ABJM Baryon Stability at Finite 't Hooft Coupling

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- Motivation: Study the stability of non-singlet baryon vertex-like configurations in ABJM

- How: Existence of a classical solution + stability analysis
  - Probe brane approx: Valid at strong 't Hooft coupling
  - Dissolve D0's $\rightarrow$ Microscopical description: Valid at finite 't Hooft coupling

- Results:
  - Non-singlet stable baryons at finite 't Hooft coupling
  - Flat B_2 required by Freed-Witten anomaly
  - New higher curvature dielectric couplings

(Based on arXiv:1105.0939 [hep-th], JHEP, with M. Picos, K. Sfetsos, K. Siampos)

1.Introduction

AdS_4/CFT_3 relates the Type IIA superstring on $AdS_4 \times CP^3$ to the $\mathcal{N}=6$ Chern-Simons matter theory with gauge group $U(N)_k \times U(N)_{-k}$ known as ABJM.

- Good description when $N^{1/5} << k$.
- Like AdS_5/CFT_4 it is a strong weak coupling duality, with 't Hooft coupling $\lambda = N/k$:
  - The string background describes the 't Hooft limit of the theory: $N, k \to \infty$ with $\lambda = N/k$ fixed
  - IIA weakly curved when $k << N$ (large 't Hooft coupling)
- 3 dimensions $\to$ Applications in condensed matter

2. Particle-like branes in ABJM

CP^3 has $H^q(CP^3) = \mathbb{Z}$ for even $q \Rightarrow$ D2, D4 and D6 particle-like branes wrapping topologically non-trivial cycles

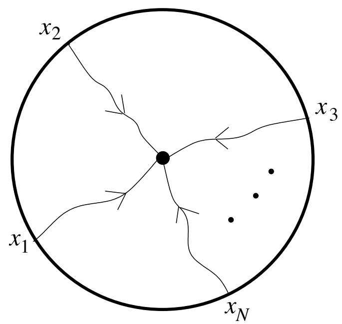
Interpretation in the dual CFT:

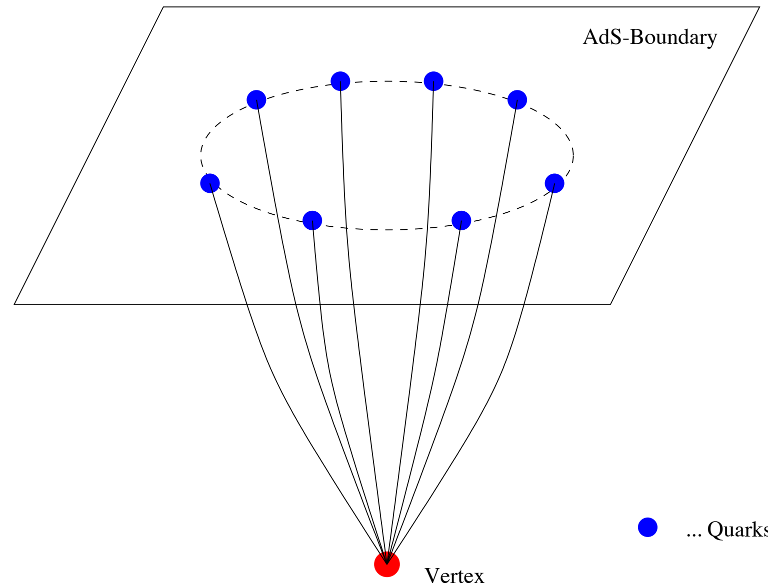
- D6 wrapped on the CP^3: Analogous to the baryon vertex in $AdS_5 \times S^5$. F_6 flux $\Rightarrow$ Tadpole that has to be cancelled with N F-strings ending on it $\leftrightarrow$ N external quarks on the boundary of AdS_4

The baryon vertex in $AdS_5 \times S^5$

Gauge invariant coupling of N external quarks

Through AdS/CFT external quarks are regarded as endpoints of F-strings in AdS





Baryon vertex in the gravity side: D5-brane wrapped on the 5-sphere (Witten'98):

$$S_{CS} = 2\pi\, T_5 \int_{\mathbb{R}\times S^5} P[F_5] \wedge A = N\, T_{F1} \int_{\mathbb{R}} dt A_t$$

N charge cancelled by N F-strings ending on the 5-brane

Dual configuration on the CFT side: N Wilson lines ending on an epsilon tensor $\longleftrightarrow$ Bound state of N quarks.

