

On the vacua of new $SO(8)$ gauged supergravity

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March 5th 2013

Based on arXiv: 1209.3003 , 1301.6919 and 1302.6057

in collaboration w/ A. Borghese, G. Dibitetto, D. Roest & O. Varela

Outlook

- 1) The old $SO(8)$ gauged SUGRA
- 2) The embedding tensor & the new $SO(8)$ gauged SUGRA
- 3) Invariant sectors of new $SO(8)$ gauged SUGRA
 - 3.2) The $SU(3)$ invariant sector
 - 3.3) The $SO(4)$ invariant sector
- 4) Collecting results & final remarks

Outlook

- 1) The old $SO(8)$ gauged SUGRA

Top-down approach

[Cremmer & Julia '79]
[de Wit & Nicolai '82, '87]
[Englert '82]

11d supergravity on the 7-sphere : $SO(8)$ gauged SUGRA with **N=8 SUSY**

Always believed to be unique !!

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The theory

- **Field content** : metric + 70 real scalars + 28 electric vectors + fermions
- **Global** E_7 symmetry & **local** $SO(8)$ gauge symmetry
- **R-symmetry** group $SU(8)$ rotating the 8 gravitini $\psi_{I=1,\dots,8}$
- The **70 = 35 (SD) + i 35 (ASD) scalars** ϕ_{IJKL} serve as coordinates in $E_7/SU(8)$ coset space
- **Unique non-trivial scalar potential**

The goal : Find **critical points** of the **scalar potential** and investigate the Physics associated : vacuum energy , residual symmetry , mass spectra , preserved SUSY . . .

The problem : It depends on **70 scalar fields !!**

The alternatives :

I) **Numerical methods** to explore certain energy ranges

[Fischbacher, '11]

II) Look at simpler (smaller) & **consistent** subsets of fields [**truncations**]

Smaller sectors

Truncation = Retain fields invariant under the action of a residual group

$$G_{res} \subset SU(8)$$

1) $G_{res} = SU(3)$: N=2 description (gravity + 1 vector + 1 hyper)

$$\mathcal{M}_{SK} = \frac{SL(2)}{SO(2)} \quad \text{and} \quad \mathcal{M}_{QK} = \frac{SU(2,1)}{SU(2) \times U(1)}$$

2) $G_{res} = SO(4)$: N=2 (gravity + 1 hyper) or N=0 descriptions

[different embeddings inside $SU(8)$]

(#) = gravitini

[R-symmetry group]

$SU(8)$ (8)

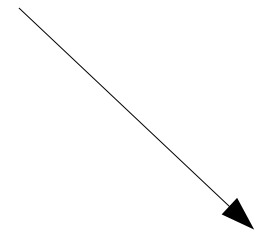
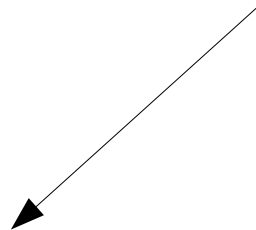


[Gauge group]

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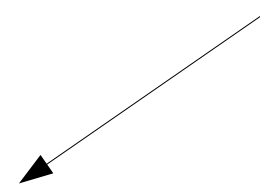
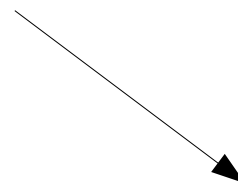
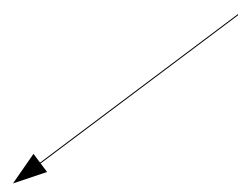
(8) $SO(7)$ (1+7)



(4+4) $SU(4)$

G_2 (1+7)

[Residual groups]



$SO(4)$

$SU(3)$

(4+4) or (1+3+1+3)

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SU(3) invariant critical points

[Warner '83, '84]

[Bobev, Halmagyi, Pilch & Warner '10]

SUSY	Symmetry	Cosm. constant	Stability
$\mathcal{N} = 8$	SO(8)	$-6 (\times 1)$	✓
$\mathcal{N} = 2$	SU(3) \times U(1)	$-\frac{9}{2}\sqrt{3} (\times 1)$	✓
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Lifting to 11d

[Freund & Rubin '80]

[Englert '82]

[Corrado, Pilch & Warner '01]

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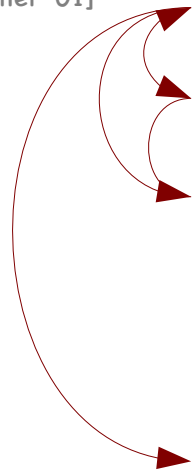
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Domain walls
&
RG-flows



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AdS/CMT applications : Holographic superconductivity

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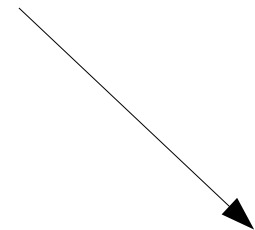
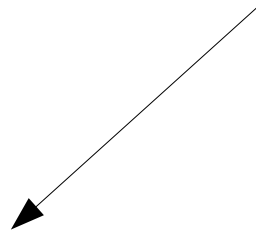


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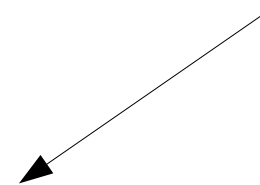
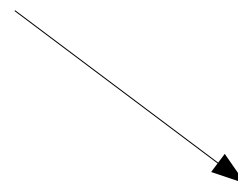
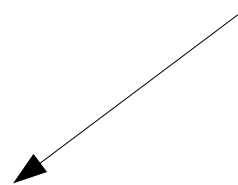
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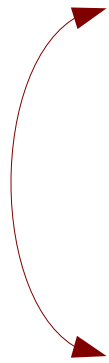
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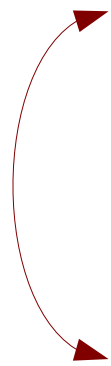
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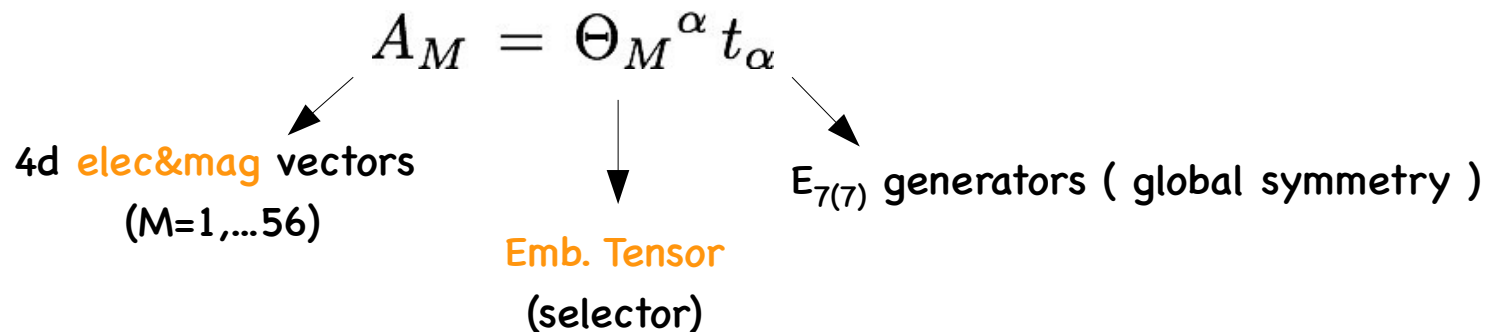
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The embedding tensor

[de Wit, Samtleben & Trigiante '07]

Most general SUGRA with N=8 SUSY in 4D : Embedding Tensor (ET) Formalism

Gauging = Promote part of the E_7 global symmetry to a local gauge symmetry



Indices : $M = 1,\dots,56$ [fundamental index of E_7 with $56 = 28 + 28'$]
 $\alpha = 1,\dots,133$ [adjoint index of E_7]
elec&mag
Sp(56)

Supersymmetry imposes linear constraints (LC) \implies ET lives in the **912** of E_7

Gauge algebra & scalar potential

Gauge algebra : Building the charges $X_{MN}{}^P = \Theta_M{}^\alpha [t_\alpha]_N{}^P$, the gauge algebra is given by

$$[A_M, A_N] = -X_{MN}{}^P A_P$$

Closure imposes quadratic constraints (QC) \implies Only 28 independent vectors

$$\Omega^{MN} \Theta_M{}^\alpha \Theta_M{}^\beta = 0$$

Scalar potential : The charges also induce a non-trivial scalar potential for the 70 scalars in the theory

$$V = \frac{g^2}{672} X_{MNP} X_{QRS} (M^{MQ} M^{NR} M^{PS} + 7 M^{MQ} \Omega^{NR} \Omega^{PS})$$

where $M = LL^t$ and $L = \exp[\vec{\phi} \vec{t}]$ is an $E_7/SU(8)$ element

The new SO(8) gauged SUGRA

[Dall'Agata, Inverso & Trigiante '12]

- Group theory

$$\text{ET decomp : } 912 = 2 \times (1 + 35_v + 35_s + 35_c + 350) \quad [\text{under SO(8)}]$$

ω -parameter family of new SO(8) gauged SUGRA's !!

