S-folds and holographic RG flows on the D3-brane

Adolfo Guarino

University of Oviedo & ICTEA

Based on 1907.04177 & 2002.03692 & 2103.12652

with Colin Sterckx and Mario Trigiante

See recent works by: Arav, Bobev, Cheung, Gauntlett, Gautason, Giambrone, Malek, Pilch, Roberts, Rosen, Samtleben, Suh, Trigiante and van Muiden.

Universidad de Oviedo
Universidá d'Uviéu
University of Oviedo

ICTEA
Instituto Universitario de Ciencias y Tecnologías del Espacio de Asturias

Outlook

- Electric-magnetic duality in maximal supergravity
- S-folds in 4D
- S-folds in 10D
- Holographic RG-flows on the D3-brane
- Concluding remarks

Electric-magnetic duality in maximal supergravity

N=8 supergravity in 4D

- SUGRA : metric + 8 gravitini + 28 vectors + 56 dilatini + 70 scalars

  $(s = 2)$ $(s = 3/2)$ $(s = 1)$ $(s = 1/2)$ $(s = 0)$

Ungauged (abelian) supergravity: Reduction of M-theory on a *torus* T^7 down to 4D produces $N = 8$ supergravity with $G = U(1)^{28}$

[$E_{7(7)}$ symmetry]

[Cremmer, Julia '79]

Gauged (non-abelian) supergravity:

❖ M-theory on S^7 produces $N = 8$ supergravity with $G = SO(8)$ [de Wit, Nicolai '82]

❖ Type IIA on S^6 produces $N = 8$ supergravity with $G = ISO(7) = SO(7) \ltimes \mathbb{R}^7$ [Hull '84]

❖ Type IIB on $\mathbb{R} \times S^5$ produces $N = 8$ supergravity with $G = [\,SO(1,1) \times SO(6)\,] \ltimes \mathbb{R}^{12}$

[Inverso, Samtleben, Trigiante '16]

✱ These supergravities believed to be unique for 30 years…

Electric-magnetic deformations

- Uniqueness historically inherited from the connection with NH geometries of branes and SCFT's

Type IIB : $AdS_5 \times S^5$ (D3-brane ~ N = 4 SYM in 4d) [Maldacena '97]

M-theory : $AdS_4 \times S^7$ (M2-brane ~ ABJM theory in 3d)

[Aharony, Bergman, Jafferis, Maldacena '08]

- N=8 supergravity in 4D admits a deformation parameter c yielding inequivalent theories. It is an electric/magnetic deformation

$$\boxed{D = \partial - g\,(A^{\text{elec}} - c\,\tilde{A}_{\text{mag}})}$$

g = 4D gauge coupling

c = deformation param.

[Dall'Agata, Inverso, Trigiante '12]

- There are two generic situations :

1) Family of $SO(8)_c$ theories : $c = [0, \sqrt{2} - 1]$ is a continuous parameter

2) Family of $CSO(p,q,r)_c$ theories : $c = 0$ or 1 is an (on/off) parameter

[Dall'Agata, Inverso, Marrani '14]

The questions arise:

- Does such an electric/magnetic deformation of 4D maximal supergravity enjoy a string/M-theory origin, or is it just a 4D feature ?

- For deformed 4D supergravities with supersymmetric AdS_4 vacua, are these AdS_4/CFT_3-dual to any identifiable 3d CFT ?

M-theory

electric/magnetic deformation	higher-dimensional origin	Holographic $\mathrm{AdS}_4/\mathrm{CFT}_3$ dual ?
✓	?	?

Obstruction for $\mathrm{SO}(8)_c$, *cf*. [de Wit, Nicolai ’13]

[Lee, Strickland-Constable, Waldram ’15]

(massive) Type IIA

electric/magnetic deformation

✓

higher-dimensional origin

✓

Holographic AdS$_4$/CFT$_3$ dual ?

✓

$$\boxed{g\,c \;=\; \hat{F}_{(0)} \;=\; k/(2\pi\ell_s)}$$

[AG, Jafferis, Varela ’15]

[AG, Varela ’15]

[AG, Tarrío, Varela ’16, ’19]

[AG, Tarrío & AG ’17]

Type IIB

electric/magnetic deformation

✓

higher-dimensional origin

✓

Holographic AdS$_4$/CFT$_3$ dual ?

✓

[this talk]

$[\, SO(1,1) \times SO(6) \,] \ltimes R^{12}$ supergravity

- Higher-dimensional origin as type IIB on $\mathbb{R}$ (or S^1) $\times$ S^5

[Dall'Agata, Inverso '11]
[Inverso, Samtleben, Trigiante '16]

- New AdS_4 vacuum with $N=4$ & $SO(4)_R$ symmetry

[Gallerati, Samtleben, Trigiante '14]

- Holographic expectation: N=4 S-fold CFT_3

[***J*-fold** = S-fold with **hyperbolic** monodromy J]

[Hull, (Çatal-Özer) '04, ('03)]
[Gaiotto, Witten '08]
[Assel, Tomasiello '18 (**N = 3 , 4**)]
[Garozzo, Lo Monaco, Mekareeya '18 '19]

➡ Singular Janus solutions : $AdS_4 \times \mathbb{R} \times M_5$

$$M_5 = S^2 \times S^2 \times I$$

[Bak, Gutperle, Hirano '03 (**N = 0**)]
[Clark, Freedman, Karch, Schnabl '04]
[D'Hoker, Ester, Gutperle '07, '07 (**N = 4**)]
[Inverso, Samtleben, Trigiante '16]
[Bobev, Gautason, Pilch, Suh, van Muiden '19, '20 (in 5D)]

- Superconformal Janus interfaces in N=4 SYM_4

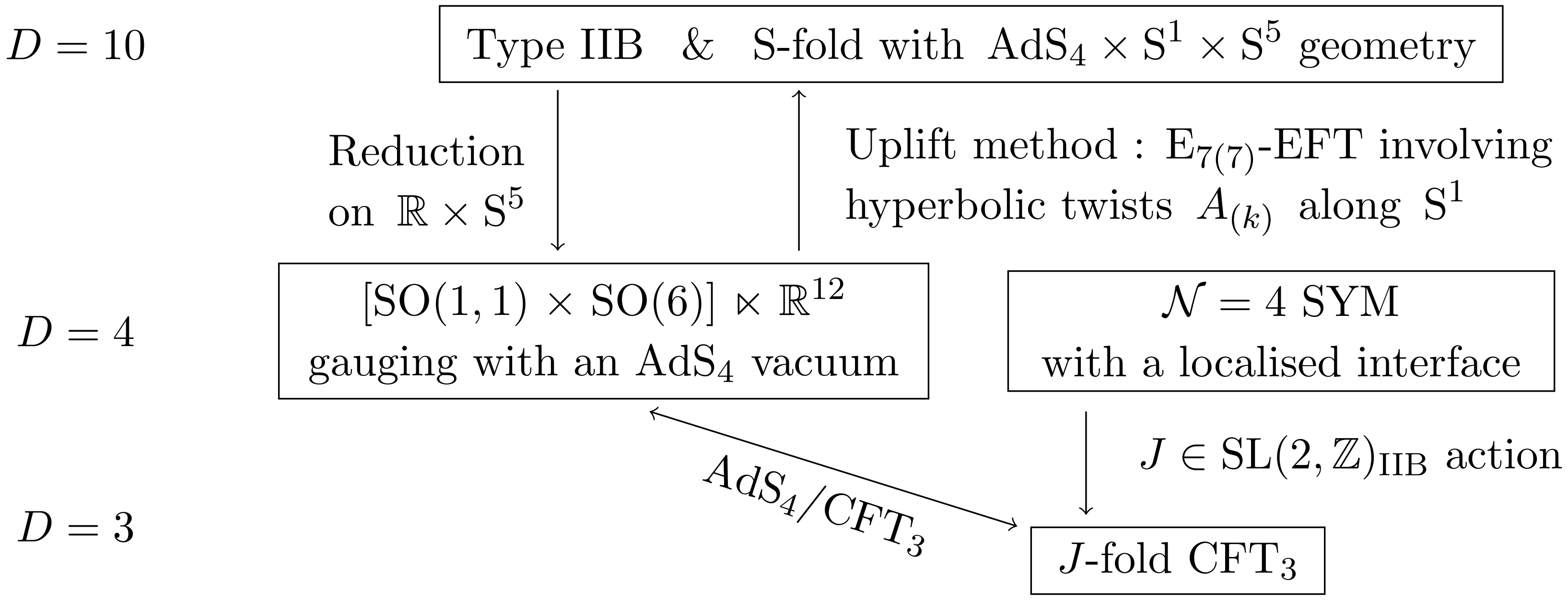
[D'Hoker, Ester, Gutperle '06 (**N = 1 , 2 , 4**)]

