Flat deformations of type IIB S-folds

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Based on 2103.12652 + work in progress with Colin Sterckx

See recent works by: Arav, Bobev, Cheung, Gauntlett, Gautason, Giambrone, Malek, Pilch, Roberts, Rosen, Samtleben, Suh, Trigiante and van Muiden.

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Outlook

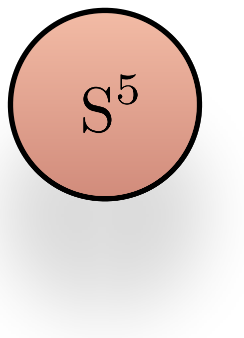
- S-folds in 10D
- S-folds in 4D
- Flat deformations
- Future directions

S-folds in 10D

Type IIB S-folds

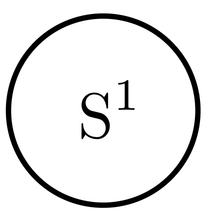
[Hull, (Çatal-Özer) ’04, (’03)]
[Inverso, Samtleben, Trigiante ’16]

10D: type IIB SUGRA solutions $\mathrm{AdS}_4 \times \mathrm{S}^1 \times \mathrm{S}^5$



5D: SO(6) gauged SUGRA **[** $\mathbb{R} \to \mathrm{S}^1 \;\Leftrightarrow$ **(hyperbolic)** $\mathrm{SO}(1,1) \subset \mathrm{SL}(2)_{\mathrm{IIB}}$ **twist = monodromy]**

$\eta \sim \eta + T$



$$A^{\alpha}{}_{\beta} = \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix}$$

$$\mathfrak{M}_{\mathrm{S}^1} = A^{-1}(\eta)A(\eta+T) = \begin{pmatrix} \cosh T & \sinh T \\ \sinh T & \cosh T \end{pmatrix}$$

No untwisted limit !!

4D: [SO(1,1) × SO(6)] ⋉ R¹² gauged SUGRA AdS_4

N=0 & SO(6) S-fold

[AG, Sterckx '19]

Flavour : SO(6) ~ S^5

$$ds^2_{10} = \frac{1}{\sqrt{2}}\, ds^2_{\mathrm{AdS}_4} + \frac{1}{2}\, d\eta^2 + d\mathring{s}^2_{\mathrm{S}^5}$$

$$\widetilde{F}_5 = 4\,(1+\star)\,\mathrm{vol}_5$$

$$\mathbb{B}^\alpha = A^\alpha{}_\beta\, \mathfrak{b}^\beta = 0$$

$$m_{\alpha\beta} = (A^{-t})_\alpha{}^\gamma\, \mathfrak{m}_{\gamma\delta}\, (A^{-1})^\delta{}_\beta$$

with $\mathfrak{m}_{\gamma\delta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

N=1 & SU(3) S-fold

[Lüst, Tsimpis '09 (local form)]
[AG, Sterckx '19]

Flavour : SU(3) ~ CP²

$$ds_{10}^2 = \frac{3\sqrt{6}}{10}\, ds^2_{\text{AdS}_4} + \frac{1}{3}\sqrt{\frac{10}{3}}\, d\eta^2 + \left[\sqrt{\frac{5}{6}}\, ds^2_{\mathbb{CP}^2} + \sqrt{\frac{6}{5}}\, \boldsymbol{\eta}^2\right]$$

$$\widetilde{F}_5 = 3\left(\frac{6}{5}\right)^{\frac{3}{4}} (1+\star)\,\text{vol}_5$$

$$\mathbb{B}^\alpha = A^\alpha{}_\beta\, \mathfrak{b}^\beta = \epsilon^{\alpha\delta}\,(A^{-t})_\delta{}^\gamma\, H_{\gamma\beta}\, \boldsymbol{\Omega}^\beta$$

[charged under $U(1)_\eta$]

$$m_{\alpha\beta} = (A^{-t})_\alpha{}^\gamma\, \mathfrak{m}_{\gamma\delta}\, (A^{-1})^\delta{}_\beta$$

with $\mathfrak{m}_{\gamma\delta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

