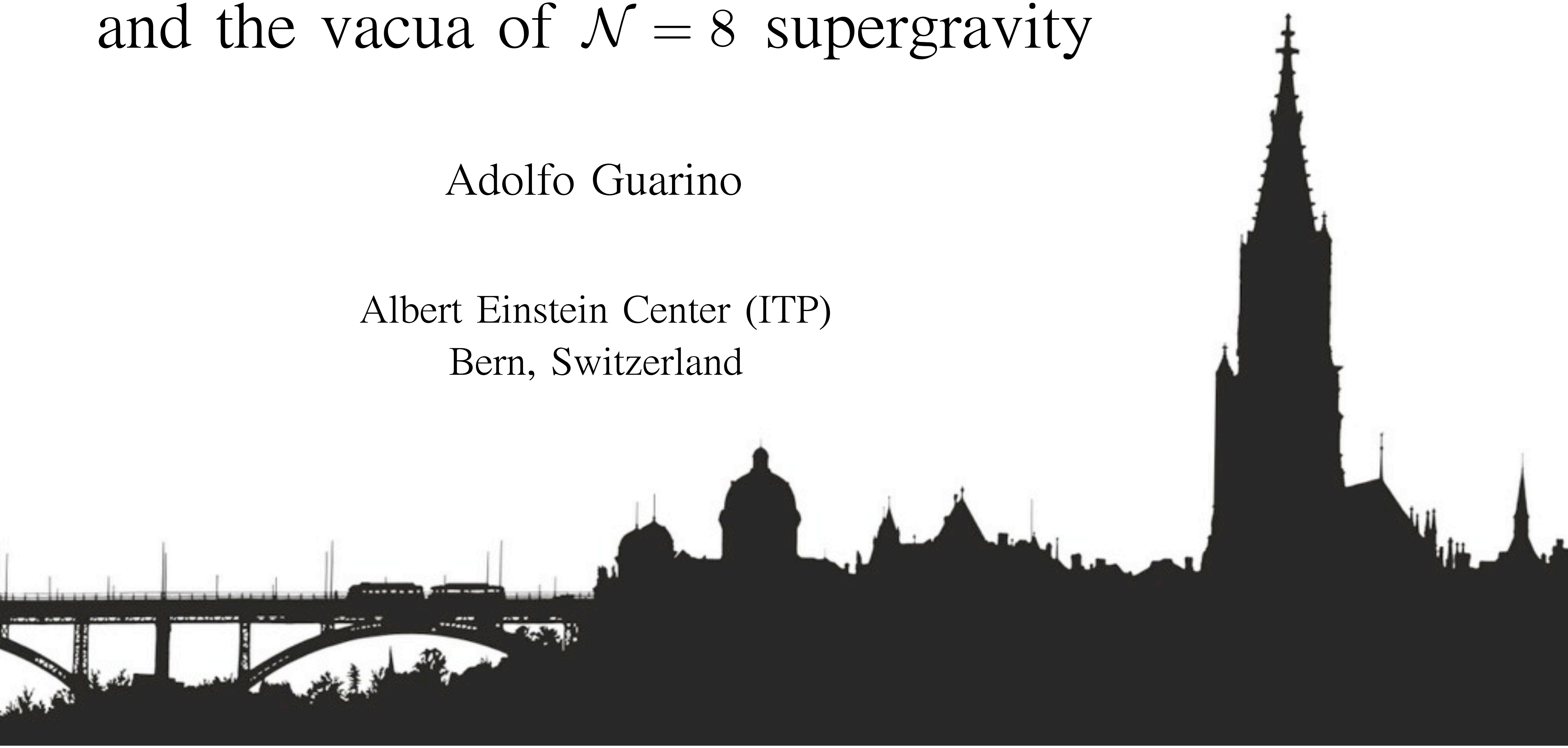
On electromagnetic duality and the vacua of $\mathcal{N} = 8$ supergravity