N=4 N=2 & SU(2) N=1 & SU(3) N=0 & SO(6)

Question : Holographic duals for N = 0, 1, 2 S-fold CFT_3 ? **[largest flavour symmetry]**

[see also Bobev, Gautason, Pilch, Suh, van Muiden '19, '20]

The picture…



$D = 10$

Type IIB & S-fold with $\mathrm{AdS}_4 \times \mathrm{S}^1 \times \mathrm{S}^5$ geometry

Reduction on $\mathbb{R} \times \mathrm{S}^5$

Uplift method : $\mathrm{E}_{7(7)}$-EFT involving hyperbolic twists $A_{(k)}$ along S^1

$D = 4$

$[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes \mathbb{R}^{12}$ gauging with an AdS_4 vacuum

$\mathcal{N} = 4$ SYM with a localised interface

$J \in \mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}$ action

$\mathrm{AdS}_4/\mathrm{CFT}_3$

$D = 3$

J-fold CFT_3

S-folds in 4D

A truncation : $\mathbb{Z}_2^3$ invariant sector

[AG, Sterckx, Trigiante '20]

- Truncation : Retaining the fields and couplings which are invariant (singlets) under a $\mathbb{Z}_2^3$ action ⇨ N = 1 supergravity coupled to 7 chiral multiplets z_i

$$\boxed{z_i = -\chi_i + i\, y_i} \qquad (y_i > 0)$$

[upper half-plane]

- The model :

$$K = -\sum_{i=1}^{7} \log[-i(z_i - \bar{z}_i)]$$

$$W = 2\, g \left[\, z_1 z_5 z_6 + z_2 z_4 z_6 + z_3 z_4 z_5 + (z_1 z_4 + z_2 z_5 + z_3 z_6)\, z_7 \,\right] + 2\, g\, c\, (1 - z_4 z_5 z_6 z_7)$$

[dyonic gauging]

- AdS₄ vacua :

N=4 & SO(4)$_R$ N=2 & SU(2) × U(1)$_R$ N=1 & SU(3) N=0 & SO(6)

⇨ Most symmetric AdS₄ vacua within **multi-parametric families** !!

N=0 family of AdS$_4$ vacua with U(1)^3 symmetry

[AG, Sterckx, Trigiante '20]

- Location : **[3 free parameters]**

$$z_{1,2,3} = c\left(-\chi_{1,2,3} + i\frac{1}{\sqrt{2}}\right) \qquad \text{and} \qquad z_4 = z_5 = z_6 = z_7 = i$$

- AdS$_4$ radius $L^2 = -3/V_0$ & scalar mass spectrum :

[BF unstable]

$$\boxed{V_0 = -2\sqrt{2}g^2c^{-1}}$$

$$\begin{aligned} m^2L^2 = \ & 6(\times 2)\,, \quad -3(\times 2)\,, \quad 0(\times 28)\,, \\ & -\frac{3}{4} + \frac{3}{2}\chi^2(\times 2)\,, \\ & -\frac{3}{4} + \frac{3}{2}\left(\chi - 2\chi_i\right)^2(\times 2) \qquad i = 1,2,3\,, \\ & -\frac{3}{4} + \frac{3}{2}\chi_i^2(\times 4) \qquad i = 1,2,3\,, \\ & -3 + 6\chi_i^2(\times 2) \qquad i = 1,2,3\,, \\ & -3 + \frac{3}{2}(\chi_i \pm \chi_j)^2(\times 2) \qquad i < j\,, \end{aligned}$$

- Flavour symmetry enhancements :

$$\boxed{\begin{array}{ccccccc} \mathrm{U}(1)^3 & \rightarrow & \mathrm{SU}(2)\times\mathrm{U}(1)^2 & \rightarrow & \mathrm{SU}(3)\times\mathrm{U}(1) & \rightarrow & \mathrm{SO}(6) \\ & & \chi_i = \chi_j & & \chi_1 = \chi_2 = \chi_3 & & \chi_{1,2,3} = 0 \end{array}}$$

N=1 family of AdS$_4$ vacua with U(1)^2 symmetry

[AG, Sterckx, Trigiante '20]

- Location : **[2 free parameters :** $\sum_{i=1}^{3} \chi_i = 0$ **]**

$$z_{1,2,3} = c \left(-\chi_{1,2,3} + i\frac{\sqrt{5}}{3} \right) \qquad \text{and} \qquad z_4 = z_5 = z_6 = z_7 = \frac{1}{\sqrt{6}}(1 + i\sqrt{5})$$

- AdS$_4$ radius $L^2 = -3/V_0$ & scalar mass spectrum :

$$\boxed{V_0 = -\frac{162}{25\sqrt{5}} g^2 c^{-1}}$$

$$\begin{aligned} m^2L^2 = {} & 0(\times 28)\,, \quad 4 \pm \sqrt{6}(\times 2)\,, \quad -2(\times 2)\,, \\ & -\frac{14}{9} + 5\chi_i^2 \pm \frac{1}{3}\sqrt{4 + 45\chi_i^2}(\times 2) \qquad i = 1,2,3\,, \\ & -\frac{14}{9} + \frac{5}{4}\chi_i^2 \pm \frac{1}{6}\sqrt{16 + 45\chi_i^2}(\times 2) \qquad i = 1,2,3\,, \\ & \frac{7}{9} + \frac{5}{4}\chi_i^2(\times 2) \qquad i = 1,2,3\,, \\ & -2 + \frac{5}{4}(\chi_i - \chi_j)^2(\times 2) \qquad i < j\,, \end{aligned}$$

- Flavour symmetry enhancements :

$$\boxed{\begin{array}{c} \mathbf{U(1)^2 \rightarrow SU(2) \times U(1) \rightarrow SU(3)} \\ \chi_i = \chi_j \qquad\qquad \chi_{1,2,3} = 0 \end{array}}$$

N=2 family of AdS$_4$ vacua with U(1) x U(1)$_R$ symmetry

[AG, Sterckx, Trigiante '20]

- Location :

[1 free parameter]

$$z_1 = -\bar{z}_3 = c\left(-\chi + i\frac{1}{\sqrt{2}}\right), \quad z_2 = ic, \quad z_4 = z_6 = i \quad \text{and} \quad z_5 = z_7 = \frac{1}{\sqrt{2}}(1+i)$$

- AdS$_4$ radius $L^2 = -3/V_0$ & scalar mass spectrum :

$$\boxed{V_0 = -3g^2c^{-1}\,,}$$

$$m^2L^2 = 0(\times 30)\,, \quad 3 \pm \sqrt{17}(\times 2)\,, \quad -2(\times 4)\,, \quad 2(\times 6)\,, \quad -2+4\chi^2(\times 6)$$
$$-1+4\chi^2 \pm \sqrt{16\chi^2+1}(\times 2)\,, \quad \chi^2 \pm \sqrt{\chi^2+2}(\times 8)\,,$$

- Flavour symmetry enhancement :

$$\boxed{\begin{array}{c}\mathbf{U(1)} \ \rightarrow \mathbf{SU(2)} \\ \chi = 0\end{array}}$$