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ω -parameter family of new SO(8) gauged SUGRA's !!

- ω -parameter : Electric (28) vs magnetic (28') vectors spanning the SO(8)

$$\Theta_{\text{elec}}^\alpha \propto \cos(\omega)$$

$$\Theta_{\text{mag}}^\alpha \propto \sin(\omega)$$



$$V(\Theta, \phi) = V(\omega, \phi)$$

ω -dependent
scalar potential !!

This is a U(1) **outside** E_7 but **inside** $Sp(56)$

- phase set to $\omega = 0$ purely electric vectors
- phase set to $\omega = \text{Pi}/2$ purely magnetic vectors
- phase set to $0 < \omega < \text{Pi}/2$ dyonic combination of vectors

The new electromagnetic phase raises some questions...

I) Do the known critical points of the standard $SO(8)$ gauged SUGRA evolve with the new phase? If so, how?

II) Do Physics (CC, mass spectra, SUSY) depends on the new phase?

III) Are there genuine critical points associated to non-vanishing values of the electromagnetic phase?

IV) Is the periodicity of the electromagnetic rotations $\pi/2$ or smaller?

V) Is there an embedding of the ω -phase into string/M-theory?

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The SU(3) group theoretical truncation

- Embedding of SU(3) inside the global & R-symmetry groups

$$E_7 \implies SL(2) \times F_4 \implies SL(2) \times SU(2,1) \times SU(3)$$

$$SU(8) \implies SU(4) \times SU(4) \implies SU(3) \times SU(3) \implies SU(3)$$

- SU(3) invariant fields

- gravitini : 8 \implies 1 + 3 + 1 + 3 \implies N=2 SUSY

- scalars : 2 [SK] + 4 [QK] real scalars \implies $z + (\zeta_1, \zeta_2) \in \mathbb{C}$

- vectors : 4 vectors (2 elec & 2 mag) in the fundam. of Sp(4) \implies $(A^{0,1}, A_{0,1})$

- N=2 theory with 1 vector + 1 hyper : $\mathcal{M}_{SK} = \frac{SL(2)}{SO(2)}$ $\mathcal{M}_{QK} = \frac{SU(2,1)}{SU(2) \times U(1)}$

$U(1)_1 \times U(1)_2$ gauging along QK isometries

The N=2 canonical formulation

[de Wit, Samtleben & Trigiante '05]

[Halmagyi, Petrini & Zaffaroni '11]

- The scalar potential with a dyonic gauging in the hypermultiplet sector

$$V = \underbrace{\Theta_M^a \Theta_N^b}_{\text{emb tens}} \left[\underbrace{4e^\kappa X^M \bar{X}^N}_{\text{SK}} \underbrace{h_{uv} k^u_a \bar{k}^v_b}_{\text{QK}} + \underbrace{P_a^x P_b^x}_{\text{QK}} \underbrace{\left(g^{i\bar{j}} f_i^M \bar{f}_{\bar{j}}^N - 3e^\kappa X^M \bar{X}^N \right)}_{\text{SK}} \right]$$

SK : data associated to the SK manifold $\implies \omega$ -independent

[**N=8 truncation**]

QK : data associated to the QK manifold $\implies \omega$ -independent

emb tens : ω -dependent vectors gauging the $U(1)_1 \times U(1)_2$ isometries dyonically

- Gauge covariant derivatives

$$Dq^u = dq^u - \left((A^0 \cos \omega - A_0 \sin \omega) k_1^u + (A^1 \cos \omega - A_1 \sin \omega) k_2^u \right)$$

involving electric ($\omega = 0$), magnetic ($\omega = \text{Pi}/2$) or dyonic ($0 < \omega < \text{Pi}/2$) vectors

- The scalar potential depends on the neutral fields z and $\zeta_{12} = \frac{|\zeta_1| + i|\zeta_2|}{1 + \sqrt{1 - |\zeta_1|^2 - |\zeta_2|^2}}$

The superpotential formulation

- The same scalar potential can be obtained as

$$V = 2 \left[\frac{4}{3} (1 - |z|^2)^2 \left| \frac{\partial \mathcal{W}}{\partial z} \right|^2 + (1 - |\zeta_{12}|^2)^2 \left| \frac{\partial \mathcal{W}}{\partial \zeta_{12}} \right|^2 - 3\mathcal{W}^2 \right]$$

from any of the **two** ω -dependent superpotentials

$$\mathcal{W}_+ = (1 - |z|^2)^{-3/2} (1 - |\zeta_{12}|^2)^{-2} [(e^{2i\omega} + z^3) (1 + \zeta_{12}^4) + 6z (1 + e^{2i\omega} z) \zeta_{12}^2]$$

$$\mathcal{W}_- = (1 - |z|^2)^{-3/2} (1 - |\zeta_{12}|^2)^{-2} [(e^{2i\omega} + z^3) (1 + \bar{\zeta}_{12}^4) + 6z (1 + e^{2i\omega} z) \bar{\zeta}_{12}^2]$$

- Setting $\omega = 0$ boils down to the **standard** SU(3) invariant superpotentials

[Bobev, Halmagyi, Pilch & Warner '10]

- Supersymmetric critical points can be extrema of only one or both superpotentials corresponding to N=1,2 respectively

Recalling the SU(3) invariant critical points at $\omega = 0$

[Warner '83, '84]

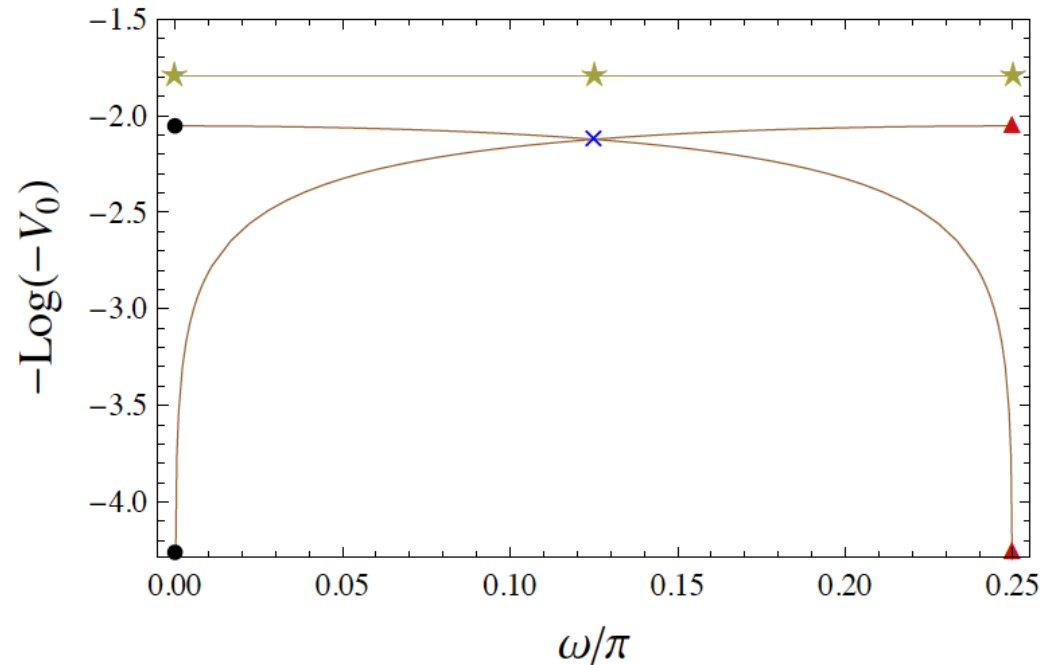
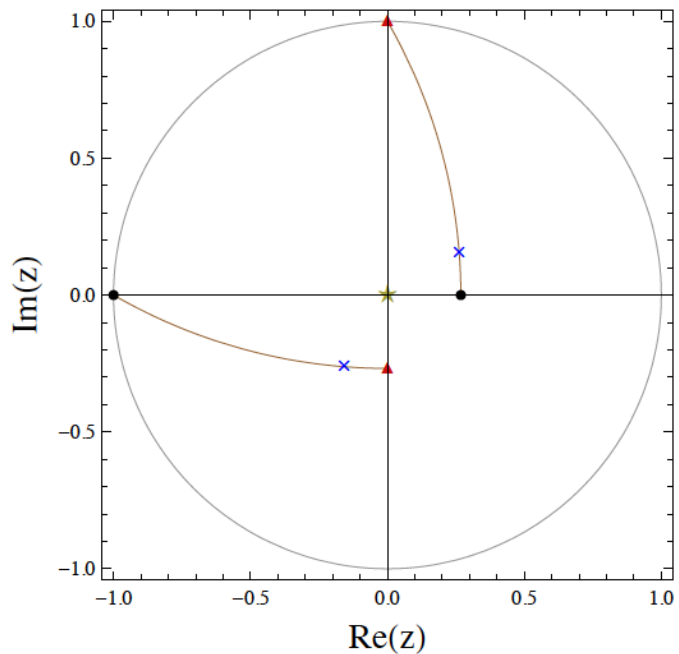
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... what happens when turning on ω ?

