Supersymmetric S-folds

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Based on 1907.04177 & 2002.03692 + work in progress

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Outlook

- Electric-magnetic duality in maximal supergravity
- S-folds in 4D
- S-folds in 10D
- Holographic RG-flows on the D3-brane
- Conclusions

Electric-magnetic duality in maximal supergravity

N=8 supergravity in 4D

- SUGRA : metric + 8 gravitini + 28 vectors + 56 dilatini + 70 scalars

  (s = 2) (s = 3/2) (s = 1) (s = 1/2) (s = 0)

Ungauged (abelian) supergravity: Reduction of M-theory on a *torus* T^7 down to 4D produces $N = 8$ supergravity with $G = U(1)^{28}$

[$E_{7(7)}$ symmetry]

[Cremmer, Julia '79]

Gauged (non-abelian) supergravity:

- M-theory on S^7 produces $N = 8$ supergravity with $G = SO(8)$ [de Wit, Nicolai '82]

- Type IIA on S^6 produces $N = 8$ supergravity with $G = ISO(7) = SO(7) \ltimes \mathbb{R}^7$ [Hull '84]

- Type IIB on $\mathbb{R} \times S^5$ produces $N = 8$ supergravity with $G = [\, SO(1,1) \times SO(6)\,] \ltimes \mathbb{R}^{12}$

[Inverso, Samtleben, Trigiante '16]

✱ These supergravities believed to be unique for 30 years…

Electric-magnetic deformations

- Uniqueness historically inherited from the connection with NH geometries of branes and SCFT's

Type IIB : $AdS_5 \times S^5$ (D3-brane ~ N = 4 SYM in 4d) [Maldacena '97]

M-theory : $AdS_4 \times S^7$ (M2-brane ~ ABJM theory in 3d)

[Aharony, Bergman, Jafferis, Maldacena '08]

- N=8 supergravity in 4D admits a deformation parameter c yielding inequivalent theories. It is an electric/magnetic deformation

$$D = \partial - g\,(A^{\text{elec}} - c\,\tilde{A}_{\text{mag}})$$

g = 4D gauge coupling
c = deformation param.

[Dall'Agata, Inverso, Trigiante '12]

- There are two generic situations :

1) Family of $SO(8)_c$ theories : $c = [0, \sqrt{2} - 1]$ is a continuous parameter

2) Family of $CSO(p,q,r)_c$ theories : $c = 0$ or 1 is an (on/off) parameter

[Dall'Agata, Inverso, Marrani '14]

The questions arise:

- Does such an electric/magnetic deformation of 4D maximal supergravity enjoy a string/M-theory origin, or is it just a 4D feature ?

- For deformed 4D supergravities with supersymmetric AdS_4 vacua, are these AdS_4/CFT_3-dual to any identifiable 3d CFT ?

M-theory

electric/magnetic deformation ✓

higher-dimensional origin ?

Holographic AdS_4/CFT_3 dual ?

Obstruction for $SO(8)_c$, *cf.* [de Wit, Nicolai '13]

[Lee, Strickland-Constable, Waldram '15]

(massive) Type IIA

electric/magnetic deformation ✓

higher-dimensional origin ✓

Holographic AdS_4/CFT_3 dual ? ✓

$$g\, c \;=\; \hat{F}_{(0)} \;=\; k/(2\pi\ell_s)$$

[AG, Jafferis, Varela '15]

[AG, Varela '15]

[AG, Tarrío, Varela '16, '19]

[AG, Tarrío & AG '17]

Type IIB

electric/magnetic deformation ✓

higher-dimensional origin ✓

Holographic AdS_4/CFT_3 dual ? ✓

[this talk]

$[\, SO(1,1) \times SO(6)\,] \ltimes R^{12}$ supergravity

- Higher-dimensional origin as type IIB on $\mathbb{R}$ (or S^1) $\times S^5$ — [Dall'Agata, Inverso '11] [Inverso, Samtleben, Trigiante '16]

- New AdS_4 vacuum with N=4 & $SO(4)_R$ symmetry — [Gallerati, Samtleben, Trigiante '14]

- Holographic expectation: N=4 S-fold CFT_3 (defects in SYM) — [Hull, (Çatal-Özer) '04, ('03)] [Gaiotto, Witten '08] [Assel, Tomasiello '18 (**N = 3 , 4**)] [Garozzo, Lo Monaco, Mekareeya '18 '19]

  [***J*-fold** = S-fold with **hyperbolic** monodromy J]

  [T[U(N)] + CS_k]

  ➡ Singular Janus-like solutions : $AdS_4 \times \mathbb{R} \times M_5$

  $M_5 = S^2 \times S^2 \times I$

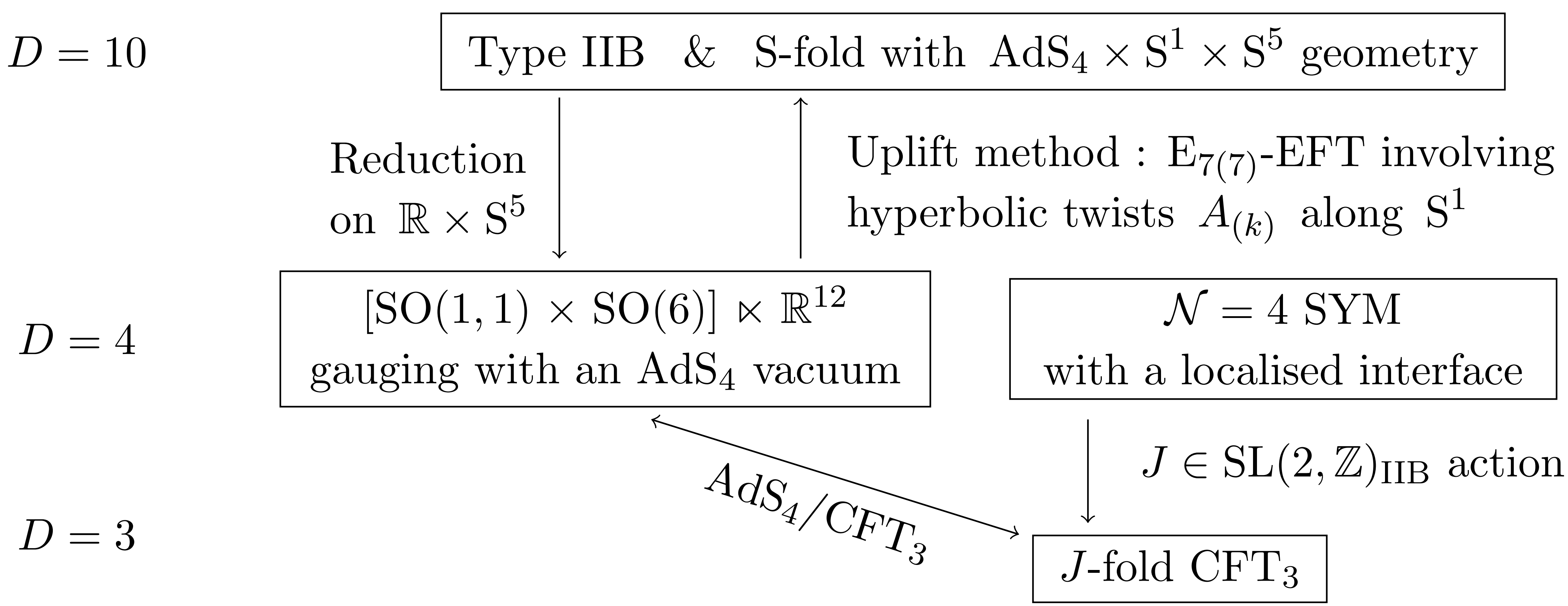
  [Bak, Gutperle, Hirano '03 (**N = 0**)] [Clark, Freedman, Karch, Schnabl '04] [D'Hoker, Ester, Gutperle '07, '07 (**N = 4**)]