Adolfo Guarino

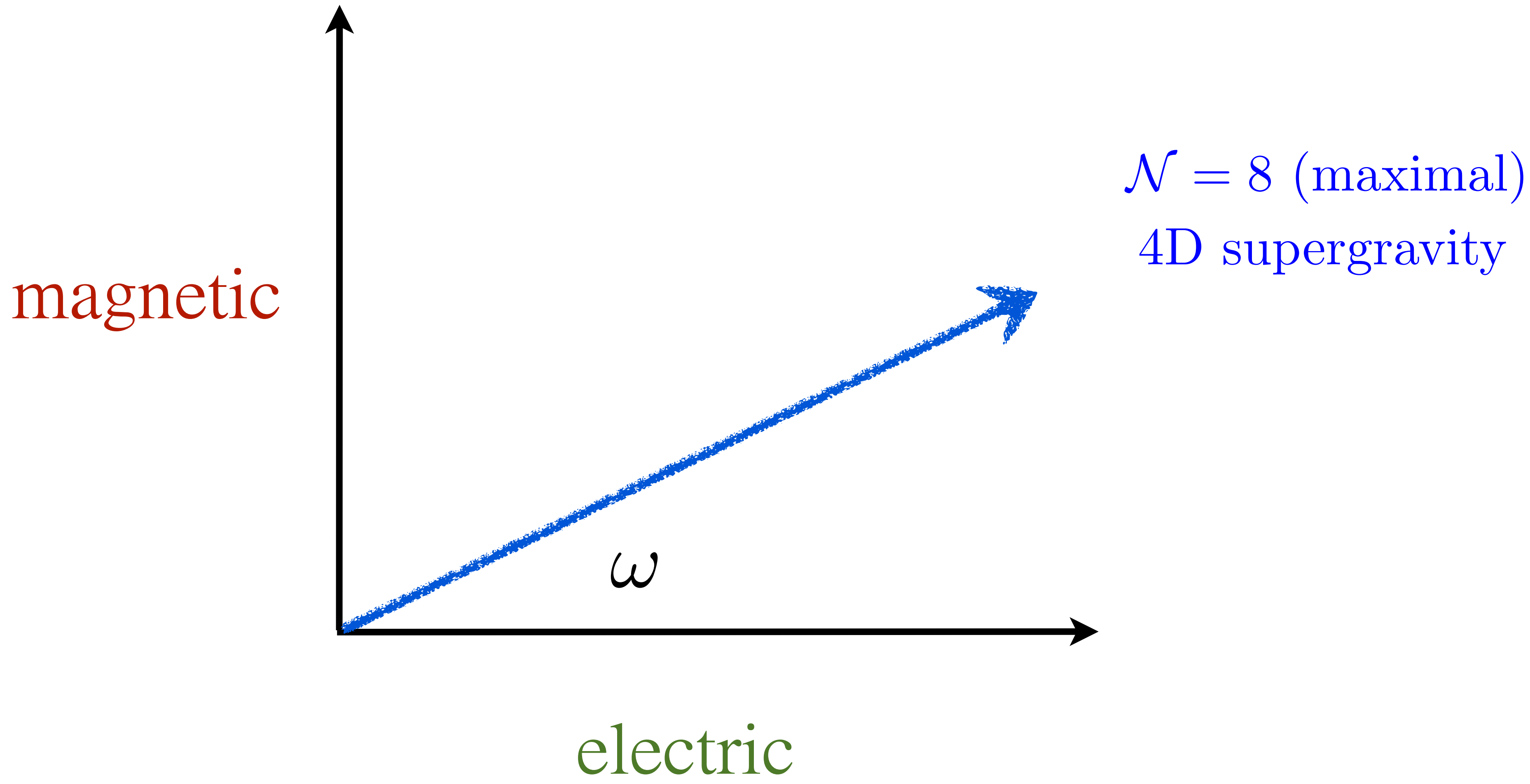
Albert Einstein Center (ITP)
Bern, Switzerland



The String Theory Universe
3 September 2013, Bern

Work in collaboration with A. Borghese & D. Roest

This talk is about the consequences of U(1)-orientating a theory...