The ω -story of the old critical points

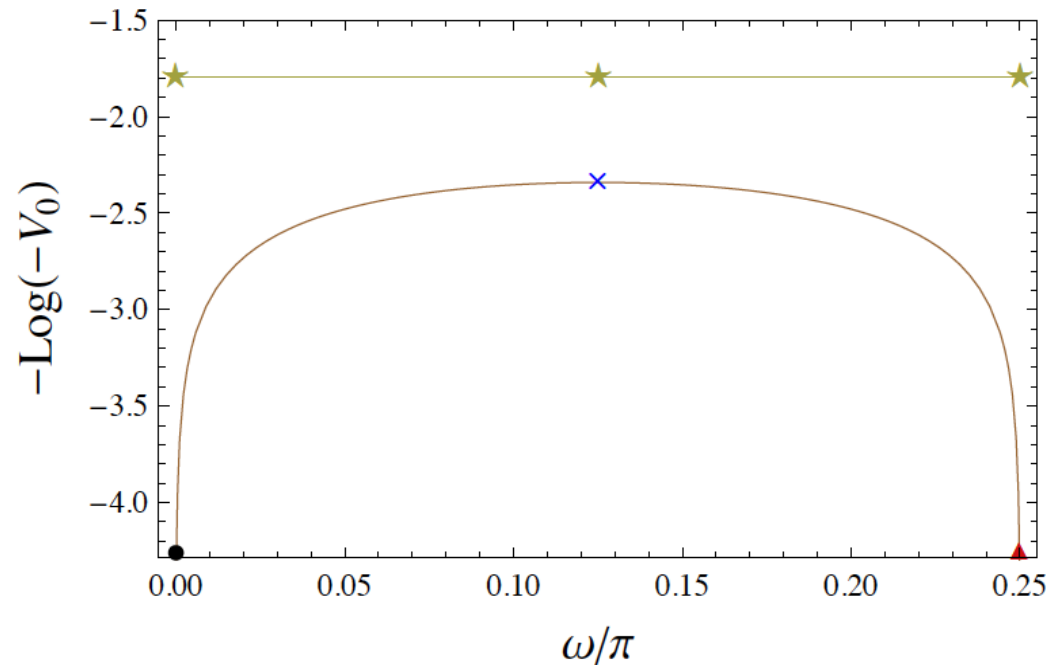
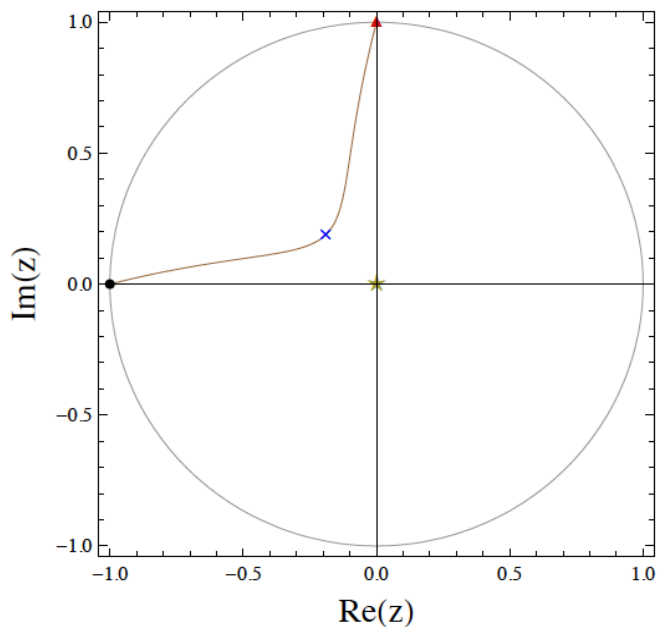
EXAMPLE : the N=8 SO(8) & N=2 SU(3)xU(1) critical points



- The normalised **mass spectra** are **insensitive** to the ω -phase \Rightarrow is G_{res} crucial ?
- There are **new solutions** which move to the field-boundary at $\omega = n \pi/4$
- Analogous ω -stories : N=1 G2-inv , N=0 SO(7)-inv , N=0 SU(4)-inv
 [**unstable**] [**unstable**]

Genuine new critical points

EXAMPLE : novel N=1 SU(3)-invariant critical points



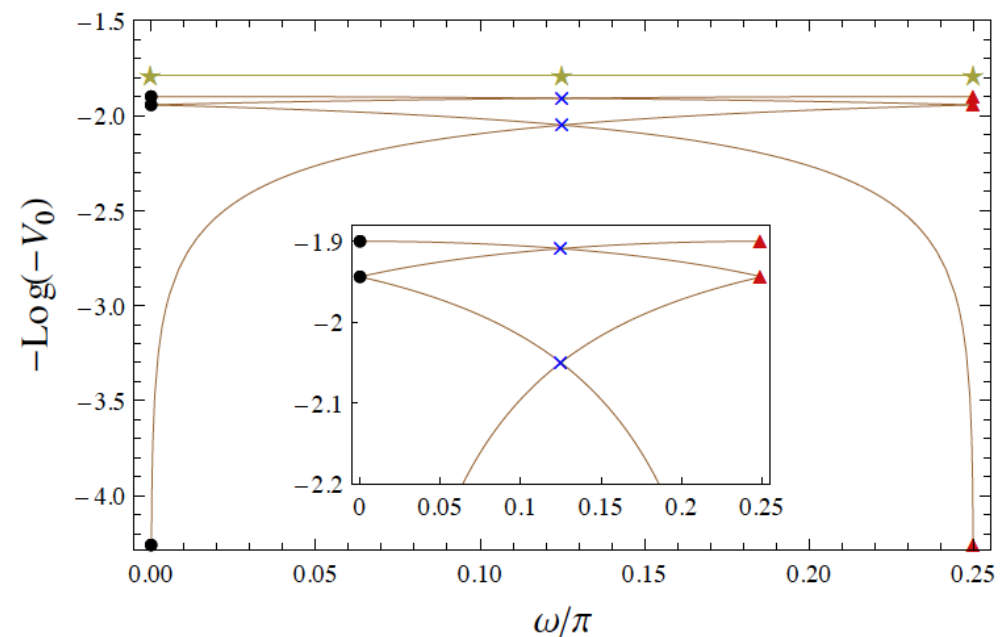
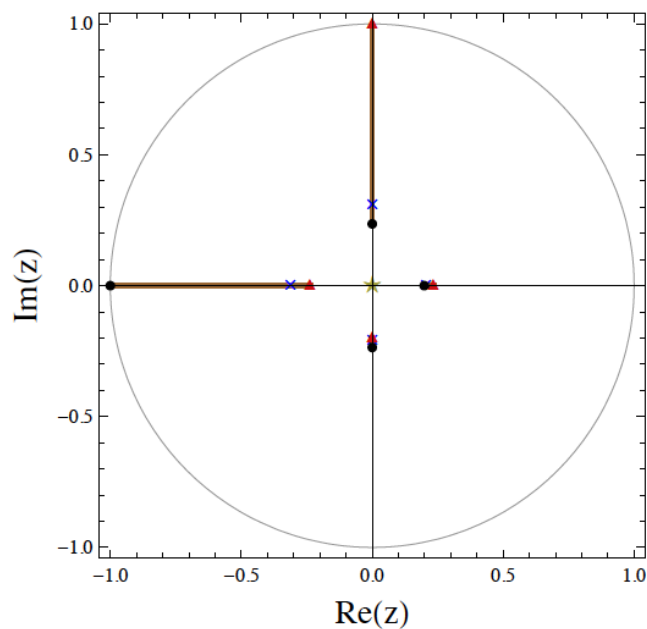
- These are **genuine solutions** which **move to the boundary** at $\omega = n \pi/4$
- Analogous ω -stories : novel N=0 G2-inv [**stable**] and N=0 SU(3)-inv [**stable**]
- **Mass spectra** of the N=0 SU(3)-inv sols **sensitive to the ω -phase** $\implies G_{\text{res}}$ is **not** all !!