- Superconformal interfaces in N=4 SYM_4 — [D'Hoker, Ester, Gutperle '06 (**N = 1 , 2 , 4**)]

N=4 N=2 & SU(2) N=1 & SU(3) N=0 & SO(6)

Question : Holographic duals for N = 0, 1, 2 S-fold CFT_3 ? **[largest flavour symmetry]**

The picture…


$D = 10$
$D = 4$
$D = 3$
Type IIB & S-fold with $\mathrm{AdS}_4 \times \mathrm{S}^1 \times \mathrm{S}^5$ geometry
Reduction on $\mathbb{R} \times \mathrm{S}^5$
Uplift method : $\mathrm{E}_{7(7)}$-EFT involving hyperbolic twists $A_{(k)}$ along S^1
$[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes \mathbb{R}^{12}$ gauging with an AdS_4 vacuum
$\mathcal{N} = 4$ SYM with a localised interface
$J \in \mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}$ action
$\mathrm{AdS}_4/\mathrm{CFT}_3$
J-fold CFT_3

S-folds in 4D

A truncation : $\mathbb{Z}_2^3$ invariant sector

[AG, Sterckx, Trigiante '20]

- Truncation : Retaining the fields and couplings which are invariant (singlets) under a $\mathbb{Z}_2^3$ action ➡ N = 1 supergravity coupled to 7 chiral multiplets z_i

$$z_i = -\chi_i + i\, y_i \qquad (y_i > 0)$$

[upper half-plane]

- The model :

$$K = -\sum_{i=1}^{7} \log[-i(z_i - \bar{z}_i)]$$

$$W = 2\,g\left[\, z_1 z_5 z_6 + z_2 z_4 z_6 + z_3 z_4 z_5 + (z_1 z_4 + z_2 z_5 + z_3 z_6)\, z_7 \,\right] + 2\,g\,c\,(1 - z_4 z_5 z_6 z_7)$$

[dyonic gauging]

- AdS_4 vacua (max. sym. solutions of N=1 BPS equations) :

N=4 & $SO(4)_R$ N=2 & $SU(2) \times U(1)_R$ N=1 & SU(3) N=0 & SO(6)

➡ Most symmetric AdS_4 vacua within **multi-parametric families** !!

N=0 family of AdS$_4$ vacua with U(1)^3 symmetry

[AG, Sterckx, Trigiante '20]

- Location : **[3 free parameters]**

$$z_{1,2,3} = c\left(-\chi_{1,2,3} + i\frac{1}{\sqrt{2}}\right) \qquad \text{and} \qquad z_4 = z_5 = z_6 = z_7 = i$$

- AdS$_4$ radius $L^2 = -3/V_0$ & scalar mass spectrum :

$$V_0 = -2\sqrt{2}g^2c^{-1}$$

[BF unstable]

$$\begin{aligned} m^2L^2 = {} & 6(\times 2)\,, \quad -3(\times 2)\,, \quad 0(\times 28)\,, \\ & -\frac{3}{4} + \frac{3}{2}\chi^2(\times 2)\,, \\ & -\frac{3}{4} + \frac{3}{2}(\chi - 2\chi_i)^2(\times 2) \qquad i = 1,2,3\,, \\ & -\frac{3}{4} + \frac{3}{2}\chi_i^2(\times 4) \qquad i = 1,2,3\,, \\ & -3 + 6\chi_i^2(\times 2) \qquad i = 1,2,3\,, \\ & -3 + \frac{3}{2}(\chi_i \pm \chi_j)^2(\times 2) \qquad i < j\,, \end{aligned}$$

- Flavour symmetry enhancements :

$$\mathrm{U}(1)^3 \;\rightarrow\; \mathrm{SU}(2)\times\mathrm{U}(1)^2 \;\rightarrow\; \mathrm{SU}(3)\times\mathrm{U}(1) \;\rightarrow\; \mathrm{SO}(6)$$

$$\chi_i = \chi_j \qquad \chi_1 = \chi_2 = \chi_3 \qquad \chi_{1,2,3} = 0$$

N=1 family of AdS$_4$ vacua with U(1)^2 symmetry

[AG, Sterckx, Trigiante '20]

- Location : **[2 free parameters :** $\sum_{i=1}^{3} \chi_i = 0$ **]**

$$z_{1,2,3} = c\left(-\chi_{1,2,3} + i\frac{\sqrt{5}}{3}\right) \quad \text{and} \quad z_4 = z_5 = z_6 = z_7 = \frac{1}{\sqrt{6}}(1 + i\sqrt{5})$$

- AdS$_4$ radius $L^2 = -3/V_0$ & scalar mass spectrum :

$$V_0 = -\frac{162}{25\sqrt{5}} g^2 c^{-1}$$

$$\begin{aligned}
m^2L^2 = 0(\times 28)\,, \quad 4 \pm \sqrt{6}(\times 2)\,, \quad -2(\times 2)\,, \\
-\frac{14}{9} + 5\chi_i^2 \pm \frac{1}{3}\sqrt{4 + 45\chi_i^2}(\times 2) \qquad i = 1,2,3\,, \\
-\frac{14}{9} + \frac{5}{4}\chi_i^2 \pm \frac{1}{6}\sqrt{16 + 45\chi_i^2}(\times 2) \qquad i = 1,2,3\,, \\
\frac{7}{9} + \frac{5}{4}\chi_i^2(\times 2) \qquad i = 1,2,3\,, \\
-2 + \frac{5}{4}(\chi_i - \chi_j)^2(\times 2) \qquad i < j\,,
\end{aligned}$$

- Flavour symmetry enhancements :

$$\mathbf{U(1)^2 \rightarrow SU(2) \times U(1) \rightarrow SU(3)}$$

$$\chi_i = \chi_j \qquad\qquad \chi_{1,2,3} = 0$$

$\mathcal{N}=2$ family of AdS_4 vacua with $U(1) \times U(1)_R$ symmetry

[AG, Sterckx, Trigiante '20]

- Location : [**1 free parameter**]

$$z_1 = -\bar{z}_3 = c\left(-\chi + i\frac{1}{\sqrt{2}}\right), \quad z_2 = ic, \quad z_4 = z_6 = i \quad \text{and} \quad z_5 = z_7 = \frac{1}{\sqrt{2}}(1+i)$$

- AdS_4 radius $L^2 = -3/V_0$ & scalar mass spectrum :

$$V_0 = -3g^2c^{-1},$$

$$m^2L^2 = 0(\times 30), \quad 3 \pm \sqrt{17}(\times 2), \quad -2(\times 4), \quad 2(\times 6), \quad -2+4\chi^2(\times 6)$$
$$-1+4\chi^2 \pm \sqrt{16\chi^2+1}(\times 2), \quad \chi^2 \pm \sqrt{\chi^2+2}(\times 8),$$

- Flavour symmetry enhancement :

$$\mathbf{U(1)} \rightarrow \mathbf{SU(2)}$$
$$\chi = 0$$

N=4 AdS_4 vacuum with $SO(4)_R$ symmetry

[Gallerati, Samtleben, Trigiante '14]

[AG, Sterckx, Trigiante '20]

- Location :

$$z_1 = z_2 = z_3 = ic \qquad \text{and} \qquad z_4 = z_5 = z_6 = -\bar{z}_7 = \frac{1}{\sqrt{2}}(1+i)$$

