HIGGS-CURVATURE COUPLING AND POST-INFLATIONARY VACUUM STABILITY

Francisco Torrentí, IFT UAM/CSIC

with Daniel G. Figueroa and Arttu Rajantie

(arXiv:1612.xxxxx)

V Postgraduate Meeting on Theoretical Physics, Oviedo, 18th November 2016

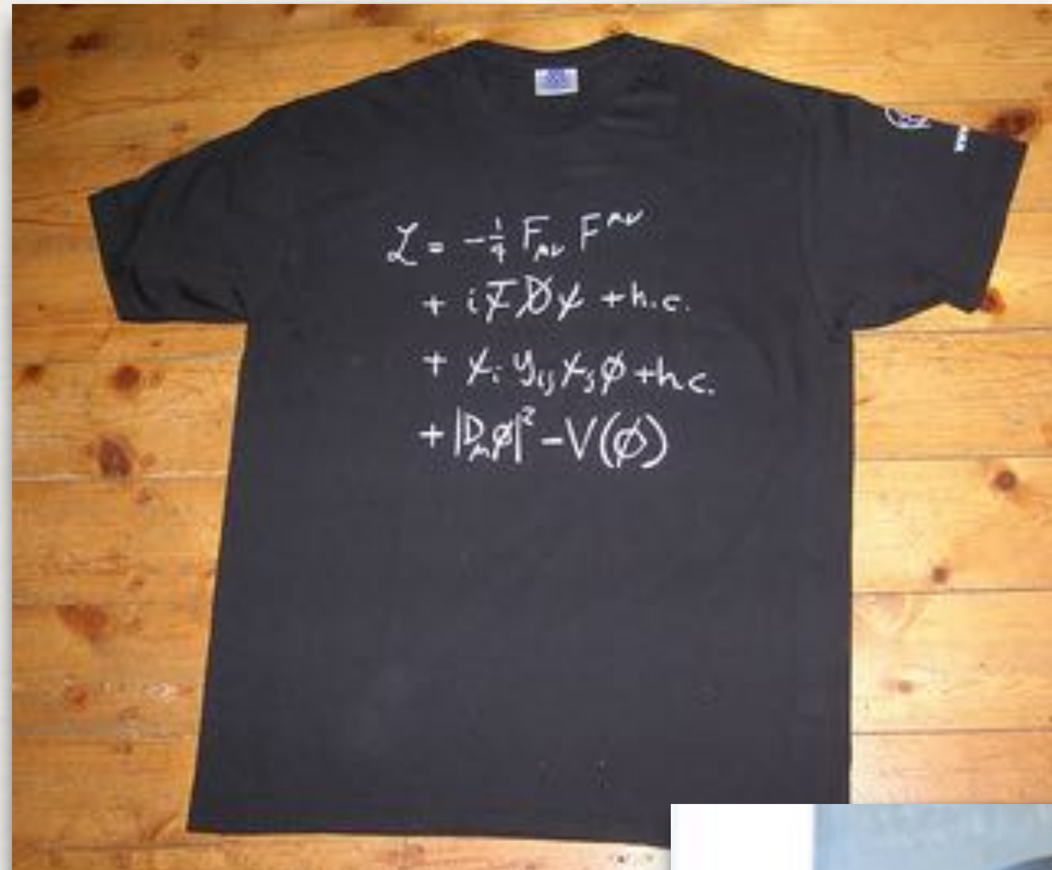
1. THE HIGGS-CURVATURE COUPLING

- The Standard Model Lagrangian in *Minkowski spacetime* (6 quarks +6 leptons+gauge bosons+Higgs) possesses **19 free parameters.**

- Since the discovery of the Higgs in the LHC (2012), we have determined all of them with **great precision**.

m_e	Electron mass	511 keV
m_μ	Muon mass	105.7 MeV
m_τ	Tau mass	1.78 GeV
m_u	Up quark mass	1.9 MeV
m_d	Down quark mass	4.4 MeV
m_s	Strange quark mass	87 MeV
m_c	Charm quark mass	1.32 GeV
m_b	Bottom quark mass	4.24 GeV
m_t	Top quark mass	172.7 GeV
θ_{12}	CKM 12-mixing angle	13.1°

θ_{23}	CKM 23-mixing angle	2.4°
θ_{13}	CKM 13-mixing angle	0.2°
δ	CKM CP-violating Phase	0.995
g_1 or g'	U(1) gauge coupling	0.357
g_2 or g	SU(2) gauge coupling	0.652
g_3 or g_s	SU(3) gauge coupling	1.221
θ_{QCD}	QCD vacuum angle	~0
v	Higgs vacuum expectation valu	246 GeV
m_H	Higgs mass	~ 125 GeV



CERN merchandising

1. THE HIGGS-CURVATURE COUPLING

- In ***curved spacetime***, there is one more possible term, required for the renormalisability of the theory:

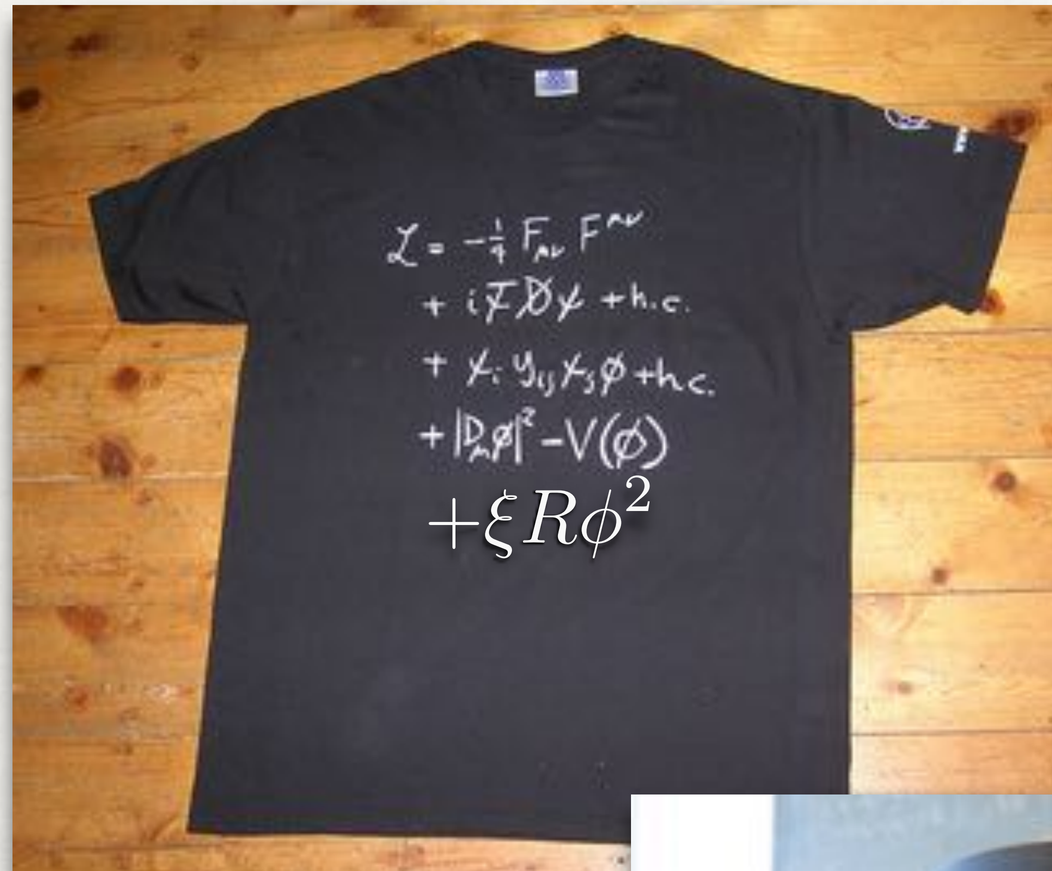
$$\boxed{\mathcal{L} = \frac{1}{2} M_{\rm pl}^2 R + \xi R \varphi^\dagger \varphi + \mathcal{L}_{\rm SM}}$$

- The Higgs-curvature coupling **ξ runs with energy**, and cannot be set to 0.

- As the Ricci scalar R **is very small today**, constraints from particle-physics experiments are very weak:

LHC: $|\xi| \lesssim 2.6 \times 10^{15}$ **(Atkins & Calmet 2012)**

But in the **early universe**, **R↑↑**, and its effects can be important.



CERN
merchandising
in curved spacetime

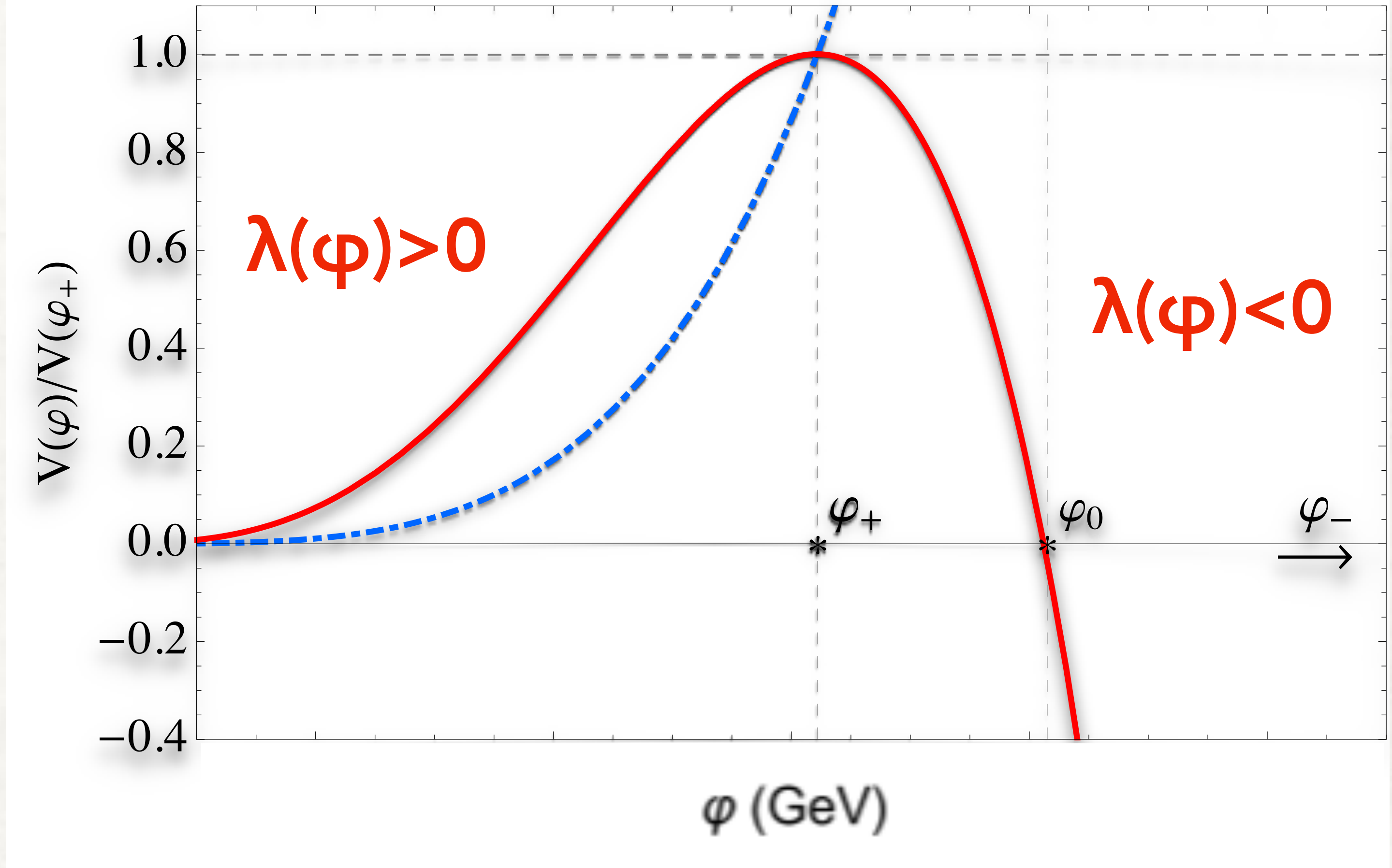
2. SM HIGGS POTENTIAL & HIGGS INSTABILITY