N=2 & SU(2) x U(1) S-fold

Flavour : SU(2) ~ S²

R-symmetry: U(1)

[AG, Sterckx, Trigiante '20]

$$ds^2 = \tfrac{1}{2}\,\Delta^{-1}\left[ds^2_{\text{AdS}_4} + d\eta^2 + d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\,\left(\sigma_2^2 + 8\,\Delta^4\,\left(\sigma_1^2+\sigma_3^2\right)\right)\right]$$

$$\Delta^{-4} = 6 - 2\cos(2\theta)$$

$$\widetilde{F}_5 = 4\,\Delta^4\,\sin\theta\,\cos^3\theta\,(1+\star)\Big[3\,d\theta\wedge d\phi\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3 - d\eta\wedge\Big(\cos(2\phi)\,d\theta - \tfrac{1}{2}\sin(2\theta)\,\sin(2\phi)\,d\phi\Big)\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3\Big]$$

$$\mathbb{B}^\alpha = A^\alpha{}_\beta\,\mathfrak{b}^\beta \quad \text{with} \quad \begin{aligned} \mathfrak{b}_1 &= \tfrac{1}{\sqrt{2}}\cos\theta\left[\Big(\sin\phi\,d\theta + \tfrac{1}{2}\sin(2\theta)\,d(\sin\phi)\Big)\wedge\sigma_2 + \sin\phi\,\frac{4\,\sin(2\theta)}{6-2\cos(2\theta)}\,\sigma_1\wedge\sigma_3\right] \\ \mathfrak{b}_2 &= \tfrac{1}{\sqrt{2}}\cos\theta\left[\Big(\cos\phi\,d\theta + \tfrac{1}{2}\sin(2\theta)\,d(\cos\phi)\Big)\wedge\sigma_2 + \cos\phi\,\frac{4\,\sin(2\theta)}{6-2\cos(2\theta)}\,\sigma_1\wedge\sigma_3\right] \end{aligned}$$

$$m_{\alpha\beta} = (A^{-t})_\alpha{}^\gamma\,\mathfrak{m}_{\gamma\delta}\,(A^{-1})^\delta{}_\beta$$

$$\text{with} \quad \mathfrak{m}_{\gamma\delta} = 2\,\Delta^2\begin{pmatrix} 1+\sin^2\theta\,\cos^2\phi & -\frac{1}{2}\,\sin^2\theta\,\sin(2\phi) \\ -\frac{1}{2}\,\sin^2\theta\,\sin(2\phi) & 1+\sin^2\theta\,\sin^2\phi \end{pmatrix}$$

N=4 & SO(4) S-fold

[Inverso, Samtleben, Trigiante '16]

R-symmetry : SO(4) ~ S² x S²

$$ds^2 = \tfrac{1}{2}\,\Delta^{-1}\left[ds^2_{\mathrm{AdS}_4} + d\eta^2 + \frac{dr^2}{1-r^2} + \Delta^4\,(3-2r^2)\,r^2 d\Omega_1^2 + \Delta^4 (1+2r^2)\,(1-r^2) d\Omega_2^2\right]$$

$$\Delta^{-4} = (1+2r^2)(3-2r^2)$$

$$\begin{aligned}\widetilde{F}_5 = \; & \frac{6\Delta^4}{8\,(1-r^2)} \mathcal{Z}^p\, d\mathcal{Y}^p \wedge d\mathcal{Y}^q \wedge d\mathcal{Y}^r \wedge d\mathcal{Z}^q \wedge d\mathcal{Z}^r \\ & + 3\,\Delta^4\, \mathcal{Z}^p \mathcal{Y}^{[p}\, d\mathcal{Y}^q \wedge d\mathcal{Y}^{r]} \wedge d\mathcal{Z}^q \wedge d\mathcal{Z}^r \wedge d\eta \\ & - \frac{1}{16}\sqrt{|g|}\,\varepsilon_{\mu\nu\rho\sigma}\, dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \wedge \left(d\eta - \frac{4}{3}\,\mathcal{Y}^p d\mathcal{Y}^p\right)\end{aligned}$$