- AdS_4 radius $L^2 = -3/V_0$ & scalar mass spectrum :

$$V_0 = -3g^2c^{-1} \, , \qquad m^2L^2 = 0(\times 48)\, , \quad 10(\times 1)\, , \quad 4(\times 10)\, , \quad -2(\times 11)$$

Next step : Uplift to type IIB on $\mathbb{R} \times S^5$ using $E_{7(7)}$-EFT

S-folds in 10D

$E_{7(7)}$-EFT

[momentum, winding, …]

- Space-time : external (D=4) + **generalised internal** ($Y^{\mathcal{M}}$ coordinates in **56** of $E_{7(7)}$)

Generalised diffs = *ordinary internal diffs* + internal *gauge transfos*

[Coimbra, Strickland-Constable, Waldram '11]

- Generalised Lie derivative built from an $E_{7(7)}$-invariant structure Y-tensor

$$\mathbb{L}_{\Lambda} U^{\mathcal{M}} = \Lambda^{\mathcal{N}} \partial_{\mathcal{N}} U^{\mathcal{M}} - U^{\mathcal{N}} \partial_{\mathcal{N}} \Lambda^{\mathcal{M}} + Y^{\mathcal{M}\mathcal{N}}{}_{\mathcal{P}\mathcal{Q}}\, \partial_{\mathcal{N}} \Lambda^{\mathcal{P}}\, U^{\mathcal{Q}}$$

[no density term]

Closure requires a **section constraint** : $Y^{\mathcal{P}\mathcal{Q}}{}_{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{P}} \otimes \partial_{\mathcal{Q}} = 0$

[**massless** theories]

Two maximal solutions : M-theory (**7** dimensional) & Type IIB (**6** dimensional)

$$y^{i=1\ldots 5} \;\; (\text{elec}) \quad , \quad \tilde{y}_1 = \sinh \eta \;\; (\text{mag})$$

[Inverso, Samtleben, Trigiante '16]

$E_{7(7)}$-EFT

[Hohm, Samtleben '13]

- $E_{7(7)}$-EFT action $\;[\; \mathcal{D}_\mu = \partial_\mu - \mathbb{L}_{A_\mu} \;]$

$$S_{\text{EFT}} = \int d^4x\, d^{56}Y\, e\, \Big[\, \hat{R} + \tfrac{1}{48}\, g^{\mu\nu}\, \mathcal{D}_\mu \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \mathcal{D}_\nu \mathcal{M}_{\mathcal{M}\mathcal{N}} - \tfrac{1}{8}\, \mathcal{M}_{\mathcal{M}\mathcal{N}}\, \mathcal{F}^{\mu\nu\mathcal{M}} \mathcal{F}_{\mu\nu}{}^{\mathcal{N}} + e^{-1}\, \mathcal{L}_{\text{top}} - V_{\text{EFT}}(\mathcal{M}, g) \,\Big]$$

with *field strengths* & *potential term* given by

$$\mathcal{F}_{\mu\nu}{}^{\mathcal{M}} = 2\, \partial_{[\mu} A_{\nu]}{}^{\mathcal{M}} - \big[A_\mu, A_\nu\big]_{\text{E}}^{\mathcal{M}} + \text{two-form terms}$$

(tensor hierarchy)

$$\begin{aligned} V_{\text{EFT}}(\mathcal{M}, g) = & -\tfrac{1}{48}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{M}} \mathcal{M}^{\mathcal{K}\mathcal{L}}\, \partial_{\mathcal{N}} \mathcal{M}_{\mathcal{K}\mathcal{L}} + \tfrac{1}{2}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{M}} \mathcal{M}^{\mathcal{K}\mathcal{L}}\, \partial_{\mathcal{L}} \mathcal{M}_{\mathcal{N}\mathcal{K}} \\ & -\tfrac{1}{2}\, g^{-1} \partial_{\mathcal{M}} g\, \partial_{\mathcal{N}} \mathcal{M}^{\mathcal{M}\mathcal{N}} - \tfrac{1}{4}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, g^{-1} \partial_{\mathcal{M}} g\, g^{-1} \partial_{\mathcal{N}} g - \tfrac{1}{4}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{M}} g^{\mu\nu}\, \partial_{\mathcal{N}} g_{\mu\nu} \end{aligned}$$

- Two-derivative potential : **ungauged** N=8 D=4 SUGRA when $\Phi(x, Y) = \Phi(x)$

Generalised Scherk-Schwarz reductions

[Hohm, Samtleben '14]
[Baguet, Hohm, Samtleben '15]
[Inverso, Samtleben, Trigiante '16]

- SL(8) twist (geometry) :

$$\rho \;=\; \mathring{\rho}(\tilde{y}_1)\; \hat{\rho}(y^i)$$

$$(U^{-1})_A{}^B = \left(\frac{\mathring{\rho}}{\hat{\rho}}\right)^{\frac{1}{2}} \begin{pmatrix} 1 & 0 & 0 & -\mathring{\rho}^{-2}\, c\, \tilde{y}_1 \\ 0 & \delta^{ij} + \hat{K}\, y^i\, y^j & -\lambda\, \hat{\rho}^2 y^i & 0 \\ 0 & -\lambda\, \hat{\rho}^2 y^j\, \hat{K} & \hat{\rho}^4 & 0 \\ -\mathring{\rho}^{-2}\, c\, \tilde{y}_1 & 0 & 0 & \mathring{\rho}^{-4}(1+\tilde{y}_1^2) \end{pmatrix}$$

- EFT fields = Twist × 4D fields :

$$g_{\mu\nu}(x,Y) = \rho^{-2}(Y) g_{\mu\nu}(x)$$

$$\mathcal{M}_{MN}(x,Y) = U_M{}^K(Y)\, U_N{}^L(Y)\, M_{KL}(x)$$

- Type IIB fields = EFT fields :

$$\begin{aligned} G^{mn} &= G^{1/2}\, \mathcal{M}^{mn} \\ \mathbb{B}_{mn}{}^{\alpha} &= G^{1/2}\, G_{mp}\, \epsilon^{\alpha\beta}\, \mathcal{M}^p{}_{n\beta} \\ m_{\alpha\beta} &= \tfrac{1}{6}\, G\, \left(\mathcal{M}^{mn}\, \mathcal{M}_{m\alpha\, n\beta} + \mathcal{M}^m{}_{k\alpha}\, \mathcal{M}^k{}_{m\beta} \right) \\ C_{klmn} &= -\tfrac{1}{4}\, G^{1/2}\, G_{k\rho}\, \mathcal{M}^\rho{}_{lmn} + \tfrac{3}{8}\, \epsilon_{\alpha\beta}\, \mathbb{B}_{k[l}{}^{\alpha}\, \mathbb{B}_{mn]}{}^{\beta} \end{aligned}$$

N=0 & SO(6) solution

[AG, Sterckx '19]

Flavour : SO(6) ~ S^5

$$ds_{10}^2 = \frac{1}{\sqrt{2}}\, ds^2_{\mathrm{AdS}_4} + \frac{1}{2}\, d\eta^2 + d\mathring{s}^2_{\mathrm{S}^5}$$

$$\widetilde{F}_5 = 4\,(1+\star)\,\mathrm{vol}_5$$

$$\mathbb{B}^\alpha = A^\alpha{}_\beta\, \mathfrak{b}^\beta = 0$$

$$m_{\alpha\beta} = (A^{-t})_\alpha{}^\gamma\, \mathfrak{m}_{\gamma\delta}\, (A^{-1})^\delta{}_\beta$$

with $\mathfrak{m}_{\gamma\delta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $A^\alpha{}_\beta \equiv \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix}$

