WdS Willem de Sitter DS

anti-De Sitter

A fake De Sitter talk

aDS anti-De Sitter

aDS De Sitter

Willem de Sitter WdS

aDS

DS DS

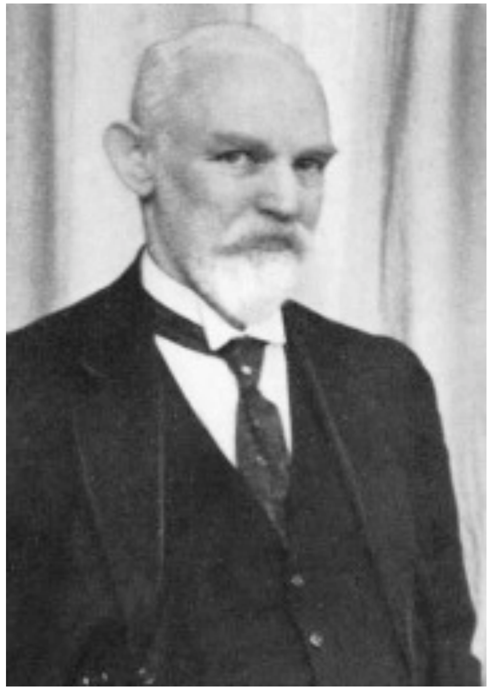
DS

anti-De Sitter De Sitter

WdS

Willem de Sitter

De Sitter aDS aDS WdS



Willem de Sitter

DS aDS

anti-De Sitter

WdS

Universidad de Oviedo
HEP Theory

De Sitter

Patrick Meessen
08/04/2010

Motivation

- Main motivation: Curiosity in the McVittie solution
  - In 1933 G.C. McVittie published an article titled "the mass-particle in an expanding universe", which should describe a non-accreting black hole in a FRW universe, i.e. it solves the Einstein equation coupled to a perfect fluid. Drawback is that it is non-accreting.
  - Should??? Well, questions about the existence of black holes is a difficult question and just 2 weeks ago Kaloper et al. proved, amongst other things, that the flat McVittie solution describes a true black hole if the late time Hubble parameter is positive and non-vanishing.

    Kaloper, Kleban & Martin: arXiv:1003.4777
  - The McVittie solution can be extended to charged multi-black holes but only if any constituent satisfies the Bogomol'nyi boud (Mass=|Charge|), and you can ask yourself how a stringy supersymmetric black hole interacts with a fluid… I mean: Don't the cosmologist tell us that we live in a phase of exponential expansion???

Motivation

- One of the most interesting characteristics of supersymmetric black holes is the attractor mechanism, which states that the entropy of the black hole and the value of the scalar fields on the horizon depend only on the electric and magnetic charges, as it is fundamental for the microscopic entropy counting. So what happens if we embed those bhs into cosmology?
- There are exact solutions describing monopoles that are globally regular and, of course, the prospect of being able to study monopoles in FRW was too tempting… Well, it still is, but the problem is hard, so
- We started with a simpler case: **fake supergravity!**
- fake sugra incarnates the main lessons learned from the BPS sols. , namely
  - that a first order spinorial eq. is easier to solve than the Einstein eqs.
  - and that if the spinorial eq. is chosen judiciously, the existence of a solution implies, under some reasonable conditions, the Einstein equations and also the ones for the scalar fields.
  - I.e. it is a solution generating technique

Motivation

- The problem is how to obtain said spinorial equation, which in this context is called the fake Killing spinor equation.
- We† followed an idea by D. Kastor & J. Traschen and took gauged supergravities and Wick-rotated the tri-holomorphic momentum maps, or if you like, we generalised the fermion-shift structure of the supersymmetry variations in such a way that a subset of the integrability conditions imply sugra-like equations of motion, but with a scalar potential that is opposite to the one in gauged sugra, so that one can avoid having to solve the major part of the equations of motion and one can enjoy oneself in choosing the necessary Special Geometry and the symplectic section, solving the stabilisation equations and some Bogomol'nyi equations in order to write down explicit solutions. And then one can try to find the horizons &c.
- It would take one hour to explain the magic words, so I'll follow the great idea by J. Gutowski and W. Sabra and talk about the simplest model which is Einstein-Maxwell-De Sitter gravity, or **fake De Sitter** for short.

