^2 - dz^2) - f(r)^{-1}(dr^2 + a^2 r^2 d\Omega_3^2)$ (black string)

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$\downarrow$ along ϕ

$5d$:
- $\rightarrow$ regular at $r = 0$;
- $\rightarrow$ if $r \rightarrow 0 \Rightarrow$ $AdS_3 \times S^2$;
- $\rightarrow$ for certain parameters, $AdS_3 \times S^2$ is a solution;
- $\rightarrow$ not asymptotically AdS.

$\downarrow$ along z

$4d$: same problematic solution as before.

Dimensional reduction: solution 2

More possibilities:

- S^3 is a $U(1)$ fibration over S^2;
- AdS_3 is a $U(1)$ fibration over AdS_2.

$$AdS_3 \times S^3 \xrightarrow{\text{along the fiber}} AdS_2 \times S^3 \xrightarrow{\text{along the fiber}} AdS_2 \times S^2$$

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- Rotate the 2 $U(1)$ fibers and reduce along one of them.

Conclusions

- Explore the space of supersymmetric solutions to understand the structure of $\mathcal{N} = 2\,, d = 4$ supergravity;
- its possible gaugings, when coupled to vector multiplets;
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