(RGI) SM Higgs potential:

$$\boxed{V(\varphi) = \frac{\lambda(\varphi)}{4}\varphi^4}$$

$$v \sim \mathcal{O}(10^2)\mathrm{GeV} \ll \varphi$$

- Potential has a maximum (barrier) at $\boldsymbol{\varphi_+}$.
- For $\boldsymbol{\varphi > \varphi_+}$, we have $\boldsymbol{\lambda(\varphi) < 0}$.
- **Higgs develops a second vacuum at $\boldsymbol{\varphi\_ >> \varphi_0, \varphi_+}$.**



Running of $\boldsymbol{\lambda(\varphi)}$ is very sensitive to ***top-quark mass.***

world average: m_t = (173.34±0.76) GeV

m_t(GeV)	φ_+(GeV)	φ_0(GeV)
172.12	7.83×10^{11}	1.01×10^{12}
172.73	5.20×10^{10}	6.70×10^{10}
173.34	7.49×10^{9}	9.65×10^{9}
173.95	1.67×10^{9}	2.15×10^{9}
174.56	4.92×10^{8}	6.34×10^{8}

(Note: For m_t<171.5GeV; $\boldsymbol{\varphi_+, \varphi_0}$ —> +∞)

G. Degrassi et al. (2012); Bezrukov et al. (2012)

2. SM HIGGS POTENTIAL & HIGGS INSTABILITY

If the Higgs had decay to the high-energy vacuum in the past, the Universe would have immediately collapsed.
This imposes strong constraints to ξ.

Constraints from INFLATION

Constraints from PREHEATING

2. SM HIGGS POTENTIAL & HIGGS INSTABILITY

If the Higgs had decay to the high-energy vacuum in the past, the Universe would have immediately collapsed.
This imposes strong constraints to ξ.

Constraints from INFLATION

Constraints from PREHEATING

- If **ξ**~0, Higgs is effectively **massless during inflation** and **fluctuates:**

Yokoyama, Starobinsky (1994)

$$\boxed{P_{\rm eq}(\varphi) = \mathcal{N} \exp\left(-\frac{2\pi^2}{3}\frac{\lambda\varphi^4}{H_*^4}\right)} \rightarrow \boxed{\varphi \sim H_* \gg \varphi_+, \varphi_0}$$

$$H_* \leq H_*^{(\rm max)} \simeq 8.4\times 10^{13}\,{\rm GeV}$$

the Higgs becomes unstable!

- Introducing a small coupling **ξ**>0 saves the day: $m_{h,\rm eff}^2 = \xi R$

Herrannen, Markannen, Nurmi & Rajantie (2014) → **Lower bound:** $\xi \gtrsim 0.06$

2. SM HIGGS POTENTIAL & HIGGS INSTABILITY

If the Higgs had decay to the high-energy vacuum in the past, the Universe would have immediately collapsed.
This imposes strong constraints to ξ.

Constraints from INFLATION

Constraints from PREHEATING

SM Higgs is excited due to **tachyonic resonance**

Upper bound

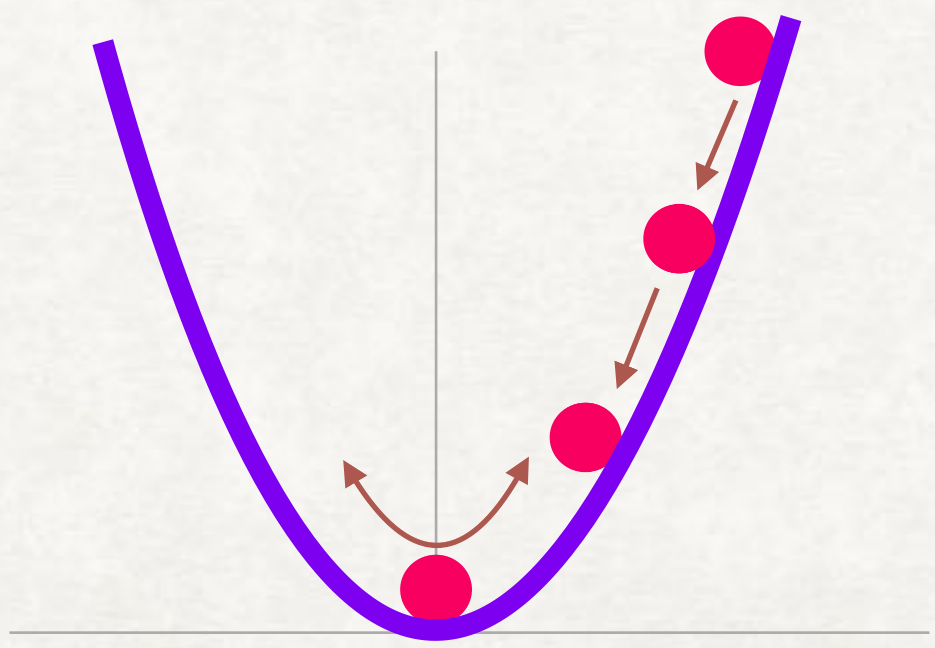
(let's see how it works!)

3. TACHYONIC RESONANCE

- We consider a chaotic inflation model with **quadratic potential**:

$$\boxed{V(\phi) = \frac{1}{2}m_\phi^2\phi^2} \qquad m_\phi \approx 6 \times 10^{-6} m_p$$

- If $\phi \gtrsim \mathcal{O}(10)m_p$, inflaton decays in a slow-roll regime, causing the exponential expansion of the Universe.
- When $\phi_* \approx 2m_p$, inflation ends, and the inflaton starts oscillating around the minimum of its potential (**preheating**).