A comment on the $\pi/4$ periodicity of the ω -phase

[Dall'Agata, Inverso & Trigiante '12]

- Argument based on the quartic E_7 -invariant \Rightarrow the period cannot be less than $\pi/4$

EXAMPLE : Transmutation of $SO(7)_+$ [x1] and $SO(7)_-$ [x2] critical points



- Scalars : $70 = 35 \text{ (SD)} + i 35 \text{ (ASD)}$ with $SD \Leftrightarrow ASD$ under $SO(8)$ triality
- What is the interrelation between ω -periodicity and triality ? \Rightarrow $SO(4)$ sectors

Outlook

- 1) The old $SO(8)$ gauged SUGRA
- 2) The embedding tensor & the new $SO(8)$ gauged SUGRA
- 3) Invariant sectors of new $SO(8)$ gauged SUGRA
 - 3.2) The $SU(3)$ invariant sector
 - 3.3) The $SO(4)$ invariant sector

Recalling the $SO(4)$ invariant critical points at $\omega = 0$

SUSY	Symmetry	Cosm. constant	Stability
$\mathcal{N} = 8$	$SO(8)$	$-6 (\times 1)$	✓
$\mathcal{N} = 0$	$SO(7)$	$-2\sqrt{5}\sqrt{5} (\times 1)$	×
		$-\frac{25}{8}\sqrt{5} (\times 2)$	×
$\mathcal{N} = 0$	$SU(4)$	$-8 (\times 1)$	×
$\mathcal{N} = 0$	$SO(4)$	$-14 (\times 2)$	✓
		$-2\sqrt{9 + 6\sqrt{3}} (\times 2)$	×

[Warner '84]

[Fischbacher, Pilch & Warner, '10]

Triality and the three embeddings $SO(4)_{v,s,c}$

- Let us take (without loss of generality) the 8 gravitini of the theory to transform as $\mathbf{8} = \mathbf{8}_v$ of $SO(8)$. Then, the 70 scalars will transform as

$$70 = 35_s \text{ (SD)} + i \, 35_c \text{ (ASD)}$$

- There are three **triality-related** embeddings of $SO(4) = SO(3)_1 \times SO(3)_2$

$N = 2$

$SO(8)_v$

$$\mathbf{8}_v \implies \mathbf{1} + \mathbf{3}_1 + \mathbf{1} + \mathbf{3}_2$$

$$\mathbf{8}_s \implies 4 + 4$$

$$\mathbf{8}_c \implies 4 + 4$$

$$35_s \implies 2 \text{ SD scalars}$$

$$35_c \implies 2 \text{ ASD scalars}$$

$N = 0$

$SO(8)_s$

$$\mathbf{8}_v \implies 4 + 4$$

$$\mathbf{8}_s \implies \mathbf{1} + \mathbf{3}_1 + \mathbf{1} + \mathbf{3}_2$$

$$\mathbf{8}_c \implies 4 + 4$$

$$35_s \implies 4 \text{ SD scalars}$$

$$35_c \implies 2 \text{ ASD scalars}$$

New $N = 0$

$SO(8)_c$

$$\mathbf{8}_v \implies 4 + 4$$

$$\mathbf{8}_s \implies 4 + 4$$

$$\mathbf{8}_c \implies \mathbf{1} + \mathbf{3}_1 + \mathbf{1} + \mathbf{3}_2$$

$$35_s \implies 2 \text{ SD scalars}$$

$$35_c \implies 4 \text{ ASD scalars}$$

- $3 = \mathbf{1} + \mathbf{2}$ splitting of the $SO(4)$ invariant sectors of $SO(8)$ gauged SUGRA

The vectorial $SO(4)_v$ embedding [$8_v = 1 + 3_1 + 1 + 3_2$]
 (1) (a) ($\hat{1}$) (\hat{a})

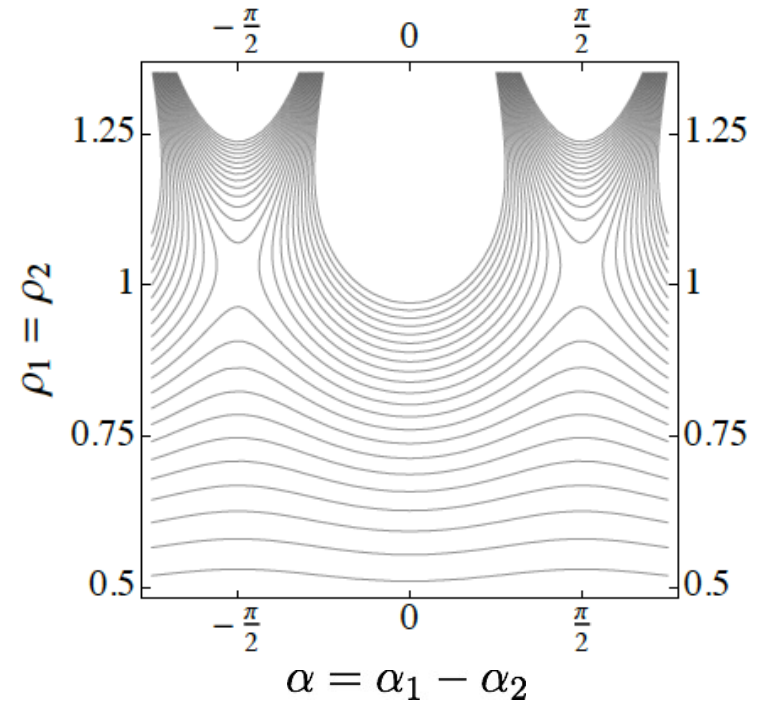
- 2 [SD] + 2 [ASD] invariant scalars

$$\phi_{1abc} = (\phi_{\hat{1}\hat{a}\hat{b}\hat{c}})^* = \rho_1 e^{i\alpha_1} \epsilon_{abc}$$

$$\phi_{\hat{1}abc} = -(\phi_{1\hat{a}\hat{b}\hat{c}})^* = \rho_2 e^{i\alpha_2} \epsilon_{abc}$$

- Scalar potential is ω -independent !!

$$V^v = -\frac{1}{8\rho^4} \left(32\rho_1^4 + 61\rho_1^2\rho_2^2 + 32\rho_2^4 \right. \\
 + 4 \cosh(2\rho) (4\rho_1^4 + 9\rho_1^2\rho_2^2 + 4\rho_2^4) \\
 \left. - \rho_1^2\rho_2^2 (\cosh(4\rho) - 8 \cos(2\alpha) \sinh^4(\rho)) \right)$$



- Stable critical points at $\alpha = \pm \frac{\pi}{2}$ and $\rho_1 = \rho_2 = \frac{1}{\sqrt{2}} \cosh^{-1}(\sqrt{5})$ with $V = -14$

- The $SO(4)_v$ solutions are **not** affected by the ω -phase !!

The spinorial $SO(4)_s$ embedding [$8_v = 4 + 4$]

(i) (\hat{i})

- 4 [SD] + 2 [ASD] invariant scalars

$$\phi_{ijkl} = (\phi_{\hat{i}\hat{j}\hat{k}\hat{l}})^* = (x_1 + iy_1) \epsilon_{ijkl}$$

$$\phi_{\hat{i}\hat{j}\hat{k}\hat{l}} = -(\phi_{ijkl})^* = \frac{1}{2} (x_2 + iy_2) \epsilon_{\hat{i}\hat{j}\hat{k}\hat{l}}$$

$$\phi_{\hat{i}\hat{j}\hat{k}\hat{l}} = x_3 \epsilon_{\hat{i}\hat{j}\hat{k}\hat{l}} + x_4 \delta_{[\hat{i}\hat{j}} \delta_{kl]}$$

- Scalar potential [modding out by a discrete D_4]

