P fluxes and exotic branes

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Based on work with D. Lombardo and F. Riccioni
and work with E. Bergshoeff, V.Penas and F. Riccioni

My talk in brief...

I focus on a particular class of non-geometric fluxes, so-called P fluxes, which belong to the (vector-spinor) **352** representation of the T-duality group SO(6,6) in $D = 4$ dimensions

- I derive how P fluxes transform under T-duality
- I discuss the role of P fluxes in a specific $\mathcal{N} = 1$ orientifold model shedding light on what happens in type IIA theory
- I derive how P fluxes modify a class of type II Bianchi identities
- I discuss the interplay between P fluxes and exotic/non-geometric branes and tadpoles

T-duality, fluxes and non-geometry

- T-duality is a symmetry between two string theories with compactified dimensions
- On a circle S^1 of radius R and coordinate X:
  - The string moves along the circle with quantized momentum $p = n/R \ (n \in \mathbb{Z})$
  - The string winds around the circle in units of $2\pi R$: $\Delta X = 2\pi R m \ (m \in \mathbb{Z})$
  - **T-duality**: $R \to 1/R$ and $n \leftrightarrow m$

T-duality, fluxes and non-geometry

$$R \to 1/R \text{ and } n \leftrightarrow m$$

- T-duality relates IIA $\leftrightarrow$ IIB string theories:

  NS-NS sectors: $g_{\mu\nu}, B_{\mu\nu}, g_{\star\mu}, B_{\star\mu}, \phi \leftrightarrow g_{\mu\nu}, B_{\mu\nu}, g_{\star\mu}, B_{\star\mu}, \phi$

  RR sectors: $C_\star,\ C_\mu \leftrightarrow C_0, C_{\star\mu}$
  $C_{\star\mu\nu}, C_{\mu\nu\rho} \leftrightarrow C_{\mu\nu},\ C_{\star\mu\nu\rho}$

- T-duality means that string theories with small and big radii are identified!
  - classical notions of geometry break down (non-geometry)
  - look for consistent exotic (non-geometric) backgrounds: globally/locally non-Riemannian

T-duality, fluxes and non-geometry

Generalization: On a torus T^d (with non-vanishing $g_{\mu\nu}$, $B_{\mu\nu}$)

- The fields can be embedded in a $2d \times 2d$ matrix
$$\mathcal{H} = \begin{bmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - bg^{-1}b \end{bmatrix}$$
- **T-duality**: $\mathcal{H} \to O\mathcal{H}O^T$, $O \in O(d, d; \mathbb{Z})$
- In Supergravity (low-energy approximation of string theory) T-duality is global $O \in O(d, d; \mathbb{R})$
- Crucial: T-duality mixes the metric g with the gauge field B in a non trivial way: we end up with a metric which is some complicated function of initial g and B (non-geometry)

T-duality, fluxes and non-geometry

Prototype of a non-geometric background: T-fold de Boer, Shigemori (2010)

- The NS5-brane is a solution of IIA/IIB supergravity, magnetically charged under B_2

	0	1	2	3	4	5	6	7	8	9
NS5	—	—	—	—	—	—				
KK5	—	—	—	—	—	—	○			
T-fold	—	—	—	—	—	—	○	○		

$$\text{NS5} \xrightarrow{T_6} \text{KK5} \xrightarrow{T_7} \text{T-fold}$$

- The T-fold turns out be globally non-geometric, geometrical well-defined only in $D = 8$, *i.e.* with isometries

T-duality, fluxes and non-geometry

- In string theory fluxes are p-forms field strengths of gauge fields, with legs along the internal manifold, integrally quantized, *e.g.*
  - IIB NS-NS sector: $B_2 \to H_3 = dB_2$ with $\int H_3 = n \in \mathbb{Z}$
  - IIB RR sector: $C_2 \to F_3 = dC_2$ with $\int F_3 = m \in \mathbb{Z}$
- Fluxes play a crucial phenomenological role in 4D compactifications inducing a potential for the scalar fields (moduli stabilisation, dS vacua, inflation...)
- In $\mathcal{N} = 1,\ D = 4$ supergravity the scalar potential is

$$V = e^{\mathcal{K}}(\mathcal{K}^{i\bar{j}} D_i W D_{\bar{j}} W - 3|W|^2)$$

  - $\mathcal{K}$ is the Kahler potential: depends on the scalars
  - W is the superpotential: contains the fluxes