G&S: arXiv:0903.0179

† PM & A. Palomo-Lozano, arXiv:0902.4814

Kastor & Traschen's Cosmological multi-bh

Start with Carter's Schwarzschild-De Sitter black hole:

$$ds^2 = f dt^2 - f^{-1} dr^2 - r^2 dS^2_{[\theta,\phi]}$$

$$f(r) = -\tfrac{\Lambda}{6}\, r^2 + 1 - \tfrac{2M}{r} + \tfrac{Q^2}{r^2} \xrightarrow[\text{Redef. } \Lambda \equiv 6\lambda^2]{\text{Extremal } M^2 = Q^2} f(r) = \frac{(r-M)^2 - \lambda^2 r^2}{r^2}$$

There is a curvature singularity at r=0 and in general there are 3 horizons

$$r = 0 < r_i = \tfrac{\sqrt{1+4\lambda M}-1}{2\lambda} < r_o = \tfrac{1-\sqrt{1-4\lambda M}}{2\lambda} < r_c = \tfrac{1+\sqrt{1-4\lambda M}}{2\lambda}$$

In order to have a black hole we must have $\boxed{M < \sqrt{\frac{3}{8\Lambda}} = \frac{1}{4\lambda}}$

Kastor & Traschen's Cosmological multi-bh

K&T realised that a multi-bh can never be spherically symmetric, so that the above coordinate system is of no use whatsoever. However, after a coordinate transformation the metric becomes

$$ds^2 = H^{-2}dt^2 - H^2 d\vec{x}^2 \quad \& \quad H = \lambda t + \frac{m}{r}$$

$$\boxed{H = \lambda t + \sum_i \frac{m_i}{|x - x_i|}}$$

The form of the solution and the occurrence of a harmonic function is typical of supersymmetric solutions, but sugra does not allow for a De Sitter-like cosmological constant!

Kastor & Traschen's Cosmological multi-bh

WAIT! ¿¿¿ Isn't there something like De Sitter supergravity ???

Yep, there is! An initial investigation into this subject was made by Pilch, Van Nieuwenhuizen and Sohnius in 1985.

Com.Math.Phys. 98(1985), 105

They found that the vector field becomes ghost-like, i.e. the kinetic term for the vector fields has the wrong sign.

This meant the end of De Sitter sugra (duh!)

In our fake De Sitter construction we won't have this problem:

★ our ``susy rule'' will be inspired by sugra, but

★ we don't need fermions! (thank God for that!)

★ the integrability condition leads to kosher eqs. of motion.

Kastor & Traschen's Cosmological multi-bh

Kastor & Traschen showed that their multi-bh admits a non-trivial solution to the following rule for a parallel/Killing spinor

Class.Quant.Grav. 13(1996), 2753

$$\nabla_a\psi \;=\; -\tfrac{1}{2}\lambda\;\gamma_a\sigma^2\psi \;+\; \lambda A_a\psi \;-\; \tfrac{1}{2}\not{F}\gamma_a\sigma^2\psi$$

this rule has the following gauge invariance

$$\left\{\begin{array}{lcl}\psi & \longrightarrow & e^{\lambda\alpha}\;\psi\\ A & \longrightarrow & A + d\alpha\end{array}\right.$$

Due to the Majorana constraint, we must have $\alpha \in \mathbb{R}$

When $\lambda = 0$ we recover the ordinary supersymmetry variation rule for the gravitino in minimal N=2 d=4 supergravity