N=4 AdS$_4$ vacuum with SO(4)$_R$ symmetry

[Gallerati, Samtleben, Trigiante '14]

[AG, Sterckx, Trigiante '20]

- Location :

$$z_1 = z_2 = z_3 = ic \qquad \text{and} \qquad z_4 = z_5 = z_6 = -\bar{z}_7 = \frac{1}{\sqrt{2}}(1+i)$$

- AdS$_4$ radius $L^2 = -3/V_0$ & scalar mass spectrum :

$$\boxed{V_0 = -3g^2c^{-1}\,;} \qquad\qquad m^2L^2 = 0(\times 48)\,, \quad 10(\times 1)\,, \quad 4(\times 10)\,, \quad -2(\times 11)$$

Next step : Uplift to type IIB on $\mathbb{R} \times \mathrm{S}^5$ using E$_{7(7)}$-EFT

S-folds in 10D

Generalised Scherk-Schwarz reductions of E7(7)-EFT

[Hohm, Samtleben '14]
[Baguet, Hohm, Samtleben '15]
[Inverso, Samtleben, Trigiante '16]

- SL(8) twist (geometry) :

$y^{i=1\dots5}$ (elec) , $\tilde{y}_1 = \sinh\eta$ (mag)

$$\rho \;=\; \mathring{\rho}(\tilde{y}_1)\;\hat{\rho}(y^i)$$

$$(U^{-1})_A{}^B = \left(\frac{\mathring{\rho}}{\hat{\rho}}\right)^{\frac{1}{2}} \begin{pmatrix} 1 & 0 & 0 & -\mathring{\rho}^{-2}\,c\,\tilde{y}_1 \\ 0 & \delta^{ij} + \hat{K}\,y^i\,y^j & -\lambda\,\hat{\rho}^2 y^i & 0 \\ 0 & -\lambda\,\hat{\rho}^2 y^j\,\hat{K} & \hat{\rho}^4 & 0 \\ -\mathring{\rho}^{-2}\,c\,\tilde{y}_1 & 0 & 0 & \mathring{\rho}^{-4}(1+\tilde{y}_1^2) \end{pmatrix}$$

- EFT fields = Twist × 4D fields :

$$g_{\mu\nu}(x,Y) \;=\; \rho^{-2}(Y)\,g_{\mu\nu}(x)$$

$$\mathcal{M}_{MN}(x,Y) \;=\; U_M{}^K(Y)\,U_N{}^L(Y)\,M_{KL}(x)$$

- Type IIB fields = EFT fields :

$$\begin{aligned} G^{mn} &= G^{1/2}\,\mathcal{M}^{mn} \\ \mathbb{B}_{mn}{}^{\alpha} &= G^{1/2}\,G_{mp}\,\epsilon^{\alpha\beta}\,\mathcal{M}^{p}{}_{n\beta} \\ m_{\alpha\beta} &= \tfrac{1}{6}\,G\,\left(\mathcal{M}^{mn}\,\mathcal{M}_{m\alpha\,n\beta} + \mathcal{M}^{m}{}_{k\alpha}\,\mathcal{M}^{k}{}_{m\beta}\right) \\ C_{klmn} &= -\tfrac{1}{4}\,G^{1/2}\,G_{k\rho}\,\mathcal{M}^{\rho}{}_{lmn} + \tfrac{3}{8}\,\epsilon_{\alpha\beta}\,\mathbb{B}_{k[l}{}^{\alpha}\,\mathbb{B}_{mn]}{}^{\beta} \end{aligned}$$

N=0 & SO(6) solution

[AG, Sterckx '19]

Flavour : SO(6) ~ S5

$$\boxed{\begin{aligned} ds^2_{10} &= \frac{1}{\sqrt{2}}\, ds^2_{\text{AdS}_4} + \frac{1}{2}\, d\eta^2 + d\mathring{s}^2_{\text{S}^5} \\ \widetilde{F}_5 &= 4\,(1+\star)\,\text{vol}_5 \\ \mathbb{B}^\alpha &= A^\alpha{}_\beta\, \mathfrak{b}^\beta = 0 \\ m_{\alpha\beta} &= (A^{-t})_\alpha{}^\gamma\, \mathfrak{m}_{\gamma\delta}\, (A^{-1})^\delta{}_\beta \end{aligned}}$$

No untwisted limit !!
(genuinely dyonic)

with $\mathfrak{m}_{\gamma\delta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $A^\alpha{}_\beta \equiv \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix}$

[$\mathbb{R} \rightarrow \text{S}^1$ ⇔ (hyperbolic) SO(1,1)-twist = monodromy]

$$\boxed{\mathfrak{M}_{\text{S}^1} = A^{-1}(\eta)A(\eta+T) = \begin{pmatrix} \cosh T & \sinh T \\ \sinh T & \cosh T \end{pmatrix}}$$

[Bak, Gutperle, Hirano '03]

unstable !!

[Inverso, Samtleben, Trigiante '16]

N=1 & SU(3) solution

[Lüst, Tsimpis '09 (local form)]
[AG, Sterckx '19]

Flavour : SU(3) ~ CP²

$$ds^2_{10} = \frac{3\sqrt{6}}{10}\, ds^2_{\mathrm{AdS}_4} + \frac{1}{3}\sqrt{\frac{10}{3}}\, d\eta^2 + \left[\sqrt{\frac{5}{6}}\, ds^2_{\mathbb{CP}^2} + \sqrt{\frac{6}{5}}\, \boldsymbol{\eta}^2\right]$$

$$\widetilde{F}_5 = 3\left(\frac{6}{5}\right)^{\frac{3}{4}} (1+\star)\, \mathrm{vol}_5$$

$$\mathbb{B}^{\alpha} = A^{\alpha}{}_{\beta}\, \mathfrak{b}^{\beta} = A^{\alpha}{}_{\beta}\left(-\tfrac{5}{12}\, H^{\beta}{}_{\gamma}(z_i)\, \boldsymbol{\Omega}^{\gamma}\right)$$

$$m_{\alpha\beta} = (A^{-t})_{\alpha}{}^{\gamma}\, \mathfrak{m}_{\gamma\delta}\, (A^{-1})^{\delta}{}_{\beta}$$

[charged under $\mathrm{U}(1)_\eta$]

with $\mathfrak{m}_{\gamma\delta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $A^{\alpha}{}_{\beta} \equiv \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix}$

[(hyperbolic) SO(1,1)-twist]

No untwisted limit !!
(genuinely dyonic)

N=2 & SU(2) x U(1)$_R$ solution

[AG, Sterckx, Trigiante '20]

Flavour : SU(2) ~ S^2

(genuinely dyonic)

$$ds^2 = \tfrac{1}{2}\,\Delta^{-1}\left[ds^2_{\text{AdS}_4} + d\eta^2 + d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\left(\sigma_2^2 + 8\,\Delta^4\,\left(\sigma_1^2 + \sigma_3^2\right)\right)\right]$$

$$\Delta^{-4} = 6 - 2\cos(2\theta)$$

$$\widetilde{F}_5 = 4\,\Delta^4\,\sin\theta\,\cos^3\theta\,\left(1+\star\right)\Big[3\,d\theta\wedge d\phi\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3 \\ - d\eta\wedge\left(\cos(2\phi)\,d\theta - \tfrac{1}{2}\sin(2\theta)\,\sin(2\phi)\,d\phi\right)\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3\Big]$$

$$\mathbb{B}^\alpha = A^\alpha{}_\beta\,\mathfrak{b}^\beta \quad \text{with} \quad \begin{aligned} \mathfrak{b}_1 &= \tfrac{1}{\sqrt{2}}\cos\theta\left[\left(\sin\phi\, d\theta + \tfrac{1}{2}\,\sin(2\theta)\,d(\sin\phi)\right)\wedge\sigma_2 + \sin\phi\,\frac{4\,\sin(2\theta)}{6-2\cos(2\theta)}\,\sigma_1\wedge\sigma_3\right] \\ \mathfrak{b}_2 &= \tfrac{1}{\sqrt{2}}\cos\theta\left[\left(\cos\phi\, d\theta + \tfrac{1}{2}\,\sin(2\theta)\,d(\cos\phi)\right)\wedge\sigma_2 + \cos\phi\,\frac{4\,\sin(2\theta)}{6-2\cos(2\theta)}\,\sigma_1\wedge\sigma_3\right] \end{aligned}$$

$$m_{\alpha\beta} = (A^{-t})_\alpha{}^\gamma\,\mathfrak{m}_{\gamma\delta}\,(A^{-1})^\delta{}_\beta \qquad \text{with} \qquad \mathfrak{m}_{\gamma\delta} = 2\,\Delta^2\begin{pmatrix} 1+\sin^2\theta\,\cos^2\phi & -\tfrac{1}{2}\sin^2(\theta)\sin(2\phi) \\ -\tfrac{1}{2}\sin^2(\theta)\sin(2\phi) & 1+\sin^2\theta\,\sin^2\phi \end{pmatrix}$$