However, within the gauge/gravity correspondence it is possible to construct bound states of l quarks with $l < N$ (non-singlets) (Brandhuber, Itzhaki, Sonnenschein, Yankielowitz'98; Imamura'98)

In $AdS_4 \times CP^3$: A D6-brane wrapped on the CP^3:

$$S_{CS} = 2\pi\, T_6 \int_{\mathbb{R}\times CP^3} P[F_6] \wedge A = N\, T_{F1} \int dt A_t$$

Cancel this charge with the charge induced by the endpoints of N open F-strings stretching between the D6 and the boundary of AdS

Non-singlets?

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- D2 wrapped on a $CP^1 \subset CP^3$: 't Hooft monopole. F_2 flux $\Rightarrow$ Tadpole that has to be cancelled with k F-strings

But k Wilson lines cannot end on an epsilon tensor

If one forms the symmetric product only the endpoint of the Wilson lines is observable and the product behaves like a 't Hooft operator creating one unit of magnetic flux at a point (ABJM) $\rightarrow$ 't Hooft monopole

- D4 wrapped on a $CP^2 \subset CP^3$: Di-baryon

  It does not capture the background fluxes.
  Same baryon charge and dimension than di-baryon:

Baryon charge N, $m_{D4} L = N \Rightarrow \Delta = \frac{m_{D4} L}{2} = \frac{N}{2}$

$\Rightarrow$ Dual configuration composed of N chirals

Di-baryon operator: $O^{D4} = \epsilon_{i_1 \dots i_N} \epsilon^{j_1 \dots j_N} A^{i_1}_{j_1} \dots A^{i_N}_{j_N}$

These configurations admit a natural generalization by allowing non-trivial worldvolume gauge fluxes:

3. Add a magnetic flux

(Gutiérrez, Y.L., Rodríguez-Gómez'10)

Candidates for holographic anyons in ABJM (Kawamoto, Lin'09)
(anyonic phase associated to the F1 attached to the baryons surrounding the D0's dissolved)

A non-trivial flux adds lower dim brane charges and modifies the way the branes capture the background fluxes.

For example, for $F = \mathcal{N} J$ the D4 captures the F_2 flux and develops a tadpole $\Rightarrow$ F-strings ending on it :

$$S_{CS} = \frac{1}{2}(2\pi)^2 T_4 \int_{\mathbb{R}\times CP^2} P[F_2] \wedge F \wedge A = \frac{k\mathcal{N}}{2} T_{F1} \int dt A_t$$

$\rightarrow$ Baryon vertex-like configurations

The magnetic flux modifies the dynamics as well:

4. Gauge/gravity calculation of the energy

(Brandhuber, Itzhaki, Sonnenschein, Yankielowitz'98; Imamura'98; Maldacena'98)

Consider a uniform distribution of strings on a $CP^{\frac{p}{2}}$ shell with $p = 2, 4, 6$

Non-SUSY but we can ignore the backreaction

In the probe brane approx, with $F = \mathcal{N} J$, $S = S_{Dp} + S_{qF1}$:

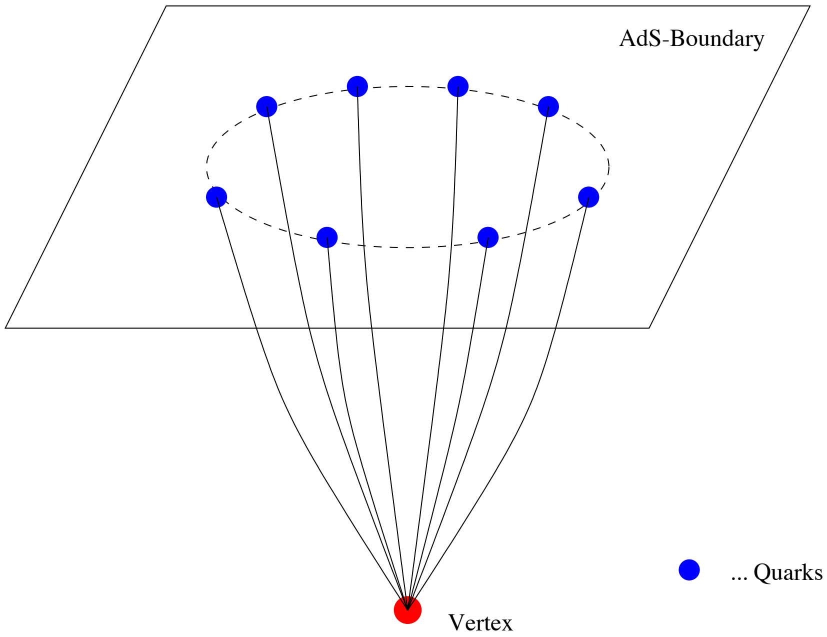
$$S_{Dp} = -Q_p \int dt \, \frac{2\rho}{L} \, , \qquad Q_p = \frac{T_p}{g_s} \mathrm{Vol}(CP^{\frac{p}{2}}) \, (L^4 + (2\pi\mathcal{N})^2)^{\frac{p}{4}}$$

$$S_{qT_{F1}} = -q \, T_{F1} \int dt \, dr \sqrt{\frac{16\rho^4}{L^4} + \rho'^2},$$

where we have taken $\tau = t$ and $\rho = \rho(r)$

$\sigma = r$

$\leftrightarrow$ Radially symmetric distribution on a circle of radius l



$$\rho(0) = \rho_0$$
$$\rho(l) = \infty$$

In fact, since the D4 wraps a non-spin manifold if must carry $F_{FW} = J$ due to the Freed-Witten anomaly (Freed, Witten'99)

$\Rightarrow$ A flat half-integer B_2 has to be switched on, such that

$$\mathcal{F} = F_{FW} + \frac{1}{2\pi} B_2 = 0$$

Then $Q_p = \frac{T_p}{g_s} \mathrm{Vol}(CP^{\frac{p}{2}}) \left(L^4 + (2\pi)^2 (\mathcal{N}-1)^2\right)^{\frac{p}{4}}$ for $p = 2, 6$

For the D6 there are new CS terms:

$$\int_{\mathbb{R}\times CP^3} P[F_2] \wedge B_2 \wedge B_2 \wedge A, \qquad \int_{\mathbb{R}\times CP^3} P[F_2] \wedge F \wedge B_2 \wedge A$$

that modify the number of F-strings. First is cancelled with

$$S^{D6}_{h.c.} = \frac{3}{2} (2\pi)^5 T_6 \int C_1 \wedge F \wedge \sqrt{\frac{\hat{A}(T)}{\hat{A}(N)}}$$

(Aharony, Hashimoto, Hirano, Ouyang'09)

In the presence of a magnetic flux, $B_2 \neq 0$ is key in order to have an integer number of F1's attached to the vertex:

D-brane	q
D2	k
D4	$k\frac{\mathcal{N}}{2}$
D6	$N + k\frac{\mathcal{N}(\mathcal{N}-2)}{8}$

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$$S_{qT_{F1}} = -q\, T_{F1} \int dt\, dr \sqrt{\frac{16\rho^4}{L^4} + \rho'^2}$$

Bulk equation of motion: $$\frac{\rho^4}{\sqrt{\frac{16\rho^4}{L^4} + \rho'^2}} = c$$

Boundary equation of motion: $$\frac{\rho_0'}{\sqrt{\frac{16\rho_0^4}{L^4} + \rho_0'^2}} = \frac{2Q_p}{L\, q\, T_{F_1}}$$

Define $\sqrt{1-\beta^2} = \frac{2Q_p}{L\,q\,T_{F1}}$ with $\beta \in [0,1]$

The two equations can be combined into:

$$\frac{\rho^4}{\sqrt{\frac{16\rho^4}{L^4} + \rho'^2}} = \frac{1}{4}\,\beta\,\rho_0^2\,L^2$$