T-duality, fluxes and non-geometry

- Non-geometric fluxes: sourced by non-geometric/exotic branes

$$\text{NS5} \xrightarrow{T_i} \text{KK5} \xrightarrow{T_j} \text{T-fold}$$

$$\Downarrow$$

parallel T-duality chain of fluxes: $H_{ijk} \xrightarrow{T_i} f^i_{jk} \xrightarrow{T_j} Q^{ij}_k$

- From the point of view of supergravity, fluxes induce a gauging in the 4D low-energy effective action.The gauging is described in terms of the embedding tensor

  de Wit, Samtleben, Trigiante (2002)

  Maximal theory in D=4: embedding tensor in the **912** of $E_{7(7)}$

NS and RR fluxes

If we decompose the **912** under T-duality $SO(6,6) \subset E_{7(7)}$ we end up with

$$\mathbf{912} = \mathbf{32} \oplus \mathbf{220} \oplus \mathbf{352} \oplus \ldots$$

The **32** rep corresponds to the RR fluxes

$$\theta_a \to \begin{cases} F_m \quad F_{mnp} \quad F_{mnpq} & \text{IIB} \\ \\ F \quad F_{mn} \quad F_{mnpq} \quad F_{mnpqrs} & \text{IIA} \end{cases}$$

...under T-duality: $F_{n_1 \ldots n_p} \xrightarrow{T_m} F_{m n_1 \ldots n_p}$

The **220** corresponds to the NS fluxes introduced before...

$$\theta_{MNP} \;\to\; H_{mnp} \quad f_{mn}^{p} \quad Q_m^{np} \quad R^{mnp}$$

...under T-duality: $H_{mnp} \xrightarrow{T^p} f_{mn}^{p} \xrightarrow{T^n} Q_m^{np} \xrightarrow{T^m} R^{mnp}$

P fluxes

P fluxes belong to the representation of the embedding tensor which is the **352** representation of $SO(6,6)$

This is the vector-spinor ('gravitino') representation θ_{Ma}

By decomposing the whole representation under $GL(6,\mathbb{R})$ one gets

$$\theta_{Ma} \to \begin{cases} P_m \quad P_m^{n_1 n_2} \quad P_m^{n_1 \dots n_4} \quad P^{m,n_1 n_2} \quad P^{m,n_1 \dots n_4} \quad P^{m,n_1 \dots n_6} \quad \text{IIA} \\ P_m^n \quad P_m^{n_1 n_2 n_3} \quad P_m^{n_1 \dots n_5} \quad P^{m,n} \quad P^{m,n_1 n_2 n_3} \quad P^{m,n_1 \dots n_5} \quad \text{IIB} \end{cases}$$

Bergshoeff, Penas, Riccioni, SR (2015)

$P^{m,n_1 \dots n_p}$ belong to mixed symmetry representations (vanishing completely antisymmetric part)

P fluxes and T-duality

What happens to a given P flux under T-duality?

We make use of the fact that P fluxes belong to a vector-spinor representation (hybrid between RR and NS)

Prescription on the indices: in the P flux treat the m upstairs and downstairs indices as forming the vector index M, while the n indices form the spinor representation

As a consequence, we derive the following T-duality rules

$$P_m^{n_1 \dots n_p} \xrightarrow{T^m} P^{m,n_1 \dots n_p m}$$
$$P_m^{n_1 \dots n_p} \xrightarrow{T^{n_p}} P_m^{n_1 \dots n_{p-1}}$$
$$P^{m,n_1 \dots n_p} \xrightarrow{T^{n_p}} P^{m,n_1 \dots n_{p-1}}$$

Lombardo, Riccioni, SR (2016)

IIA/IIB orientifold models

We consider T-duality and P fluxes (NS, RR) in a specific $\mathcal{N}=1$ model: IIA/IIB $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold with O3 and O6-planes.