$$\begin{aligned}\mathbb{H}^\alpha = A^\alpha{}_\beta\, \mathfrak{h}^\beta = \; & \frac{(V_- A)^\alpha}{1+2r^2}\,\varepsilon_{pqr}\, d\mathcal{Y}^p \wedge d\mathcal{Y}^q \wedge \left(\frac{(3+2r^2)}{3\,(1+2r^2)}\, d\mathcal{Y}^r - \mathcal{Y}^r\, d\eta\right) \\ & - \frac{(V_+ A)^\alpha}{3-2r^2}\,\varepsilon_{pqr}\, d\mathcal{Z}^p \wedge d\mathcal{Z}^q \wedge \left(\frac{(5-2r^2)}{3\,(3-2r^2)}\, d\mathcal{Z}^r + \mathcal{Z}^r\, d\eta\right)\end{aligned}$$

$$m_{\alpha\beta} = (A^{-t})_\alpha{}^\gamma\, \mathfrak{m}_{\gamma\delta}\, (A^{-1})^\delta{}_\beta$$

with $\mathfrak{m}_{\gamma\delta} = \dfrac{\Delta^2}{\sqrt{3}} \begin{pmatrix} 3+2r^2 & -4r^2 \\ -4r^2 & 3+2r^2 \end{pmatrix}$

S-folds as limiting Janus solutions

[Bak, Gutperle, Hirano '03 (**N = 0**)]
[Clark, Freedman, Karch, Schnabl '04]
[D'Hoker, Ester, Gutperle '07, '07 (**N = 4**)]
[Bobev, Gautason, Pilch, Suh, van Muiden '19, '20]

↓

Janus solutions AdS_4/CFT_3 dual to interfaces of SYM_4

[D'Hoker, Ester, Gutperle '06 (**N = 1 , 2 , 4**)]

↓

SUSY interfaces of SYM_4 are classified !!

N=1 & SU(3) N=2 & SU(2) × $U(1)_R$ N=4 & $SO(4)_R$

> Within a category with fixed amount of supersymmetry (i.e. fixed r), we shall principally be interested in the theory which has *maximal internal symmetry*, since other theories with the same amount of supersymmetry but less internal symmetries may be viewed as perturbations of the former by BPS operators that further break the internal symmetry.

S-folds in 4D

$[\,SO(1,1) \times SO(6)\,] \ltimes R^{12}$ gauged maximal supergravity

- N=8 SUGRA : metric + 8 gravitini + 28 vectors + 56 dilatini + 70 scalars

metric	8 gravitini	28 vectors	56 dilatini	70 scalars
(s = 2)	(s = 3/2)	(s = 1)	(s = 1/2)	(s = 0)

$E_{7(7)}$ / SU(8)
coset space

- Deformation parameter c yielding inequivalent theories: electric/magnetic

$$D = \partial - g\,(A^{\text{elec}} - c\,\tilde{A}_{\text{mag}})$$

g = 4D gauge coupling
c = deformation param.

[Dall'Agata, Inverso, Trigiante '12]

- Two inequivalent cases :

$$c = 0 \; : \quad A^{\alpha}{}_{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{or} \qquad c \neq 0 \; : \quad A^{\alpha}{}_{\beta} = \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix}$$

A truncation : $\mathbb{Z}_2^3$ invariant sector

[AG, Sterckx, Trigiante '20]

- Truncation : Retaining the fields and couplings which are invariant (singlets) under a $\mathbb{Z}_2^3$ action ➡ N = 1 supergravity coupled to 7 chiral multiplets z_i

$$z_i = -\chi_i + i\, y_i \qquad (y_i > 0)$$

[upper half-plane]

- The model :

$$K = -\sum_{i=1}^{7} \log[-i(z_i - \bar{z}_i)]$$

$$W = 2\,g\,\big[\, z_1 z_5 z_6 + z_2 z_4 z_6 + z_3 z_4 z_5 + (z_1 z_4 + z_2 z_5 + z_3 z_6)\, z_7 \,\big] + 2\,g\,c\,(1 - z_4 z_5 z_6 z_7)$$

[dyonic gauging]

- AdS$_4$ vacua :

N=0 & SO(6) N=1 & SU(3) N=2 & SU(2) × U(1) N=4 & SO(4)

➡ Most symmetric AdS$_4$ vacua within **multi-parametric families** !!