Bilinears, Fierz identities & 2 cases 2b considered

Use chiral spinors:

$$2\ \epsilon_I \equiv (1-\gamma_5)\psi_I$$

$$\epsilon^I \equiv \ (\epsilon_I)^\star$$

$$\bar{\epsilon}_I \epsilon_J = \varepsilon_{IJ}\ X$$

$$i\bar{\epsilon}^I \gamma_a \epsilon_J \ = \ \tfrac{1}{2}\ V_a\ \delta^I{}_J \ + \ \tfrac{1}{2}\ V^x_a\ \ (\sigma^x)^I{}_J$$

$$\bar{\epsilon}_I \gamma_{ab} \epsilon_J \ = \ \Phi^x_{ab}\ \tfrac{i}{2}\,(\sigma^x)_{IJ}$$

$$\imath_V V^x \ = \ 0 \ \ , \ \ g\,(V,V) \ = \ 4|X|^2 \ \ , \ \ g\,(V^x,V^y) \ = \ -4|X|^2\ \delta^{xy}$$

$$g_{ab} = \frac{1}{4|X|^2}\,[V_a V_b \ - \ V^x_a V^x_b]$$

2 cases:

I)Timelike case, $g(V,V) > 0$

II) Null case, $g(V,V) = 0$

$$\overline{X}\ \Phi^x = \frac{1}{2i}\ [V\wedge V^x \ + \ i\ \star(V\wedge V^x)] \ \longrightarrow \ \star\Phi^x = -i\Phi^x$$

Integrability condition & Equations of motion

Using the known result $\gamma^b\ \nabla_{[a}\nabla_{b]}\psi\ = \frac{1}{4}\ R_{ab}\gamma^b\psi$, we can derive:

$$\underbrace{\left(R_{ab}\ -\ 3\lambda^2\ \eta_{ab}\ -\ 2\left[F_a{}^c F_{bc}\ -\ \tfrac{1}{4}\eta_{ab}F^2\right]\right)}_{\sim 0\ ::\ \text{Einstein equation}}\ \gamma^b\psi\ =\ \underbrace{-\nabla_a F^{bc}}_{\sim 0\ ::\ \text{Maxwell eq.}}\gamma_c\gamma_a\sigma^2\psi$$

These eoms can be derived from EM-DS gravity: $\int_4 \left[\ R\ -\ 6\lambda^2\ -\ F^2\right]$

A careful analysis shows that in the

- Timelike case the Einstein equation is solved iff you have the spinor and you solved the Maxwell equation, and in the
- Null case you need to solve the Maxwell equation and only one component of the Einstein equation!

This is an effort and time saving result that works generically in (fake) sugra.

Differential constraint on the bilinears

As we have a rule for the parallel propagation of the spinor, we can calculate the parallel propagation of the bilinears:

$$\mathtt{D}X = dX - 2\lambda A\ X = \ + \tfrac{\lambda}{2}\ V\ +\ \imath_V F^+$$

$$\mathtt{D}V^x\ =\ -2\lambda\ \mathrm{Im}\,(\ \Phi^x)$$

$$\nabla_a V_b = 2\lambda\ A_a V_b\ +\ 2\lambda \mathrm{Re}(X)\ \eta_{ab}\ -\ 4\mathrm{Re}\left(\overline{X} F^+_{ab}\right)$$

Observe the when $\lambda = 0$, we have $\nabla_{(a}V_{b)} = 0$ so that the solutions have a timelike or null Killing vector: this is generic for supersymmetric solutions.

In fake De Sitter there is no such Killing vector.

We can introduce a coordinate system and analyse the implications of the differential constraints, the Fierz identities and give the most general expressions for the metric and the gauge field, which is then the most general solutions of EM-DS admitting a fake Killing spinor.

I won't enlighten you with the details...