3. TACHYONIC RESONANCE

- To obtain the post-inflationary dynamics of the system, we solve the **field** and **Friedmann** equations:

$$\ddot{\phi} + 3H(t)\dot{\phi} + m_\phi^2\phi = 0 \qquad H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{(\dot{\phi}^2 + m_\phi^2\phi^2)}{6m_p^2}$$

- The **inflaton** solution is:

$$\boxed{\phi(t) \simeq \Phi(t)\sin(m_\phi t) \qquad \Phi(t) = \sqrt{\frac{8}{3}}\frac{m_p}{m_\phi t}}$$

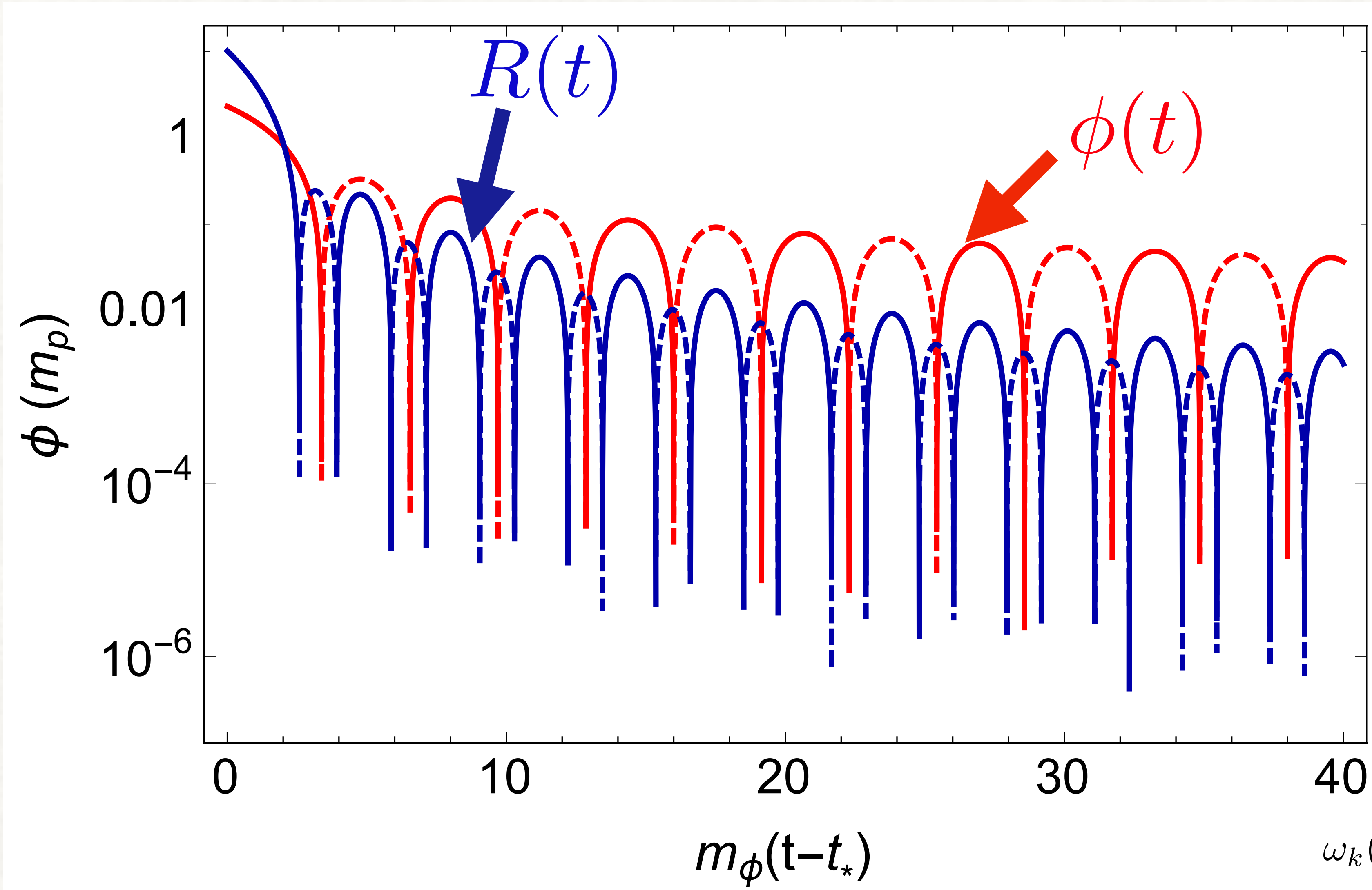
decaying amplitude

And the **Ricci scalar** and **scale factor**:

ε(t): small oscillating function

$$R(t) \equiv 6\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a}\right] = \frac{1}{m_p^2}(2m_\phi^2\phi^2 - \dot{\phi}^2) \quad \Rightarrow \quad \boxed{a(t) = t^{2/3(1+\epsilon(t))}}$$