Integrating: Size of the configuration:

$$\ell = \frac{L^2}{4\rho_0}\int_1^\infty dz\,\frac{\beta}{z^2\sqrt{z^4-\beta^2}}$$

Same form for the baryon vertex in $AdS_5 \times S^5$

Same dependence on L^2 in $AdS_5 \times S^5$: Prediction of AdS/CFT for the strong coupling behavior of the CS theory

On-shell energy:

$$E = E_{Dp} + E_{qF1} = qT_{F_1}\rho_0\Big(\sqrt{1-\beta^2} + \int_1^\infty dz \frac{z^2}{\sqrt{z^4-\beta^2}}\Big)$$

Binding energy:

$$E_{\rm bin} = q\,T_{F_1}\,\rho_0\Big(\sqrt{1-\beta^2} + \int_1^\infty dz \Big[\frac{z^2}{\sqrt{z^4-\beta^2}} - 1\Big] - 1\Big)$$

where we have substracted the energy of the constituents (when the brane is located in $\rho_0 = 0$ the strings become radial and correspond to free quarks)

- $E_{\rm bin}$ negative and decreases monotonically with β
- $E_{\rm bin} = 0$ for $\beta = 0$ (q free radial strings stretching from ρ_0 to ∞ plus a Dp-brane at ρ_0) (only for non-zero magnetic flux)

As a function of ℓ :

$$E_{\rm bin} = -f(\beta)\frac{(g_s N)^{2/5}}{\ell} \quad \text{with} \quad f(\beta) \geq 0$$

$\Rightarrow$ - The configuration is stable

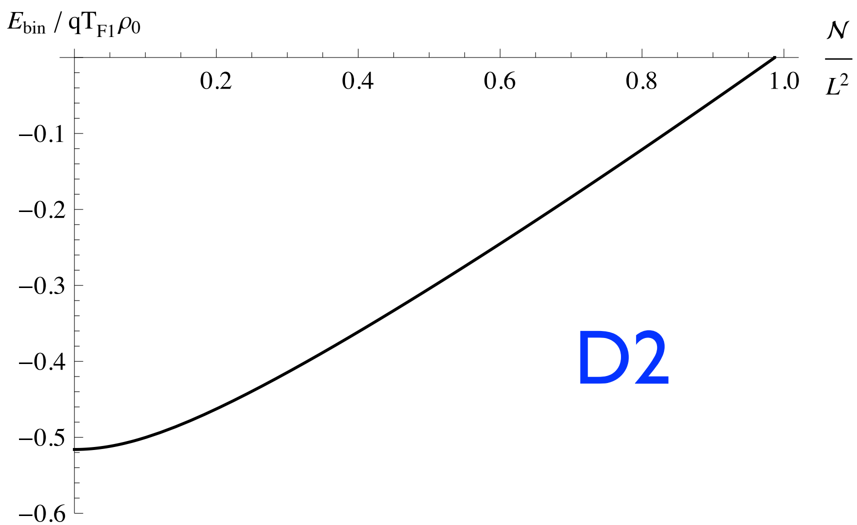
- $E_{\rm bin} \sim 1/\ell$ dictated by conformal invariance

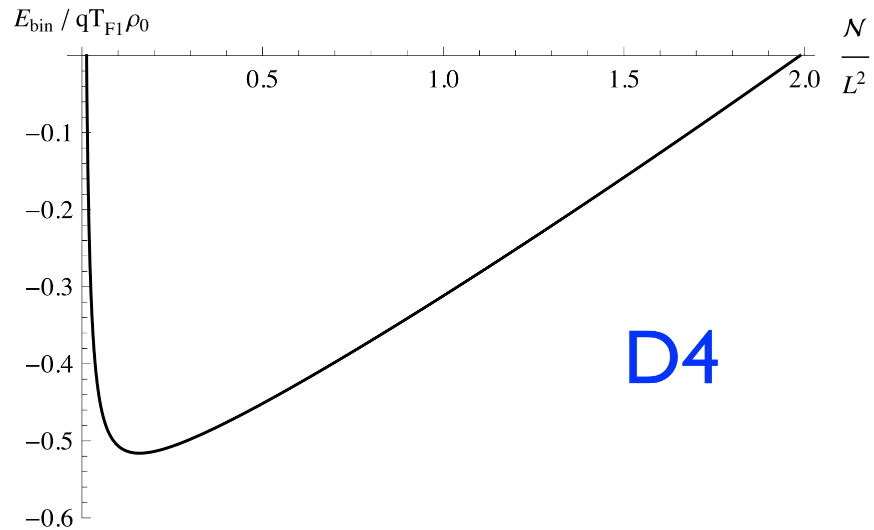
- As a function of the 't Hooft coupling, $\lambda = N/k$,