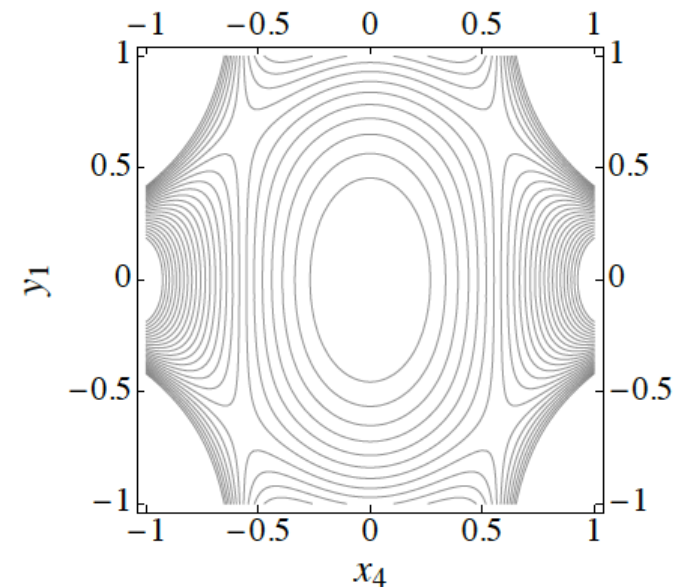
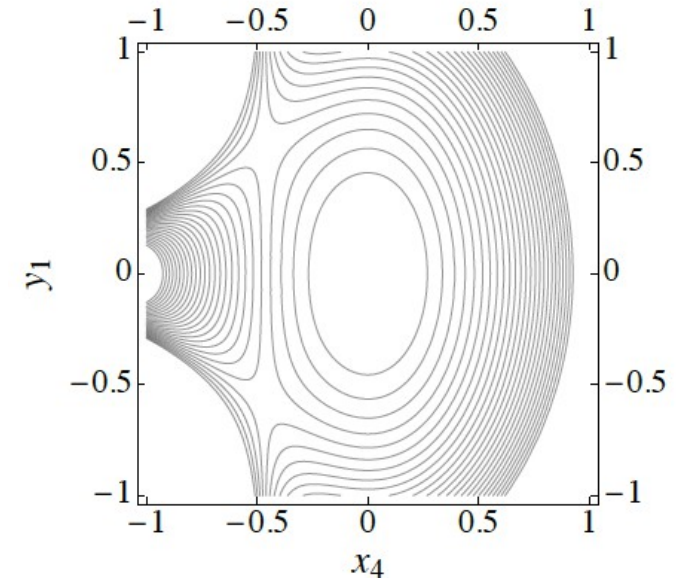
$$V^s = \cos^2(\omega) V_0^s(x_4, y_1) + \sin^2(\omega) V_0^s(-x_4, y_1)$$

with the $\omega = 0$ potential

$$V_0^s = \frac{1}{2} \sinh^2(y_1) (\cosh(6x_4) - 4 \sinh^3(2x_4)) - \frac{3}{4} \cosh(2x_4) (3 \cosh(2y_1) + 5) .$$

- i) $\omega = 0 \implies$ 2 unstable sols with $V = -2\sqrt{9 + 6\sqrt{3}}$
- ii) $\omega = \text{Pi}/4 \implies$ 4 unstable sols with $V = -6\sqrt{3}$

[standard SUGRA]



- $\text{Pi}/4$ periodicity is **broken** unless a new $SO(4)$ sector comes to the rescue

The new $SO(4)_c$ embedding [$8_v = 4 + 4$]

(i) (\hat{i})

- 2 [SD] + 4 [ASD] invariant scalars

$$\phi_{ijkl} = (\phi_{\hat{i}\hat{j}\hat{k}\hat{l}})^* = (x_1 + iy_1) \epsilon_{ijkl}$$

$$\phi_{\hat{i}\hat{j}\hat{k}\hat{l}} = -(\phi_{ijkl})^* = \frac{1}{2} (x_2 + iy_2) \epsilon_{\hat{i}\hat{j}\hat{k}\hat{l}}$$

$$\phi_{i\hat{j}\hat{k}\hat{l}} = i y_3 \epsilon_{i\hat{j}\hat{k}\hat{l}} + i y_4 \delta_{[i\hat{j}} \delta_{k\hat{l}]}$$

- Scalar potential [modding out by a discrete D_4]

$$V^c = V_0^c + 4 \sin \omega \cos \omega \sinh^2(x_1) \sinh^3(2y_4)$$

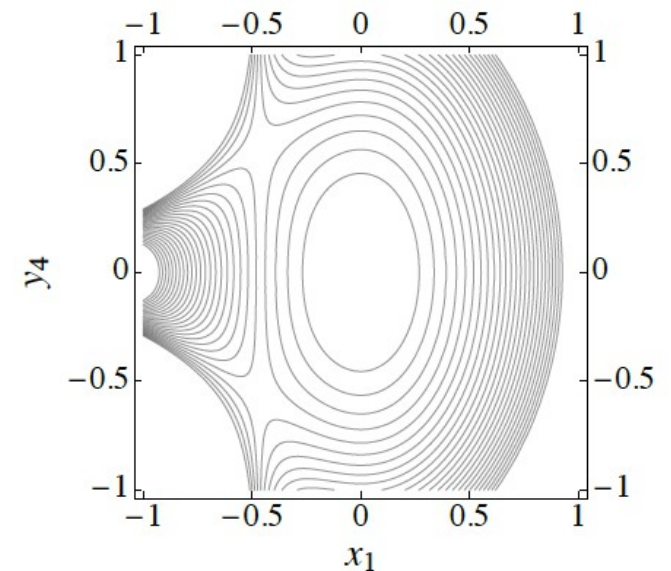
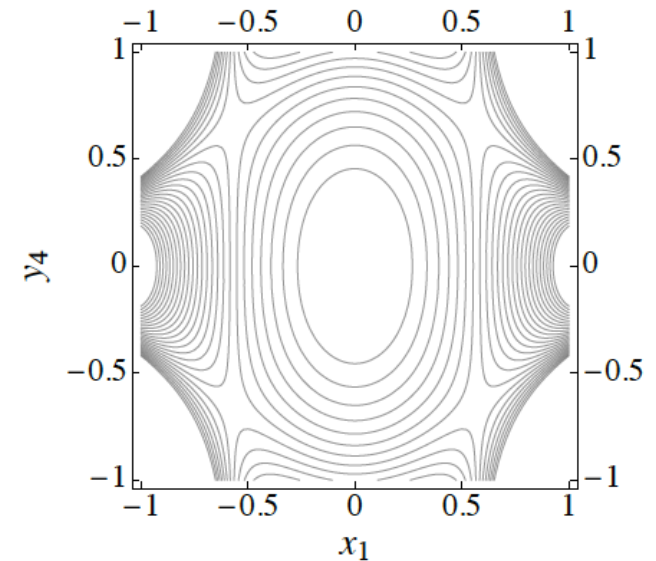
with the $\omega = 0$ potential

$$V_0^c = \frac{1}{2} \sinh^2(x_1) \cosh(6y_4) - \frac{3}{4} \cosh(2y_4) (3 \cosh(2x_1) + 5)$$

- i) $\omega = 0 \implies$ 4 unstable sols with $V = -6\sqrt{3}$
- ii) $\omega = \text{Pi}/4 \implies$ 2 unstable sols with $V = -2\sqrt{9 + 6\sqrt{3}}$

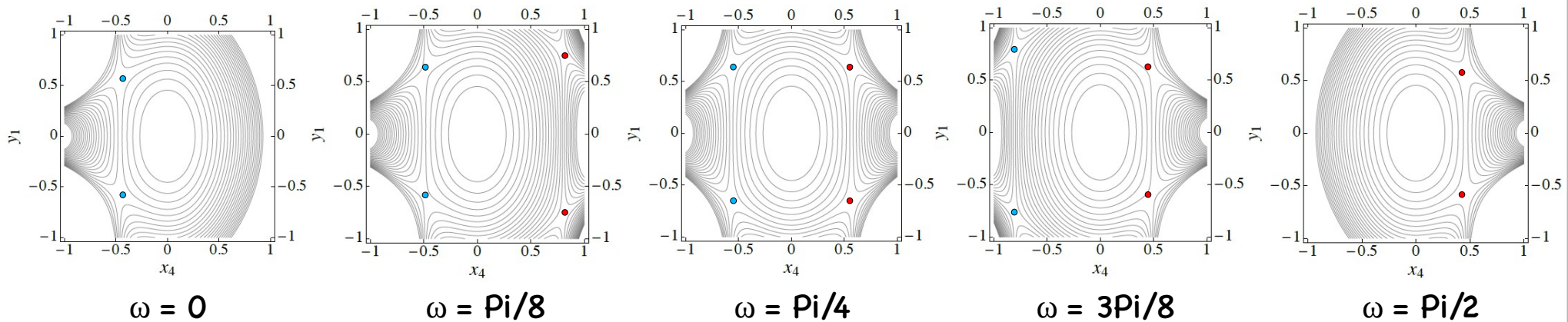
- The combination of the $SO(4)_s$ & $SO(4)_c$ sectors restores $\text{Pi}/4$ periodicity !!

[standard SUGRA]



Singular limits in the $SO(4)_s$ sector [analogous for $SO(4)_c$]

- One pair of solutions **runs away** when ω approaches 0 and $\pi/2$



- What happens to these solutions? Do they abandon the $SO(8)$ theory and that's why we see them disappearing? If so, where do they go to?
- Fortunately, there is a way of sitting on top of a solution and travel with it along the different theories (gaugings) which are compatible with it . . .