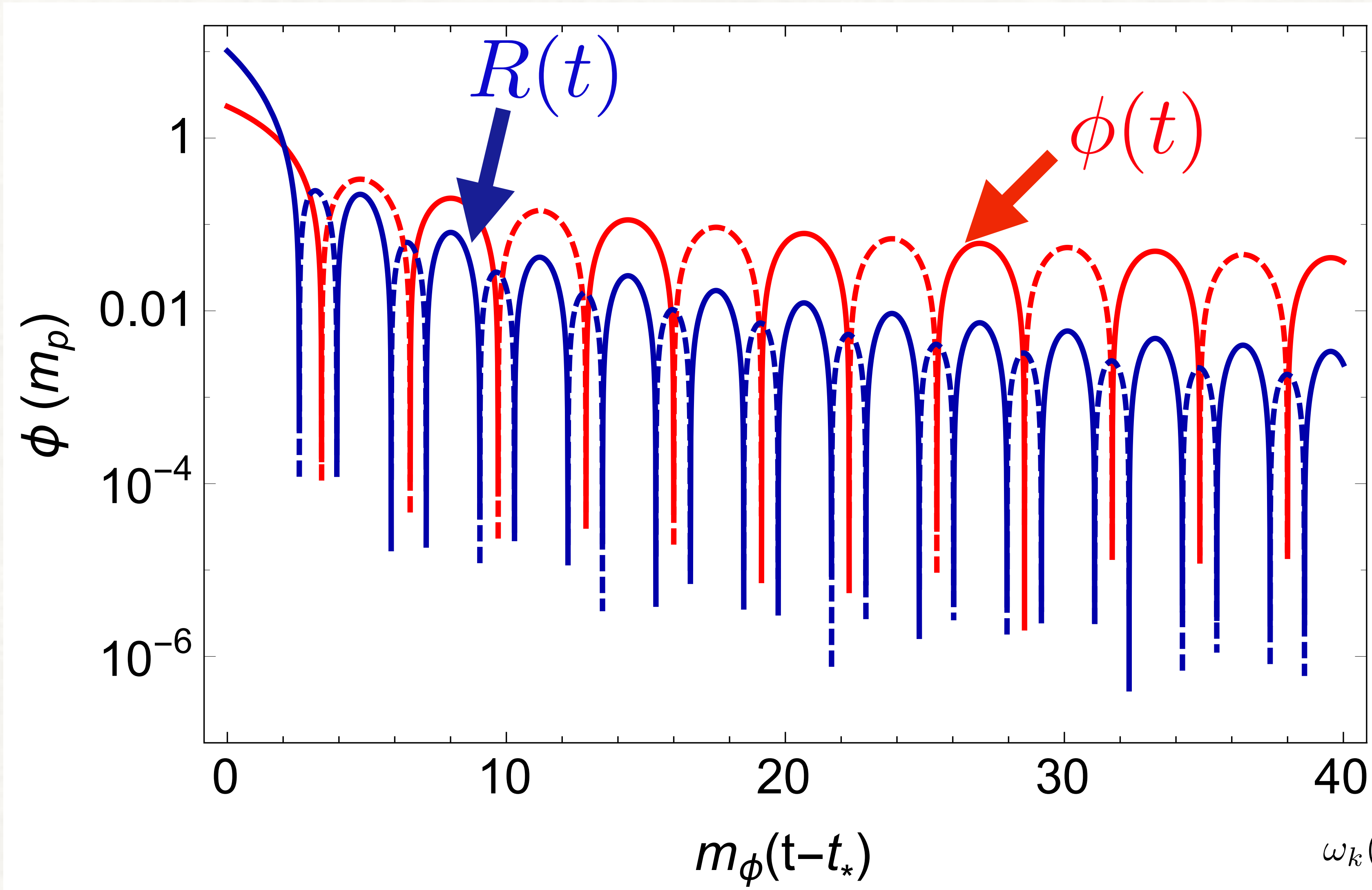
3. TACHYONIC RESONANCE



$\omega_k(t) = \sqrt{(k/a(t))^2 + \xi R(t)}$

$\phi(t) \approx 0 \rightarrow R(t) < 0 \rightarrow m^2_{h,\mathrm{eff}}(t) \equiv \xi R(t) < 0 \rightarrow \boxed{h_k \sim e^{|\omega_k|t}}$

3. TACHYONIC RESONANCE



$\omega_k(t) = \sqrt{(k/a(t))^2 + \xi R(t)}$

$$\phi(t) \approx 0 \rightarrow R(t) < 0 \rightarrow m^2_{h,\mathrm{eff}}(t) \equiv \xi R(t) < 0 \rightarrow \boxed{h_k \sim e^{|\omega_k| t}}$$

The presence of the negative effective mass induces a strong excitation of the Higgs field modes: **tachyonic resonance**.

3. TACHYONIC RESONANCE

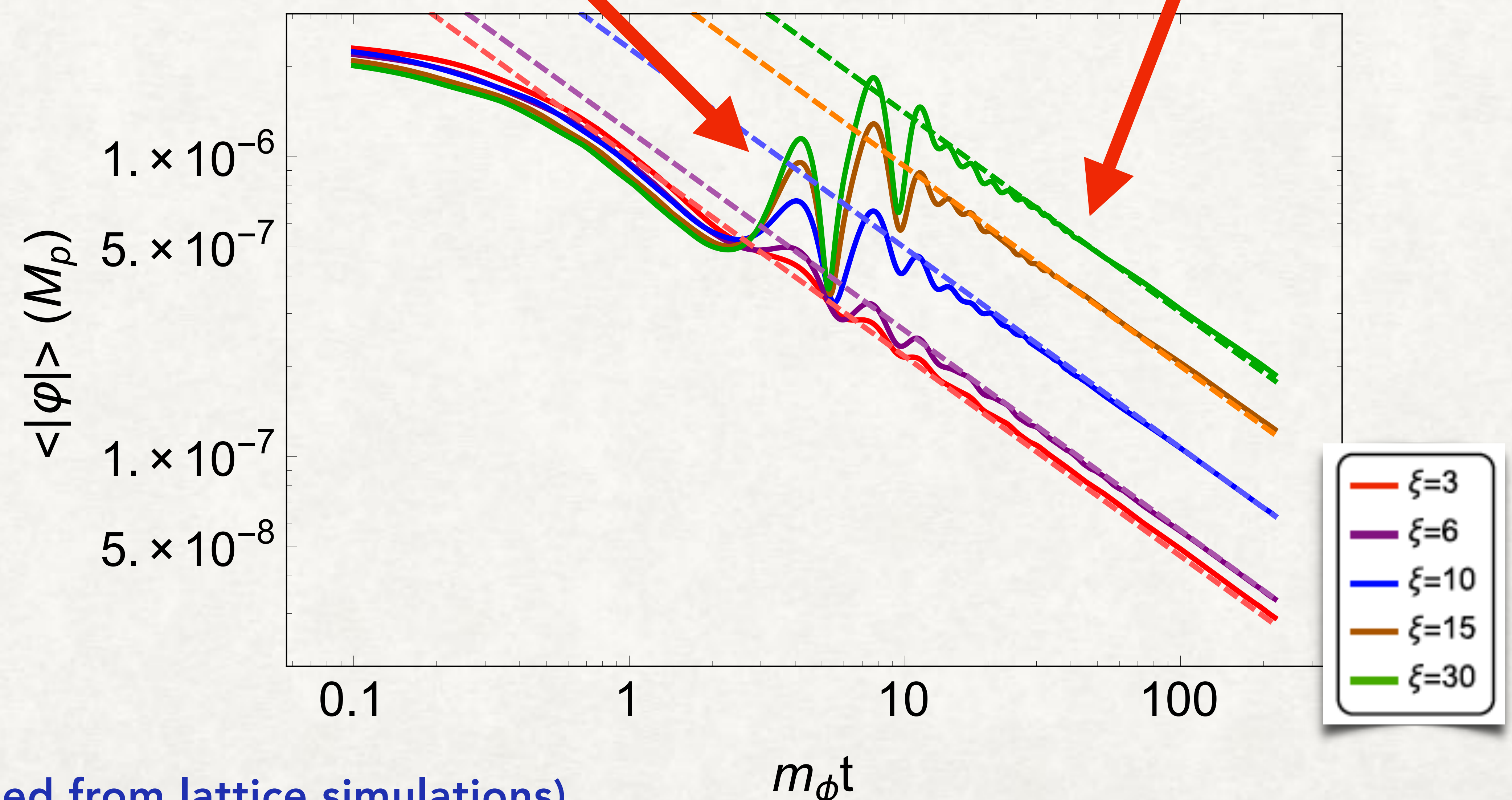
TWO REGIMES IN THE HIGGS TIME-EVOLUTION (λ = 0):