Aldazabal, Cámara, Font, Ibáñez (2006)

$T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$:

- T^6 is factorized: $T^6 = \bigotimes_{i=1}^{3} T^2_{(i)}$

  each subtorus has coordinates (x^i, y^i)
- Basis of closed 2-forms: $\omega_i = dx^i \wedge dy^i$
- Kahler form: $J = \sum_i A_i \omega_i$
- Holomorphic 3-form:

  $\Omega = (dx^1 + i\tau_1 dy^1) \wedge (dx^2 + i\tau_2 dy^2) \wedge (dx^3 + i\tau_3 dy^3)$

IIB/O3

IIB/O3: IIB modded out by $\Omega_P(-1)^{F_L}\sigma_B$ where

$$\sigma_B(x^i) = -x^i \qquad \sigma_B(y^i) = -y^i$$

- 7 complex moduli (U_i, T_i, S):
  - complex structure moduli

    $U_i = \tau_i$
  - complex Kahler moduli T_i

    $\mathcal{J}_c = C_4 + \frac{i}{2}e^{-\phi} J \wedge J = i\sum_i T_i \tilde{\omega}_i$
  - axion-dilaton

    $S = e^{-\phi} + iC_0$

IIA/O6

IIA/O6 obtained from IIB/O3 by performing three T-dualities along $x^1, \quad x^2, \quad x^3$
Involution is now

$$\sigma_A(x^i) = x^i \qquad \sigma_A(y^i) = -y^i$$

Complex scalars embedded in:

- Complexified holomorphic 3-form is

$$\Omega_c = C_3 + i\mathrm{Re}(C\Omega) = iS(dx^1 \wedge dx^2 \wedge dx^3) + iU_i(dx \wedge dy \wedge dy)^i$$

- Complex Kahler moduli are

  $J_c = B + iJ = i\sum_i T_i\omega_i$

  IIB and IIA moduli related by T-duality as $T_i \leftrightarrow U_i$

Allowed RR fluxes

IIB RR fluxes	IIA RR fluxes
$F_{x^1x^2x^3}$	F
$F_{y^ix^jx^k}$	$F_{x^iy^i}$
$F_{x^iy^jy^k}$	$F_{x^jy^jx^ky^k}$
$F_{y^1y^2y^3}$	$F_{x^1y^1x^2y^2x^3y^3}$

IIB/O3: only F_3 turned on
IIA/O6: F, F_2, F_4, F_6

Allowed NS fluxes

IIB NS fluxes	IIA NS fluxes
$H_{x^1x^2x^3}$	$R^{x^1x^2x^3}$
$H_{y^ix^jx^k}$	$-Q_{y^i}^{x^jx^k}$
$H_{x^iy^jy^k}$	$-f_{y^jy^k}^{x^i}$
$H_{y^iy^jy^k}$	$H_{y^iy^jy^k}$
$Q_{x^i}^{x^jx^k}$	$f_{x^jx^k}^{x^i}$
$Q_{y^j}^{x^ky^i}$	$f_{y^jx^k}^{y^i}$
$Q_{y^i}^{x^jx^k}$	$-H_{y^ix^jx^k}$
$Q_{x^j}^{x^iy^k}$	$Q_{x^i}^{y^kx^j}$
$Q_{x^i}^{y^jy^k}$	$R^{x^iy^ky^j}$
$Q_{y^i}^{y^jy^k}$	$Q_{y^i}^{y^jy^k}$

IIA/IIB superpotentials

In IIB/O3 H_3 and F_3 turned on determine the superpotential

$$W_B = \int (F_3 - iSH_3) \wedge \Omega \qquad \text{Gukov, Vafa, Witten (2001)}$$

with all NS fluxes turned on, generalises to:

$$W_B = \int (F_3 - iSH_3 + Q \cdot \mathcal{J}_c) \wedge \Omega \qquad \text{Shelton, Taylor, Wecht (2005)}$$

$$= P_1(U) + SP_2(U) + TP_3(U)$$

The IIA/O6 superpotential is

$$W_A = \int [e^{J_c} \wedge F_{RR} + \Omega_c \wedge (H_3 + fJ_c + QJ_c^{(2)} + RJ_c^{(3)})]$$

which has the form

$$W_A = P_1(T) + SP_2(T) + UP_3(T)$$

Consistently, W_A and W_B match under T-duality ($U \leftrightarrow T$)

IIA/IIB superpotentials with P fluxes

In IIB/O3 the P flux P_m^{np} has been already introduced as the S-dual of Q_m^{np}