N=0 family of AdS_4 vacua with $U(1)^3$ symmetry

- Location : **[3 free parameters]**

$$z_{1,2,3} = c\left(-\chi_{1,2,3} + i\frac{1}{\sqrt{2}}\right) \qquad \text{and} \qquad z_4 = z_5 = z_6 = z_7 = i$$

- AdS_4 radius $L^2 = -3/V_0$ & scalar mass spectrum :

$$V_0 = -2\sqrt{2}g^2c^{-1}$$

[BF unstable]

$$\begin{aligned}
m^2L^2 = {} & 6(\times 2)\,, \quad -3(\times 2)\,, \quad 0(\times 28)\,, \\
& -\frac{3}{4} + \frac{3}{2}\chi^2(\times 2)\,, \\
& -\frac{3}{4} + \frac{3}{2}(\chi - 2\chi_i)^2(\times 2) \qquad i = 1,2,3\,, \\
& -\frac{3}{4} + \frac{3}{2}\chi_i^2(\times 4) \qquad i = 1,2,3\,, \\
& -3 + 6\chi_i^2(\times 2) \qquad i = 1,2,3\,, \\
& -3 + \frac{3}{2}(\chi_i \pm \chi_j)^2(\times 2) \qquad i < j\,,
\end{aligned}$$

- Flavour symmetry enhancements :

$$\begin{array}{ccccccc}
U(1)^3 & \rightarrow & SU(2)\times U(1)^2 & \rightarrow & SU(3)\times U(1) & \rightarrow & SO(6) \\
 & & \chi_i = \chi_j & & \chi_1 = \chi_2 = \chi_3 & & \chi_{1,2,3} = 0
\end{array}$$

N=1 family of AdS$_4$ vacua with U(1)^2 symmetry

- Location : **[2 free parameters :** $\sum_{i=1}^{3} \chi_i = 0$ **]**

$$z_{1,2,3} = c \left(-\chi_{1,2,3} + i \frac{\sqrt{5}}{3} \right) \quad \text{and} \quad z_4 = z_5 = z_6 = z_7 = \frac{1}{\sqrt{6}} (1 + i\sqrt{5})$$

- AdS$_4$ radius $L^2 = -3/V_0$ & scalar mass spectrum :

$$V_0 = -\frac{162}{25\sqrt{5}} g^2 c^{-1}$$

$$\begin{aligned} m^2 L^2 = \; & 0(\times 28)\,, \quad 4 \pm \sqrt{6}(\times 2)\,, \quad -2(\times 2)\,, \\ & -\frac{14}{9} + 5\chi_i^2 \pm \frac{1}{3}\sqrt{4 + 45\chi_i^2}(\times 2) \qquad i = 1,2,3\,, \\ & -\frac{14}{9} + \frac{5}{4}\chi_i^2 \pm \frac{1}{6}\sqrt{16 + 45\chi_i^2}(\times 2) \qquad i = 1,2,3\,, \\ & \frac{7}{9} + \frac{5}{4}\chi_i^2(\times 2) \qquad i = 1,2,3\,, \\ & -2 + \frac{5}{4}(\chi_i - \chi_j)^2(\times 2) \qquad i < j\,, \end{aligned}$$

- Flavour symmetry enhancements :

$$\mathbf{U(1)^2 \rightarrow SU(2) \times U(1) \rightarrow SU(3)}$$

$$\chi_i = \chi_j \qquad\qquad \chi_{1,2,3} = 0$$

N=2 family of AdS_4 vacua with U(1) x U(1) symmetry

- Location : **[1 free parameter]**

$$z_1 = -\bar{z}_3 = c\left(-\chi + i\frac{1}{\sqrt{2}}\right), \quad z_2 = ic, \quad z_4 = z_6 = i \quad \text{and} \quad z_5 = z_7 = \frac{1}{\sqrt{2}}(1+i)$$