Timelike case: generalising K&T's solution

adapt coordinates:

$$V = \sqrt{2}\partial_\tau$$

$$V^\sharp = 2\sqrt{2}|X|^2\,(d\tau + \omega)$$

The sols. are built out of

$$\mathcal{R} = \mathrm{Re}\,(1/X) \equiv \sqrt{2}\lambda\,\tau + \tilde{\mathcal{R}}$$

$$\mathcal{I} = \mathrm{Im}\,(1/X)$$

$$\mathsf{A} = \mathsf{A}_i dy^i$$

$$V^x = V^x_i dy^i$$

τ-independent

In terms of these, we can express the fields as

$$A = -\tfrac{1}{2}\mathcal{R}\,V^\sharp \;+\; \mathsf{A}$$

$$ds^2 = 2|X|^2\,\left(d\tau \;-\; 2\lambda\,\tau\mathsf{A} + \tilde{\omega}\right)^2 \;-\; \frac{1}{2|X|^2}\,V^x_i V^x_j dy^i dy^j$$

where:

$$\frac{1}{|X|^2} = \mathcal{R}^2 + \mathcal{I}^2$$

$$\star_{(3)}\tilde{\mathsf{D}}\tilde{\omega} = \tilde{\mathcal{R}}\tilde{d}\mathcal{I} - \mathcal{I}\tilde{d}\tilde{\mathcal{R}} + \sqrt{2}\lambda\,\mathcal{I}\tilde{\omega}$$

Timelike case: generalising K&T's solution

the triple $(\mathcal{I}, \mathsf{A}, V^x)$ must satisfy the following equation

$$\tilde{d}V^x = 2\lambda \mathsf{A} \wedge V^x + \tfrac{\lambda}{2}\mathcal{I}\ \varepsilon^{xyz} V^y \wedge V^z$$

which implies the integrability condition

$$-\sqrt{2}\star_{(3)} \tilde{d}\tilde{\mathsf{A}} = \tilde{d}\mathcal{I} + 2\lambda \mathsf{A}\ \mathcal{I}$$

we can use the gauge invariance $\left\{\begin{array}{lcl} \mathsf{A} & \longrightarrow & \mathsf{A} - \tilde{d}\phi(y) \\ \mathcal{I} & \longrightarrow & e^{2\lambda\phi}\mathcal{I} \end{array}\right.$

and take $\mathcal{I}$ to be constant.

The K&T case reduces to $\mathcal{I} = 0\ ,\ \mathsf{A} = 0\ ,\ V^x = dy^x$

Timelike case: generalising K&T's solution

In 1998 P. Gauduchon and K.P. Tod showed that the condition
J.Geom.Phys.25(1998), 291

$$\tilde{d}V^x = 2\lambda \mathsf{A} \wedge V^x + \tfrac{\lambda}{2}\mathcal{I}\ \varepsilon^{xyz}V^y \wedge V^z$$

Implies that the 3-dimensional space is a restricted subclass of Einstein-Weyl spaces, and these spaces are called Gauduchon-Tod.

Furthermore, they classified all the compact ones and also the ones that admit a A that is divergenceless.

In a related development, V. Buchholz showed that if a Weyl connection on a 3-dimensional manifold is to admit a weight zero spinor, which is what we are having here, then the manifold must be Gauduchon-Tod.
math.DG/9901125

Prime example of a GT space: the Berger sphere

$$ds^2_{(3)} = d\phi^2 \ + \ \sin^2(\phi)d\varphi^2 \ + \ \cos^2(\mu)\left[d\chi \ + \ \cos(\phi)d\varphi\right]^2$$

$$\theta = \sin(\mu)\cos(\mu) \ \left[d\chi \ + \ \cos(\phi)d\varphi\right]$$

$$\kappa = \cos(\mu)$$

$$\mu \in [0, \pi/2)$$

Remarks:

• when $\mu = 0$ we are dealing with the 3-sphere, albeit in the Hodge fibration form

• we physicist would call it a squashed 3-sphere.