Aldazabal, Cámara, Font, Ibáñez (2005)

By requiring that the superpotential transforms properly under S-duality, one obtains

$$W_B = \int [(F_3 - iSH_3) + (Q - iSP)\mathcal{J}_c] \wedge \Omega$$

which adds a new ST term

$$W_B = P_1(U) + SP_2(U) + TP_3(U) + STP_4(U)$$

We now use T-duality rules to find all possible P fluxes, to find W_A in a covariant form (for this particular model) and to generalise W_B to all P fluxes

P fluxes in IIA/IIB orientifolds

(Part of) IIA P fluxes found performing 3 T-dualities from IIB to IIA along the three x directions

IIB P fluxes	IIA P fluxes
$P^{x^k x^j}_{y^i}$	$P^{x^i}_{y^i}$
$P^{y^j x^k}_{y^i}$	$P^{x^i x^j y^j}_{y^i}$
$P^{y^j y^k}_{y^i}$	$P^{x^i x^j x^k y^j y^k}_{y^i}$
$P^{x^k x^j}_{x^i}$	P^{x^i, x^i}
$P^{y^j x^k}_{x^i}$	$P^{x^i, x^i x^j y^j}$
$P^{y^j y^k}_{x^i}$	$P^{x^i, x^i x^j x^k y^j y^k}$

Not the whole story... according to the symmetries: $P^{x_i, x_i} \Rightarrow P^{y_i, y_i}$, $P^{x_i}_{y_i} \Rightarrow P^{y_i}_{x_i}$, $P^{x_i, x_i x_j y_j} \Rightarrow P^{y_i, y_i x_j y_j}$, and so on ...

IIB P fluxes	IIA P fluxes
$P^{x^k x^j}_{y^i}$	$P^{x^i}_{y^i}$
$P^{y^j x^k}_{y^i}$	$P^{x^i x^j y^j}_{y^i}$
$P^{y^j y^k}_{y^i}$	$P^{x^i x^j x^k y^j y^k}_{y^i}$
$P^{x^k x^j}_{x^i}$	P^{x^i, x^i}
$P^{y^j x^k}_{x^i}$	$P^{x^i, x^i x^j y^j}$
$P^{y^j y^k}_{x^i}$	$P^{x^i, x^i x^j x^k y^j y^k}$
$P^{x^i, x^i x^j x^k y^i}$	$P^{y^i}_{x^i}$
$P^{x^i, x^i x^j y^i y^k}$	$P^{y^i x^k y^k}_{x^i}$
$P^{x^i, x^i y^i y^j y^k}$	$P^{y^i x^j y^j x^k y^k}_{x^i}$
$P^{y^i, x^i x^j x^k y^i}$	P^{y^i, y^i}
$P^{y^i, y^i x^i y^j x^k}$	$P^{y^i, y^i x^j y^j}$
$P^{y^i, y^i y^j y^k x^i}$	$P^{y^i, y^i x^j x^k y^j y^k}$

IIA/IIB superpotentials with all P fluxes

Start from

$$W_B = \int [(F_3 - iSH_3) + (Q - iSP_1^2)\mathcal{J}_c] \wedge \Omega$$

by T-duality we find the IIA superpotential, completed by including all the possible IIA P fluxes in the following covariant form:

$$W_A = \int [e^{J_c} F_{RR} + \Omega_c(H_3 + fJ_c + QJ_c^2 + RJ_c^3 - P_1^1\Omega_c$$
$$+(P^{1,1} - P_1^3)\Omega_c J_c - (P^{1,3} + P_1^5)\Omega_c J_c^2 - (P^{1,5}\Omega_c J_c^3))]$$

We come back to IIB/O3 determining W_B with all IIB P-fluxes

$$W_B = \int [(F_3 - iSH_3) + (Q - iSP_1^2)J_c - P^{1,4}\mathcal{J}_c^2] \wedge \Omega$$
$$= P_1(U) + SP_2(U) + TP_3(U) + STP_4(U) + T^2 P_5(U)$$

this agrees with the IIB superpotential originally proposed in
Aldazabal, Andrés, Cámara, Graña (2010)

