M-theory beyond twisted tori

Adolfo Guarino

Albert Einstein Center (ITP)
Bern, Switzerland

Recent Developments in String Theory
24 July 2014, Ascona

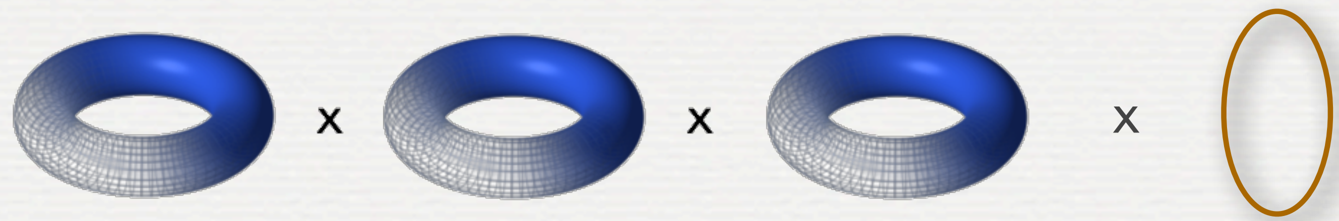
In collaboration with Jean-Pierre Derendinger [arXiv:1406.6930]

... one-minute summary

Twisted torus $X_7 = T^7/(Z_2 \times Z_2 \times Z_2)$

[Dall'Agata & Prezas '05]
[Derendinger & A.G '14]

- Factorisation :



(η^1, η^2) (η^3, η^4) (η^5, η^6) η^7

- Twist specified by a metric flux

$$d\eta^A + \frac{1}{2}\,\omega_{BC}{}^A \eta^B \wedge \eta^C = 0 \qquad [\, A = 1\,, \dots, 7\,]$$

- Background gauge fluxes

$$\frac{1}{2}\,G_{(4)} = -\,a_1\,\eta^{3456} + \dots \qquad \text{and} \qquad \frac{1}{4}\,G_{(7)} = a_0\,\eta^{1234567}$$

- G_2-structure : 7 moduli fields

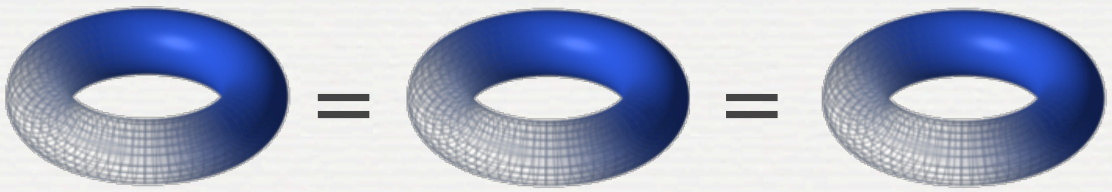
$$\frac{1}{2}(A_{(3)} + i\Phi_{(3)}) = U_1\,\eta^{127} + U_2\,\eta^{347} + U_3\,\eta^{567} + S\,\eta^{135} - T_1\,\eta^{146} - T_2\,\eta^{362} - T_3\,\eta^{524}$$

Twisted torus $X_7 = T^7/(Z_2 \times Z_2 \times Z_2)$

[Dall'Agata & Prezas '05]
[Derendinger & A.G '14]

- Cyclic plane-exchange-symmetry

  [Derendinger, Kounnas, Petropoulos & Zwirner '04]



(η^1, η^2) (η^3, η^4) (η^5, η^6)

$$U_1 = U_2 = U_3 \equiv U \qquad \text{and} \qquad T_1 = T_2 = T_3 \equiv T$$

- M-theory flux-induced superpotential as an STU-model

$$\begin{aligned} W_{\text{M-theory}} &= a_0 - b_0\, S + 3\, c_0\, T - 3\, a_1\, U + 3\, a_2\, U^2 + 3\,(2\, c_1 - \tilde{c}_1)\, U\, T + 3\, b_1\, S\, U \\ &- 3\, c_3'\, T^2 - 3\, d_0\, S\, T \end{aligned}$$

  - Couplings : $G_{(7)} =$ cte , $G_{(4)} =$ linear and metric = quadratic

M-theory *vs* Type IIA interpretation

[Dall'Agata & Prezas '05]
[Derendinger & A.G '14]

- M-theory ➡ Type IIA orientifold upon reduction along η^7

M-theory origin	Type IIA origin	Flux/coupling
$\omega_{bc}{}^{a}$	$\omega_{bc}{}^{a}$	$\tilde{c}_1$
$\omega_{ka}{}^{j}$	$\omega_{ka}{}^{j}$	c_1
$\omega_{jk}{}^{a}$	$\omega_{jk}{}^{a}$	b_1
$-\omega_{ai}{}^{7}$	F_{ai}	a_2
$-\omega_{7i}{}^{a}$	non-geometric	d_0
$-\omega_{a7}{}^{i}$	non-geometric	c'_3
$-\frac{1}{2}\,G_{aibj}$	$-F_{aibj}$	a_1
$\frac{1}{2}\,G_{ijk7}$	H_{ijk}	b_0
$\frac{1}{2}\,G_{ibc7}$	H_{ibc}	c_0
$\frac{1}{4}\,G_{aibjck7}$	F_{aibjck}	a_0

- Index splitting A $\rightarrow$ ($a = 1,3,5$) + ($i = 2,4,6$) + 7

M-theory *vs* Type IIA interpretation

[Dall'Agata & Prezas '05]
[Derendinger & A.G '14]