[(hyperbolic) SO(1,1)-twist]

[Bak, Gutperle, Hirano '03] **unstable !!**

No untwisted limit !!
(genuinely dyonic)

N=1 & SU(3) solution

[Lüst, Tsimpis '09 (local form)]
[AG, Sterckx '19]

Flavour : SU(3) ~ CP²

$$ds^2_{10} = \frac{3\sqrt{6}}{10}\, ds^2_{\mathrm{AdS}_4} + \frac{1}{3}\sqrt{\frac{10}{3}}\, d\eta^2 + \left[\sqrt{\frac{5}{6}}\, ds^2_{\mathbb{CP}^2} + \sqrt{\frac{6}{5}}\, \boldsymbol{\eta}^2\right]$$

$$\widetilde{F}_5 = 3\left(\frac{6}{5}\right)^{\frac{3}{4}} (1+\star)\, \mathrm{vol}_5$$

$$\mathbb{B}^\alpha = A^\alpha{}_\beta\, \mathfrak{b}^\beta = A^\alpha{}_\beta \left(-\tfrac{5}{12}\; H^\beta{}_\gamma(z_i)\, \boldsymbol{\Omega}^\gamma\right)$$

$$m_{\alpha\beta} = (A^{-t})_\alpha{}^\gamma\, \mathfrak{m}_{\gamma\delta}\, (A^{-1})^\delta{}_\beta$$

[charged under $U(1)_\eta$]

with $\mathfrak{m}_{\gamma\delta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $A^\alpha{}_\beta \equiv \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix}$

[(hyperbolic) SO(1,1)-twist]

No untwisted limit !!
(genuinely dyonic)

N=2 & SU(2) x U(1)$_R$ solution

[AG, Sterckx, Trigiante '20]

Flavour : SU(2) ~ S²

(genuinely dyonic)

$$ds^2 = \tfrac{1}{2}\,\Delta^{-1}\left[ds^2_{\text{AdS}_4} + d\eta^2 + d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\left(\sigma_2^2 + 8\,\Delta^4\,\left(\sigma_1^2 + \sigma_3^2\right)\right)\right]$$

$$\Delta^{-4} = 6 - 2\cos(2\theta)$$

$$\widetilde{F}_5 = 4\,\Delta^4\,\sin\theta\,\cos^3\theta\,(1+\star)\,\Big[3\,d\theta\wedge d\phi\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3 - d\eta\wedge\left(\cos(2\phi)\,d\theta - \tfrac{1}{2}\sin(2\theta)\,\sin(2\phi)\,d\phi\right)\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3\Big]$$

$$\mathbb{B}^\alpha = A^\alpha{}_\beta\,\mathfrak{b}^\beta \quad \text{with} \quad \begin{aligned} \mathfrak{b}_1 &= \tfrac{1}{\sqrt{2}}\cos\theta\left[\left(\sin\phi\,d\theta + \tfrac{1}{2}\sin(2\theta)\,d(\sin\phi)\right)\wedge\sigma_2 + \sin\phi\,\frac{4\,\sin(2\theta)}{6-2\cos(2\theta)}\,\sigma_1\wedge\sigma_3\right] \\ \mathfrak{b}_2 &= \tfrac{1}{\sqrt{2}}\cos\theta\left[\left(\cos\phi\,d\theta + \tfrac{1}{2}\sin(2\theta)\,d(\cos\phi)\right)\wedge\sigma_2 + \cos\phi\,\frac{4\,\sin(2\theta)}{6-2\cos(2\theta)}\,\sigma_1\wedge\sigma_3\right] \end{aligned}$$

$$m_{\alpha\beta} = (A^{-t})_\alpha{}^\gamma\,\mathfrak{m}_{\gamma\delta}\,(A^{-1})^\delta{}_\beta \qquad \text{with} \qquad \mathfrak{m}_{\gamma\delta} = 2\,\Delta^2\begin{pmatrix} 1+\sin^2\theta\,\cos^2\phi & -\tfrac{1}{2}\sin^2(\theta)\sin(2\phi) \\ -\tfrac{1}{2}\sin^2(\theta)\sin(2\phi) & 1+\sin^2\theta\,\sin^2\phi \end{pmatrix}$$

Singular Janus & S-fold interpretation

[Inverso, Samtleben, Trigiante '16]
[AG, Sterckx, (Trigiante) '19, ('20)]

- Singular Janus (linear dilaton) : $\Phi(\eta, y^i) = -2\,\eta + f(y^i)$ **[S^5 coordiantes y^i]**

- $\mathbb{R} \to \mathrm{S}^1 \;\Leftrightarrow\;$ **hyperbolic** monodromy : $\mathfrak{M}_{\mathrm{S}^1} = A^{-1}(\eta) A(\eta + T) = \begin{pmatrix} \cosh T & \sinh T \\ \sinh T & \cosh T \end{pmatrix}$

- Generalising the A-twist to a k-family ($k > 2$) :

$$A_{(k)} = A g(k) \qquad \text{with} \qquad g(k) = \begin{pmatrix} \frac{(k^2-4)^{\frac{1}{4}}}{\sqrt{2}} & 0 \\ \frac{k}{\sqrt{2}(k^2-4)^{\frac{1}{4}}} & \frac{\sqrt{2}}{(k^2-4)^{\frac{1}{4}}} \end{pmatrix}$$

Then

$$\mathfrak{M}(k) = A^{-1}_{(k)}(\eta)\;\; A_{(k)}\big(\eta + T(k)\big) = \begin{pmatrix} k & 1 \\ -1 & 0 \end{pmatrix} = -\mathcal{S}\mathcal{T}^k \in \mathrm{SL}(2,\mathbb{Z})_{\mathrm{IIB}}$$

with $T(k) = \log(k + \sqrt{k^2-4}) - \log(2)$ and $\mathrm{Tr}\mathfrak{M}(k) > 2.$ **[hyperbolic]**

Holographic RG flows on the D3-brane

[In progress…]

D3-brane and SYM_4

[AdS_5 factor]

- $c = 0$ **:**

D3-brane

$$ds^2_{10} = \tfrac{1}{2}\, g^2\, \Delta^{-1}(z_i)\, \left(e^{2A(z)}\, \eta_{\alpha\beta}\, dx^\alpha dx^\beta + dz^2\right) + \Delta^2(z_i)\, d\eta^2 + d\mathring{s}^2_{S^5}\ ,$$

$$\widetilde{F}_5 = 4\, L_5^{-1}\, (1+\star)\, \text{vol}_5 \quad , \quad \mathbb{B}^\alpha = 0\ ,$$

$$m_{\alpha\beta} = \begin{pmatrix} e^{-\Phi_0} & 0 \\ 0 & e^{\Phi_0} \end{pmatrix} .$$

DW_4 domain-wall (SYM_4)

$$z_{1,2,3} = -\chi_{1,2,3} + i\, \frac{(g\, z)^2}{8} \quad , \quad z_4 = z_5 = z_6 = z_7 = i\, e^{-\frac{1}{2}\Phi_0} \quad \text{and} \quad e^A = (g\, z)^3\ .$$