$$\langle\varphi(t)\rangle \sim e^{\alpha t}$$

initial tachyonic resonance

$$\langle\varphi(t)\rangle \propto t^{-2/3}$$

late-time dynamics



(Obtained from lattice simulations)

4. HIGGS INSTABILITY

We now introduce the Higgs potential $V(\varphi) = \lambda(\varphi)\varphi^4$:

Higgs EOM:
($h = \varphi a^{3/2}$)

$$\ddot{h}_k + \left[\frac{k^2}{a^2} + \xi R(t) + \Delta + \lambda(\varphi)\langle\varphi^2\rangle\right] h_k = 0$$

$R(t)<0$: *Destabilizing* effect
$R(t)>0$: *Stabilizing* effect

$\lambda(\varphi)<0$: *Destabilizing* effect
$\lambda(\varphi)>0$: *Stabilizing* effect

- Analytically and/or numerically, it is difficult to determine the values of ξ for which the Higgs becomes unstable. **(Herranen et al., 2015) (Kohri & Matsui, 2016)**

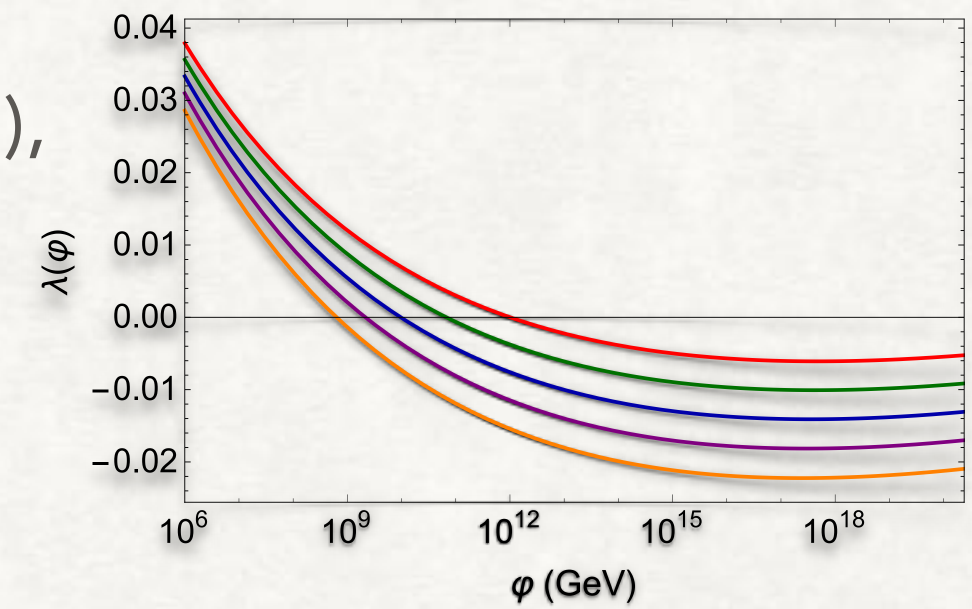
- Tachyonic resonance is a <u>non-perturbative process</u>, which must be studied with <u>lattice simulations</u> (*i.e. solving the differential equations of motion in a discrete finite box*). **(Ema, Mukaida & Nakayama, 2016) (Figueroa, Rajantie & F.T., t.b.p.)**

4. LATTICE SIMULATIONS

1. **$N^3=256^3$** points, **$L\approx(0.03\ m_\phi)^{-1}$** → Momenta captured: **$0.18\ m_\phi < p < 40\ m_\phi$**

2. The running of **$\lambda(\varphi)$** is introduced in the lattice as a **local function of the lattice point (*not a constant*).**

3. We consider different runnings of **$\lambda(\varphi)$**, corresponding to different values of the top-quark mass.
   (Note: We modify the running at high energies for numerical stability)