- M-theory ➡ Type IIA orientifold upon reduction along η^7

M-theory origin	Type IIA origin	Flux/coupling
${\omega_{bc}}^{a}$	${\omega_{bc}}^{a}$	$\tilde{c}_1$
${\omega_{ka}}^{j}$	${\omega_{ka}}^{j}$	c_1
${\omega_{jk}}^{a}$	${\omega_{jk}}^{a}$	b_1
$-{\omega_{ai}}^{7}$	F_{ai}	a_2
$-{\omega_{7i}}^{a}$	non-geometric	d_0
$-{\omega_{a7}}^{i}$	non-geometric	c'_3
$-\frac{1}{2}\,G_{aibj}$	$-F_{aibj}$	a_1
$\frac{1}{2}\,G_{ijk7}$	H_{ijk}	b_0
$\frac{1}{2}\,G_{ibc7}$	H_{ibc}	c_0
$\frac{1}{4}\,G_{aibjck7}$	F_{aibjck}	a_0

7d twist in M-theory ➡ *6d* twist in IIA

- Index splitting $A \to (\, a = 1,3,5 \,) + (\, i = 2,4,6 \,) + 7$

M-theory *vs* Type IIA interpretation

[Dall'Agata & Prezas '05]
[Derendinger & A.G '14]

- M-theory ➡ Type IIA orientifold upon reduction along η^7

M-theory origin	Type IIA origin	Flux/coupling
${\omega_{bc}}^a$	${\omega_{bc}}^a$	$\tilde{c}_1$
${\omega_{ka}}^j$	${\omega_{ka}}^j$	c_1
${\omega_{jk}}^a$	${\omega_{jk}}^a$	b_1
$-{\omega_{ai}}^7$	F_{ai}	a_2
$-{\omega_{7i}}^a$	non-geometric	d_0
$-{\omega_{a7}}^i$	non-geometric	c'_3
$-\frac{1}{2}\,G_{aibj}$	$-F_{aibj}$	a_1
$\frac{1}{2}\,G_{ijk7}$	H_{ijk}	b_0
$\frac{1}{2}\,G_{ibc7}$	H_{ibc}	c_0
$\frac{1}{4}\,G_{aibjck7}$	F_{aibjck}	a_0

7d twist in M-theory ➡ $F_{(2)}$ in IIA

- Index splitting $A \rightarrow (\,a=1,3,5\,) + (\,i=2,4,6\,) + 7$

M-theory *vs* Type IIA interpretation

[Dall'Agata & Prezas '05]
[Derendinger & A.G '14]

- M-theory ➡ Type IIA orientifold upon reduction along η^7

M-theory origin	Type IIA origin	Flux/coupling
${\omega_{bc}}^a$	${\omega_{bc}}^a$	$\tilde{c}_1$
${\omega_{ka}}^j$	${\omega_{ka}}^j$	c_1
${\omega_{jk}}^a$	${\omega_{jk}}^a$	b_1
$-{\omega_{ai}}^7$	F_{ai}	a_2
$-{\omega_{7i}}^a$	non-geometric	d_0
$-{\omega_{a7}}^i$	non-geometric	c'_3
$-\frac{1}{2}\,G_{aibj}$	$-F_{aibj}$	a_1
$\frac{1}{2}\,G_{ijk7}$	H_{ijk}	b_0
$\frac{1}{2}\,G_{ibc7}$	H_{ibc}	c_0
$\frac{1}{4}\,G_{aibjck7}$	F_{aibjck}	a_0

7d twist in M-theory ➡ non-geom in IIA

[Shelton, Taylor & Wecht '05]
[Aldazabal, Cámara, Font & Ibáñez '06]

- Index splitting A $\rightarrow$ ($a = 1,3,5$) + ($i = 2,4,6$) + 7

M-theory *vs* Type IIA interpretation

[Dall'Agata & Prezas '05]
[Derendinger & A.G '14]

- M-theory ➡ Type IIA orientifold upon reduction along η^7

M-theory origin	Type IIA origin	Flux/coupling
${\omega_{bc}}^a$	${\omega_{bc}}^a$	$\tilde{c}_1$
${\omega_{ka}}^j$	${\omega_{ka}}^j$	c_1
${\omega_{jk}}^a$	${\omega_{jk}}^a$	b_1
$-{\omega_{ai}}^7$	F_{ai}	a_2
$-{\omega_{7i}}^a$	non-geometric	d_0
$-{\omega_{a7}}^i$	non-geometric	c'_3
$-\frac{1}{2}\,G_{aibj}$	$-F_{aibj}$	a_1
$\frac{1}{2}\,G_{ijk7}$	H_{ijk}	b_0
$\frac{1}{2}\,G_{ibc7}$	H_{ibc}	c_0
$\frac{1}{4}\,G_{aibjck7}$	F_{aibjck}	a_0

$G_{(4)}$ in M-theory ➡ $F_{(4)}$ in IIA

- Index splitting $A \rightarrow (\,a = 1,3,5\,) + (\,i = 2,4,6\,) + 7$

M-theory *vs* Type IIA interpretation

[Dall'Agata & Prezas '05]
[Derendinger & A.G '14]