- $c \neq 0$ **:**

non-removable but **sub-leading corrections** in $1/(gz)$

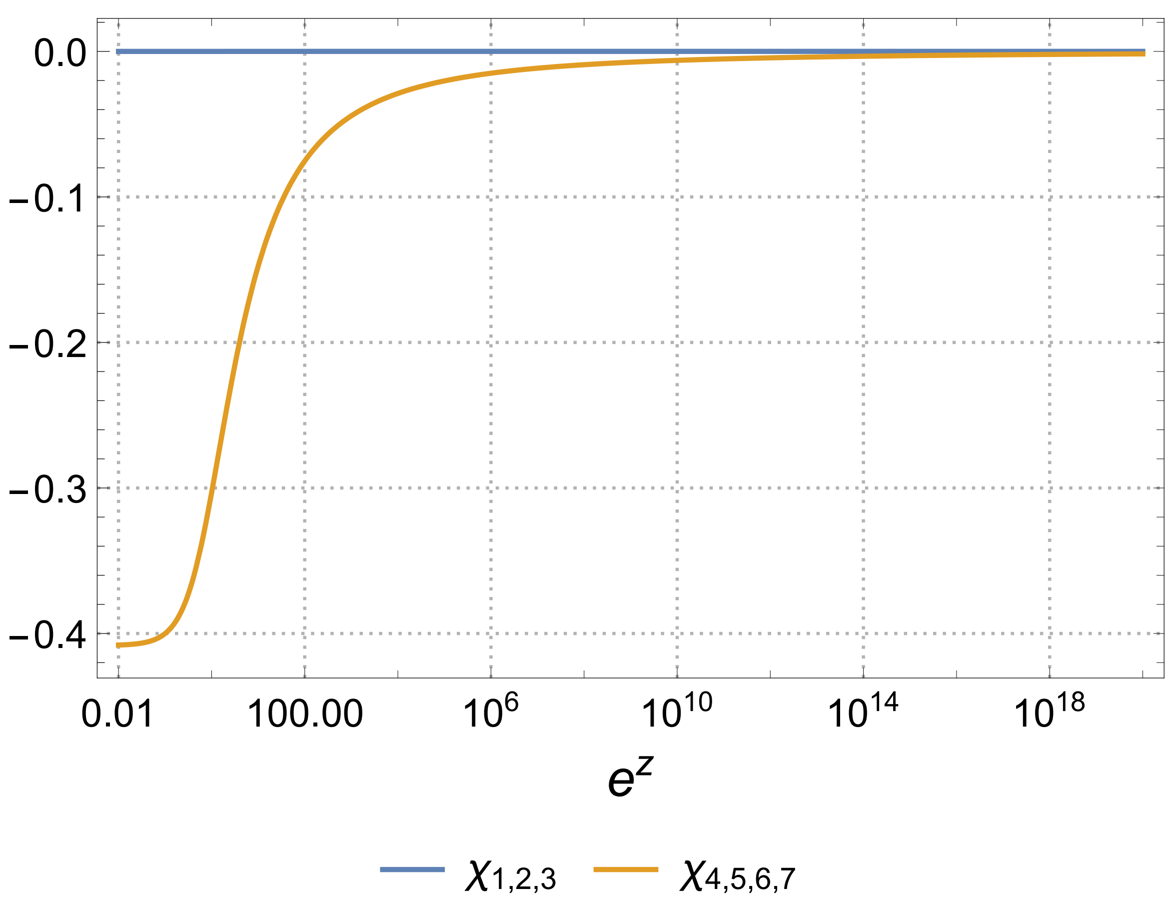
$$e^A = (g\, z)^3 \left[1 + c\, \left(\lambda_0 + \frac{3}{2}\frac{\lambda_1}{gz}\right) + c^2\, \left(\epsilon_0 + \frac{3}{2}\frac{\lambda_1'}{gz} + \frac{3}{4}\frac{\lambda_1^2}{(gz)^2} + \cosh^2\Phi_0\, \frac{16}{(gz)^4} + \ldots\right)\right]$$

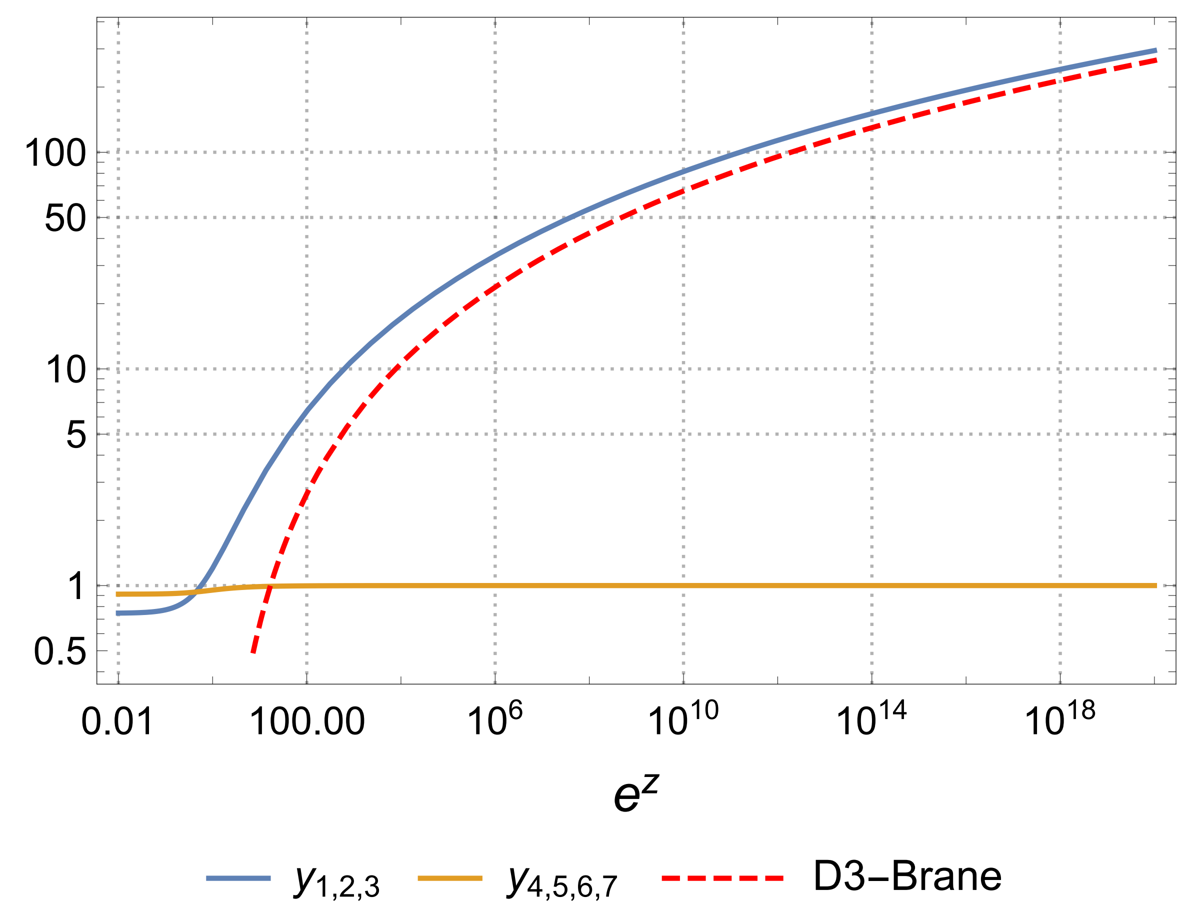
$$\text{Im} z_{4,5,6,7} = e^{-\frac{1}{2}\Phi_0} \left[1 + c\, \tau + c^2\, \left(\delta_0 + \cosh\Phi_0\, \left(\sinh\Phi_0 - \tfrac{1}{4}\cosh\Phi_0\right) \frac{32}{(gz)^4} + \ldots\right)\right]$$