. . . the so-called **“Go To The Origin”** (GTTO) approach

The GTTO approach

Idea : Looking for the theories (gaugings) compatible with a given critical point instead of looking for the critical points compatible with a given theory

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Going To The Origin : If a critical point is found at $\phi = \phi_0$ with a residual symmetry G_{res} , it can always be brought to $\phi = 0$ via an E_7 transformation. After this, the quantities in the theory (e.g. fermion mass terms) will adopt a form compatible with G_{res} [$E_7/SU(8)$ is an **homogeneous space**]

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Applicability : Ansatz for the fermion masses compatible with $G_{\text{res}} = SO(4)_s \times D_4$

- gravitino-gravitino mass terms : $\mathcal{A}^{ij} = \alpha \delta^{ij}$, $\mathcal{A}^{\hat{i}\hat{j}} = \alpha \delta^{\hat{i}\hat{j}}$

- gravitino-dilatino mass terms : $\mathcal{A}_i{}^{jkl} = \beta \epsilon_i{}^{jkl}$, $\mathcal{A}_i{}^{\hat{j}\hat{k}\hat{l}} = -\delta \epsilon_i{}^{\hat{j}\hat{k}\hat{l}} + \gamma \delta_i^{[\hat{j}} \delta^{\hat{k}]l}$
 $\mathcal{A}_{\hat{i}}{}^{j\hat{k}\hat{l}} = -\beta \epsilon_{\hat{i}}{}^{j\hat{k}\hat{l}}$, $\mathcal{A}_{\hat{i}}{}^{jkl} = \delta \epsilon_{\hat{i}}{}^{jkl} + \gamma \delta_{\hat{i}}^{[j} \delta^{k]l}$

It allows for four free parameters $\alpha, \beta, \delta, \gamma \in \mathbb{C}$

Fermion masses for $G_{\text{res}} = \text{SO}(4)_s \times \text{D}_4$

- **Solve the QC & EOM** in order to find **all** the theories compatible with a critical point preserving $G_{\text{res}} = \text{SO}(4)_s \times \text{D}_4 \implies$ There is a **one-parameter** family of theories !!

- gravitino-gravitino mass terms :

$$\begin{aligned} \text{Re}[\alpha(\theta)] &= A \sin(\theta) (2\sqrt{2} \cos^2(\theta) + B) \\ \text{Im}[\alpha(\theta)] &= A \cos(\theta) (2\sqrt{2} \sin^2(\theta) - B) \end{aligned}$$

- gravitino-dilatino mass terms :

$$\begin{aligned} \text{Re}[\beta(\theta)] &= \cos(\theta) \left(1 - \cos(2\theta) - \frac{B}{\sqrt{2}} \right) \\ \text{Im}[\beta(\theta)] &= \sin(\theta) \left(1 + \cos(2\theta) + \frac{B}{\sqrt{2}} \right) \\ \delta(\theta) &= e^{i\theta} \quad \gamma(\theta) = -i 2 \sqrt{2} A e^{-i\theta} \end{aligned}$$

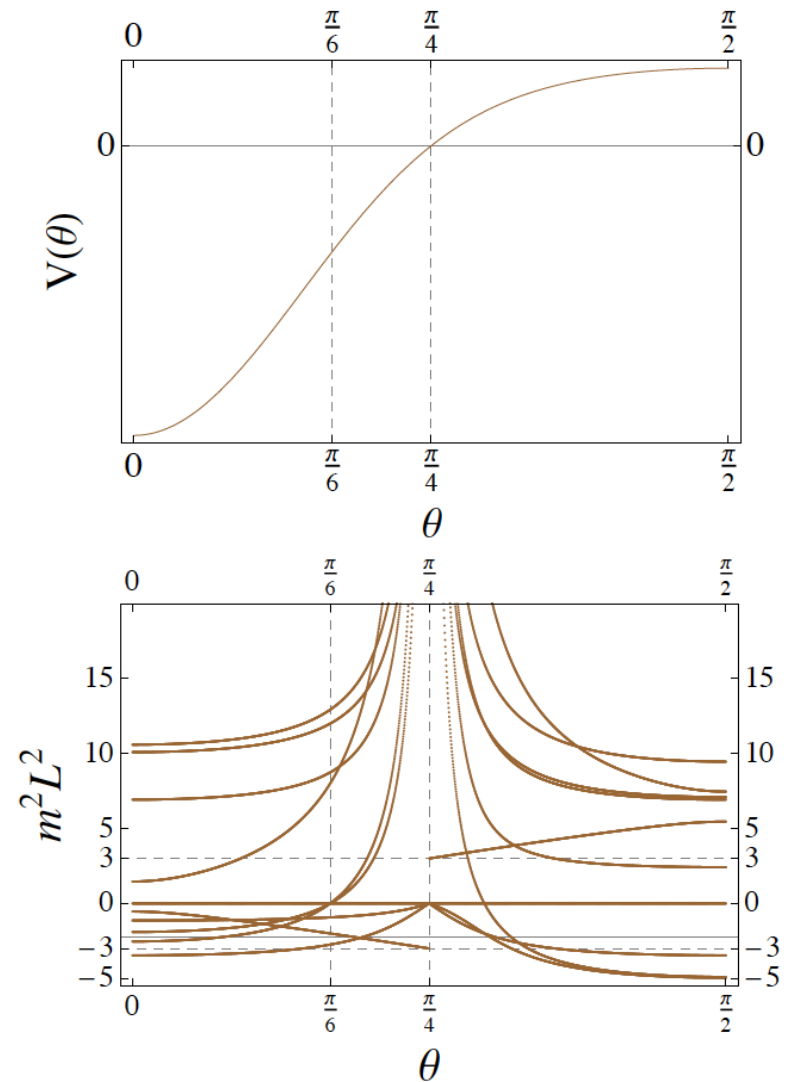
with $A = \frac{1}{2}(\sqrt{2} \cos(2\theta)B + \cos(4\theta) + 3)^{1/2}$ and $B = (\cos(4\theta) + 5)^{1/2}$

- The **scalar potential** interpolates between AdS and dS solutions for $\theta = [0, \text{Pi}/2]$

$$V(\theta) = -6 (1 + \cos(4\theta) + \sqrt{2} \cos(2\theta) B)$$

Gauge group & AdS/Mkw/dS transitions

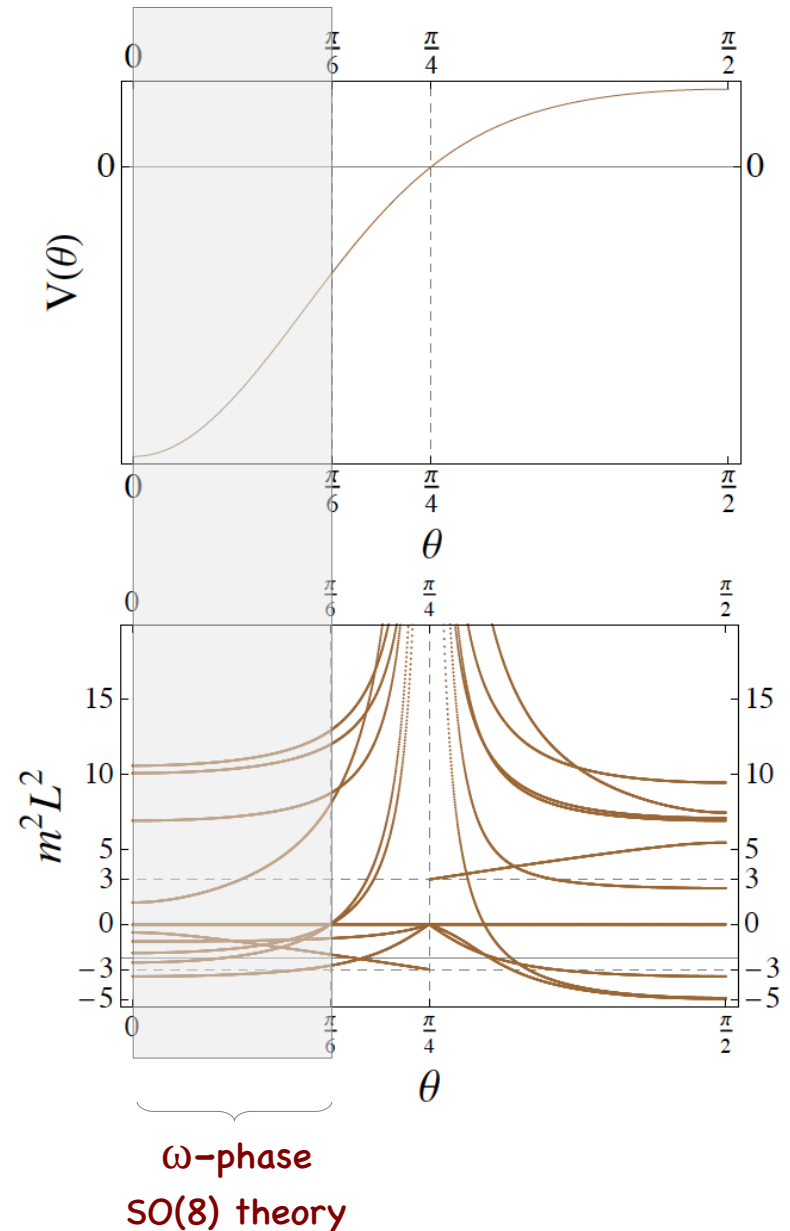
- The whole story of the solution preserving $G_{\text{res}} = SO(4)_s \times D_4$ can be tracked