- M-theory ➡ Type IIA orientifold upon reduction along η^7

M-theory origin	Type IIA origin	Flux/coupling
$\omega_{bc}{}^{a}$	$\omega_{bc}{}^{a}$	$\tilde{c}_1$
$\omega_{ka}{}^{j}$	$\omega_{ka}{}^{j}$	c_1
$\omega_{jk}{}^{a}$	$\omega_{jk}{}^{a}$	b_1
$-\omega_{ai}{}^{7}$	F_{ai}	a_2
$-\omega_{7i}{}^{a}$	non-geometric	d_0
$-\omega_{a7}{}^{i}$	non-geometric	c'_3
$-\frac{1}{2}\, G_{aibj}$	$-F_{aibj}$	a_1
$\frac{1}{2}\, G_{ijk7}$	H_{ijk}	b_0
$\frac{1}{2}\, G_{ibc7}$	H_{ibc}	c_0
$\frac{1}{4}\, G_{aibjck7}$	F_{aibjck}	a_0

$G_{(4)}$ in M-theory ➡ $H_{(3)}$ in IIA

- Index splitting A $\rightarrow$ (a = 1,3,5) + (i = 2,4,6) + 7

M-theory *vs* Type IIA interpretation

[Dall'Agata & Prezas '05]
[Derendinger & A.G '14]

- M-theory ➡ Type IIA orientifold upon reduction along η^7

M-theory origin	Type IIA origin	Flux/coupling
${\omega_{bc}}^{a}$	${\omega_{bc}}^{a}$	$\tilde{c}_1$
${\omega_{ka}}^{j}$	${\omega_{ka}}^{j}$	c_1
${\omega_{jk}}^{a}$	${\omega_{jk}}^{a}$	b_1
$-{\omega_{ai}}^{7}$	F_{ai}	a_2
$-{\omega_{7i}}^{a}$	non-geometric	d_0
$-{\omega_{a7}}^{i}$	non-geometric	c'_3
$-\frac{1}{2}\,G_{aibj}$	$-F_{aibj}$	a_1
$\frac{1}{2}\,G_{ijk7}$	H_{ijk}	b_0
$\frac{1}{2}\,G_{ibc7}$	H_{ibc}	c_0
$\frac{1}{4}\,G_{aibjck7}$	F_{aibjck}	a_0

$G_{(7)}$ in M-theory ➡ $F_{(6)}$ in IIA

- Index splitting $\mathrm{A} \rightarrow (\, a = 1,3,5 \,) + (\, i = 2,4,6 \,) + 7$

Question :

What are the consequences of turning on the two genuine M-theory metric fluxes (c_3', d_0) being non-geometric in type IIA ?

$$W_{\text{M-theory}} \;=\; W_{\text{IIA}} \; - 3\,c_3'\,T^2 - 3\,d_0\,S\,T$$

- Recall : $c_3' = {\omega_{7a}}^i$ and $d_0 = {\omega_{i7}}^a$

Twisted tori as Scherk-Schwarz (SS) reductions

[Scherk & Schwarz '79]

- An ordinary SS reduction of M-theory (32 supercharges) requires

[Dall'Agata & Prezas '05]

$$\omega_{[AB}{}^{F}\,\omega_{C]F}{}^{D} = 0 \qquad \text{and} \qquad \underbrace{\omega_{[AB}{}^{F}\,G_{CDE]F}}_{} = 0$$

guaranteed by the $Z_2 \times Z_2 \times Z_2$ symmetries !!

- In terms of the flux parameters

$$\begin{array}{rlcr}
i) & \omega_{[ai}{}^{D}\,\omega_{c]D}{}^{k} = 0 & \rightarrow & -a_2\,c_3' + c_1\,(c_1 - \tilde{c}_1) = 0 \\
ii) & \omega_{[ai}{}^{D}\,\omega_{k]D}{}^{c} = 0 & \rightarrow & -d_0\,a_2 + (c_1 - \tilde{c}_1)\,b_1 = 0 \\
iii) & \omega_{[ib}{}^{D}\,\omega_{c]D}{}^{7} = 0 & \rightarrow & a_2\,(2\,c_1 - \tilde{c}_1) = 0 \\
iv) & \omega_{[ij}{}^{D}\,\omega_{k]D}{}^{7} = 0 & \rightarrow & 3\,b_1\,a_2 = 0 \\
v) & \omega_{[7a}{}^{D}\,\omega_{b]D}{}^{k} = 0 & \rightarrow & (2\,c_1 - \tilde{c}_1)\,c_3' = 0 \\
vi) & \omega_{[7a}{}^{D}\,\omega_{j]D}{}^{c} = 0 & \rightarrow & b_1 c_3' + (c_1 - \tilde{c}_1)\,d_0 = 0 \\
vii) & \omega_{[7i}{}^{D}\,\omega_{j]D}{}^{k} = 0 & \rightarrow & b_1 c_3' + 2\,c_1\,d_0 = 0
\end{array}$$

NO moduli stabilisation if all of them are imposed !!