R-symmetry : U(1) yes or no?

R-symmetry : U(1) yes or no?

- Dimensional reduction of 10D SYM produces N=4 SYM

[Brink, Scherk & Schwarz '76]

$$L_{10\mathrm{D}} = -\frac{1}{2}F^2 + \frac{i}{2}\,\bar{\lambda}\,\not{D}\lambda$$

$i = 1, .., 4$

$$\begin{aligned} L_{4\mathrm{D}} \;\; &= \;\; -\frac{1}{2}F^2 + i\,\bar{\lambda}_i\,\not{D}\lambda^i + \frac{1}{2}(D\phi_{ij})^2 \\ &\quad -\frac{i}{2}g\,(f\,\phi^{ij}\bar{\lambda}_i\lambda_j + c.c) \\ &\quad -\frac{1}{4}g^2\,(f\,\phi_{ij}\phi_{kl})^2 \end{aligned}$$

reduction = fermi masses + scalar potential

R-symmetry : U(1) yes or no?

- Dimensional reduction of 10D SYM produces N=4 SYM [Brink, Scherk & Schwarz '76]

$$L_{10D} = -\frac{1}{2}F^2 + \frac{i}{2}\,\bar{\lambda}\,\not{D}\lambda \quad \xrightarrow{i=1,..,4}$$

$$\begin{aligned} L_{4D} \;=\; & -\frac{1}{2}F^2 + i\,\bar{\lambda}_i\,\not{D}\lambda^i + \frac{1}{2}(D\phi_{ij})^2 \\ & -\frac{i}{2}g\,(f\,\phi^{ij}\bar{\lambda}_i\lambda_j + c.c) \\ & -\frac{1}{4}g^2\,(f\,\phi_{ij}\phi_{kl})^2 \end{aligned}$$

reduction = fermi masses + scalar potential

> Reality condition on the 6 scalars :

$$\phi^*_{ij} = \phi^{ij} = \frac{1}{2}\,\epsilon^{ijkl}\phi_{kl}$$

R-symmetry group is SU(4) and not U(4) !!

R-symmetry : U(1) yes or no?

- Dimensional reduction of 10D SYM produces N=4 SYM

[Brink, Scherk & Schwarz '76]

$$L_{10D} = -\frac{1}{2}F^2 + \frac{i}{2}\bar{\lambda}\,\not{D}\lambda \quad \xrightarrow{i=1,..,4} \quad \begin{aligned} L_{4D} \;=\; & -\tfrac{1}{2}F^2 + i\,\bar{\lambda}_i\,\not{D}\lambda^i + \tfrac{1}{2}(D\phi_{ij})^2 \\ & -\tfrac{i}{2}g\,(f\,\phi^{ij}\bar{\lambda}_i\lambda_j + c.c) \\ & -\tfrac{1}{4}g^2\,(f\,\phi_{ij}\phi_{kl})^2 \end{aligned}$$

reduction = fermi masses + scalar potential

> Reality condition on the 6 scalars :

$$\phi^*_{ij} = \phi^{ij} = \frac{1}{2}\,\epsilon^{ijkl}\phi_{kl}$$

R-symmetry group is SU(4) and not U(4) !!

[Cremmer & Julia '78, '79]

- Analogous results for N=8 gauged SUGRAs from M/Type II reductions with fluxes

$[f \leftrightarrow H_3, F_p, \omega, \ldots]$

> Reality condition on the 70 scalars :

$$\phi^*_{IJKL} = \phi^{IJKL} = \frac{1}{24}\,\epsilon^{IJKLMNPQ}\phi_{MNPQ}$$

$I = 1, \ldots, 8$

R-symmetry group is SU(8) and not U(8) !!