[AG, Sterckx ’21]

Holographic RG flows on the D3-brane

RG flows = (flat sliced) domain-walls :

$$ds^2_{\mathrm{DW}_4} = e^{2A(z)}\,\eta_{\alpha\beta}\,dx^\alpha dx^\beta + dz^2$$

BPS flow equations :

$$\boxed{\partial_z A = \mp\,|\mathcal{W}| \qquad \text{and} \qquad \partial_z \Sigma^I = \pm\, K^{IJ}\,\partial_{\Sigma^J}|\mathcal{W}|}$$

with $\mathcal{W} = e^{\frac{K}{2}}\,W = m_{3/2}$

D3-brane and N=4 super Yang-Mills

- $c = 0$ **:** D3-brane ⇹ AdS$_5$ ⇹ DW$_4$

[DW$_4$ metric] **[AdS$_5$ metric]**

$$\begin{aligned} ds^2_{10} &= \tfrac{1}{2}\, g^2\, \Delta^{-1}(z_i)\, \left(e^{2A(z)}\, \eta_{\alpha\beta}\, dx^\alpha dx^\beta + dz^2 \right) + \Delta^2(z_i)\, d\eta^2 + d\mathring{s}^2_{\text{S}^5} \ , \\ \widetilde{F}_5 &= 4\, g\, (1+\star)\, \text{vol}_5 \qquad , \qquad \mathbb{B}^\alpha = 0 \ , \\ m_{\alpha\beta} &= \begin{pmatrix} e^{-\Phi_0} & 0 \\ 0 & e^{\Phi_0} \end{pmatrix} . \end{aligned}$$

Note: $\Delta(z_i) = |z_{4,5,6,7}|^{-1}\ \text{Im} z_{4,5,6,7}\ \text{Im} z_{1,2,3}$

DW$_4$ domain-wall
(SYM$_4$)

$$z_{1,2,3} = -\chi^{(0)}_{1,2,3} + i\, \frac{(g\, z)^2}{8} \quad , \quad z_4 = z_5 = z_6 = z_7 = i\, e^{-\frac{1}{2}\Phi_0} \quad \text{and} \quad e^A = (g\, z)^3$$

[3 free parameters]

BPS equations require

$$\sum_{i=1}^{3} \text{Re} z_i = -\sum_{i=1}^{3} \chi^{(0)}_i = 0$$

Deformed D3-brane

- **$c \neq 0$:** Expansion of BPS equations in powers of $\dfrac{c}{(gz)^2}$

[quadratic order]

$$\begin{aligned}
z_{1,2,3} &= \tfrac{1}{3}\, c \sinh\Phi_0 \left(1 - 384 \cosh^2\Phi_0 \, \frac{c^2}{(g\,z)^4} \, \log(gz)\right) \\
&+ i\, \frac{(g\,z)^2}{8} \left(1 + 32 \cosh^2\Phi_0 \, \frac{c^2}{(g\,z)^4}\right) \,, \\
z_{4,5,6,7} &= 4\, e^{-\frac{1}{2}\Phi_0} \cosh\Phi_0 \, \frac{c}{(gz)^2} \left(1 + 64 \left(1 - 3\cosh(2\Phi_0)\right) \frac{c^2}{(g\,z)^4} \log(gz)\right) \\
&+ i\, e^{-\frac{1}{2}\Phi_0} \left(1 - 8\, \left(\cosh^2\Phi_0 - 2\sinh(2\,\Phi_0)\right) \frac{c^2}{(g\,z)^4}\right) \,, \\
e^{A} &= (g\,z)^3 \left(1 + 16 \cosh^2\Phi_0 \, \frac{c^2}{(g\,z)^4}\right) \,.
\end{aligned}$$

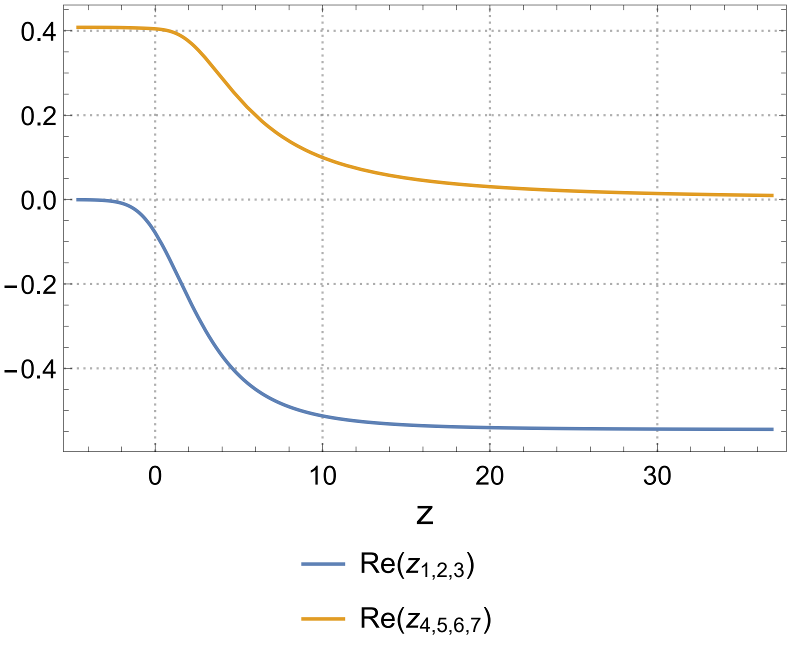
… so this time

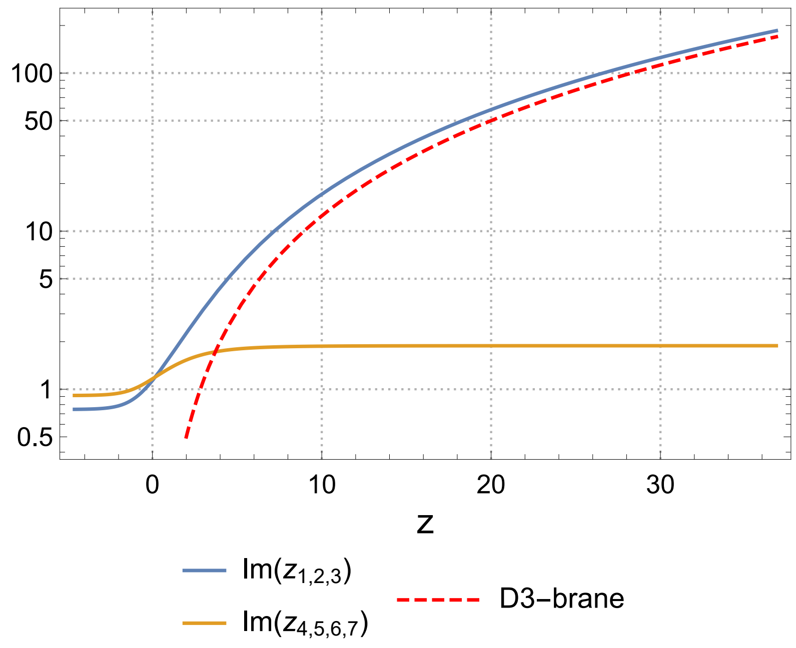
$$\sum_{i=1}^{3} \mathrm{Re} z_i = c \sinh\Phi_0 \left(1 - 384 \cosh^2\Phi_0 \, \frac{c^2}{(g\,z)^4} \, \log(gz)\right)$$