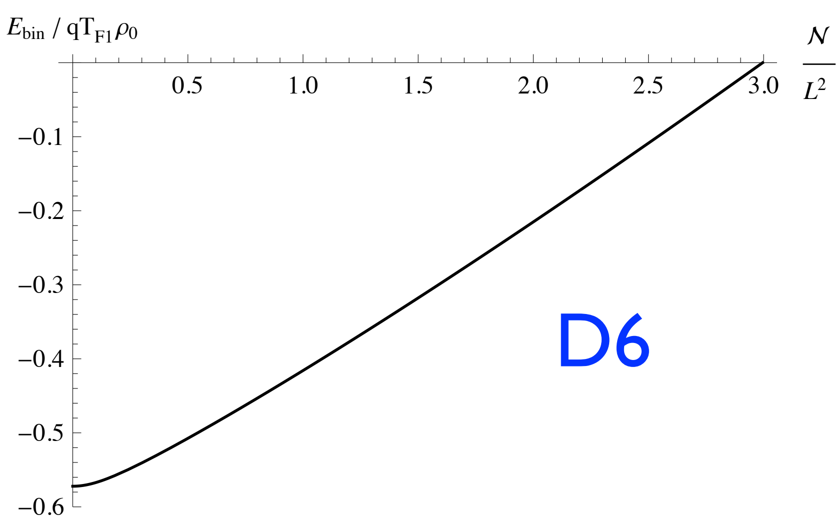
$E_{\rm bin} \sim \sqrt{\lambda}$, as in AdS_5 $\Rightarrow$ Non-trivial prediction for the non-perturbative regime of the CS theory

(Mariño, Putrov'09)

$\frac{2Q_p}{L\,q\,T_{F1}} \leq 1 \Rightarrow$ Bound on the magnetic flux







Binding energy per string

q : Number of strings

D-brane	q
D2	k
D4	$k\dfrac{\mathcal{N}}{2}$
D6	$N + k\dfrac{\mathcal{N}(\mathcal{N}-2)}{8}$

5. Reduce the number of fundamental strings

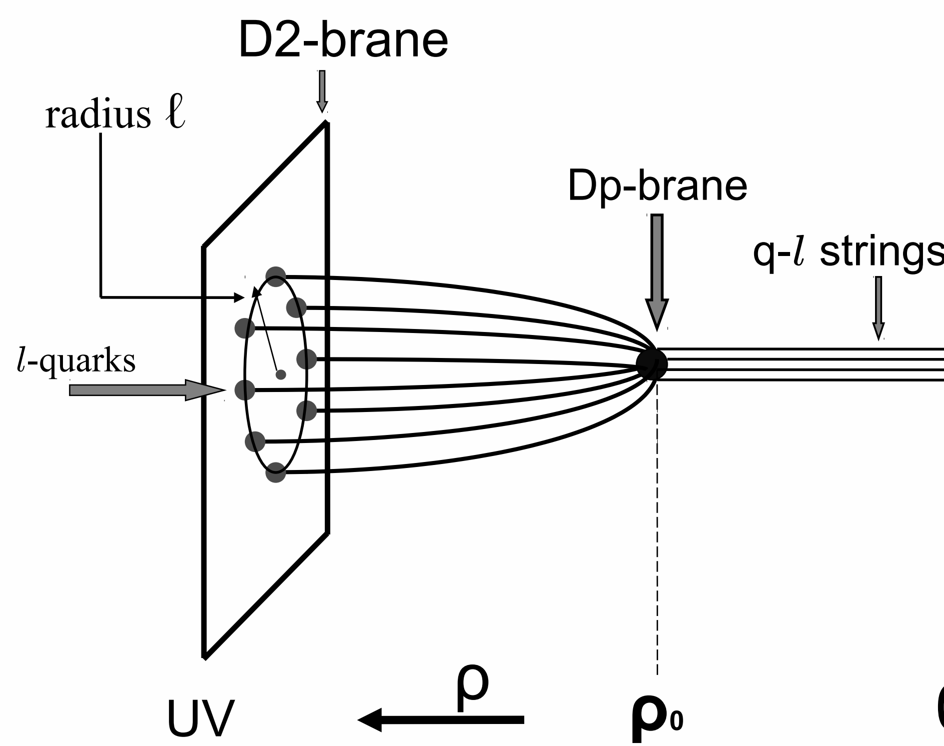
In $AdS_5 \times S^5$: Baryon vertex classical solutions with number of quarks $5N/8 \leq l \leq N$ (non-singlet)