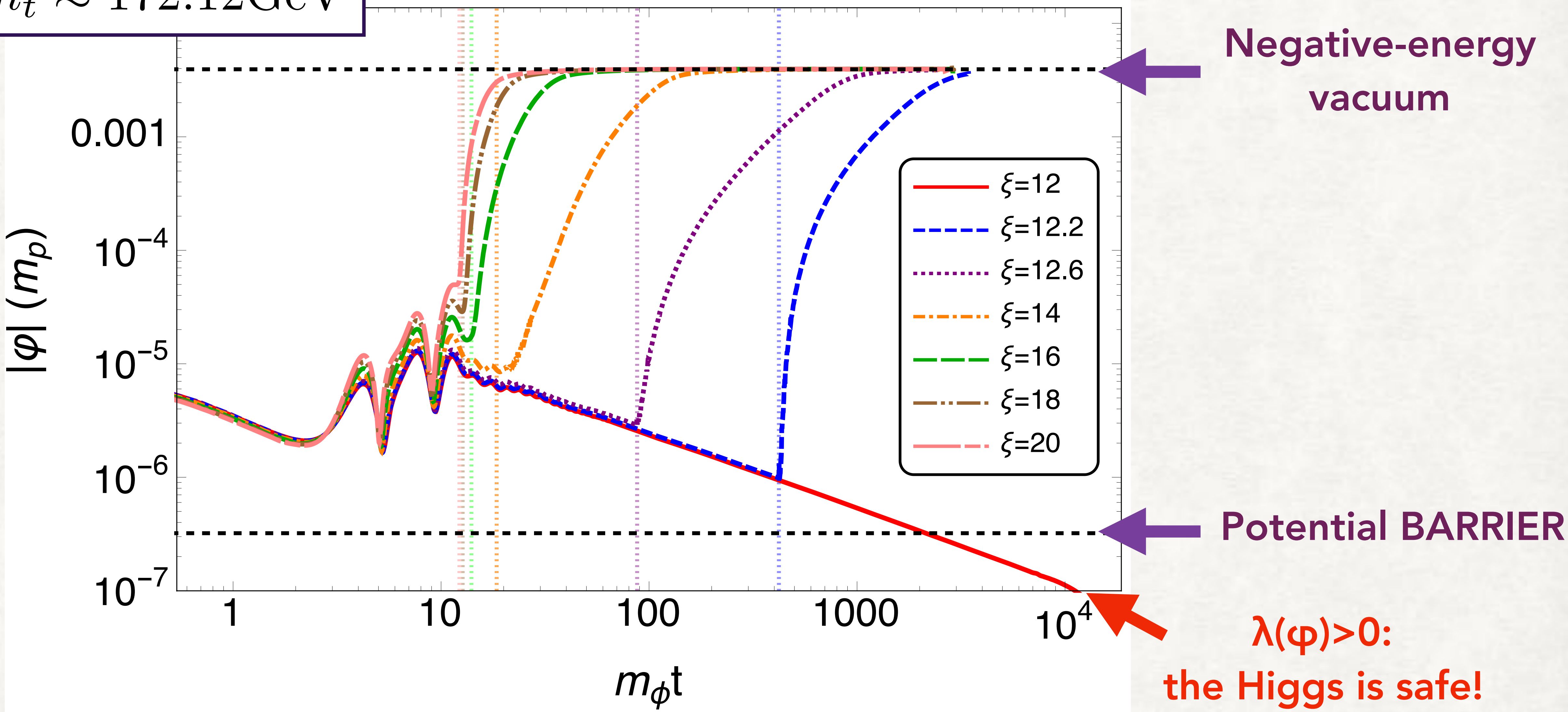
We determine ξ_c

- $\xi>\xi_c$: The Higgs goes to negative-energy vacuum at time **ti**.
- $\xi<\xi_c$: The Higgs goes to EW vacuum.

(Figueroa, Rajantie & F.T., arXiv:1612.xxxxx)