(IR) N=1 & SU(3) *J*-fold CFT_3 ⟺ SYM_4 (UV)

$$
\begin{array}{rcccccccccccc}
m^2L^2 & = & -\frac{20}{9}\,(\times 3) & , & -2\,(\times 2) & , & -\frac{8}{9}\,(\times 3) & ; & 0\,(\times 2) & ; & 4-\sqrt{6}\,(\times 2) & , & 4+\sqrt{6}\,(\times 2) \\
\Delta_+ & = & \mathbf{\frac{5}{3}}\,(\times 3) & , & \mathbf{2}\,(\times 2) & , & \frac{8}{3} & ; & 3 & ; & \mathbf{1+\sqrt{6}}\,(\times 2) & , & 2+\sqrt{6} \\
\Delta_- & = & \frac{4}{3} & , & 1 & , & \mathbf{\frac{1}{3}}\,(\times 3) & ; & \mathbf{0}\,(\times 2) & ; & 2-\sqrt{6} & , & \mathbf{1-\sqrt{6}}\,(\times 2)
\end{array}
$$

[2 irrelevant operators]

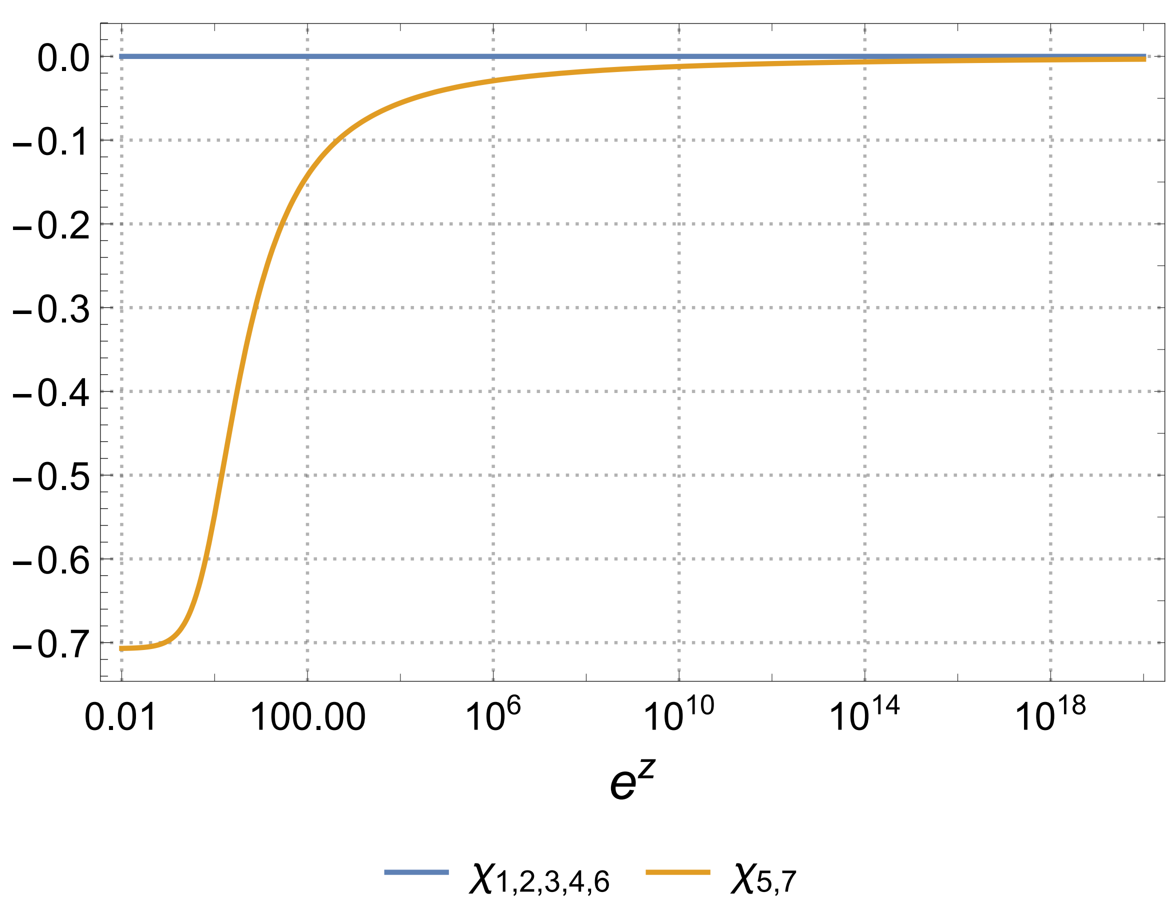


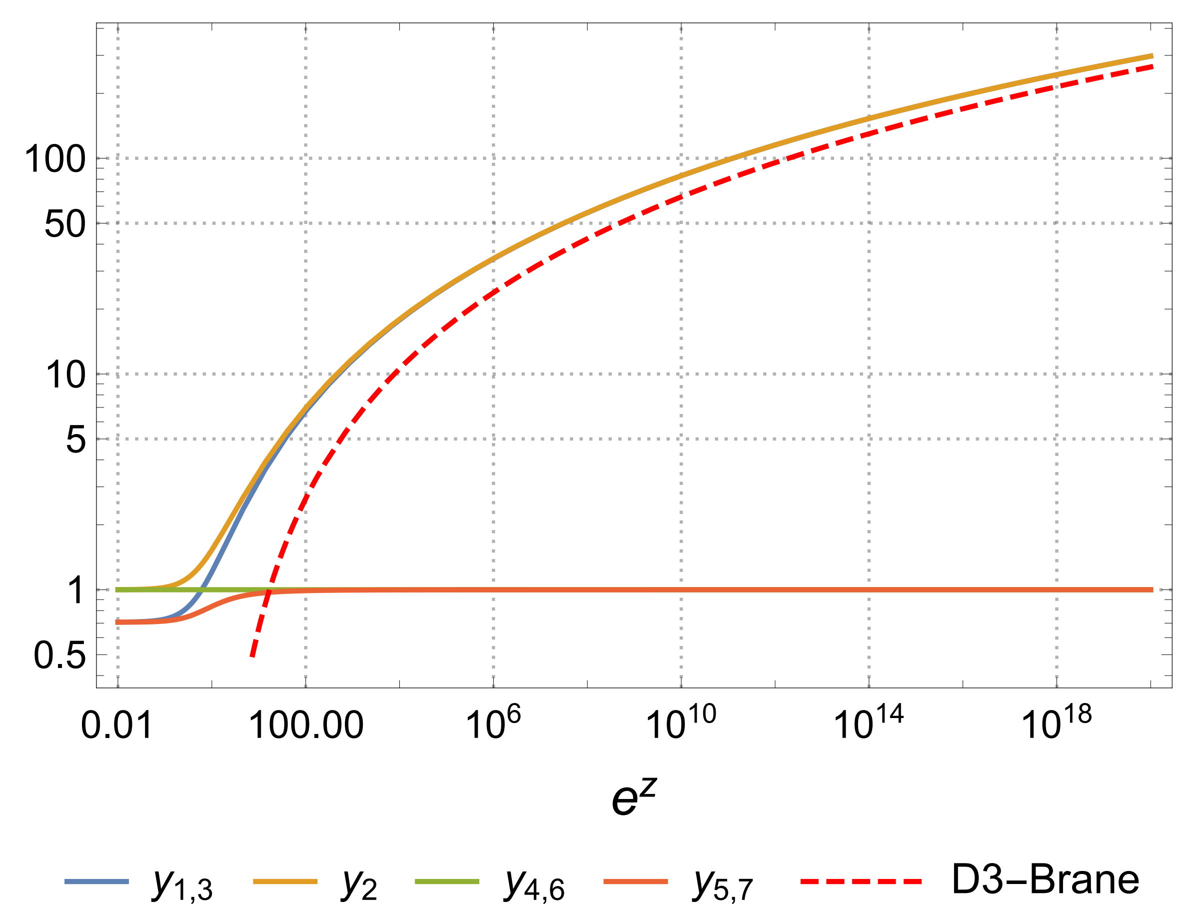


(IR) N=2 & SU(2) J-fold CFT$_3$ ⟺ SYM$_4$ (UV)