(IR) N=1 & SU(3) *J*-fold CFT$_3$ ⟷ SYM$_4$ (UV)

$$\begin{array}{rcccccccccccc}
m^2L^2 & = & -\frac{20}{9}\,(\times 3) & , & -2\,(\times 2) & , & -\frac{8}{9}\,(\times 3) & ; & 0\,(\times 2) & ; & 4-\sqrt{6}\,(\times 2) & , & 4+\sqrt{6}\,(\times 2) \\
\Delta_{+} & = & \mathbf{\frac{5}{3}}\,(\times 3) & , & \mathbf{2}\,(\times 2) & , & \frac{8}{3} & ; & 3 & ; & \mathbf{1+\sqrt{6}}\,(\times 2) & , & 2+\sqrt{6} \\
\Delta_{-} & = & \frac{4}{3} & , & 1 & , & \mathbf{\frac{1}{3}}\,(\times 3) & ; & \mathbf{0}\,(\times 2) & ; & 2-\sqrt{6} & , & \mathbf{1-\sqrt{6}}\,(\times 2)
\end{array}$$

[2 irrelevant operators]

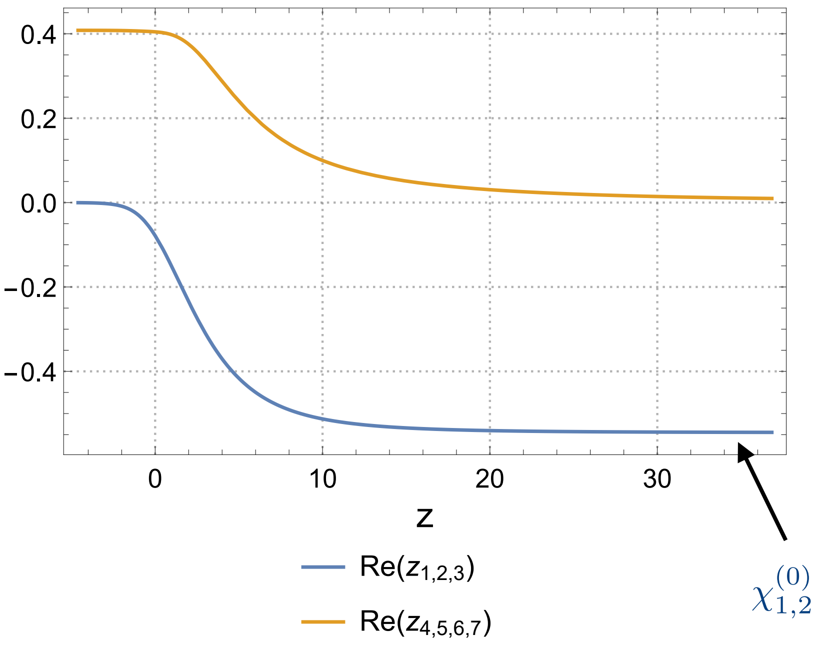




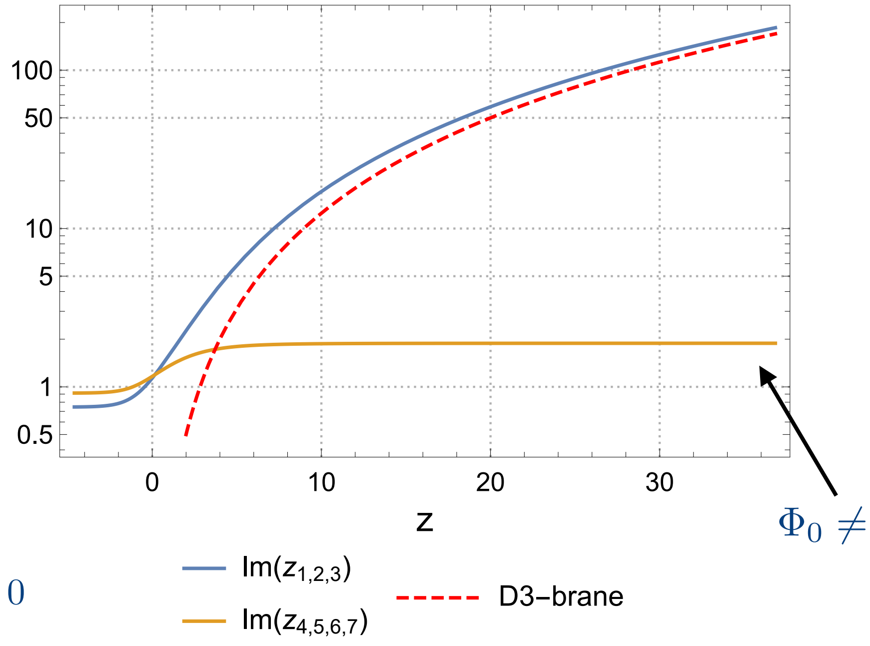
(IR) N=1 & SU(3) *J*-fold CFT$_3$ ⟺ SYM$_4$ (UV)

$$\begin{array}{rcccccccccccc}
m^2L^2 & = & -\frac{20}{9}\,(\times 3) & , & -2\,(\times 2) & , & -\frac{8}{9}\,(\times 3) & ; & 0\,(\times 2) & ; & 4-\sqrt{6}\,(\times 2) & , & 4+\sqrt{6}\,(\times 2) \\
\Delta_+ & = & \mathbf{\frac{5}{3}}\,(\times 3) & , & \mathbf{2}\,(\times 2) & , & \frac{8}{3} & ; & 3 & ; & \mathbf{1+\sqrt{6}}\,(\times 2) & , & 2+\sqrt{6} \\
\Delta_- & = & \frac{4}{3} & , & 1 & , & \mathbf{\frac{1}{3}}\,(\times 3) & ; & \mathbf{0}\,(\times 2) & ; & 2-\sqrt{6} & , & \mathbf{1-\sqrt{6}}\,(\times 2)
\end{array}$$

[2 irrelevant operators]



$\chi^{(0)}_{1,2,3} \neq 0$



$\Phi_0 \neq 0$

10D flows

Geometry

axion here !!

$$\boldsymbol{\eta} = d\beta + \mathrm{Re}z_{1,2,3}\, d\eta + \boldsymbol{A}_1$$

$$ds^2_{10} = \tfrac{1}{2}\,\Delta^{-1}\,\left(ds^2_{\mathrm{DW}_4} + 2\,(gc)^{-2}\,\Delta\,H(z_i)\,d\eta^2\,\right) + g^{-2}\,F(z_i)\,\left[ds^2_{\mathbb{CP}_2} + F(z_i)^{-2}\,\boldsymbol{\eta}^2\right]$$

with $F(z_i) = |z_{4,5,6,7}|^{-1}\ \mathrm{Im}z_{4,5,6,7}$ $\qquad H(z_i) = F(z_i)^{-1}\ (\mathrm{Im}z_{1,2,3})^2$ $\qquad \Delta = F(z_i)\ \mathrm{Im}z_{1,2,3}$

$$\mathbb{B}^\alpha = A^\alpha{}_\beta\,\mathfrak{b}^\beta \qquad \text{with} \qquad \mathfrak{b}^2 + i\,|z_{4,5,6,7}|^2\,\mathfrak{b}^1 = -i\,g^{-2}\,\mathrm{Re}z_{4,5,6,7}\ \boldsymbol{\Omega}$$

