# Warped $AdS_3$ black holes in higher derivative gravity theories

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# Introduction

Context: Holographic dualities in 2+1 dimensions
*in this talk: warped duality*

Main question: Bulk/Boundary entropy matching in arbitrary higher derivative theory of gravity

# $AdS_3/CFT_2$

In general relativity (GR), Strominger '97 shows

$$S_{BH}^{BTZ} = S_{\text{Cardy}}^{CFT}\,.$$

Indeed,

$$S_{BH}^{BTZ} = \frac{\pi r_+}{2} = \frac{\pi}{2}\left(\sqrt{2\ell(\ell M + J)} + \sqrt{2\ell(\ell M - J)}\right)$$

$$S_{\text{Cardy}}^{CFT} = 2\pi\sqrt{\frac{c\,L_0}{6}} + 2\pi\sqrt{\frac{c\,\bar{L}_0}{6}}$$

One has $M = \frac{1}{\ell}(L_0 + \bar{L}_0)$ and $J = L_0 - \bar{L}_0$.

# Higher Derivative (HD) or Higher Curvature (HC)

General Lagrangian (diffeomorphism invariant without gravitational anomalies):

$$\mathbf{L} = \star f(g_{ab}, R_{abcd}, \nabla_{e_1} R_{abcd}, \nabla_{(e_1}\nabla_{e_2)} R_{abcd}, ..., \nabla_{(e_1}...\nabla_{e_n)} R_{abcd})$$

Example: New Massive Gravity

$$L_{NMG} = \frac{1}{16\pi}\int d^3x\sqrt{-g}\left((R-2\Lambda)+\frac{1}{m^2}\left(R_{\mu\nu}R^{\mu\nu}-\frac{3}{8}R^2\right)\right)$$

Why?
GR = low energy effective action of an UV compete theory (ex:ST)
Corrections to the Einstein-Hilbert action are expected.

# Match of the entropies in HD?

Modification of the entropies

- Bulk: Bekenstein-Hawking $\rightarrow$ Iyer-Wald
  The entropy law is modified to keep the first law valid.
- Boundary Entropy formula depends of the charges who are theory dependant.

A priori no reason that the match is still preserved.

# $AdS_3/CFT_2$ in a HD theory

Effect of higher curvature terms boils down in a global multiplicative renormalization of all charges of the theory.
Ref: Saida-Soda '99, Kraus-Larsen '05

Bulk:

$$S^{HD} = \alpha S^{GR}$$

Boundary

$$S^{CFT,HC} = 2\pi \left( \sqrt{\frac{c^{HC}\, L_0^{HC}}{6}} + \sqrt{\frac{c^{HC}\, \bar{L}_0^{HC}}{6}} \right)$$
$$= 2\pi\alpha \left( \sqrt{\frac{c\, L_0}{6}} + 2\pi \sqrt{\frac{c\, \bar{L}_0}{6}} \right) = \alpha S^{CFT,GR}$$

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Boundary

$$S^{CFT,HD} = \alpha S^{CFT,GR}$$

Consequently,

$$S^{HD} = S^{CFT,HD}\,.$$

# Plan

1. Method
2. Warped duality
3. Entropies in the warped duality

# Method

# Method

- The methods used in $AdS_3/CFT_2$ are not transposable to our case of interest.
- Inspired by the work of Azeyanagi, Compère, Ogawa, Tachikawa and Terashima (arXiv: 0903.4176) on Bulk/Boundary entropy match for 4D Extremal BH

  Formalism: Covariant phase formalism

# Covariant Phase Space Formalism

$\mathbf{L}$: Lagrangian $n$-form

- The EOM $\mathcal{E} = 0$ are determined through

$$\delta \mathbf{L}(\Phi) = \mathcal{E}(\Phi)\delta\Phi + d\mathbf{\Theta}[\delta\Phi, \Phi]$$

  with $\mathbf{\Theta}[\delta\Phi, \Phi]$: symplectic potential ($n-1$-form).