- AdS_4 radius $L^2 = -3/V_0$ & scalar mass spectrum :

$$V_0 = -3g^2c^{-1},$$

$$m^2L^2 = 0(\times 30), \quad 3 \pm \sqrt{17}(\times 2), \quad -2(\times 4), \quad 2(\times 6), \quad -2+4\chi^2(\times 6)$$
$$-1+4\chi^2 \pm \sqrt{16\chi^2+1}(\times 2), \quad \chi^2 \pm \sqrt{\chi^2+2}(\times 8),$$

- Flavour symmetry enhancement :

$$\mathbf{U(1)} \rightarrow \mathbf{SU(2)}$$
$$\chi = 0$$

N=4 AdS_4 vacuum with SO(4) symmetry

[Gallerati, Samtleben, Trigiante '14]

[AG, Sterckx, Trigiante '20]

- Location :

$$z_1 = z_2 = z_3 = ic \qquad \text{and} \qquad z_4 = z_5 = z_6 = -\bar{z}_7 = \frac{1}{\sqrt{2}}(1+i)$$

- AdS_4 radius $L^2 = -3/V_0$ & scalar mass spectrum :

$$V_0 = -3g^2c^{-1} \,, \qquad m^2L^2 = 0(\times 48)\,, \quad 10(\times 1)\,, \quad 4(\times 10)\,, \quad -2(\times 11)$$

- **No free parameters** within the $\mathbb{Z}_2^3$-invariant sector

Flat deformations

Axion-like deformations in 10D

[AG, Sterckx '21]

S^5 metric : $$d\mathring{s}^2_{S^5} = d\alpha^2 + \cos^2\alpha\, d\theta_1^2 + \sin^2\alpha\,\left(d\beta^2 + \cos^2\beta\, d\theta_2^2 + \sin^2\beta\, d\theta_3^2\right)$$

$$\theta_1' = \theta_1 + g\,\chi_1\,\eta \quad , \quad \theta_2' = \theta_2 + g\,\chi_2\,\eta \quad , \quad \theta_3' = \theta_3 + g\,\chi_3\,\eta$$

… so axions induce a fibration on S^5 when moving around S^1 characterised by a **non-trivial monodromy**

$$h(T) = \begin{pmatrix} h_1 & & \\ & h_2 & \\ & & h_3 \end{pmatrix} \quad \text{with} \quad h_i = \exp\left(i\,\chi_i\,\sigma_2\,T\right) \in \mathrm{SO}(2)$$

symmetry

$$\chi_i \rightarrow \chi_i + n_i\,\frac{2\pi}{T}$$

Global breaking of symmetries of S^5 ➡ matching patterns of symmetry breaking @ AdS_4 vacua !

- KK spectrum at the N=2 S-fold periodic under $\chi \rightarrow \chi + 2\pi/T$

[see Malek's talk]