$$m_{\alpha\beta} = (A^{-t})_\alpha{}^\gamma\,\mathfrak{m}_{\gamma\delta}\,(A^{-1})^\delta{}_\beta \qquad \text{with} \qquad \mathfrak{m}_{\gamma\delta} = \begin{pmatrix} |z_{4,5,6,7}|^2 & 0 \\ 0 & |z_{4,5,6,7}|^{-2} \end{pmatrix}$$

$$\begin{aligned}
\widetilde{F}_5 \;=\;& g\,(1+\star)\Big[\Big(4 - 6\,(1 - F(z_i)^2)\Big)\,\mathrm{vol}_{\mathbb{CP}_2} \wedge (\boldsymbol{\eta} - \mathrm{Re}z_{1,2,3}\,d\eta) \\
&+ \Big(4\,\mathrm{Re}z_{1,2,3} + (\mathrm{Re}z_{4,5,6,7})^2\ (1 - |z_{4,5,6,7}|^{-4})\Big)\,\mathrm{vol}_{\mathbb{CP}_2} \wedge d\eta \\
&- d\mathrm{Re}z_{1,2,3} \wedge d\eta \wedge \boldsymbol{J} \wedge (\boldsymbol{\eta} - \mathrm{Re}z_{1,2,3}\,d\eta)\Big]\,.
\end{aligned}$$

Note: $\theta_\gamma{}^\beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

10D flows

Geometry

$$\boldsymbol{\eta} = d\beta + \mathrm{Re}z_{1,2,3}\, d\eta + \boldsymbol{A}_1$$

$$ds^2_{10} = \tfrac{1}{2}\,\Delta^{-1}\,\left(ds^2_{\mathrm{DW}_4} + 2\,(gc)^{-2}\,\Delta\, H(z_i)\, d\eta^2\,\right) + g^{-2}\, F(z_i)\,\left[ds^2_{\mathbb{CP}_2} + F(z_i)^{-2}\,\boldsymbol{\eta}^2\right]$$

with $F(z_i) = |z_{4,5,6,7}|^{-1}\ \mathrm{Im}z_{4,5,6,7}$ $\quad H(z_i) = F(z_i)^{-1}\ (\mathrm{Im}z_{1,2,3})^2$ $\quad \Delta = F(z_i)\ \mathrm{Im}z_{1,2,3}$

$$\mathbb{H}^\alpha = A^\alpha{}_\beta\,\left(d\eta\wedge\mathfrak{b}^\gamma\,\theta_\gamma{}^\beta + d\mathfrak{b}^\beta\right) \qquad \text{with} \qquad \mathfrak{b}^2 + i\,|z_{4,5,6,7}|^2\,\mathfrak{b}^1 = -i\,g^{-2}\,\mathrm{Re}z_{4,5,6,7}\ \boldsymbol{\Omega}$$

$$dm_{\alpha\beta} = -(|z_{4,5,6,7}|^2 + |z_{4,5,6,7}|^{-2})\,(A^{-t}\,\theta\,A^{-1})_{\alpha\beta}\ d\eta + \dots$$

$$\begin{aligned}\widetilde{F}_5 \ &= \ g\,(1+\star)\Big[\Big(4-6\,(1-F(z_i)^2)\Big)\,\mathrm{vol}_{\mathbb{CP}_2}\wedge(\boldsymbol{\eta}-\mathrm{Re}z_{1,2,3}\,d\eta)\\ &+ \ \Big(4\,\mathrm{Re}z_{1,2,3} + (\mathrm{Re}z_{4,5,6,7})^2\ (1-|z_{4,5,6,7}|^{-4})\Big)\,\mathrm{vol}_{\mathbb{CP}_2}\wedge d\eta\\ &- \ d\mathrm{Re}z_{1,2,3}\wedge d\eta\wedge\boldsymbol{J}\wedge(\boldsymbol{\eta}-\mathrm{Re}z_{1,2,3}\,d\eta)\Big]\ .\end{aligned}$$

[anisotropy in SYM$_4$]

[see Mateos, Trancanelli ’11, Hoyos, Jokela, Penín, Ramallo ’20]

Axions in 10D

SU(3) symmetry : $\boxed{\beta' = \beta - \chi^{(0)}_{1,2,3}\,\eta}$ [= coordinate redefinition (locally)]

General case : $\boxed{\theta'_1 = \theta_1 + g\,\chi^{(0)}_1\,\eta \quad , \quad \theta'_2 = \theta_2 + g\,\chi^{(0)}_2\,\eta \quad , \quad \theta'_3 = \theta_3 - g\,\chi^{(0)}_3\,\eta}$

S^5 metric : $d\mathring{s}^2_{\mathrm{S}^5} = d\alpha^2 + \cos^2\alpha\, d\theta_1^2 + \sin^2\alpha\,\left(d\beta^2 + \cos^2\beta\, d\theta_2^2 + \sin^2\beta\, d\theta_3^2\right)$

… so axions induce a fibration of S^5 over S^1 characterised by a **non-trivial monodromy**

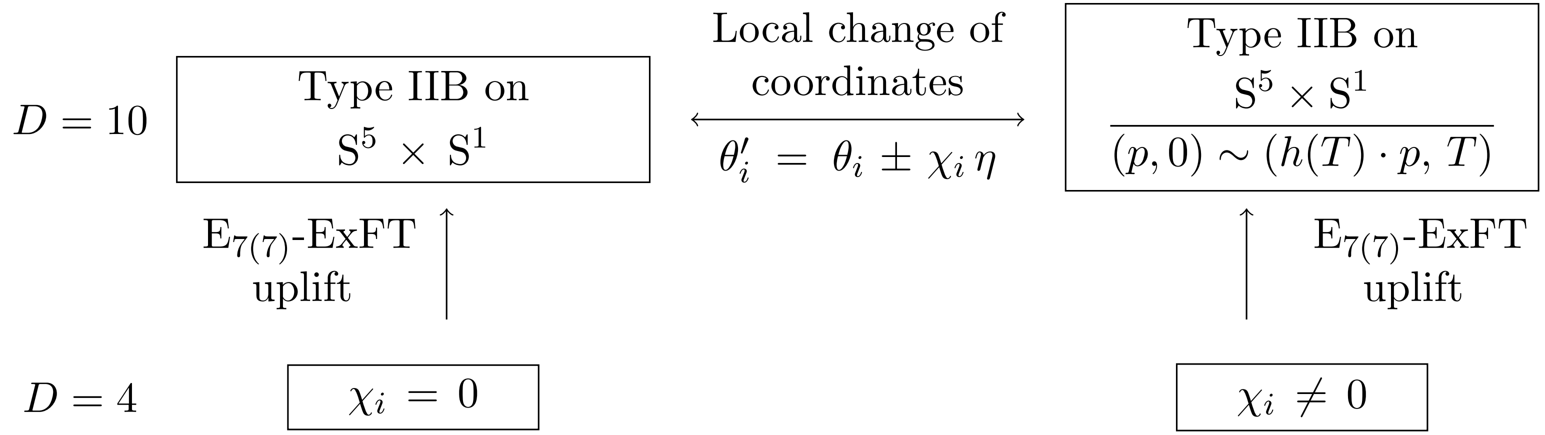
$$\boxed{h(T) = \begin{pmatrix} h_1 & & \\ & h_2 & \\ & & h_3 \end{pmatrix} \qquad \text{with} \qquad h_i = \exp\left(i\,\chi^{(0)}_i\,\sigma_2\,T\right) \in \mathrm{SO}(2)}$$

symmetry

$$\chi_i \to \chi_i + n_i\,\frac{2\pi}{T}$$

Global breaking of symmetries of S^5 ➡ matching patterns of symmetry breaking @ AdS$_4$ vacua !