An extra U(1) in N=8 gauged supergravity

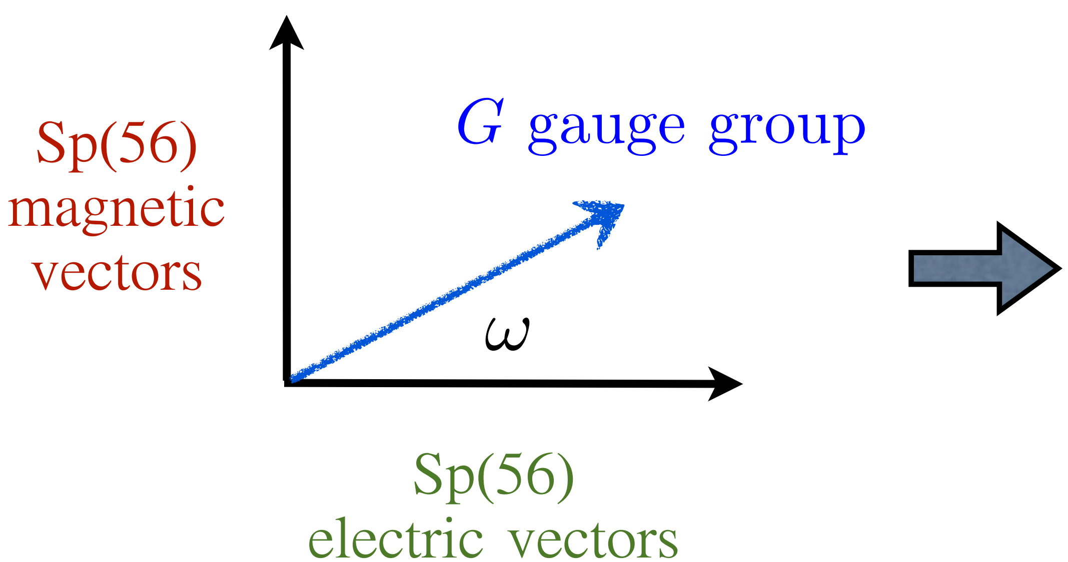
Gauge fields : The theory contains 56 = 28 (electric) + 28 (magnetic) vectors spanning a 28-dimensional gauge symmetry group $G \subset E_7$

An extra U(1) in N=8 gauged supergravity

Gauge fields : The theory contains 56 = 28 (electric) + 28 (magnetic) vectors spanning a 28-dimensional gauge symmetry group $G \subset E_7$

[Dall' Agata, Inverso & Trigiante '12]

- Recently, an extra U(1) rotation outside the R-symmetry group SU(8) has been identified and used to orientate G inside the Sp(56) group of electromagnetic transf.



Covariant derivative :

$$D_\mu \phi = \partial_\mu \phi + \left(\cos\omega \, A_\mu^{(\text{electric})} + \sin\omega \, A_\mu^{(\text{magnetic})} \right) \phi$$

> Therefore : $\omega = 0$ (electric) , $\omega = \frac{\pi}{2}$ (magnetic) and $0 < \omega < \frac{\pi}{2}$ (dyonic)

GOALS :

1) Use the embedding-tensor formalism to compute the ω-dependent scalar potential and analyse its critical points

[de Wit, Samtleben & Trigiante ’07]

[Dall’ Agata, Inverso & Trigiante ‘12]

[Borghese, A.G , & Roest ‘13]

2) Compute fermion masses in order to... (the answer in 8 min)

[Borghese, A.G , & Roest ’12, ‘13]

Gaugings, embedding tensor & scalar potential

Gauging procedure : Part of the global E_7 symmetry group is promoted to a local symmetry group G (gauging)

Gaugings, embedding tensor & scalar potential

Gauging procedure : Part of the global E_7 symmetry group is promoted to a local symmetry group G (gauging)

Embedding tensor : It is a "selector" specifying which generators of E_7 become gauge symmetries G and then will have an associated gauge field

$$A_\mu^M = \Theta^M{}_\alpha \, t^\alpha \quad \Rightarrow \quad [A^M, A^N] = X^{MN}{}_P \, A^P \quad \text{with} \quad X^{MN}{}_P = \Theta^M{}_\alpha \, [t^\alpha]^N{}_P$$