(Brandhuber, Itzhaki, Sonnenschein, Yankielowitz'98; Imamura'98)

Stable against fluctuations for $0.813N \leq l \leq N$

(Sfetsos, Siampos'08)

In $AdS_4 \times CP^3$:



5.1. The classical solution

The boundary equation of motion changes:

$$\frac{\rho_0'}{\sqrt{\frac{16\rho_0^4}{L^4}+\rho_0'^2}} = \frac{2Q_p}{L\,l\,T_{F1}} + \frac{q-l}{l} \leq 1$$

$$\Rightarrow \ \frac{q}{2}(1+\sqrt{1-\beta^2}) \leq l \leq q$$

$\frac{2Q_p}{L\,q\,T_{F1}} \leq 1 \Rightarrow$ Bound on the magnetic flux

$(l, \mathcal{N})$ parameter space bounded by the values for which the baryon vertex reduces to free quarks

5.2. Stability analysis

Important in establishing the physical parameter space (Avramis, Sfetsos, Siampos'06-08)

Ansatz for the fluctuations (for the strings):

$$\delta x^\mu(t,\rho) = \delta x^\mu(\rho) e^{-i\omega t} \quad \text{for} \quad x^\mu = r, \theta$$

Expand the Nambu-Goto action to quadratic order and study the zero mode problem $\leftrightarrow$ Critical curve in the parametric space separating the stable and unstable regions

Stability reduced to an eigenvalue problem of the general Sturm-Liouville type

Instabilities emerge from longitudinal fluctuations of the strings

Bound for the number of F-strings coming from stability:

$$l \geq \frac{q}{1+\gamma_c}(1+\sqrt{1-\beta^2}) \qquad \gamma_c = 0.538$$

More restrictive than the bound imposed by the existence of a classical solution:

$$l \geq \frac{q}{2}(1+\sqrt{1-\beta^2})$$

$\Rightarrow$ Non-singlet states exist for $\lambda >> 1$

Can we reach the finite 't Hooft coupling region?

Microscopical description in terms of fuzzy $CP^{\frac{p}{2}}$ built up out of n dielectric D0-branes valid when $N << n^{\frac{4}{p}} k$

6. The microscopical description

A non-trivial magnetic flux induces D0-brane charge on the Dp

$\Rightarrow$ Complementary description in terms of multiple D0 expanded into fuzzy $CP^{\frac{p}{2}}$ by Myers dielectric effect

- Macroscopical description valid in the sugra limit:

$$L >> 1 \;\Leftrightarrow N >> k$$

- Micro when

$$\frac{\mathrm{Vol}(CP^{\frac{p}{2}})}{n} << l_s^p \;\Leftrightarrow\; N << n^{\frac{4}{p}} k$$