[Derendinger & A.G '14]

- Index splitting A $\rightarrow$ ($a = 1,3,5$) + ($i = 2,4,6$) + 7

Beyond twisted tori by including sources

- Could some of the previous SS conditions be relaxed ? [Villadoro & Zwirner '07]

$$\omega_{[\bullet\bullet}{}^{D}\,\omega_{\bullet]D}{}^{\psi} \neq 0 \;\Rightarrow\; \text{Non-vanishing KK6 (KKO6) charge}$$

- The inclusion of KK6 sources will break some of the 32 supercharges

Type	x^0	x^1	x^2	x^3	η^a	η^i	η^b	η^j	η^c	η^k	η^7	KK6 $\rightarrow$ type IIA
$i)$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		ψ			$\times$	KK5 (KKO5)
$ii)$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	ψ				$\times$	$\widetilde{\text{KK5}}$ ($\widetilde{\text{KKO5}}$)
$iii)$	$\times$	$\times$	$\times$	$\times$	$\times$			$\times$		$\times$	ψ	$\text{D6}_\perp$ ($\text{O6}_\perp$)
$iv)$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$		$\times$		ψ	$\text{D6}_\parallel$ ($\text{O6}_\parallel$)
$v)$	$\times$	$\times$	$\times$	$\times$	$\times$	ψ		$\times$		$\times$		$\text{KK6}_\perp$ ($\text{KKO6}_\perp$)
$vi)$	$\times$	$\times$	$\times$	$\times$	$\times$		ψ	$\times$		$\times$		$\widetilde{\text{KK6}}_\perp$ ($\widetilde{\text{KKO6}}_\perp$)
$vii)$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$		$\times$	ψ		$\text{KK6}_\parallel$ ($\text{KKO6}_\parallel$)

- Index splitting A $\rightarrow$ ($a = 1,3,5$) + ($i = 2,4,6$) + 7

Beyond twisted tori by including sources

- Could some of the previous SS conditions be relaxed ? [Villadoro & Zwirner '07]

$$\omega_{[\bullet\bullet}{}^{D}\,\omega_{\bullet]D}{}^{\psi} \neq 0 \;\Rightarrow\; \text{Non-vanishing KK6 (KKO6) charge}$$

- The inclusion of KK6 sources will break some of the 32 supercharges

Type	x^0	x^1	x^2	x^3	η^a	η^i	η^b	η^j	η^c	η^k	η^7	KK6 $\rightarrow$ type IIA
i)	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		ψ			$\times$	KK5 (KKO5)
ii)	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	ψ				$\times$	$\widetilde{\text{KK5}}$ ($\widetilde{\text{KKO5}}$)
iii)	$\times$	$\times$	$\times$	$\times$	$\times$			$\times$		$\times$	ψ	$\text{D6}_{\perp}$ ($\text{O6}_{\perp}$)
iv)	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$		$\times$		ψ	$\text{D6}_{\parallel}$ ($\text{O6}_{\parallel}$)
v)	$\times$	$\times$	$\times$	$\times$	$\times$	ψ		$\times$		$\times$		$\text{KK6}_{\perp}$ ($\text{KKO6}_{\perp}$)
vi)	$\times$	$\times$	$\times$	$\times$	$\times$		ψ	$\times$		$\times$		$\widetilde{\text{KK6}}_{\perp}$ ($\widetilde{\text{KKO6}}_{\perp}$)
vii)	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$		$\times$	ψ		$\text{KK6}_{\parallel}$ ($\text{KKO6}_{\parallel}$)

KK6 $\Rightarrow$ KK5 in IIA

[Villadoro & Zwirner '07]

- Index splitting A $\rightarrow$ ($a = 1,3,5$) + ($i = 2,4,6$) + 7

Beyond twisted tori by including sources

- Could some of the previous SS conditions be relaxed ? [Villadoro & Zwirner '07]

$$\omega_{[\bullet\bullet}{}^{D}\,\omega_{\bullet]D}{}^{\psi} \neq 0 \;\Rightarrow\; \text{Non-vanishing KK6 (KKO6) charge}$$

- The inclusion of KK6 sources will break some of the 32 supercharges

Type	x^0	x^1	x^2	x^3	η^a	η^i	η^b	η^j	η^c	η^k	η^7	KK6 $\rightarrow$ type IIA
$i)$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		ψ			$\times$	KK5 (KKO5)
$ii)$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	ψ				$\times$	$\widetilde{\text{KK5}}$ ($\widetilde{\text{KKO5}}$)
$iii)$	$\times$	$\times$	$\times$	$\times$	$\times$			$\times$		$\times$	ψ	$\text{D6}_\perp$ ($\text{O6}_\perp$)
$iv)$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$		$\times$		ψ	$\text{D6}_\parallel$ ($\text{O6}_\parallel$)
$v)$	$\times$	$\times$	$\times$	$\times$	$\times$	ψ		$\times$		$\times$		$\text{KK6}_\perp$ ($\text{KKO6}_\perp$)
$vi)$	$\times$	$\times$	$\times$	$\times$	$\times$		ψ	$\times$		$\times$		$\widetilde{\text{KK6}}_\perp$ ($\widetilde{\text{KKO6}}_\perp$)
$vii)$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$		$\times$	ψ		$\text{KK6}_\parallel$ ($\text{KKO6}_\parallel$)

KK6 $\rightarrow$ D6 in IIA

[Villadoro & Zwirner '07]

- Index splitting A $\rightarrow$ (a = 1,3,5) + (i = 2,4,6) + 7

Beyond twisted tori by including sources

- Could some of the previous SS conditions be relaxed ? [Villadoro & Zwirner '07]

$$\omega_{[\bullet\bullet}{}^{D}\,\omega_{\bullet]D}{}^{\psi} \neq 0 \;\Rightarrow\; \text{Non-vanishing KK6 (KKO6) charge}$$