P fluxes and BI

Another interesting application: generalise the following NS-NS BI including all P fluxes (crucial to concrete models):

$$
\begin{gathered}
f^{r}_{[mn}H_{pq]r} = 0 \\
Q^{mr}_{[n}H_{pq]r} + f^{r}_{[np}f^{m}_{q]r} = 0 \\
4Q^{[m|r}_{[p}f^{n]}_{q]r} + f^{r}_{pq}Q^{mn}_{r} + R^{mnr}H_{pqr} = 0 \\
R^{[mn|r}f^{p]}_{qr} + Q^{[m|r}_{q}Q^{np]}_{r} = 0 \\
R^{[mn|r}Q^{pq]}_{r} = 0
\end{gathered}
$$

Shelton, Taylor, Wecht (2005)

Aldazabal, Cámara, Font, Ibáñez (2006)

Ihl, Robbins, Wrase (2007)

P fluxes and BI

In IIB we get:

$$6f^{r}_{[mn}H_{pq]r} + 4F_{[mnp}P_{q]} + 2P^{rs}_{[m}F_{npq]rs} = 0$$

$$3Q^{mr}_{[n}H_{pq]r} + 3f^{r}_{[np}f^{m}_{q]r} - 3P^{mr}_{[n}F_{pq]r} - P^{m,mr}F_{npqmr} = 0$$

$$-Q^{mn}_{r}f^{r}_{pq} - 4Q^{[m|r}_{[p}f^{n]}_{q]r} - R^{mnr}H_{pqr} + 2F_{[p}P^{mn}_{q]} + P^{m,mn}F_{pqm} +$$
$$P^{n,mn}F_{pqn} + P^{mnrs}_{[p}F_{q]rs} + \frac{1}{2}P^{m,nmrs}F_{pqmrs} - \frac{1}{2}P^{n,mnrs}F_{pqnrs} = 0$$

$$3R^{[mn|r}f^{p]}_{qr} + 3Q^{[mn}_{r}Q^{p]r}_{q} + P^{mnpr}_{q}F_{r} - P^{m,mnpr}F_{mqr} - P^{n,mnpr}F_{nqr} -$$
$$P^{p,mnpr}F_{pqr} = 0$$

$$6R^{[mn|r}Q^{pq]}_{r} + F_{m}P^{m,mnpq} + F_{n}P^{n,mnpq} + F_{p}P^{p,mnpq} +$$
$$F_{q}P^{q,mnpq} + \frac{1}{2}P^{m,npqmrs}F_{mrs} + \frac{1}{2}P^{n,npqmrs}F_{nrs} + \frac{1}{2}P^{p,npqmrs}F_{prs} +$$
$$\frac{1}{2}P^{q,npqmrs}F_{qrs}=0$$

P fluxes and BI

...and in IIA we get:

$$6f^r_{[mn}H_{pq]r} + 4P^r_{[m}F_{npq]r} = 0$$

$$3Q^{mr}_{[n}H_{pq]r} + 3f^r_{[np}f^m_{q]r} - 3P^m_{[n}F_{pq]} - P^{m,m}F_{npqm} + \frac{1}{2}P^{m,mrs}F_{npqmrs} + \frac{3}{2}P^{mrs}_{[n}F_{pq]rs} = 0$$

$$-Q^{mn}_r f^r_{pq} - 4Q^{[m|r}_{[p}f^{n]}_{q]r} - R^{mnr}H_{pqr} + 2P^{mnr}_{[p}F_{q]r} - P^{m,mnr}F_{pqmr} - P^{n,mnr}F_{pqnr} = 0$$

$$3R^{[mn|r}f^{p]}_{qr} + 3Q^{[mn}_r Q^{p]r}_q + FP^{mnp}_q - P^{m,mnp}F_{mq} - P^{n,mnp}F_{nq} - P^{p,mnp}F_{pq} - \frac{1}{2}P^{mnprs}_q F_{rs} - P^{m,npmrs}F_{qmrs} - P^{n,npmrs}F_{qnrs} - P^{p,npmrs}F_{qprs} = 0$$

$$6R^{[mn|r}Q^{pq]}_r - F_{qr}P^{q,mnpqr} - F_{pr}P^{p,mnpqr} - F_{nr}P^{n,mnpqr} - F_{mr}P^{m,mnpqr} = 0$$

P fluxes and exotic branes

The final thing... we study the interplay between P fluxes, exotic branes and tadpoles...