- The symplectic structure of the configuration phase

$$\mathbf{\Omega}[\delta_1\Phi, \delta_2\Phi; \Phi] = \int_C \omega[\delta_1\Phi, \delta_2\Phi; \Phi]$$

  in terms of the symplectic current

$$\omega = \delta\mathbf{\Theta}\,.$$

- Invariance of the Lagrangian under diffeomorphisms

$$\delta_\xi \mathbf{L} = \mathcal{L}_\xi \mathbf{L} = d(i_\xi \mathbf{L}) \,.$$

Thus,

$$\mathcal{E}(\Phi)\delta\Phi = d(\mathbf{\Theta}[\delta\Phi, \Phi] - i_\xi \mathbf{L}) \,.$$

- The Noether current is defined as

$$\mathbf{J}(\Phi) := \mathbf{\Theta}[\delta\Phi, \Phi] - i_\xi \mathbf{L} \,.$$

J is closed on-shell: $d\mathbf{J} \approx 0$.
Thus, it exists a $n$-form Q, called the Noether charge s.t.

$$\mathbf{J}_\xi(\Phi) := -d\mathbf{Q}_\xi(\Phi) \,.$$

One defines

$$k_\xi(\delta\Phi, \Phi) := -i_\xi \mathbf{\Theta}[\delta\Phi, \Phi] - \delta \mathbf{Q}_\xi(\Phi)\,.$$

If $\Phi$ satisfy EOM, $\delta\Phi$ the linearized EOM,

$$\omega[\delta_\xi \Phi, \delta\Phi; \Phi] = dk_\xi(\delta\Phi, \Phi)\,.$$

Integrating that equation,

$$\delta H_\xi = \mathbf{\Omega}[\delta_\xi \Phi, \delta\Phi; \Phi] = \int_{\Sigma = \partial C} k_\xi(\delta\Phi, \Phi)$$

where $H_\xi$ is the Hamiltonian generating the flow $\Phi \to \delta_\xi \Phi$.

One can show that the representation of asymptotic symmetry algebra by a Dirac bracket is

$$\delta_\xi H_\zeta := \{H_\zeta, H_\xi\} = H_{[\zeta,\xi]} + \int_{\Sigma=\partial C} k_\zeta(\delta_\xi \Phi, \Phi)\,.$$

So the central charge is $\int_{\Sigma=\partial C} k_\zeta(\delta_\xi \Phi, \Phi)$.

Ambiguities in the definitions :
In the literature, it was advocated that the so-called invariant symplectic structure, based on cohomological argument is the one to be used.

$$\omega^{inv} = \omega - dE$$

$$k_\xi^{inv}(\delta\Phi, \Phi) = k_\xi(\delta\Phi, \Phi) - E(\delta\Phi, \Phi)$$

In our cases, this E term will always be zero. We can only considered the symplectic formulation.

# Higher curvature Lagragian (without derivative)

Higher curvature Lagragian

$$\mathbf{L} = \star f(g_{ab}, R_{abcd})$$

Rewritten in terms of auxiliary fields $Z$ and $\mathbb{R}_{abcd}$

$$\mathbf{L} = \star[f(g_{ab}, \mathbb{R}_{abcd}) + Z^{abcd}(R_{abcd} - \mathbb{R}_{abcd})]$$

The EOM for the auxiliary fields are

$$Z^{abcd} = \frac{\partial f(g_{ab}, \mathbb{R}_{abcd})}{\partial \mathbb{R}_{abcd}}\,, \qquad \mathbb{R}_{abcd} = R_{abcd}\,.$$

$$\mathbf{L} = \star[f(g_{ab}, \mathbb{R}_{abcd}) + Z^{abcd}(R_{abcd} - \mathbb{R}_{abcd})]$$

No higher than the second derivative of $g_{ab}$ in the Lagrangian, so we can derive, for example in 3D

$$(Q_\xi)_c = \left(-Z^{abcd}\nabla_c\xi_d - 2\xi_c\nabla_d Z^{abcd}\right)\epsilon_{abc}$$
$$\Theta_{ef} = -2\left(Z^{abcd}\nabla_d\delta g_{bc} - \delta g_{bc}\nabla_d Z^{abcd}\right)\epsilon_{aef}\ .$$

# Exact charges

For an axisymmetric and stationary BH, the mass and the angular momentum are defined

$$\delta M^{HC} := \delta H_{\partial_t} = \int_\infty \delta \mathbf{Q}_{\partial_t} + \int_\infty i_{\partial_t} \mathbf{\Theta}$$
$$\delta J^{HC} := \delta H_{\partial_\phi} = -\int_\infty \delta \mathbf{Q}_{\partial_\phi}$$

And the Iyer-Wald entropy,

$$S^{HC} = S^{IW} = -2\pi \int_{\text{horizon}} dA\, Z^{abcd} \epsilon_{ab} \epsilon_{cd}$$

where $\epsilon_{ab}$ is the binormal to the horizon.