Gauge group & AdS/Mkw/dS transitions

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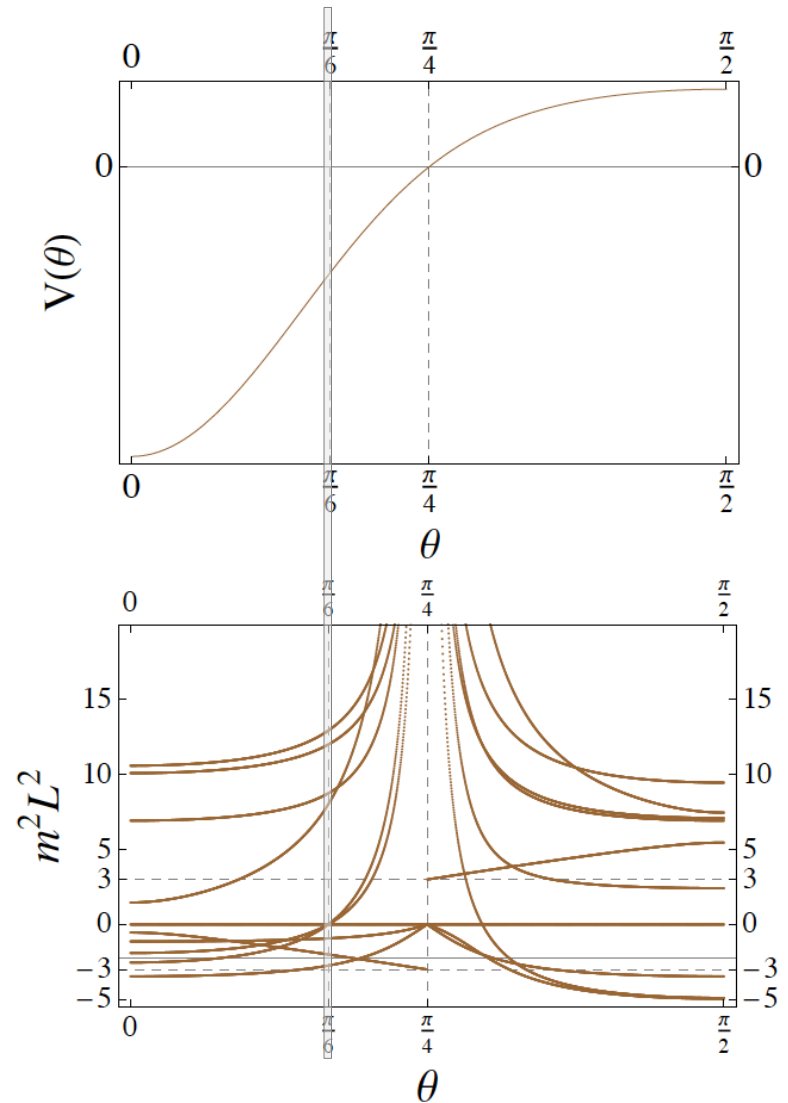
i) $\theta = [0, \pi/6)$ \implies $SO(8)$ gauging
 [unstable AdS solutions]



Gauge group & AdS/Mkw/dS transitions

- The whole story of the solution preserving $G_{\text{res}} = SO(4)_s \times D_4$ can be tracked

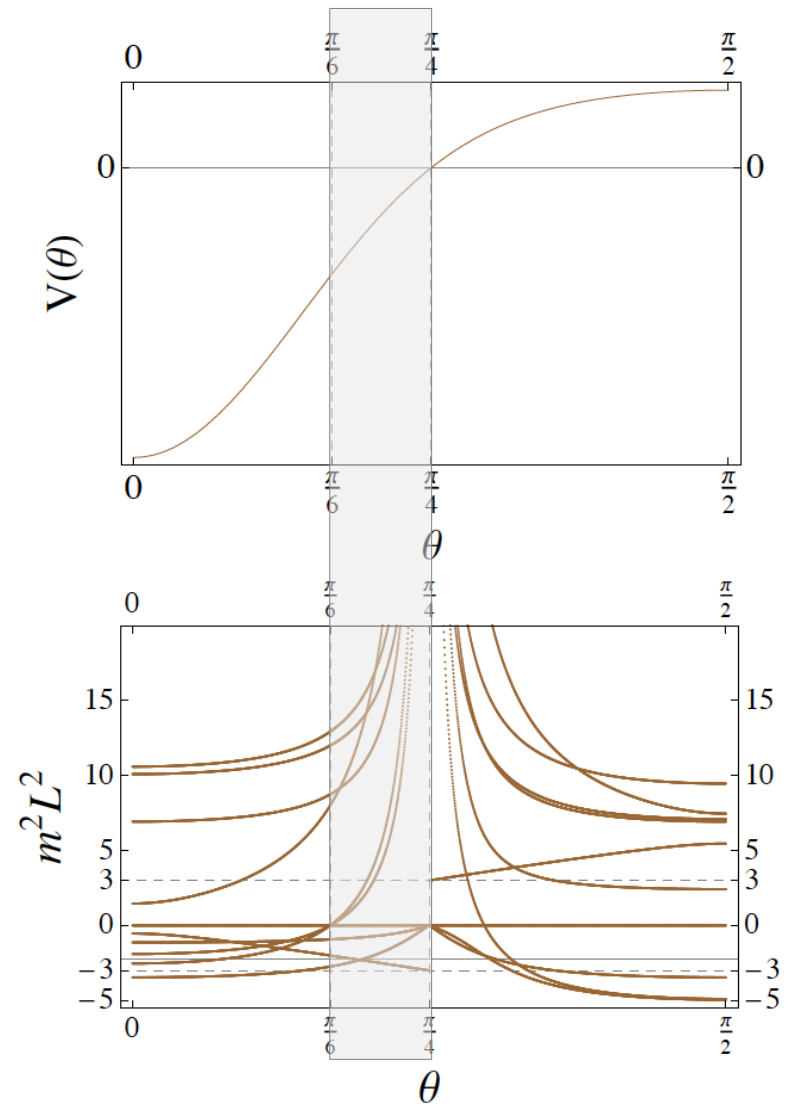
ii) $\theta = \pi/6 \implies SO(2) \times SO(6) \times_s T^{12}$ gauging
 [unstable AdS solution]



Gauge group & AdS/Mkw/dS transitions

- The whole story of the solution preserving $G_{\text{res}} = SO(4)_s \times D_4$ can be tracked

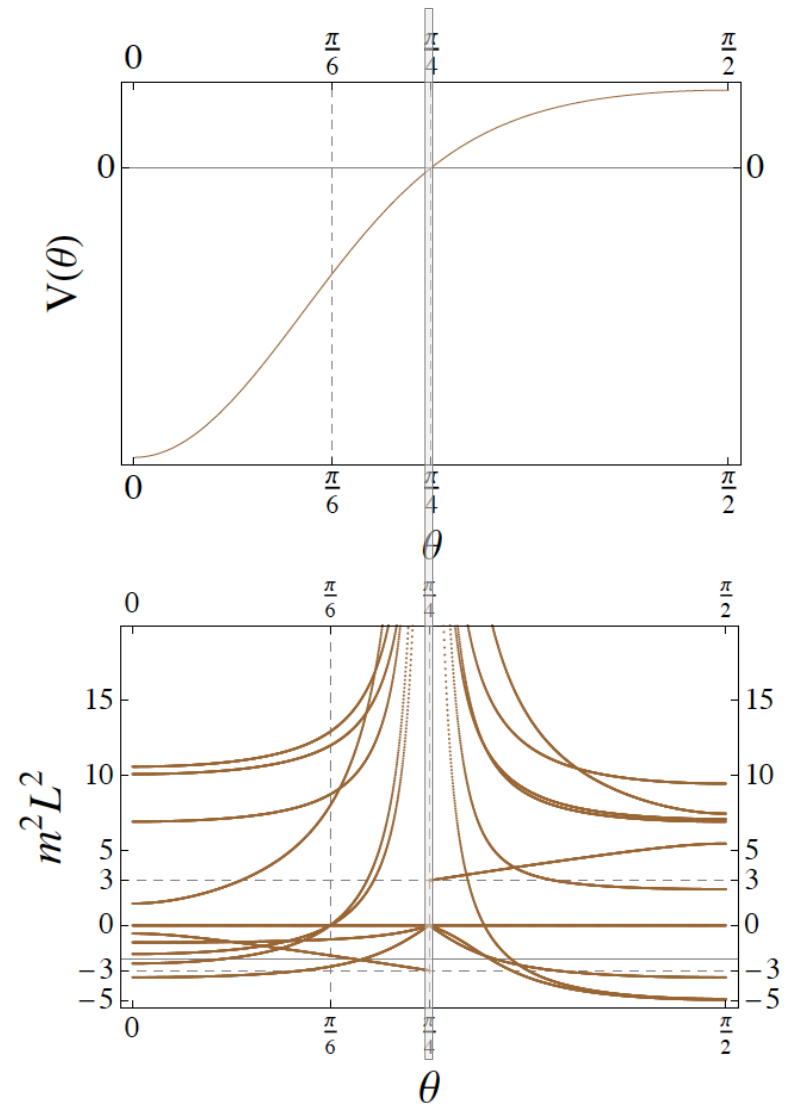
iii) $\theta = (\pi/6, \pi/4) \implies SO(6,2)$ gauging
[unstable AdS solutions]