- The inclusion of KK6 sources will break some of the 32 supercharges

Type	x^0	x^1	x^2	x^3	η^a	η^i	η^b	η^j	η^c	η^k	η^7	KK6 $\rightarrow$ type IIA
$i)$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$		ψ			$\times$	KK5 (KKO5)
$ii)$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	ψ				$\times$	$\widetilde{\text{KK5}}$ ($\widetilde{\text{KKO5}}$)
$iii)$	$\times$	$\times$	$\times$	$\times$	$\times$			$\times$		$\times$	ψ	$D6_{\perp}$ ($O6_{\perp}$)
$iv)$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$		$\times$		ψ	$D6_{\parallel}$ ($O6_{\parallel}$)
$v)$	$\times$	$\times$	$\times$	$\times$	$\times$	ψ		$\times$		$\times$		$KK6_{\perp}$ ($KKO6_{\perp}$)
$vi)$	$\times$	$\times$	$\times$	$\times$	$\times$		ψ	$\times$		$\times$		$\widetilde{KK6}_{\perp}$ ($\widetilde{KKO6}_{\perp}$)
$vii)$	$\times$	$\times$	$\times$	$\times$	$\times$		$\times$		$\times$	ψ		$KK6_{\parallel}$ ($KKO6_{\parallel}$)

KK6 ➡ Exotic in IIA

[absent if $(c_3', d_0) = 0$]

[Derendinger & A.G '14]

‣ Index splitting A $\rightarrow$ ($a = 1,3,5$) + ($i = 2,4,6$) + 7

[Derendinger & A.G '14]

An EFT to describe M-theory / strings backgrounds

- The embedding tensor formalism (ET) provides an EFT to describe *4d* effective actions irrespective of their higher-dimensional origin

[Nicolai, Samtleben, de Wit, Trigiante, ...]

- Fluxes in M-theory / strings $=$ Parameters in the EFT

$$f_{\alpha MNP} \in \mathrm{SL}(2) \times \mathrm{SO}(6,6)$$

[Schön & Weidner '06]

- Backgrounds preserving 16 ($N = 4$) or 32 ($N = 8$) supercharges

$N = 4$

[Schön & Weidner '06]

$$\left\{ \begin{array}{lll} f_{\alpha R[MN}\, f_{\beta PQ]}{}^{R} = 0 & \rightarrow & \text{Conditions } i) \text{ , } iii) \text{ and } v) \\ \epsilon^{\alpha\beta}\, f_{\alpha MNR}\, f_{\beta PQ}{}^{R} = 0 & \rightarrow & \text{Conditions } ii) \text{ and } vi) \end{array} \right.$$

$N = 8$ (extra)

[Dibitetto, A.G & Roest '11]

$$\left\{ \begin{array}{lll} \epsilon^{\alpha\beta}\, f_{\alpha[MNP}\, f_{\beta QRS]}\big|_{\mathrm{SD}} = 0 & \rightarrow & \text{Conditions } iv) \text{ and } vii) \\ f_{\alpha MNP}\, f_{\beta}{}^{MNP} = 0 & \rightarrow & \text{No additional conditions} \end{array} \right.$$

[Derendinger & A.G '14]

An EFT to describe M-theory / strings backgrounds

- The embedding tensor formalism (ET) provides an EFT to describe *4d* effective actions irrespective of their higher-dimensional origin

[Nicolai, Samtleben, de Wit, Trigiante, ...]

- Fluxes in M-theory / strings $=$ Parameters in the EFT

$$f_{\alpha MNP} \in \mathrm{SL}(2) \times \mathrm{SO}(6,6)$$

[Schön & Weidner '06]

- Backgrounds preserving 16 ($N = 4$) or 32 ($N = 8$) supercharges

$N = 4$

[Schön & Weidner '06]

$$\left\{ \begin{array}{lcl} f_{\alpha R[MN}\, f_{\beta PQ]}{}^{R} = 0 & \rightarrow & \text{Conditions } i) \;,\; iii) \text{ and } v) \\ \epsilon^{\alpha\beta}\, f_{\alpha MNR}\, f_{\beta PQ}{}^{R} = 0 & \rightarrow & \text{Conditions } ii) \text{ and } vi) \end{array} \right.$$

can be relaxed !!

$N = 8$ (extra)

[Dibitetto, A.G & Roest '11]

$$\left\{ \begin{array}{lcl} \epsilon^{\alpha\beta}\, f_{\alpha[MNP}\, f_{\beta QRS]}\big|_{\mathrm{SD}} = 0 & \rightarrow & \text{Conditions } iv) \text{ and } vii) \\ f_{\alpha MNP}\, f_{\beta}{}^{MNP} = 0 & \rightarrow & \text{No additional conditions} \end{array} \right.$$

Backgrounds preserving 16 supercharges

[Derendinger & A.G '14]

- SS conditions *iv)* and *vii)* can be relaxed if demanding only 16 supercharges

$$iv) \qquad \omega_{[ij}{}^{D}\,\omega_{k]D}{}^{7} \neq 0 \quad \rightarrow \quad 3\,b_1\,a_2 \neq 0$$

$$vii) \qquad \omega_{[7i}{}^{D}\,\omega_{j]D}{}^{k} \neq 0 \quad \rightarrow \quad b_1 c_3' + 2\,c_1\,d_0 \neq 0$$

  - KK6 $\rightarrow$ D6 in IIA (iv) & KK6 $\rightarrow$ Exotic in IIA (vii) can be included