$[\, M = 1, \ldots, 56 \,]$

$[\, \alpha = 1, \ldots, 133 \,]$

Gaugings, embedding tensor & scalar potential

Gauging procedure : Part of the global E_7 symmetry group is promoted to a local symmetry group G (gauging)

Embedding tensor : It is a "selector" specifying which generators of E_7 become gauge symmetries G and then will have an associated gauge field

$$A_\mu^M = \Theta^M{}_\alpha\, t^\alpha \quad \Rightarrow \quad [A^M, A^N] = X^{MN}{}_P\, A^P \qquad \text{with} \qquad X^{MN}{}_P = \Theta^M{}_\alpha\, [t^\alpha]^N{}_P$$

$[\, M = 1, \ldots, 56\,]$

$[\, \alpha = 1, \ldots, 133\,]$

Scalar potential : Straightforward once the embbeding tensor $\Theta^M{}_\alpha(\omega)$ is known

$$V = \frac{1}{672}\, X_{MNP}\, X_{QRS}\, \left(M^{MQ}\, M^{NR}\, M^{PS} + 7\, M^{MQ}\, \Omega^{NR}\, \Omega^{PS} \right)$$

where $M(\phi) \in \frac{E_7}{SU(8)}$ contains the 70 scalar fields of the theory

Example 1 : G_2-invariant sector of G = SO(8)

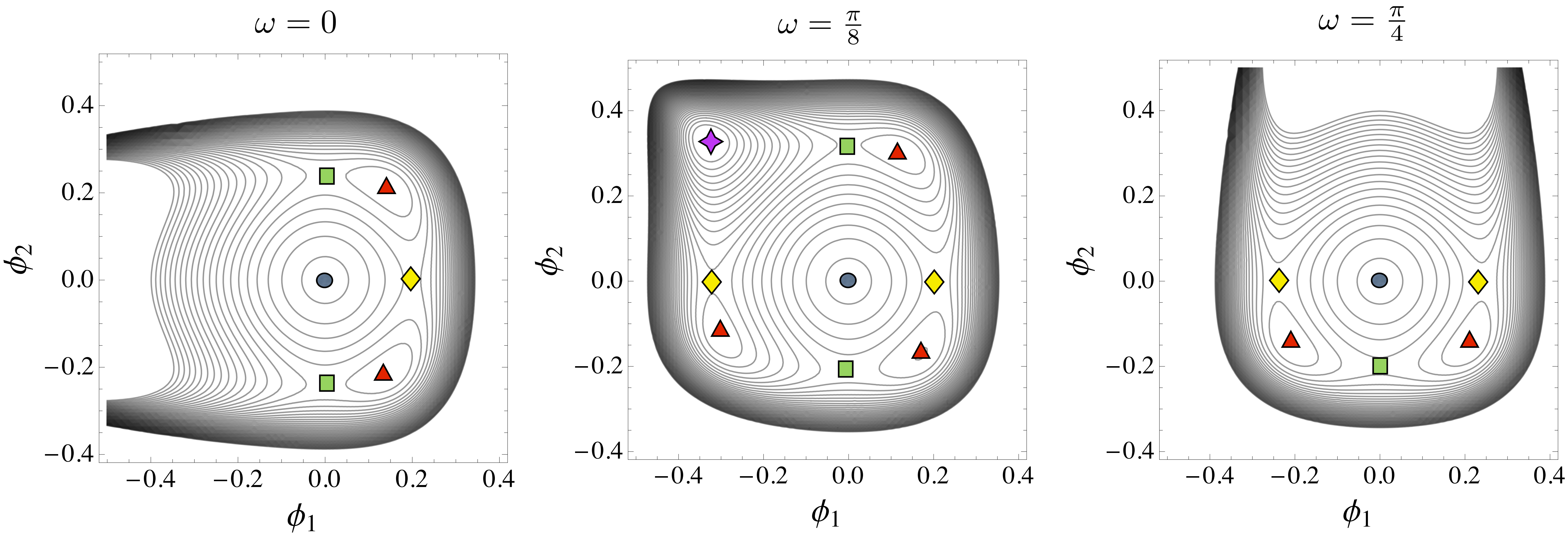
[Dall' Agata, Inverso & Trigiante '12]
[Borghese, A.G & Roest '12]