- KK spectrum at the N=2 S-fold periodic under $\chi \to \chi + 2\pi/T$ [Giambrone, Malek, Samtleben, Trigiante ’21]



$D=10$

Type IIB on $S^5 \times S^1$

Local change of coordinates

$\theta_i' = \theta_i \pm \chi_i\,\eta$

Type IIB on $\dfrac{S^5 \times S^1}{(p,0)\sim(h(T)\cdot p,\,T)}$

$E_{7(7)}$-ExFT uplift

$E_{7(7)}$-ExFT uplift

$D=4$

$\boxed{\chi_i = 0}$

$\boxed{\chi_i \neq 0}$

Ex : N=1 S-folds:

[CP² factor]

$$\phi \to \phi - (\chi_1 - \chi_2)\,\eta$$

$$\psi \to \psi - (\chi_1 + \chi_2 - 2\,\chi_3)\,\eta$$

[U(1) fiber]

$$\beta \to \beta - \chi_3\,\eta$$

$$\sum_{i=1}^{3} \chi_i = 0$$

[SU(3) monodromy to break SU(3) symmetry]

Ex : D3-brane:

$$\theta_i \to \theta_i + \chi_i^{(0)}\ \eta$$

$$\sum_{i=1}^{3} \chi_i^{(0)} \neq 0$$

[SU(4) monodromy to break SU(4) symmetry]

4D *vs* 5D

Relevant branching : $\mathrm{SU}(8) \supset \mathrm{USp}(8) \supset \mathrm{SU}(4)\times\mathrm{U}(1)_S$

4D **5D**

$\mathrm{SU}(4)\times\mathrm{U}(1)_S$	$\supset$	$\mathrm{SU}(3)\times\mathrm{U}(1)\times\mathrm{U}(1)_S$
$\mathbf{1}_4 + \mathbf{1}_{-4}$	$\rightarrow$	$\mathbf{1}_{(0,4)} + \mathbf{1}_{(0,-4)}$
$\mathbf{10}_{-2} + \overline{\mathbf{10}}_2$	$\rightarrow$	$\left(\bar{\mathbf{6}}_{(2,-2)} + \mathbf{3}_{(-2,-2)} + \mathbf{1}_{(-6,-2)}\right) + \left(\mathbf{6}_{(-2,2)} + \bar{\mathbf{3}}_{(2,2)} + \mathbf{1}_{(6,2)}\right)$
$\mathbf{20'}_0$	$\rightarrow$	$\bar{\mathbf{6}}_{(-4,0)} + \mathbf{8}_{(0,0)} + \mathbf{6}_{(4,0)}$
$\mathbf{15}_0$	$\rightarrow$	$\mathbf{8}_{(0,0)} + \mathbf{3}_{(4,0)} + \bar{\mathbf{3}}_{(-4,0)} + \mathbf{1}_{(0,0)}$
$\mathbf{6}_2 + \mathbf{6}_{-2}$	$\rightarrow$	$\left(\mathbf{3}_{(-2,2)} + \bar{\mathbf{3}}_{(2,2)}\right) + \left(\mathbf{3}_{(-2,-2)} + \bar{\mathbf{3}}_{(2,-2)}\right)$
$\mathbf{1}_0$	$\rightarrow$	$\mathbf{1}_{(0,0)}$

5D scalars
$\mathrm{E}_{6(6)}/\mathrm{USp}(8)$

5D vectors/tensors

5D metric

4D scalars
$\mathrm{E}_{7(7)}/\mathrm{SU}(8)$

4D *vs* 5D

Relevant branching : $\mathrm{SU}(8) \supset \mathrm{USp}(8) \supset \mathrm{SU}(4) \times \mathrm{U}(1)_S$

4D **5D**

$\mathrm{SU}(4) \times \mathrm{U}(1)_S$	$\supset$	$\mathrm{SU}(3) \times \mathrm{U}(1) \times \mathrm{U}(1)_S$	
$\mathbf{1}_4 + \mathbf{1}_{-4}$	$\rightarrow$	$\mathbf{1}_{(0,4)} + \mathbf{1}_{(0,-4)}$	
$\mathbf{10}_{-2} + \overline{\mathbf{10}}_2$	$\rightarrow$	$\left(\bar{\mathbf{6}}_{(2,-2)} + \mathbf{3}_{(-2,-2)} + \mathbf{1}_{(-6,-2)}\right) + \left(\mathbf{6}_{(-2,2)} + \bar{\mathbf{3}}_{(2,2)} + \mathbf{1}_{(6,2)}\right)$	**5D scalars** $\mathrm{E}_{6(6)}/\mathrm{USp}(8)$
$\mathbf{20'}_0$	$\rightarrow$	$\bar{\mathbf{6}}_{(-4,0)} + \mathbf{8}_{(0,0)} + \mathbf{6}_{(4,0)}$	
$\mathbf{15}_0$	$\rightarrow$	$\mathbf{8}_{(0,0)} + \mathbf{3}_{(4,0)} + \bar{\mathbf{3}}_{(-4,0)} + \overset{\chi^{(0)}_{1,2,3}}{\mathbf{1}_{(0,0)}}$	**5D vectors/tensors**
$\mathbf{6}_2 + \mathbf{6}_{-2}$	$\rightarrow$	$\left(\mathbf{3}_{(-2,2)} + \bar{\mathbf{3}}_{(2,2)}\right) + \left(\mathbf{3}_{(-2,-2)} + \bar{\mathbf{3}}_{(2,-2)}\right)$	
$\mathbf{1}_0$	$\rightarrow$	$\mathbf{1}_{(0,0)}$	**5D metric**

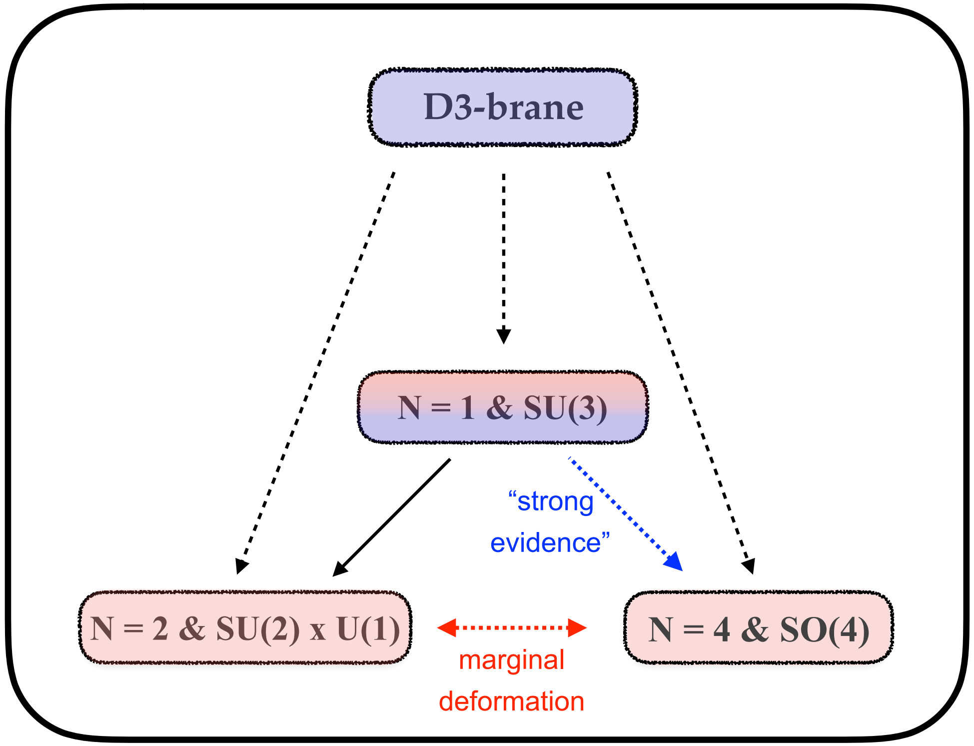
4D scalars
$\mathrm{E}_{7(7)}/\mathrm{SU}(8)$

[coupling N = 4 SYM$_4$ to off-shell conformal supergravity]

⇒ SU(3)-invariant one-form V_μ deformation of N=4 SYM$_4$

[Festuccia, Seiberg '11] [Maxfield '16]

Concluding remarks…



[Bobev, Gautason, Pilch, Suh, van Muiden '19, '20]
[Giambrone, Malek, Samtleben, Trigiante '21]
[Arav, (Cheung), Gauntlett, Roberts, Rosen '21]
[Bobev, Gautason, van Muiden '21]

- New families of N=1,2 S-folds and RG flows
- Conformal manifold of 3d N=2 S-fold CFT's
- Global aspects of axions ? , 5D picture ? , KK spectra ? , Brane set-ups ? ….