- KK6 sources induce new terms in the scalar potential

  - Situation 1 : Only KK6 $\rightarrow$ D6 in IIA ➡ No moduli stabilisation
  - Situation 2 : Only KK6 $\rightarrow$ exotic in IIA ➡ Full moduli stabilisation
  - Situation 3 : Both types of KK6 sources ➡ Full moduli stabilisation

- **KKO6 sources crucial to stabilise moduli in backgrounds with 16 supercharges**

  - Unique gauging : $G = \mathrm{SO}(3) \ltimes \mathrm{Nil}_9$

Taxonomy of M-theory flux vacua

[Derendinger & A.G '14]

ID	$D6_{\parallel}$ ($O6_{\parallel}$) / $KK6_{\parallel}$ ($KKO6_{\parallel}$)	Stable	Flat dir.	SUSY	$\dim(G_{\text{res}})$	$\widetilde{W}_{27}$
vac 0	yes / no	✓	yes	$\mathcal{N}=0$	3	$\neq 0$
vac 1	no / yes	✓	yes	$\mathcal{N}=3$	3	$\neq 0$
vac 2	no / yes	✓	yes	$\mathcal{N}=0$	3	$\neq 0$
vac 3	no / yes	✓	no	$\mathcal{N}=0$	3	0
vac 4	no / yes	✓	no	$\mathcal{N}=1$	3	0
vac 5	no / yes	✓	no	$\mathcal{N}=0$	3	0
vac 6	no / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 7	no / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 8	no / yes	✓	no	$\mathcal{N}=0$	3	$\neq 0$
vac 9	yes / yes	✓	yes	$\mathcal{N}=3$	6	$\neq 0$
vac 10	yes / yes	✓	no	$\mathcal{N}=0$	6	$\neq 0$
vac 11	yes / yes	✓	no	$\mathcal{N}=1$	6	0
vac 12	yes / yes	✓	no	$\mathcal{N}=0$	6	0
vac 13	yes / yes	×	no	$\mathcal{N}=0$	6	0
vac 14	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 15	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 16	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 17	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$

Taxonomy of M-theory flux vacua

[Derendinger & A.G '14]

ID	$D6_{\parallel}$ ($O6_{\parallel}$) / $KK6_{\parallel}$ ($KKO6_{\parallel}$)	Stable	Flat dir.	SUSY	$\dim(G_{\text{res}})$	$\widetilde{W}_{27}$
vac 0	yes / no	✓	yes	$\mathcal{N}=0$	3	$\neq 0$
vac 1	no / yes	✓	yes	$\mathcal{N}=3$	3	$\neq 0$
vac 2	no / yes	✓	yes	$\mathcal{N}=0$	3	$\neq 0$
vac 3	no / yes	✓	no	$\mathcal{N}=0$	3	0
vac 4	no / yes	✓	no	$\mathcal{N}=1$	3	0
vac 5	no / yes	✓	no	$\mathcal{N}=0$	3	0
vac 6	no / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 7	no / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 8	no / yes	✓	no	$\mathcal{N}=0$	3	$\neq 0$
vac 9	yes / yes	✓	yes	$\mathcal{N}=3$	6	$\neq 0$
vac 10	yes / yes	✓	no	$\mathcal{N}=0$	6	$\neq 0$
vac 11	yes / yes	✓	no	$\mathcal{N}=1$	6	0
vac 12	yes / yes	✓	no	$\mathcal{N}=0$	6	0
vac 13	yes / yes	×	no	$\mathcal{N}=0$	6	0
vac 14	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 15	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 16	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 17	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$

- *Weak* G_2-holonomy (vac 3–5)
- *Weak* G_2-holonomy (vac 11–13)

Taxonomy of M-theory flux vacua

[Derendinger & A.G '14]

- $N = 3$ SUSY (vac 1)
- $N = 1$ SUSY (vac 4)
- $N = 3$ SUSY (vac 9)
- $N = 1$ SUSY (vac 11)

ID	$D6_{\parallel}$ ($O6_{\parallel}$) / $KK6_{\parallel}$ ($KKO6_{\parallel}$)	Stable	Flat dir.	SUSY	$\dim(G_{\text{res}})$	$\widetilde{W}_{27}$
vac 0	yes / no	✓	yes	$\mathcal{N}=0$	3	$\neq 0$
vac 1	no / yes	✓	yes	$\mathcal{N}=3$	3	$\neq 0$
vac 2	no / yes	✓	yes	$\mathcal{N}=0$	3	$\neq 0$
vac 3	no / yes	✓	no	$\mathcal{N}=0$	3	0
vac 4	no / yes	✓	no	$\mathcal{N}=1$	3	0
vac 5	no / yes	✓	no	$\mathcal{N}=0$	3	0
vac 6	no / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 7	no / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 8	no / yes	✓	no	$\mathcal{N}=0$	3	$\neq 0$
vac 9	yes / yes	✓	yes	$\mathcal{N}=3$	6	$\neq 0$
vac 10	yes / yes	✓	no	$\mathcal{N}=0$	6	$\neq 0$
vac 11	yes / yes	✓	no	$\mathcal{N}=1$	6	0
vac 12	yes / yes	✓	no	$\mathcal{N}=0$	6	0
vac 13	yes / yes	×	no	$\mathcal{N}=0$	6	0
vac 14	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 15	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 16	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 17	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$