• The Berger sphere is the unique compact Gauduchon-Tod space,

• and it is complete.

• In order to use it in order to construct a solution, we must rescale the metric as to have $\kappa = 2\sqrt{3}\lambda$

A curious solution based on the Berger sphere

$$ds^2 = \left(dt + \xi^{-1}\tanh(\xi t)\theta\ \right)^2 - \frac{\cos^2(\mu)}{4\xi^2}\cosh^2(\xi t)\ dB_\mu$$

$$A = \xi^{-1}\left[1 - 2\tanh^2(\xi t)\right]\ \theta$$

Some properties:

a) when $\mu = 0$ we recover 4-dimensional De Sitter space,

b) metric well-behaved for all $t \in \mathbb{R}$, we have global coordinates

c) All the curvature invariants are finite but non-vanishing,

d) In order to avoid closed timelike loops we must have $\mu \in [0, \pi/4)$

e) The space is causally-geodesically complete

f) Charged geodesics fly off to infinity in a finite time (of course!)

Null Case: Deforming the Nariai Cosmos

Recall that the null case means $X = 0$, so that the vector V is a null-vector, which we will rebaptise as $L(\text{ightlike})$.

The fundamental equation follows from adapting the equation

$$\nabla_a V_b = 2\lambda\ A_a V_b\ +\ 2\lambda \mathrm{Re}(X)\ \eta_{ab}\ -\ 4\mathrm{Re}\left(\overline{X} F^+_{ab}\right)$$

to the case at hand:

$$\boxed{\nabla_a L_b = 2\lambda\ A_a L_b}$$

This system was studied by Gibbons & Pope and we will briefly outline the implications

Class.Quant.Grav. 25(2009), 125015

Metrical spaces with a recurrent vector

$$g(L, L) = 0$$

$$\nabla L = \mathsf{B} \otimes L$$

B is called a recurrent vector

From the definition we can derive:

$$R(X,Y)L = d\mathsf{B}(X,Y)\ L$$

Is a generator of
$\mathfrak{hol}(\nabla) \subseteq \mathfrak{so}(1, d-1)$ $\longrightarrow$

$$\boxed{\mathfrak{hol}(\nabla) \subseteq \mathfrak{sim}(d-2)}$$

FYI: $R(X,Y)L \equiv [\nabla_X, \nabla_Y]L - \nabla_{[X,Y]}L$

$\mathfrak{sim}(d-2) \subseteq \mathfrak{so}(1,d-1)$

Introduce the covariant generators and take the metric to be lightcone, then

$$[M_{ab}, M_{cd}] = \eta_{bc} M_{ad} - \eta_{ac} M_{bd} + \eta_{bd} M_{ca} - \eta_{ad} M_{cb}$$

$$\eta_{+-} = 1 \quad , \quad \eta_{ij} = \delta_{ij}$$

It is then easy to see that $\mathfrak{so}(1,d-1)$ admits the 3-grading $L_- + L_0 + L_+$, where the subalgebras are

$$\begin{aligned} L_\pm &\equiv \text{span}\,[M_{\pm i}] \\ L_0 &\equiv \text{span}\,[M_{+-}, M_{ij}] \end{aligned}$$

$$\boxed{\mathfrak{sim}(d-2) \simeq L_0 \ltimes L_+}$$

Titbit: it is the maximal subgroup of the Lorentz group

Titbit: it can be generalised to SO(t,s) straightforwardly

Titbit: it is the algebraic structure underlying Very Special Relativity

Metrical spaces with a recurrent vector

$\uparrow\downarrow$ The system has a 'gauge invariance': $\left\{ \begin{array}{l} L \to e^{\theta} L \\ B \to B + d\theta \end{array} \right.$