$$
\begin{array}{rclcccccccc}
m^2L^2 & = & -2\,(\times 4) & , & 3-\sqrt{17}\,(\times 2) & ; & 0\,(\times 2) & ; & 2\,(\times 4) & , & 3+\sqrt{17}\,(\times 2) \\
\Delta_+ & = & \mathbf{2}\,(\times 2) & , & \tfrac{\mathbf{1}}{\mathbf{2}}(\mathbf{1}+\sqrt{\mathbf{17}})\,(\times 2) & ; & 3 & ; & \tfrac{\mathbf{1}}{\mathbf{2}}(\mathbf{3}+\sqrt{\mathbf{17}})\,(\times 2) & , & \tfrac{1}{2}(5+\sqrt{17}) \\
\Delta_- & = & \mathbf{1}\,(\times 2) & , & \tfrac{1}{2}(5-\sqrt{17}) & ; & \mathbf{0}\,(\times 2) & ; & \tfrac{\mathbf{1}}{\mathbf{2}}(\mathbf{3}-\sqrt{\mathbf{17}})\,(\times 2) & , & \tfrac{\mathbf{1}}{\mathbf{2}}(\mathbf{1}-\sqrt{\mathbf{17}})\,(\times 2)
\end{array}
$$

[4 irrelevant operators]