$\Rightarrow$ It allows to explore the region of finite λ

Complementary for finite n

Should agree in the large n limit

6.1. Fuzzy CP

- $CP^{\frac{p}{2}}$: coset manifold $\dfrac{SU(\frac{p}{2}+1)}{U(\frac{p}{2})}$. Submanifold of $\mathbb{R}^{\frac{p^2}{4}+p}$ determined by the constraints:

$$\sum_{i=1}^{\frac{p^2}{4}+p}(x^i)^2 = 1\,, \qquad \sum_{j,k=1}^{\frac{p^2}{4}+p} d^{ijk}x^j x^k = \frac{\frac{p}{2}-1}{\sqrt{\frac{p}{4}(\frac{p}{2}+1)}}x^i$$

$\to p$ dimensional manifold

Fubini-Study metric of the $CP^{\frac{p}{2}}$ given by

$$ds^2_{CP^{\frac{p}{2}}} = \frac{p}{4(\frac{p}{2}+1)}\sum_{i=1}^{\frac{p^2}{4}+p}(dx^i)^2$$

- Matrix level definition $\leftrightarrow$ Fuzzy $CP^{\frac{p}{2}}$:

$X^i = \dfrac{1}{\sqrt{C_n}}T^i$, T^i : generators of $SU(\dfrac{p}{2}+1)$ in the $(m,0)$ irrep

Substituting in Myers action for D0-branes:

$$S_{DBI} = -\int \mathrm{STr}\Big\{e^{-\phi}\sqrt{|\det\Big(P[E_{\mu\nu}+E_{\mu i}(Q^{-1}-\delta)^i_j E^{jk}E_{k\nu}]\Big)\det Q|}\Big\}$$

$$Q^i_j = \delta^i_j + \frac{i}{2\pi}[X^i, X^k]E_{kj}$$

$$\Rightarrow \quad S^{DBI}_{nD0} = -\frac{n}{g_s}\Big(1+\frac{L^4}{16\pi^2 m(m+\frac{p}{2}+1)}\Big)^{\frac{p}{4}}\int dt\,\frac{2\rho}{L}$$

$$n = \dim(m,0)\,,\; n = \frac{\mathcal{N}^{\frac{p}{2}}}{2^{\frac{p}{2}}(\frac{p}{2})!}+\ldots \;\Rightarrow\; m \sim \frac{\mathcal{N}}{2}$$ for large n

and S^{DBI}_{nD0} exactly reproduces the macroscopical result:

$$S_{Dp} = -Q_p\int dt\,\frac{2\rho}{L}\,, \qquad Q_p = \frac{T_p}{g_s}\mathrm{Vol}(CP^{\frac{p}{2}})\,(L^4+(2\pi\mathcal{N})^2)^{\frac{p}{4}}$$

6.2. The F-strings from the Dp to the boundary of AdS

CS action for coincident branes:

$$S_{CS} = \int \mathrm{STr}\Big\{P\Big(e^{\frac{i}{2\pi}(i_X i_X)} \sum_q C_q \; e^{B_2}\Big)e^{2\pi F}\Big\}$$

Dependence of the background potentials on the non-Abelian scalars:

$$C_q(t,X) = C_q(t) + X^k \partial_k C_q(t) + \frac{1}{2} X^l X^k \partial_l \partial_k C_q(t) + \dots$$

For example:

$$S_{CS_2} = -\frac{i}{(2\pi)^2} \int \mathrm{STr}\{(i_X i_X)^3 F_6 \wedge A\} =$$

$$= N\Big(m(m+4)\Big)^{-3/2} \frac{(m+3)!}{m!} \int dt\, A_t \to N \int dt\, A_t \;\; \text{for } m >> 1$$

$$S_{CS_1} = i \int STr\{(i_X i_X) F_2 \wedge A\}$$ gives the F1 charge prop. to k

6.3. The flat half-integer NS-NS 2-form

Introduced macroscopically to cancel the flux of the Freed-Witten vector field, such that $\mathcal{F} = F_{FW} + \frac{1}{2\pi} B_2 = 0$

Microscopically: We should find an obstacle to the expansion of the D0 into a fuzzy CP^2 when $B_2 = 0$.

However, F_{FW} does not couple in the action for D0

How precisely $B_2 \neq 0$ allows the construction of the CP^2 ?