Taxonomy of M-theory flux vacua

[Derendinger & A.G '14]

ID	$D6_{\parallel}$ ($O6_{\parallel}$) / $KK6_{\parallel}$ ($KKO6_{\parallel}$)	Stable	Flat dir.	SUSY	$\dim(G_{\text{res}})$	$\widetilde{W}_{27}$
vac 0	yes / no	✓	yes	$\mathcal{N}=0$	3	$\neq 0$
vac 1	no / yes	✓	yes	$\mathcal{N}=3$	3	$\neq 0$
vac 2	no / yes	✓	yes	$\mathcal{N}=0$	3	$\neq 0$
vac 3	no / yes	✓	no	$\mathcal{N}=0$	3	0
vac 4	no / yes	✓	no	$\mathcal{N}=1$	3	0
vac 5	no / yes	✓	no	$\mathcal{N}=0$	3	0
vac 6	no / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 7	no / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 8	no / yes	✓	no	$\mathcal{N}=0$	3	$\neq 0$
vac 9	yes / yes	✓	yes	$\mathcal{N}=3$	6	$\neq 0$
vac 10	yes / yes	✓	no	$\mathcal{N}=0$	6	$\neq 0$
vac 11	yes / yes	✓	no	$\mathcal{N}=1$	6	0
vac 12	yes / yes	✓	no	$\mathcal{N}=0$	6	0
vac 13	yes / yes	×	no	$\mathcal{N}=0$	6	0
vac 14	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 15	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 16	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$
vac 17	yes / yes	×	no	$\mathcal{N}=0$	3	$\neq 0$

- $\dim(G_{res}) = 6$

Final remarks

• Moduli stabilisation can be achieved upon twisted reductions of *massless* M-theory if KK6 (KKO6) sources are included.

• Using the ET formalism (*4d*) as a guiding principle, the minimal setup corresponds to $N = 4$ backgrounds (16 supercharges) violating some of the SS conditions

$$\omega_{[\bullet\bullet}{}^{D}\,\omega_{\bullet]D}{}^{\psi} \neq 0 \;\Rightarrow\; \text{Non-vanishing KK6 (KKO6) charge}$$

- *New* situation compared to IIA orientifolds (*7d vs 6d* isometries within SO(6,6))

• M-theory interpretation of *non-geometric* fluxes in a type IIA incarnation of the effective STU-models ➡ KK6 (KKO6) corresponding to exotic IIA sources

In progress [with Uppsala group] :

- understand the *7d / 6d* interplay at the level of SU(3)-structures
- *11d/10d* lifting of 1/2-BPS backgrounds corresponding to KK6/exotic IIA sources

Thanks !!

Extra material...

M-theory on G_2-manifolds (X_7) with fluxes

- *7d* manifolds with G_2-structure possess an invariant 3-form

$$\Phi_{(3)} = \eta^{127} + \eta^{347} + \eta^{567} + \eta^{135} - \eta^{146} - \eta^{362} - \eta^{524}$$

- Co-calibrated G_2-structure

[Friedrich '02]
[Bryant '03]

$$\begin{aligned} d\Phi_{(3)} &= \widetilde{W}_1 \star_7 \Phi_{(3)} + 2\,\widetilde{W}_{27} \\ d \star_7 \Phi_{(3)} &= 0 \end{aligned}$$

- Enhancements to *weak* G_2-holonomy $(\widetilde{W}_{27} = 0)$ or G_2-holonomy $(\widetilde{W}_{27} = \widetilde{W}_1 = 0)$

- $N = 1$ supergravity in terms of a complex 3-form (moduli fields in *4d*) :

$$W_{\text{M-theory}} = \frac{1}{4}\int_{X_7} G_{(7)} + \frac{1}{4}\int_{X_7} (A_{(3)} + i\Phi_{(3)}) \wedge \left[G_{(4)} + \frac{1}{2} d(A_{(3)} + i\Phi_{(3)}) \right]$$

[House & Micu '04]
[Dall'Agata & Prezas '05]

M-theory fluxes / ET dictionary

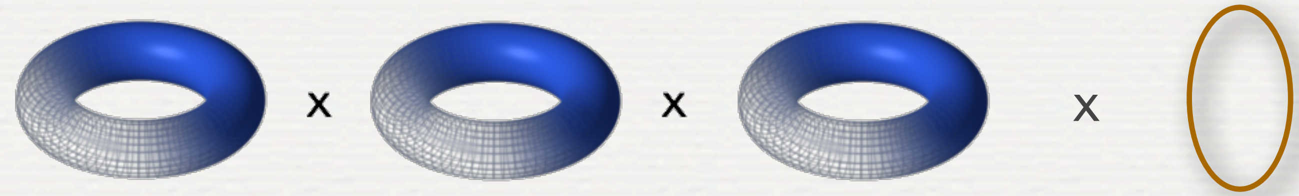
M-theory origin	Type IIA origin	Fluxes	Embedding tensor
${\omega_{bc}}^{a}$	${\omega_{bc}}^{a}$	$\tilde{c}_1^{(I)}$	${f_{+}}^{bc}{}_{a}$
${\omega_{aj}}^{k}$	${\omega_{aj}}^{k}$	$\hat{c}_1^{(I)}$	${f_{+}}^{aj}{}_{k}$
${\omega_{ka}}^{j}$	${\omega_{ka}}^{j}$	$\check{c}_1^{(I)}$	${f_{+}}^{ka}{}_{j}$
${\omega_{jk}}^{a}$	${\omega_{jk}}^{a}$	$b_1^{(I)}$	${f_{-}}^{ibc}$
$-{\omega_{ai}}^{7}$	F_{ai}	$a_2^{(I)}$	$-{f_{+}}^{ajk}$
$-{\omega_{7i}}^{a}$	non-geometric	$d_0^{(I)}$	${f_{-}}^{bc}{}_{i}$
$-{\omega_{a7}}^{i}$	non-geometric	$c_3'^{(I)}$	${f_{+jk}}^{a}$
$-\frac{1}{2}\,G_{aibj}$	$-F_{aibj}$	$a_1^{(I)}$	${f_{+}}^{abk}$
$\frac{1}{2}\,G_{ijk7}$	H_{ijk}	b_0	$-{f_{-}}^{abc}$
$\frac{1}{2}\,G_{ibc7}$	H_{ibc}	$c_0^{(I)}$	${f_{+}}^{bc}{}_{i}$
$\frac{1}{4}\,G_{aibjck7}$	F_{aibjck}	a_0	$-{f_{+}}^{abc}$
non-geometric	$-F_{(0)}$ (Romans mass)	a_3	${f_{+}}^{ijk}$