- Truncate most of the 70 scalars and look for critical points of $V(\phi)$ with large residual symmetry groups $G_0 \subset G$

Example 1 : G_2-invariant sector of G = SO(8)

[Dall' Agata, Inverso & Trigiante '12]
[Borghese, A.G & Roest '12]

- Truncate most of the 70 scalars and look for critical points of $V(\phi)$ with large residual symmetry groups $G_0 \subset G$

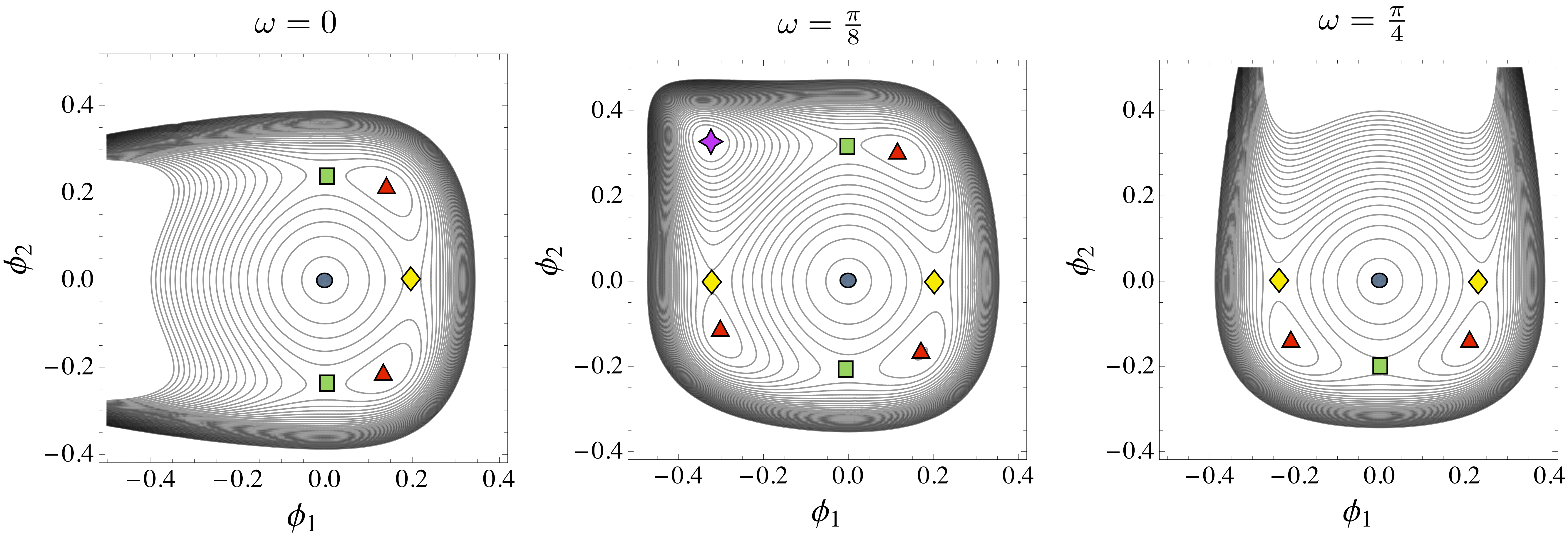


critical point	residual sym G_0	SUSY	Stability
●	$SO(8)$	$\mathcal{N} = 8$	✓
■	$SO(7)_-$	$\mathcal{N} = 0$	×
◆	$SO(7)_+$	$\mathcal{N} = 0$	×
▲	G_2	$\mathcal{N} = 1$	✓
✦	G_2	$\mathcal{N} = 0$	✓