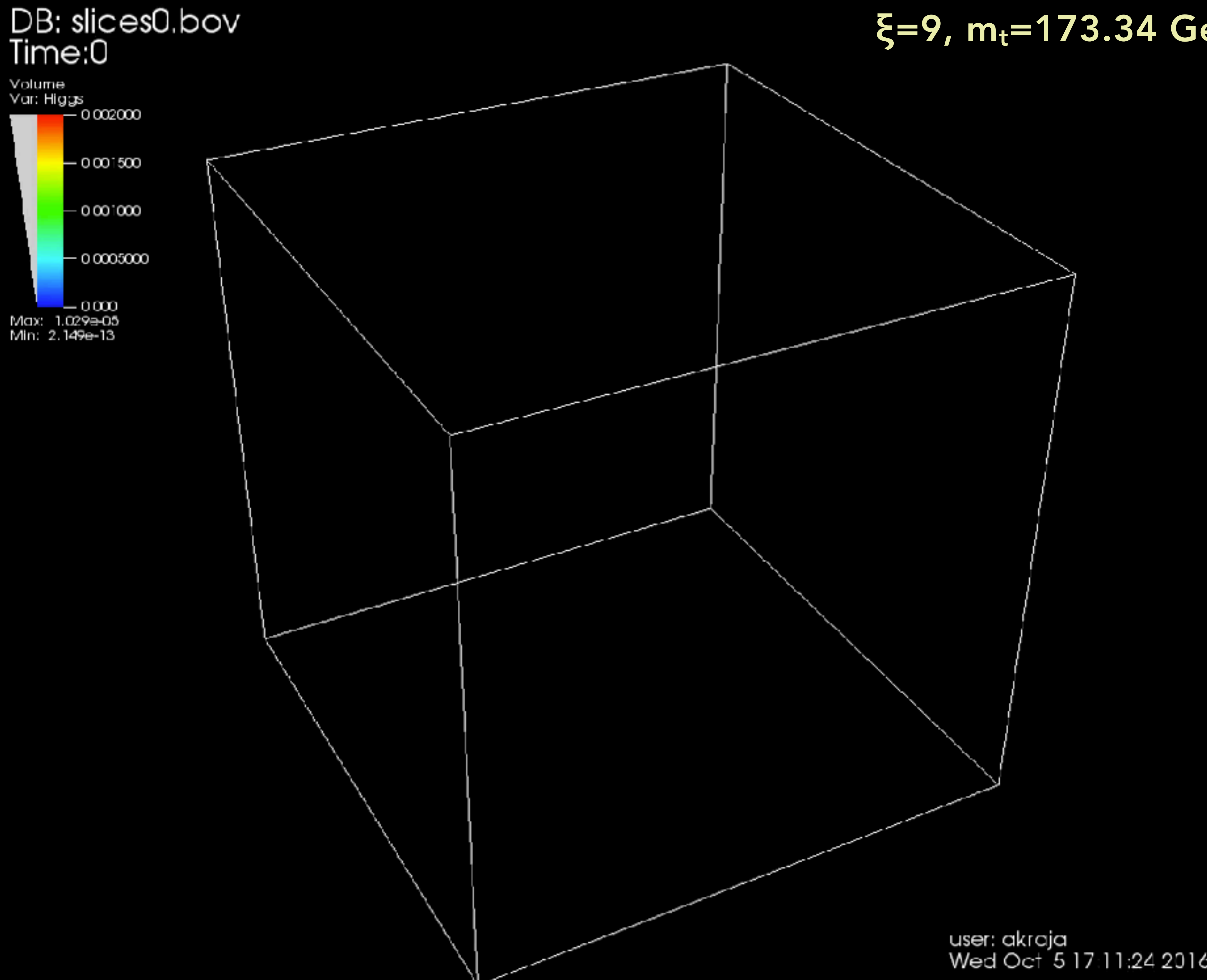
4. LATTICE SIMULATIONS: RESULTS

$m_t \approx 172.12\mathrm{GeV}$

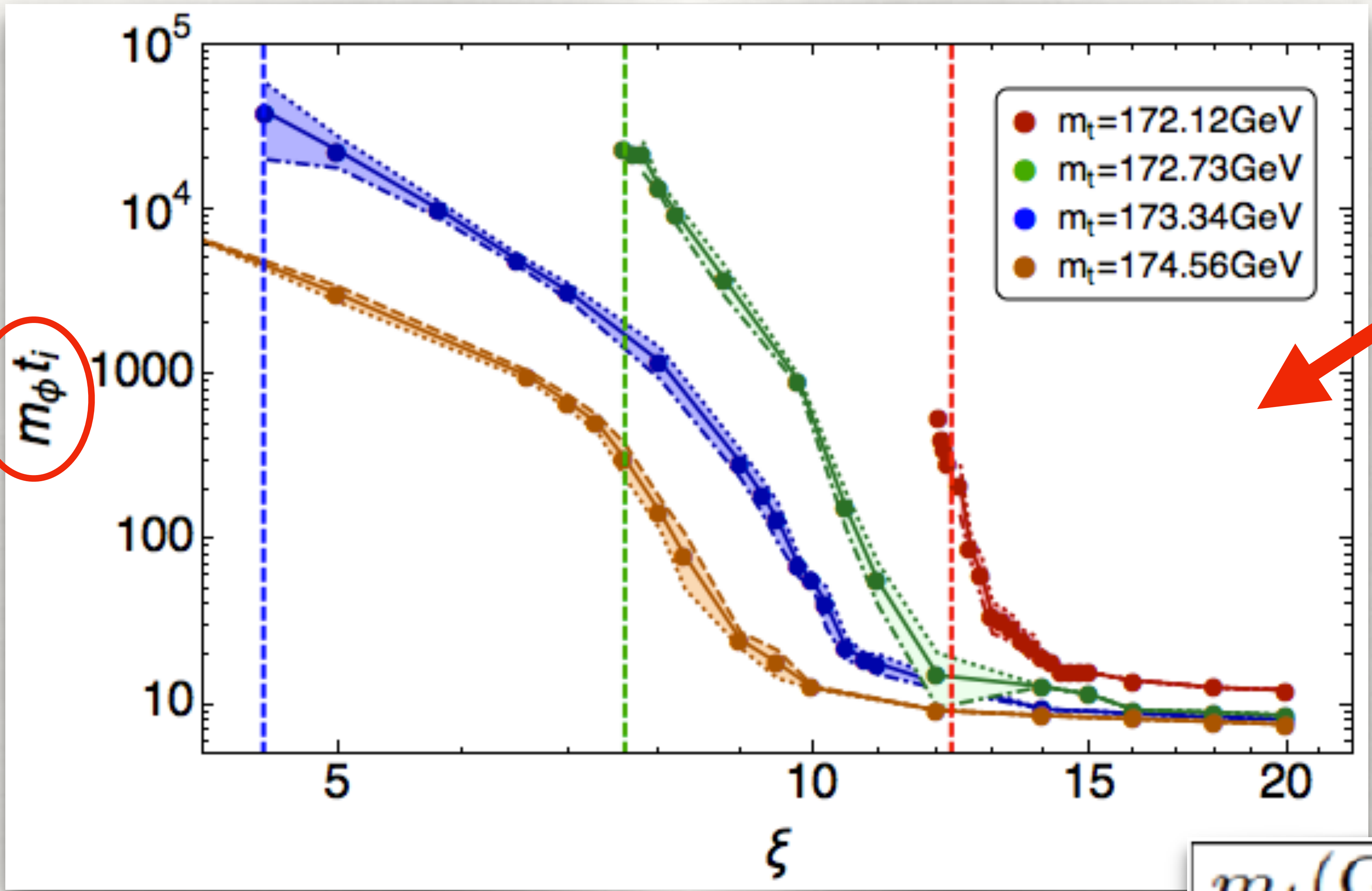


For **ξ ≳ ξ_c ≈12.1**, the Higgs field becomes unstable at a time **$t_i(\xi)$**

$\xi=9$, $m_t=173.34$ GeV



4. LATTICE SIMULATIONS: RESULTS



Instability time t_i depends strongly on ξ

Dependence on initial (random) quantum fluctuations

m_t(GeV)	ξ_c
172.12	12.2 ± 0.2
172.73	7.7 ± 0.1
173.34	4.3 ± 0.2
173.95	< 4.0

Note: For values $\xi<4$, $m_t>173.34$ GeV, lattice approach is not valid.

4. LATTICE SIMULATIONS: RESULTS

- The SM Higgs is coupled to other SM particles: gauge bosons and fermions. They may affect the post-inflationary Higgs dynamics.

- Dominant decay products: **electroweak gauge bosons:**

$$S = -\int a^3(t) d^4x \left(\frac{1}{g^2} \sum_{a=1}^{3} W^a_{\mu\nu} W_a^{\mu\nu} + \frac{1}{g'^2} Y_{\mu\nu} Y^{\mu\nu} + (D_\mu \Phi)(D^\mu \Phi) + \lambda (\Phi^\dagger \Phi)^2 \right)$$

- We introduce an **Abelian-Higgs** model in the lattice, mimicking the full non-Abelian structure of the Standard Model. **Their effect is not very relevant**.

$m_t(\mathrm{GeV})$	WITH g. bosons: ξ_c	WITHOUT g. bosons: ξ_c
172.12	13 ± 0.3	12.2 ± 0.2
172.73	8 ± 0.8	7.7 ± 0.1
173.34	4.5 ± 0.8	4.3 ± 0.2
173.95	< 4.0	< 4.0

5. CONCLUSIONS

- As the Ricci scalar was much greater in the past than now, **early-universe cosmology can provide tight constraints for the Higgs-curvature coupling**.

- With lattice simulations, one can determine **upper bounds** for **ξ**:

$m^2\varphi^2$ inflation &
m_t=173.34 GeV

$$\boxed{0.06 \lesssim \xi \lesssim 4}$$

- Bounds are **dependent on inflationary model** and **running of λ(φ)**. Lower-energy models can widen this range.