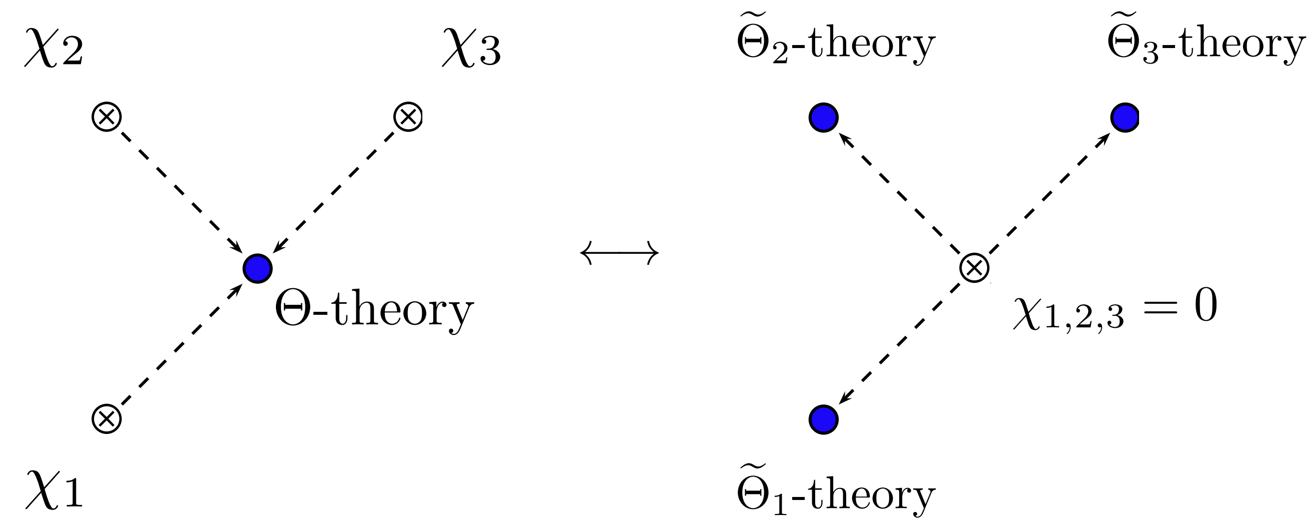
[Giambrone, Malek, Samtleben, Trigiante '21]

Axion-like deformations in 4D

[AG, Sterckx (in progress]

- $E_{7(7)}$ / SU(8) scalar coset :

  [homogeneous space]



$$\widetilde{\Theta} \;=\; \Theta^{[\mathrm{SO}(1,1)\times \mathrm{SO}(6)]\ltimes \mathbb{R}^{12}} \;+\; (\delta\Theta)^{\mathrm{CSS}}(\chi_1\,,\,\chi_2\,,\,\chi_3\,,\,0)$$

- CSS twist from 5D to 4D :

$$\chi^{ij} = \begin{pmatrix} 0 & \chi_1 & & & & \\ -\chi_1 & 0 & & & & \\ & & 0 & \chi_2 & & \\ & & -\chi_2 & 0 & & \\ & & & & 0 & \chi_3 \\ & & & & -\chi_3 & 0 \end{pmatrix} \in \mathfrak{so}(2)^3 \subset \mathfrak{so}(6) \;\subset \mathfrak{usp}(8)$$

  [“Flat” gauging : V = 0]