# Warped duality

# Bulk Warped $AdS_3$

$$ds^2 = \frac{\ell^2}{\nu^2+3}\left[-\cosh^2(\sigma)d\tau^2 + d\sigma^2 + \frac{4\nu^2}{\nu^2+3}\Big(du + \sinh(\sigma)d\tau\Big)^2\right]$$

Isometry group: $SL(2,R)\times U(1)$

$\nu = 1$: AdS

# Warped BHs

Spacelike stretched warped black holes are

$$ds^2 = dt^2 + \frac{dr^2}{\frac{(\nu+3)}{\ell} r^2 - 12mr + \frac{4j\ell}{\nu}} + dt\, d\phi \left(-\frac{4\nu r}{\ell}\right)$$
$$d\phi^2 \left(\frac{3(\nu^2-1)}{\ell^2} r^2 + 12mr - \frac{4j\ell}{\nu}\right)$$

with $m, j$: parameters characterising the BH ($j < \frac{9\ell m^2 \nu}{3+\nu^2}$)
$\ell$: original AdS radius
$\nu^2 > 1\,.$

Locally warped $AdS_3$

Solutions of New Massive Gravity (not GR)

# Boundary Warped CFT (WCFT)

- Chiral scaling symmetry on 2D FT → extended local algebra Virasoro-Kac-Moody $U(1)$

$$i[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}$$
$$i[L_m, P_n] = -nP_{m+n}$$
$$i[P_m, P_n] = \frac{k}{2}\, m\delta_{m+n,0}\,.$$

- Partition function

$$Z(\beta,\theta) = \mathrm{Tr} e^{-\beta P_0 - \beta\Omega L_0}$$

Modular covariance → Cardy-like entropy formula

$$S^{WCFT} = \frac{2\pi i}{\Omega} P_0^{vac} - \frac{8\pi^2}{\beta\Omega} L_0^{vac}$$

# Entropies in the warped duality

# Form of a generic tensor made out of the metric

Property of maximally symmetric spacetimes (ex: $AdS_3$), all the tensors made out of the curvature and its covariant derivatives can be expressed in terms of the metric.

Ex: $R_{\mu\nu} = -\frac{2}{\ell^2} g_{\mu\nu}$

# Entropy of a warped BH

Warped BHs are NOT locally maximally symmetric.

Nevertheless, we prove using the symmetries that

scalar: $C(\nu, \ell)$
symm. 2-tensor: $S_{\mu\nu} = C_1(\nu, \ell) g_{\mu\nu} + C_2(\nu, \ell) R_{\mu\nu}$

$$Z^{abcd} = A\left(g^{ac}g^{bd} - g^{ad}g^{bc}\right) + B\left(g^{ac}R^{bd} - g^{ad}R^{bc} + g^{bd}R^{ac} - g^{bc}R^{ad}\right)$$

where $A, B$ fonctions of $\nu, \ell$ and the coupling constants of the theory.

Moreover, computing the charges is an on-shell procedure, and it imposes the relation

$$B = -\frac{A\ell^2}{2(-3+5\nu^2)}\,.$$

Bulk

$$S^{HC} = \left(\frac{8\pi(-3+20\nu^2)}{(-3+5\nu^2)}A\right) S^{NMG}$$

# Entropy in warped CFT

The Cardy-like formula takes the form

$$S^{WCFT} = \frac{2\pi i}{\Omega} P_0^{vac} - \frac{8\pi^2}{\beta\Omega} L_0^{vac}$$

where the vacuum is given by $m = i/6$ and $j = 0$.
One has

$$(L_0, P_0, k, c)^{HC} = \left( \frac{8\pi(-3+20\nu^2)}{(-3+5\nu^2)} A \right) (L_0, P_0, k, c)^{NMG} .$$

Boundary

$$S^{HC,WCFT} = \left( \frac{8\pi(-3+20\nu^2)}{(-3+5\nu^2)} A \right) S^{NMG,WCFT}$$

# Match

The match between the entropies was proved in NMG.
Ref: Donnay, Giribet arXiv-1504.05640.

It is still true in higher derivative theory

$$S^{HC} = S^{WCFT} .$$

# Conclusion

# Conclusion

Warped duality

- enough symmetries to encode the full specificity of the theory in two constants (for the charges - not true for any tensor)
- on-shell condition leaves us with only one constant

This renormalization leads to the conservation of the entropy matching in any theory of gravity.

Powerful formalism to compute charges using the symmetries.

# Related works

- BTZ with Compère-Song-Strominger boundary conditions
  ASG: Virasoro - Kac-Moody U(1) algebra AdS/WCFT
- Flat Space Cosmologies (FSC)
  ASG: $BMS_3$ $FSC/BMS_3$

Ref: CZ arXiv-1604.02120

For locally maximally symmetric spacetimes (BTZ and FSC), higher derivative effects are encoded into a renormalization of the charges.