Example 1 : G_2-invariant sector of G = SO(8)

[Dall' Agata, Inverso & Trigiante '12]
[Borghese, A.G & Roest '12]

- Truncate most of the 70 scalars and look for critical points of $V(\phi)$ with large residual symmetry groups $G_0 \subset G$



> Mass spectra insensitive to ω

> $\frac{\pi}{4}$-periodicity with transmutation of $SO(7)_{\pm}$

> Runaway of points at $\omega = n\,\frac{\pi}{4}$

critical point	residual sym G_0	SUSY	Stability
●	$SO(8)$	$\mathcal{N}=8$	✓
■	$SO(7)_-$	$\mathcal{N}=0$	×
◆	$SO(7)_+$	$\mathcal{N}=0$	×
▲	G_2	$\mathcal{N}=1$	✓
✦	G_2	$\mathcal{N}=0$	✓

Example 2 : SO(4) invariant sectors of G = SO(8)

[Borghese, A.G & Roest '13]

- Three embeddings $SO(4)_{v,s,c}$ related by Triality :

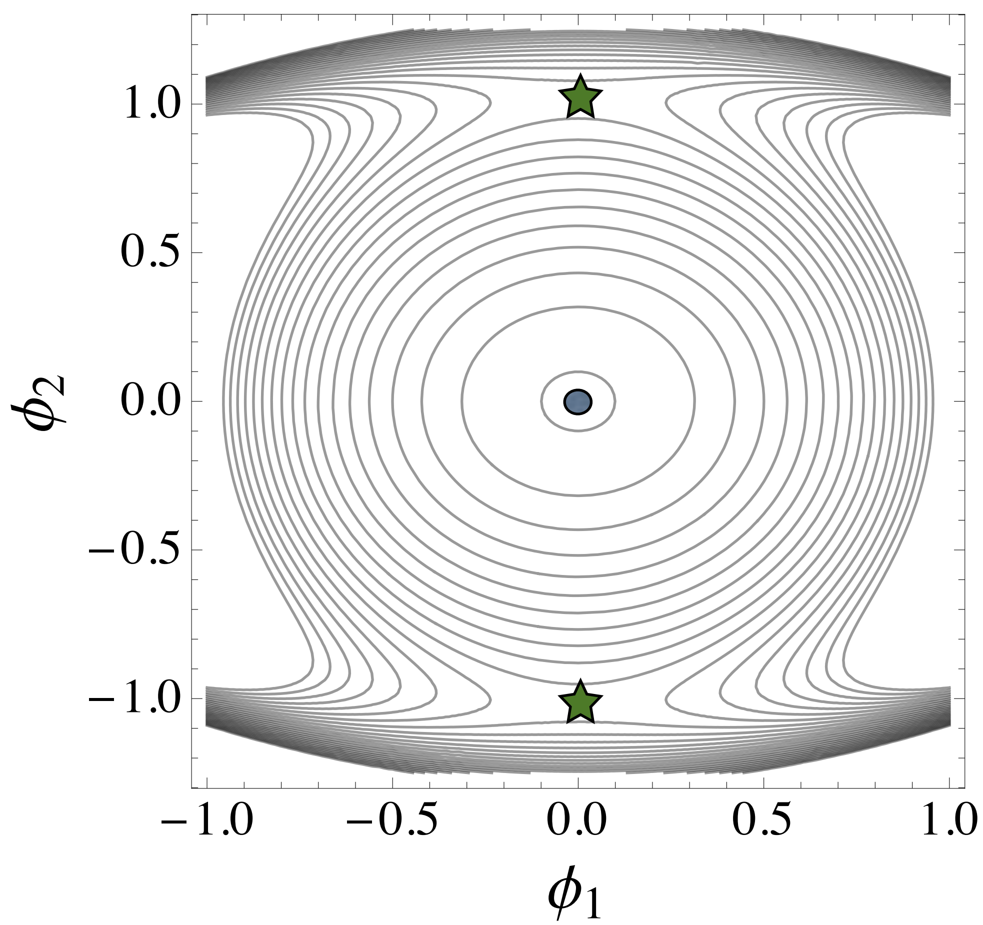
Example 2 : SO(4) invariant sectors of G = SO(8)