Macroscopically B_2 modifies the BI action such that
$\mathcal{N} \to \mathcal{N} - 1$ (for the D2 and D6)

$\Rightarrow$ Microscopically we should include $\frac{1}{m}$ corrections

To this order we find for $B_2 = 0$:

$\mathcal{N} = 2m + \frac{p}{2} + 1 \;\Rightarrow\; \mathcal{N} \in 2\mathbb{Z}$ for p=2,6

Whereas for $B_2 \neq 0$:

$\mathcal{N}_{D2,D6} = 2m + \frac{p}{2} + 1$ and $\mathcal{N} \to \mathcal{N} - 1$

$\mathcal{N}_{D4} = 2m + \frac{p}{2} \;\Rightarrow\; \mathcal{N} \in 2\mathbb{Z}$ for all p

$\Rightarrow B_2 \neq 0$ to have $\mathcal{N}$ properly quantized

Confirmed by the CS action (include as well

$$S_{CS_3} = -\frac{1}{2\pi}\int \mathrm{STr}\Big\{(i_X i_X)^2 F_2 \wedge B_2 \wedge A\Big\}$$)

Higher curvature couplings are needed in order to cancel the contribution of

$$S_{CS_4} = -\frac{i}{2}\frac{1}{(2\pi)^2}\int \mathrm{STr}\Big\{(i_X i_X)^3 F_2 \wedge B_2 \wedge B_2 \wedge A\Big\}$$

$$S_{h.c.} = -\frac{1}{2(2\pi)^2}\int_{\mathbb{R}} P[(i_X i_X)^2 C_1 \wedge \sqrt{\frac{\hat{A}(T)}{\hat{A}(N)}}] =$$

$$= -\frac{i}{(2\pi)^2}\int_{\mathbb{R}} [(i_X i_X)^3 (F_2 \wedge \sqrt{\frac{\hat{A}(T)}{\hat{A}(N)}})]A$$

In general:

$$S_{h.c.} = T_p \int d^{p+1}\xi \, \mathrm{STr} \left[P \left(e^{\frac{i}{2\pi}(i_X i_X)} \sum_q C_q \, e^{B_2} \sqrt{\frac{\hat{A}(T)}{\hat{A}(N)}} \right) e^{2\pi F} \right]_{p+1}$$

$\Rightarrow$ New dielectric couplings of RR-fields to derivatives of B_2 and the metric through T-duality

(Becker, Guo, Robbins'10) (Garousi'10)

Similar stability analysis

7. Conclusions

Stability of baryon vertex like configurations in $AdS_4 \times CP^3$:

- Condition for existence of a classical solution:

$$l \geq \frac{q}{2}(1+\sqrt{1-\beta^2})$$

- Stable when $l \geq \frac{q}{1+\gamma_c}(1+\sqrt{1-\beta^2})$ $\quad \gamma_c = 0.538$

$\Rightarrow$ More restrictive

- Probe brane approx $\Rightarrow$ Valid in the SUGRA limit $\lambda >> 1$
- For non-zero magnetic flux: D0-brane charge dissolved $\Rightarrow$

Alternative description in terms of D0-branes expanded into fuzzy $CP^{\frac{p}{2}}$ by Myers dielectric effect

Microscopical description valid for finite 't Hooft coupling

- Expansion caused by a purely gravitational dielectric effect
- CS terms indicate the need to introduce F-strings
- Non-singlet classical stable solutions for finite λ
- Prediction of new dielectric higher curvature couplings, with further implications through T and S duality

(Becker, Guo, Robbins'10) (Garousi'10)

8. Open questions

- New dielectric higher curvature couplings confirmed from string amplitudes?
- Can we extrapolate the micro results to $\mathcal{N} \to 0$?
- What happens in theories with less susy, like $AdS_5 \times T^{1,1}$?
- Include the backreaction $\longleftrightarrow$ Look for supersymmetric spike solutions (partial studies in Kawamoto, Lin'09)

  Marginal bound states?

  Non-singlet states?

- Explore the finite temperature case

Thanks!