The NS and RR fluxes induce RR tadpoles

In IIB/O3 we have a D3/O3 tadpole induced by

$$\int C_4 \wedge H_3 \wedge F_3$$

and a D7 tadpole also

$$\int C_8 \wedge QF_3$$

In IIA/O6 we have the D6/O6 term

$$\int C_7 \wedge (-H_3 F_0 + \omega F_2 - QF_4 + RF_6)$$

P fluxes and exotic branes

Precisely like the NS and RR fluxes, also the P fluxes induce tadpoles (that must be cancelled by introducing branes).

P_p^{mn} induces a charge for the 7-brane which is the S-dual of the D7-brane

$$\int E_8 \wedge P_1^2 H_3$$

What is the corresponding tadpole in IIA? We need to know what happens to E_8 under T-duality $\rightarrow$ spectrum analysis: look at classification of all branes in any dimension

P fluxes and exotic branes

The branes in the maximal theories in any dimension have been classified according to how their tension scales with the dilaton in the string frame, $T \sim g_S^{\alpha}$

Bergshoeff, Riccioni (2011)

Bergshoeff, Marrani, Riccioni (2012)

- $\alpha = 0$: fundamental branes
- $\alpha = -1$: D-branes
- $\alpha = -2$: NS (solitonic) -branes
- $\alpha = -3$: S-dual of D7-brane

P fluxes and exotic branes

e.g. to find all the NS branes ($\alpha = -2$) you need to compactify the mixed-symmetry potentials

$$D_6 \quad D_{7,1} \quad D_{8,2} \quad D_{9,3} \quad D_{10,4}$$

The extra indices encode the fact that the corresponding brane solutions must have isometries

Lozano-Tellechea, Ortín (2001)

Bergshoeff, Ortín, Riccioni (2011)

$D_{7,1} \rightarrow D_{6x,x}$ KK monopole

$D_{8,2} \rightarrow D_{6xy,xy}$ T-fold

These are the exotic branes seen before

P fluxes and exotic branes

For the $\alpha = -3$ brane one needs to compactify the potentials

$$E_{4,MNa} \to \begin{cases} E_8 \ E_{8,2} \ E_{8,4} \ E_{9,2,1} \ E_{8,6} \ E_{9,4,1} \ E_{10,2,2} \ E_{10,4,2} \ E_{10,6,2} & \text{IIB} \\ E_{8,1} \ E_{8,3} \ E_{9,1,1} \ E_{8,5} \ E_{9,3,1} \ E_{9,5,1} \ E_{10,3,2} \ E_{10,5,2} & \text{IIA} \end{cases}$$

In our last paper we find how these fields transform under T-duality:

- $\alpha = -1$: $\quad 0 \longleftrightarrow 1 \quad\quad C_{\ldots} \xrightarrow{T^x} C_{\ldots x}$
- $\alpha = -2$: $\quad 0 \longleftrightarrow 1,1 \quad\quad D_{\ldots} \xrightarrow{T^x} D_{\ldots x,x}$

  $1 \longleftrightarrow 1 \quad\quad D_{\ldots x} \xrightarrow{T^x} D_{\ldots x}$
- $\alpha = -3$: $\quad 0 \longleftrightarrow 1,1,1 \quad\quad E_{\ldots} \xrightarrow{T^x} E_{\ldots x,x,x}$

  $1 \longleftrightarrow 1,1 \quad\quad E_{\ldots x} \xrightarrow{T^x} E_{\ldots x,x}$

P fluxes and exotic branes

Back to our $\mathcal{N} = 1$ model

Using the T-duality rules for fluxes and branes, we can figure out what are the tadpoles induced by all the fluxes and which branes can be included to cancel them

We find a class of $\alpha = -3$ exotic branes that can be included both in IIB and in IIA, giving the non-trivial constraints:

$$
\begin{gathered}
P_1^2 \cdot H_3 \leftrightarrow E_8 \\
P_1^2 \cdot Q \leftrightarrow E_{8,4},\ E_{9,2,1} \\
P^{1,4} \cdot Q \leftrightarrow E_{10,4,2}
\end{gathered}
$$