(Non-iso) example : Twisted torus $X_7 = T^7/(Z_2 \times Z_2 \times Z_2)$

- Factorisation :

[7 = 2 + 2 + 2 + 1]

[A = ($i = 1,3,5$, $a = 2,4,6$) + 7]



(η^1, η^2) (η^3, η^4) (η^5, η^6) η^7

- Twist specified by a metric flux

$$d\eta^A + \frac{1}{2}\,\omega_{BC}{}^A \eta^B \wedge \eta^C = 0$$

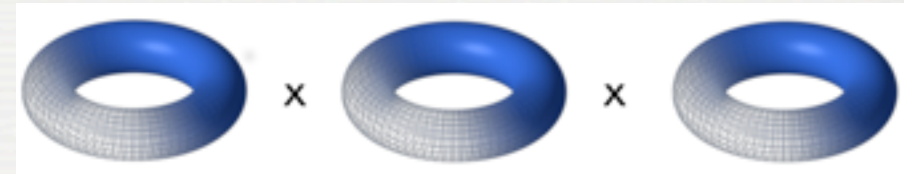
- Background gauge fluxes

$$\frac{1}{2}\,G_{(4)} = -\sum_{I=1}^{3} a_1{}^{(I)}\,\tilde{\omega}^I + b_0\,\beta^0 + \sum_{I=1}^{3} c_0{}^{(I)}\,\alpha_I \qquad \text{and} \qquad \frac{1}{4}\,G_{(7)} = a_0\,\eta^{1234567}$$

- Moduli fields

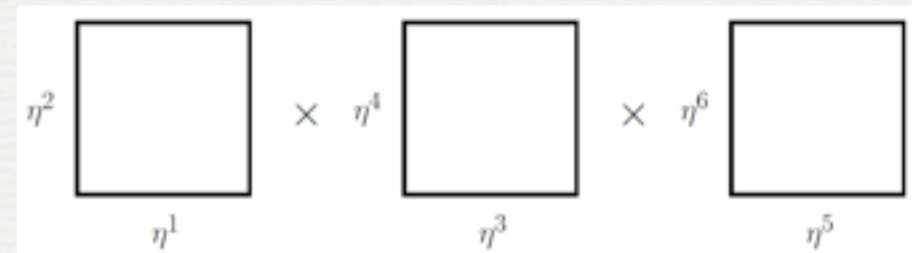
$$\frac{1}{2}(A_{(3)} + i\Phi_{(3)}) = \sum_{I=1}^{3} U_I\,\omega_I + S\,\alpha_0 - \sum_{I=1}^{3} T_I\,\beta^I$$

Geometry of the $Z_2 \times Z_2$ orbifold of T^6



- Orbifold action

$$\theta_1 \; : \; (\eta^1, \eta^2, \eta^3, \eta^4, \eta^5, \eta^6) \rightarrow (\eta^1, \eta^2, -\eta^3, -\eta^4, -\eta^5, -\eta^6)$$
$$\theta_2 \; : \; (\eta^1, \eta^2, \eta^3, \eta^4, \eta^5, \eta^6) \rightarrow (-\eta^1, -\eta^2, \eta^3, \eta^4, -\eta^5, -\eta^6)$$



- Invariant forms

0-forms ➡ points

1-forms ➡ none

2-forms ➡ $\omega_1 = \eta^{12} \quad , \quad \omega_2 = \eta^{34} \quad , \quad \omega_3 = \eta^{56}$

3-forms ➡
$$\alpha_0 = \eta^{135} \quad , \quad \alpha_1 = \eta^{235} \quad , \quad \alpha_2 = \eta^{451} \quad , \quad \alpha_3 = \eta^{613}$$
$$\beta^0 = \eta^{246} \quad , \quad \beta^1 = \eta^{146} \quad , \quad \beta^2 = \eta^{362} \quad , \quad \beta^3 = \eta^{524}$$

4-forms ➡ $\tilde{\omega}^1 = \eta^{3456} \quad , \quad \tilde{\omega}^2 = \eta^{1256} \quad , \quad \tilde{\omega}^3 = \eta^{1234}$

5-forms ➡ none

6-forms ➡ internal volume