Gauge group & AdS/Mkw/dS transitions

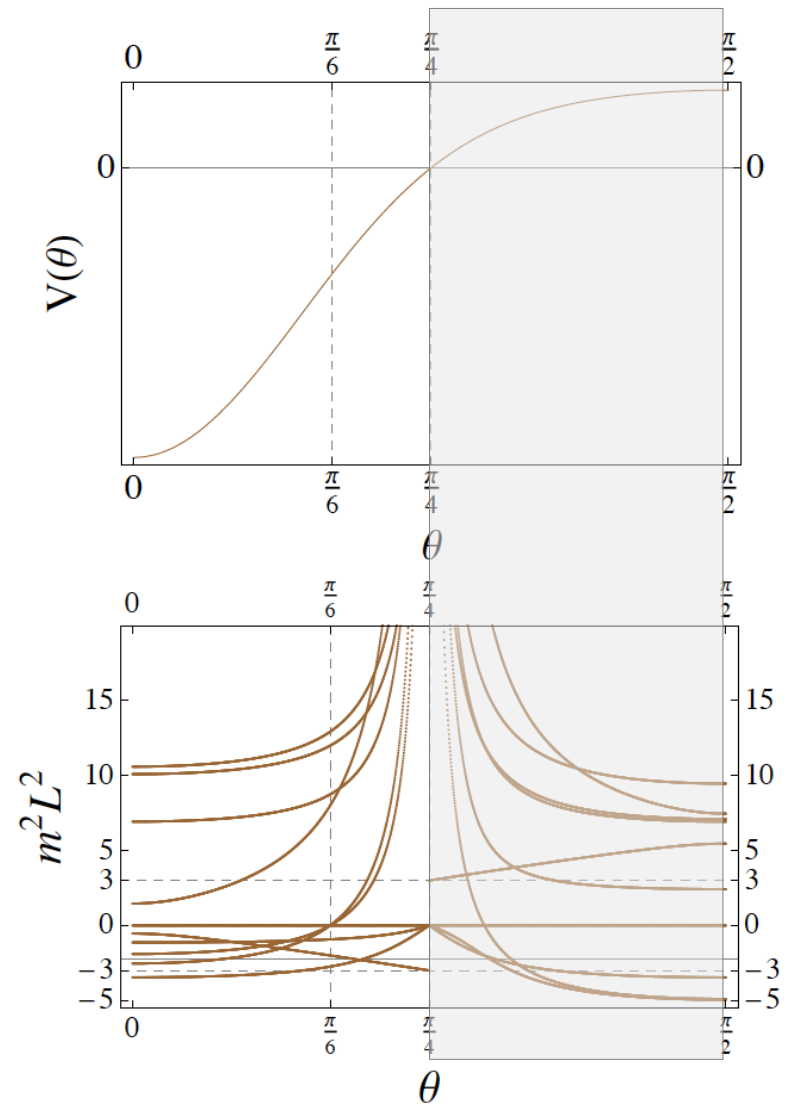
- The whole story of the solution preserving $G_{\text{res}} = SO(4)_s \times D_4$ can be tracked

iv) $\theta = \text{Pi}/4 \implies SO(3,1)^2 \times_s T^{16}$ gauging
 [Mkw solution without tachyons]



Gauge group & AdS/Mkw/dS transitions

- The whole story of the solution preserving $G_{\text{res}} = SO(4)_s \times D_4$ can be tracked



v) $\theta = (\pi/4, \pi/2]$ \implies $SO(4,4)$ gauging
 [unstable dS solutions]

[Dall'Agata & Inverso '12]

Tachyon
 dilution

Outlook

- 1) The old $SO(8)$ gauged SUGRA
- 2) The embedding tensor & the new $SO(8)$ gauged SUGRA
- 3) Invariant sectors of new $SO(8)$ gauged SUGRA :
 - 3.2) The $SU(3)$ invariant sector
 - 3.3) The $SO(4)$ invariant sector
- 4) Collecting results & final remarks

SU(3) & SO(4) invariant critical points of new SO(8) gauged SUGRA

SUSY	Symmetry	CC ($\omega = 0$)	Stability	CC ($\omega = \pi/8$)	Stability	ω -dep masses
$\mathcal{N} = 8$	SO(8)	-6 ($\times 1$)	✓	-6 ($\times 1$)	✓	×
$\mathcal{N} = 2$	SU(3) \times U(1)	-7.794 ($\times 1$)	✓	-8.354 ($\times 2$)	✓	×
$\mathcal{N} = 1$	G_2	-7.192 ($\times 2$)	✓	-7.943 ($\times 2$)	✓	×
		-	-	-7.040 ($\times 1$)	✓	×
$\mathcal{N} = 1$	SU(3)	-	-	-10.392 ($\times 1$)	✓	×
$\mathcal{N} = 0$	SO(7)	-6.687 ($\times 1$)	×	-6.748 ($\times 2$)	×	×
		-6.988 ($\times 2$)	×	-7.771 ($\times 2$)	×	×
$\mathcal{N} = 0$	SU(4)	-8 ($\times 1$)	×	-8.581 ($\times 2$)	×	×
$\mathcal{N} = 0$	G_2	-	-	-10.170 ($\times 1$)	✓	×
$\mathcal{N} = 0$	SU(3)	-	-	-10.237 ($\times 2$)	✓	✓
$\mathcal{N} = 0$	SO(4)	-14 ($\times 2$)	✓	-14 ($\times 2$)	✓	×
		-8.807 ($\times 2$)	×	-9.110 ($\times 2$)	×	✓
		-	-	-15.599 ($\times 2$)	×	✓
		-10.392 ($\times 4$)	×	-15.599 ($\times 2$)	×	✓
				-9.110 ($\times 2$)	×	✓



Standard supergravity



New supergravity

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		-	×	-9.110 ($\times 2$)	×	✓

[genuine]

Standard supergravity

New supergravity

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$\mathcal{N} = 0$	SO(4)	-14 ($\times 2$)	✓	-14 ($\times 2$)	✓	×
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		-	×	-15.599 ($\times 2$)	×	✓
		-10.392 ($\times 4$)	×	-15.599 ($\times 2$)	×	✓
				-9.110 ($\times 2$)	×	✓

[ω -dep]



Standard supergravity



New supergravity

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		-8.807 ($\times 2$)	\times	-9.110 ($\times 2$)	\times	✓
		-	-	-15.599 ($\times 2$)	\times	✓
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	[standard]	-10.392 ($\times 4$)	\times	-9.110 ($\times 2$)	\times	✓



Standard supergravity



New supergravity

Final remarks

Mass spectra

- One has to go to small $SU(3)$ & $SO(4)$ residual groups to start seeing ω -dependent mass spectra \implies The residual symmetry does not uniquely determine masses !!

Periodicity

- **Triality** enters the game to restore $\pi/4$ periodicity

Tachyon amelioration

- Tachyons can get diluted around AdS/Mkw/dS transitions \implies stable dS in $N=8$?

Domain-walls and RG flows

- ω -dependent $N=2$ superpotential for the $SU(3)$ -inv sector \implies New domain-wall solutions at $\omega \neq 0$ \implies Prediction of the free energy F_{IR}/F_{UV}

[rely on M-theory embedding]

Lifting to 11d SUGRA?

- It seems to require vectors from dimensional reduction of A_3 and A_6 , so ...

... thank you all !!