IIB		IIA	
potential	internal component	internal component	potential
E_8	$E_{x_i y_i x_j y_j}$	$E_{x_i y_i x_j y_j x_k, x_i x_j x_k, x_k}$	$E_{9,3,1}$
$E_{8,4}$	$E_{x_i y_i x_j x_k, x_i y_i x_j x_k}$	$E_{x_i y_i x_j x_k, y_i}$	$E_{8,1}$
	$E_{x_i y_i x_j y_j, x_i y_i x_j y_j}$	$E_{x_i y_i x_j y_j x_k, y_i y_j x_k, x_k}$	$E_{9,3,1}$
	$E_{x_i y_i x_j y_k, x_i y_i x_j y_k}$	$E_{x_i y_i x_j y_j x_k, y_i x_k y_k, x_k}$	$E_{9,3,1}$
	$E_{x_i y_i y_j y_k, x_i y_i y_j y_k}$	$E_{x_i y_i x_j y_j x_k y_k, y_i x_j y_j x_k y_k, x_j x_k}$	$E_{10,5,2}$
$E_{9,2,1}$	$E_{x_i y_i x_j y_j x_k, x_i x_k, x_i}$	$E_{x_i x_j y_j x_k, x_j}$	$E_{8,1}$
	$E_{x_i y_i x_j y_j y_k, x_i y_k, x_i}$	$E_{y_i x_j y_j x_k y_k, x_j x_k y_k, x_k}$	$E_{9,3,1}$
	$E_{x_i y_i x_j y_j x_k, y_i x_k, y_i}$	$E_{x_i y_i x_j y_j x_k, x_i y_i x_j, y_i}$	$E_{9,3,1}$
	$E_{x_i y_i x_j y_j y_k, y_i y_k, y_i}$	$E_{x_i y_i x_j y_j x_k y_k, x_i y_i x_j x_k y_k, y_i x_k}$	$E_{10,5,2}$
$E_{10,4,2}$	$E_{x_1 y_1 x_2 y_2 x_3 y_3, x_i y_i x_j y_j, x_i y_i}$	$E_{y_i x_j y_j x_k y_k, y_i y_j x_k, y_i}$	$E_{9,3,1}$
	$E_{x_1 y_1 x_2 y_2 x_3 y_3, x_i y_i x_j x_k, x_j x_k}$	$E_{x_i y_i y_j y_k, y_i}$	$E_{8,1}$
	$E_{x_1 y_1 x_2 y_2 x_3 y_3, x_i y_j x_k y_k, x_i y_j}$	$E_{y_i x_j y_j x_k y_k, x_j y_j y_k, y_k}$	$E_{9,3,1}$
	$E_{x_1 y_1 x_2 y_2 x_3 y_3, x_i y_i y_j y_k, y_j y_k}$	$E_{x_1 y_1 x_2 y_2 x_3 y_3, y_i x_j y_j x_k y_k, y_j y_k}$	$E_{10,5,2}$

P fluxes and exotic branes

E_8 maps to $E_{9,3,1}$ in IIA/O6. In components, the tadpole is induced by:

$$\int E_{[9|,mnp,m} \times (NS \cdot P)^{mnp,m}_{|1]},$$

with $(NS \cdot P)^{mnp,m}_q =$
$-2P^{m,m[n|r} f^{p]}_{|q]r} + f^m_{pq} P^{p,mnp} + f^m_{nq} P^{n,mnp} + P^{m,m} Q^{np}_q + Q^{mr}_q P^{mnp}_r -$
$2P^{m[n|r}_q Q^{m|p]}_r + \frac{1}{2} P^{mnprs}_q f^m_{rs} + P^m_q R^{mnp} + \frac{1}{2} P^{m,mnprs} H_{qrs}$

$$\int E_{9,3,1} \wedge (P^{1,4} f + P^{1,1} Q + P^3_1 Q + P^5_1 f + P^1_1 R + P^{1,5} H_3)$$

This nice structure between fields and P fluxes generalise to all the other $\alpha = -3$ fields.

Conclusions

- We have a T-duality rule for P fluxes (and branes)
- We have an explicit T-dual expression for the superpotential with P fluxes included
- We have a generalized expression for tadpole conditions including exotic branes

Conclusions

Next steps:

- Extend to other fluxes and branes (and models)
- Study moduli stabilisation
- Dynamics of exotic branes