$\downarrow\uparrow$ L is hypersurface orhogonal, i.e. $L \wedge dL = 0$, which by the Frobenius theorem implies the existence of two functions Υ and u such that: $L = \Upsilon \, du$

$$\downarrow\uparrow - \uparrow\downarrow \quad \Rightarrow \quad \boxed{\begin{array}{lcl} L & = & du \\ \mathrm{B} & = & \aleph \, L \end{array}}$$

we can then introduce another lightlike coordinate by $L = \partial_v$ and use complex coordinates z for the remaining directions. The analisis is the reasonably straightforward and gives

Null Case: Deforming the Nariai Cosmos

We limited ourselves to the minimal solution compatible admitting a fake Killing spinor

JHEP05(2009)042

$$
\begin{aligned}
A &= -\lambda \; v \; du \\
ds^2 &= 2du\left(dv - \lambda^2 v^2 du\right) - \frac{dz d\bar{z}}{\lambda^2(1+|z|^2)^2}
\end{aligned}
$$

2d De Sitter

A 2-sphere of radius $4\lambda^2$

This solution is known as the electrically charged Nariai cosmos and its holonomy is $\mathfrak{so}(1,1) \oplus \mathfrak{so}(2) \subset \mathfrak{sim}(2)$

Sci.Rept.Tohoku U. 34(1950), 160

Null Case: Deforming the Nariai Cosmos

Gutowski & Sabra were more thorough and went all the way, resulting in
arXiv:0903.0179

$$
\begin{aligned}
A &= -\lambda\, v\, du \\
ds^2 &= 2du\left(dv - \left[\lambda^2 v^2 + \Upsilon(z,\bar{z})\right] du\right) - \tfrac{dz d\bar{z}}{\lambda^2(1+|z|^2)^2}
\end{aligned}
$$

✴it's a deformation of the Nariai cosmos; the deformation has to satisfy

$$\partial\bar{\partial}\Upsilon = 0$$

✴the deformation has proper $\mathfrak{sim}(2)$ holonomy

Conclusions & Discussion

- We used fake supergravity techniques to find solutions to Einstein-Maxwell-De Sitter gravity.
- Interesting mathematical structures arise, such as Gauduchon-Tod spaces and spaces of sim-holonomy.
- Why are sim-holonomy spaces interesting?
  - Coley, Gibbons, Hervik & Pope showed that metrical sim-holomy spaces are spaces with constant scalar curvature invariants and have therefore improved quantum properties.
  - experience with pp-waves shows indicates that the quantum properties of the general sim-holonmy spaces should also be better-behaved.

Conclusions & Discussion

- More general d=4 theories can be, and have been, treated
  - In general a potential for the scalars is created which makes the theories less interesting
  - In some cases said potential vanishes and one constructs non-BPS solutions to kosher stringy sugras
  - The asymptotic cosmological spacetime is generically Kasner, i.e. power-like, not De Sitter.
  - The location of the event horizons is non-trivial as there is no Killing horizon, and so the question of the attractor mechanism is out of reach
  - but also apparent horizons satisfy the laws of thermodynamics… and as soon as I have sorted out the details I'll send you an e-mail :)

Conclusions & Discussion

- In higher dimensions even less work has been done...
- London emulated K&T's work for higher dimensional black holes and also showed that the solutions satisfied a fake parallel spinor condition
  NPB434(1995), 709
- In d=5 the only work that was done is on the minimal sugra, i.e. without scalar fields and only one gauge field
  - in the timelike case the solutions have a standard susy form and the 4-dimensional basespace has to be Hyper-Kähler-Torsion
    Grover, Gutowski, Herdeiro & Sabra, NPB809(2009), 406
  - in the null-case one finds Kundt spacetimes, and all have a torsionful holonomy contained in
    Grover, Gutowski, Herdeiro, Meessen, Palomo-Lozano & Sabra, JHEP07(2009)069
- The fake solutions in d=5 induce fake solutions in d=4, in more or less the same way as happens in ordinary sugra.