Thank you !

Extra material

Singular Janus & S-fold interpretation

[Inverso, Samtleben, Trigiante '16]
[AG, Sterckx, (Trigiante) '19, ('20)]

- Singular Janus (linear dilaton) : $\Phi(\eta, y^i) = -2\,\eta + f(y^i)$ **[S^5 coordiantes y^i]**

- $\mathbb{R} \to \mathrm{S}^1 \Leftrightarrow$ **hyperbolic** monodromy : $\mathfrak{M}_{\mathrm{S}^1} = A^{-1}(\eta)A(\eta+T) = \begin{pmatrix} \cosh T & \sinh T \\ \sinh T & \cosh T \end{pmatrix}$

- Generalising the A-twist to a k-family ($k > 2$) :

$$A_{(k)} = A g(k) \qquad \text{with} \qquad g(k) = \begin{pmatrix} \frac{(k^2-4)^{\frac{1}{4}}}{\sqrt{2}} & 0 \\ \frac{k}{\sqrt{2}(k^2-4)^{\frac{1}{4}}} & \frac{\sqrt{2}}{(k^2-4)^{\frac{1}{4}}} \end{pmatrix}$$

Then

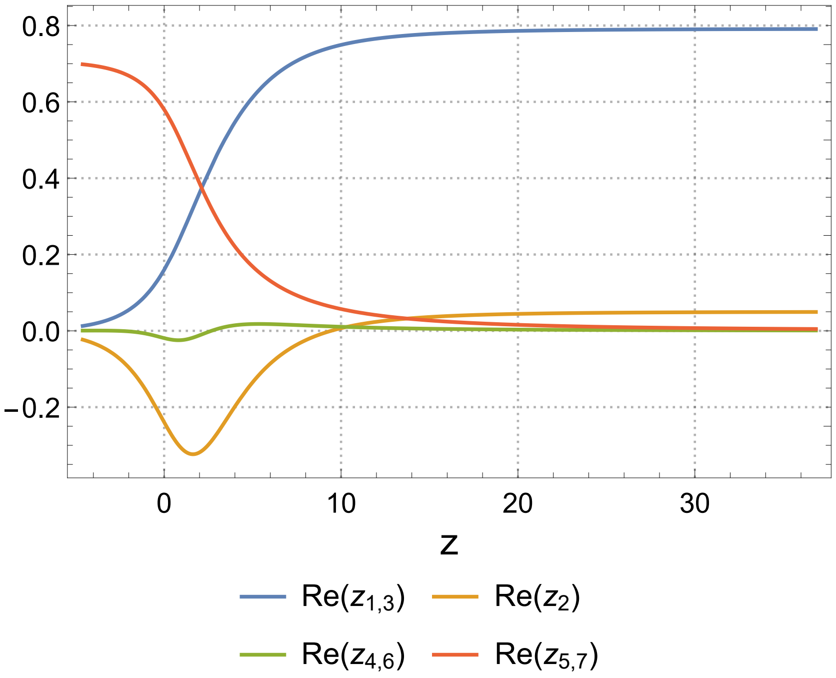
$$\boxed{\mathfrak{M}(k) = A_{(k)}^{-1}(\eta)\;\; A_{(k)}\big(\eta + T(k)\big) = \begin{pmatrix} k & 1 \\ -1 & 0 \end{pmatrix} = -\mathcal{S}\mathcal{T}^k \in \mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}}$$

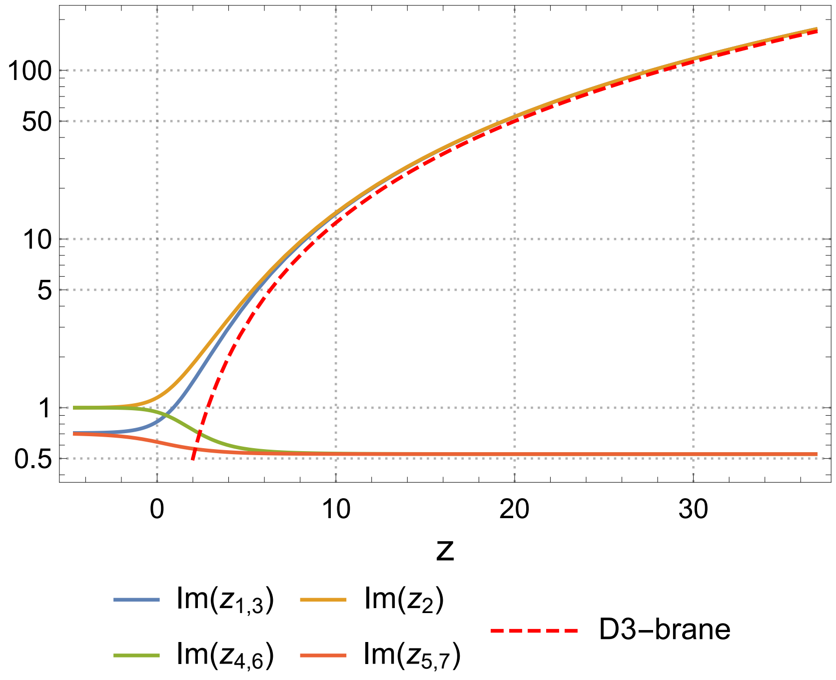
with $T(k) = \log(k + \sqrt{k^2-4}) - \log(2)$ and $\mathrm{Tr}\mathfrak{M}(k) > 2.$ **[hyperbolic]**

(IR) N=2 & SU(2) *J*-fold CFT$_3$ ⟺ SYM$_4$ (UV)

$$\begin{array}{rclcrcrcrcrcr}
m^2L^2 & = & -2\,(\times 4) & , & 3-\sqrt{17}\,(\times 2) & ; & 0\,(\times 2) & ; & 2\,(\times 4) & , & 3+\sqrt{17}\,(\times 2) \\
\Delta_+ & = & \mathbf{2}\,(\times 2) & , & \mathbf{\tfrac{1}{2}(1+\sqrt{17})}\,(\times 2) & ; & 3 & ; & \mathbf{\tfrac{1}{2}(3+\sqrt{17})}\,(\times 2) & , & \tfrac{1}{2}(5+\sqrt{17}) \\
\Delta_- & = & \mathbf{1}\,(\times 2) & , & \tfrac{1}{2}(5-\sqrt{17}) & ; & \mathbf{0}\,(\times 2) & ; & \mathbf{\tfrac{1}{2}(3-\sqrt{17})}\,(\times 2) & , & \mathbf{\tfrac{1}{2}(1-\sqrt{17})}\,(\times 2)
\end{array}$$

[4 irrelevant operators]





(IR) N=4 *J*-fold CFT$_3$ ⟷ SYM$_4$ (UV)

$$
\begin{array}{rcrcrcrcr}
m^2L^2 & = & -2\,(\times 3) & ; & 0\,(\times 6) & ; & 4\,(\times 4) & , & 10\,(\times 1) \\
\Delta_+ & = & \mathbf{2}\,(\times 3) & ; & \mathbf{3}\,(\times 3) & ; & \mathbf{4}\,(\times 1) & , & 5 \\
\Delta_- & = & 1 & ; & \mathbf{0}\,(\times 3) & ; & -\mathbf{1}\,(\times 3) & , & -\mathbf{2}\,(\times 1)
\end{array}
$$

[4 irrelevant operators]