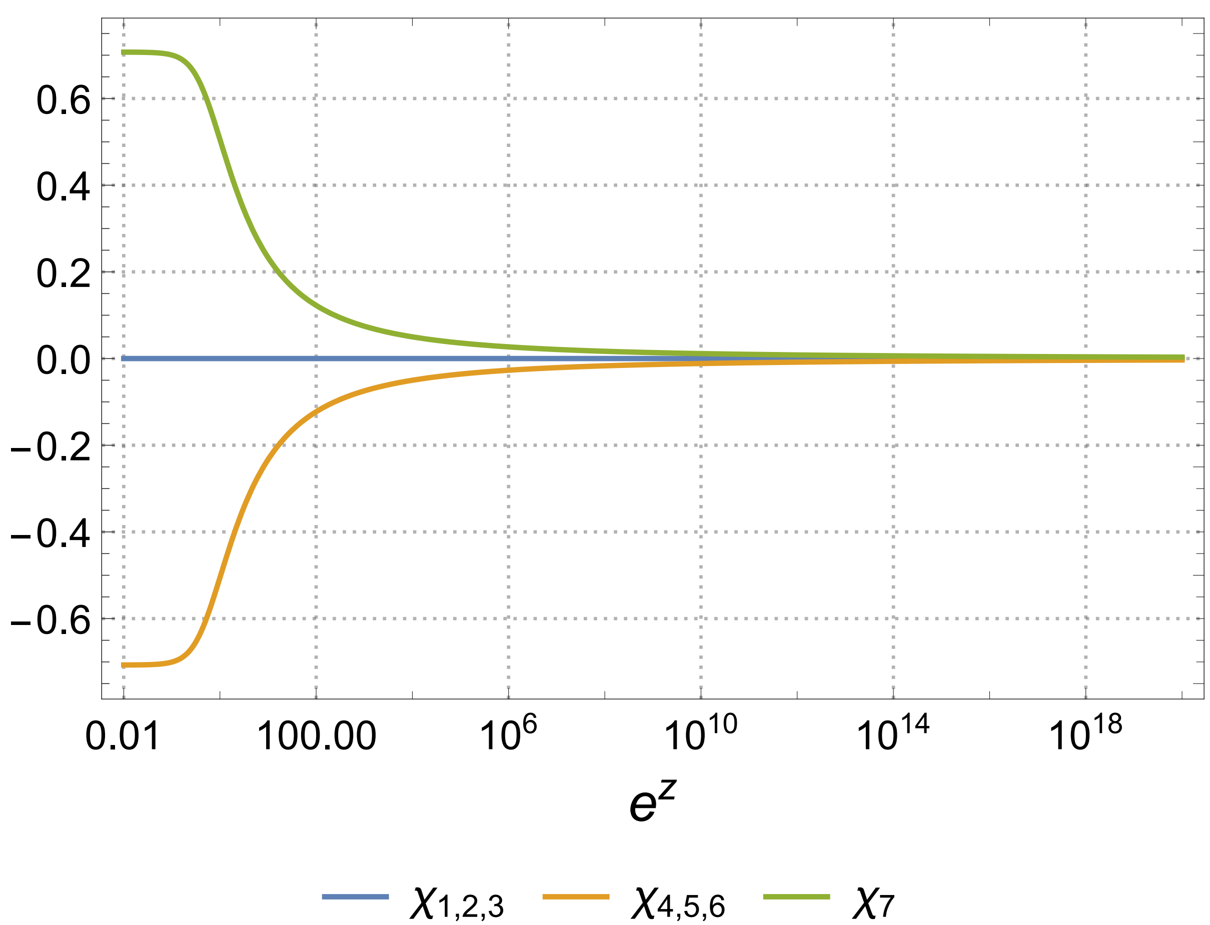


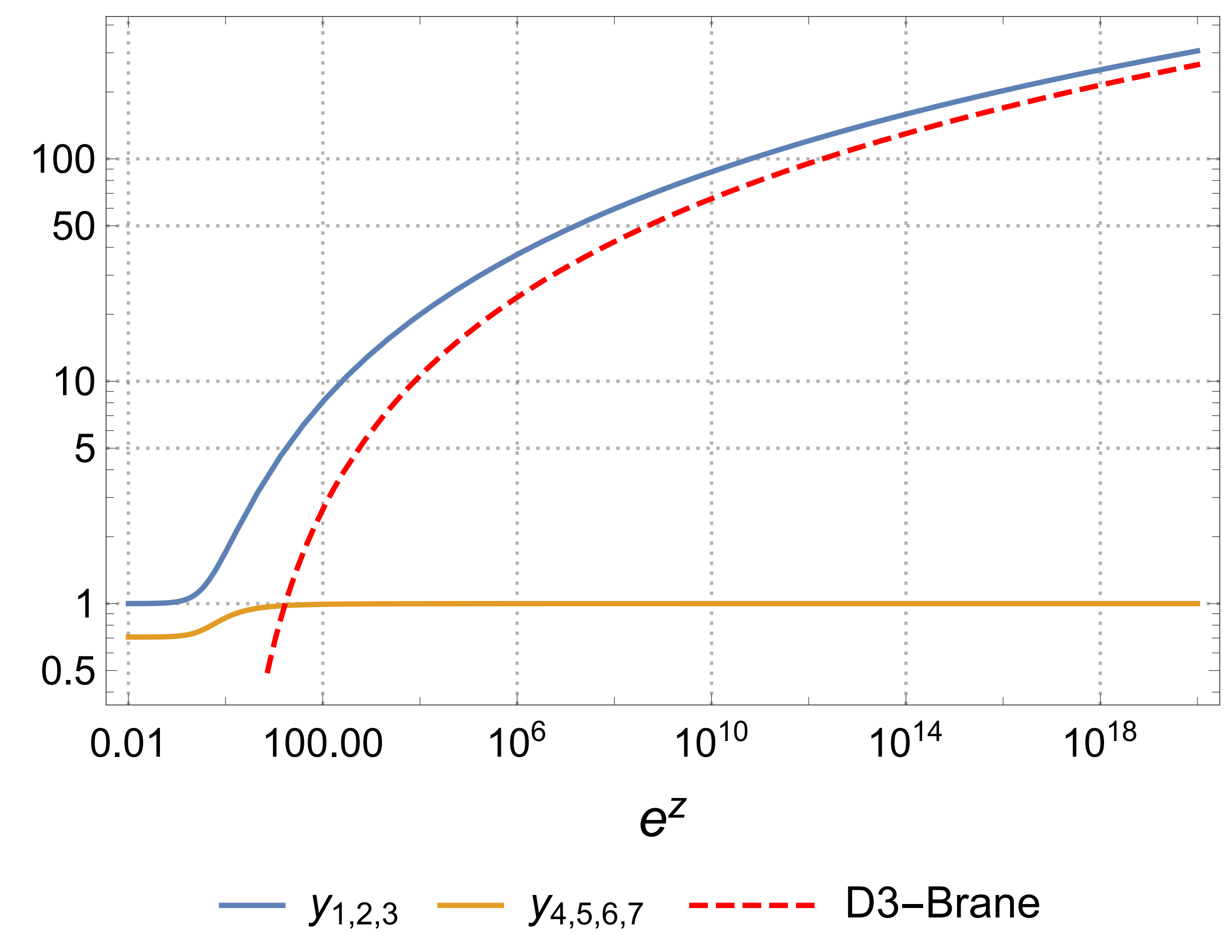


(IR) N=4 *J*-fold CFT_3 ⟺ SYM_4 (UV)

$$
\begin{array}{rcccccccc}
m^2L^2 & = & -2\,(\times 3) & ; & 0\,(\times 6) & ; & 4\,(\times 4) & , & 10\,(\times 1) \\
\Delta_+ & = & \mathbf{2}\,(\times 3) & ; & \mathbf{3}\,(\times 3) & ; & \mathbf{4}\,(\times 1) & , & 5 \\
\Delta_- & = & 1 & ; & \mathbf{0}\,(\times 3) & ; & -\mathbf{1}\,(\times 3) & , & -\mathbf{2}\,(\times 1)
\end{array}
$$

[4 irrelevant operators]

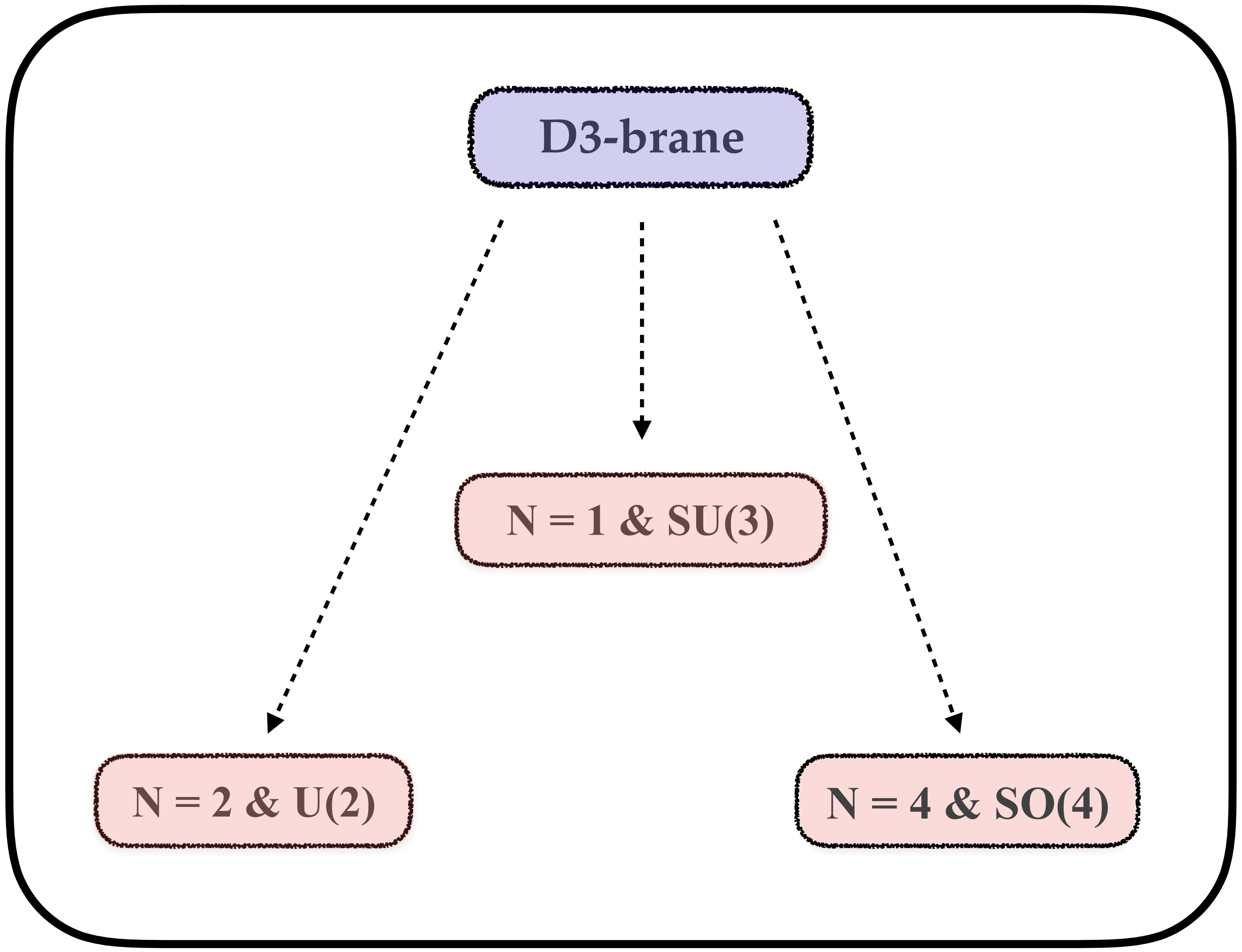




Holographic RG-flows :

IR	UV
$AdS_4 \times \mathbb{R}$ (or S^1) $\times S^5$	$AdS_5 \times S^5$
$ds^2_{10} = \frac{1}{2}\, g^2\, \Delta_{IR}^{-1} \left(ds^2_{AdS_4} + f(z_i)\, d\eta^2 \right) + ds^2_{S^5}$	$ds^2_{10} = ds^2_{AdS_5} + d\mathring{s}^2_{S^5}$

[anisotropic SYM_4]



Question : CFT_3 to CFT_3 flows ?

Summary

[Dall'Agata, Inverso '11]
[Inverso, Samtleben, Trigiante '16]

- Dyonic $[SO(1,1) \times SO(6)] \ltimes R^{12}$ gauging connected with type IIB on $\mathbb{R}$ (or S^1) $\times S^5$

- Type IIB (S-folds) : Gravity duals of J-fold CFT_3's with various (super) symmetries

  [$J = - ST^k$ monodromies ($k > 2$)]

$\mathcal{N} = 0$ & $SO(6)$ **unstable !!**	$\mathcal{N} = 1$ & $SU(3)$	$\mathcal{N} = 2$ & $SU(2) \times U(1)_R$

[see also Bobev, Gautason, Pilch, Suh, van Muiden '19, '20 (5D approach)]

- Holographic RG-flows on the D3-brane : (deformed) SYM_4 to J-fold CFT_3's

- Brane set-ups ? , axions $\chi_{1,2,3}$ (flavour sym breaking) ? , non-abelian T-duals ? ....

Thank you !