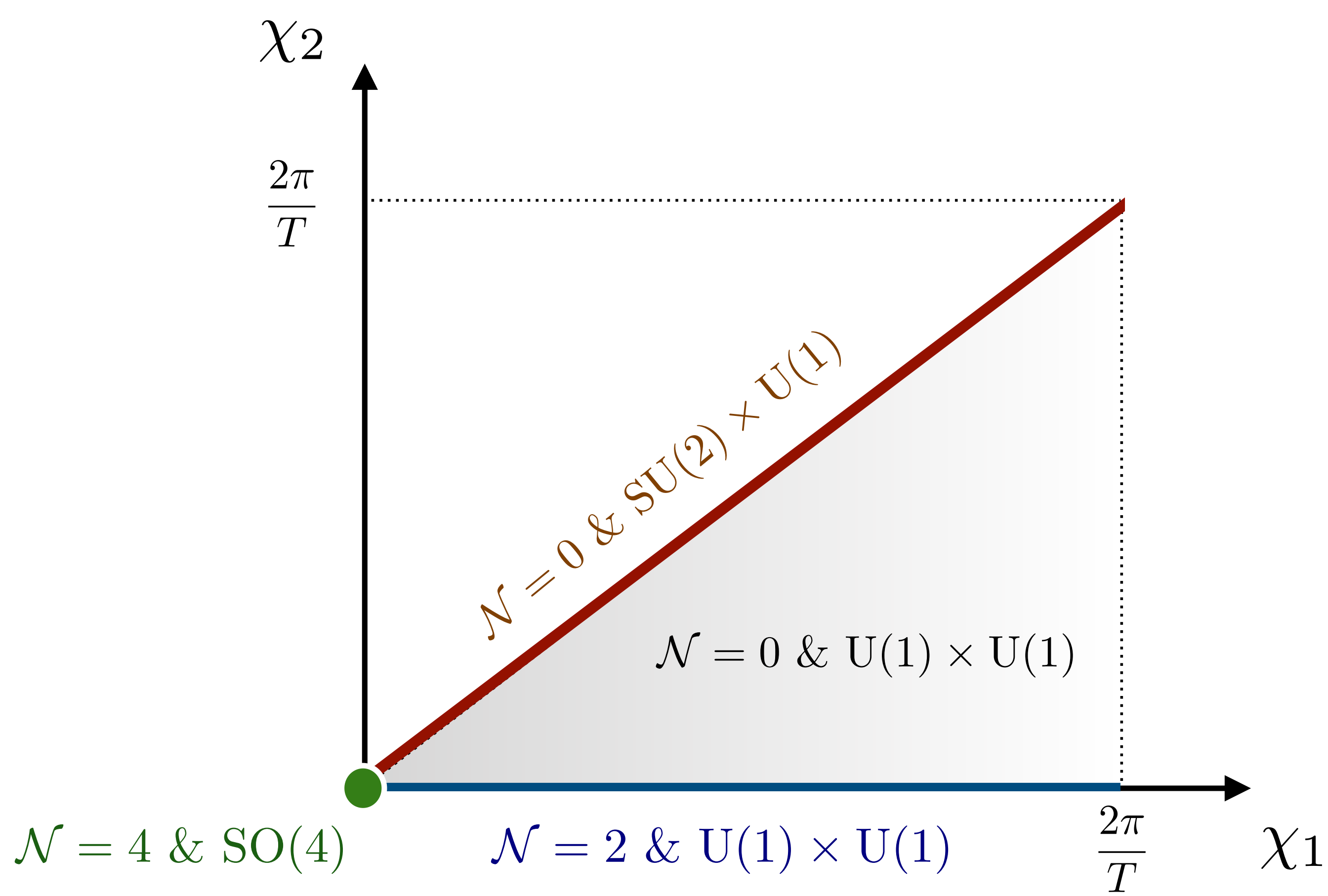
[Cremmer, Scherk, Schwarz ’79 (ungauged 5D)]

Flat deformations of the N=4 & SO(4) S-fold

- CSS twist from 5D to 4D : $\chi^{ij} = 12\sqrt{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \chi_1 & \chi_2 \\ 0 & 0 & 0 & 0 & \chi_2 & \chi_1 \\ 0 & 0 & -\chi_1 & -\chi_2 & 0 & 0 \\ 0 & 0 & -\chi_2 & -\chi_1 & 0 & 0 \end{pmatrix} \in \mathfrak{so}(2)^2 \subset \mathfrak{so}(4)$

- Two-parameter family of S-folds :

[not $\mathbb{Z}_2^3$-invariant]



[Bobev, Gautason, van Muiden '21]

Discrete sym: $\chi_i \leftrightarrow -\chi_i \quad , \quad \chi_1 \leftrightarrow \chi_2$

Non-susy & perturbatively **stable**

(at the lower-dimensional supergravity level)

Future directions

- Compactness of $\chi_{1,2}$ → Type IIB uplift : S^5 monodromies
  → KK spectra at the flat-deformed N=4 & SO(4) S-folds

- Higher-dimensional stability *vs* AdS Swampland Conjecture [Ooguri, Vafa '17]

- Oxidation of AdS_4 vacua with $\chi^{ij} \neq 0$ to 5D & one-form deformations of SYM

[Bobev, Gautason, Pilch, Suh, van Muiden '19, '20]

[Arav, (Cheung), Gauntlett, Roberts, Rosen '21]

- Application to other (half-)maximal supergravities ευχαριστώ

thanks !!