[Borghese, A.G & Roest '13]

- Three embeddings $SO(4)_{v,s,c}$ related by Triality :

  i) the vectorial embedding : $8_v = (1,1) + (3,1) + (1,1) + (1,3)$

$\omega = 0$



[Warner '84]
[Fischbacher, Pilch & Warner '10]

critical point	residual sym G_0	SUSY	Stability
●	$SO(8)$	$\mathcal{N} = 8$	✓
★	$SO(4)_v$	$\mathcal{N} = 0$	✓

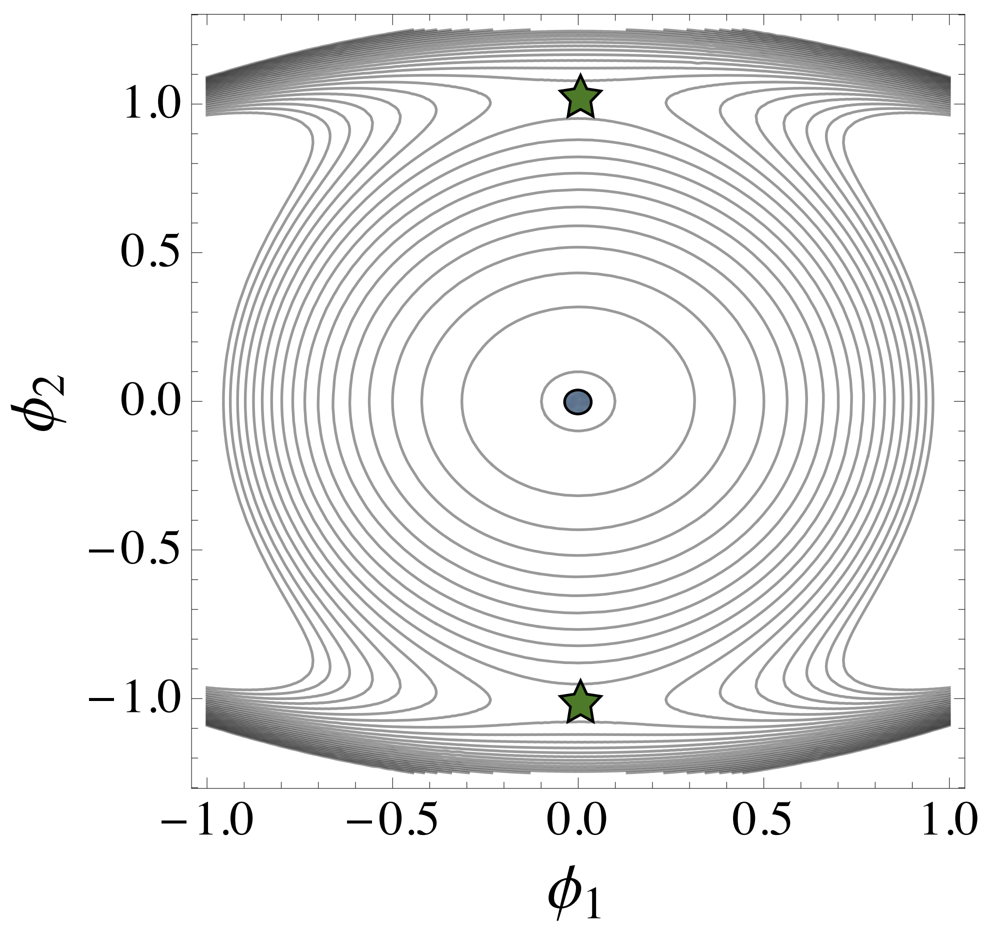
Example 2 : SO(4) invariant sectors of G = SO(8)

[Borghese, A.G & Roest '13]

- Three embeddings $SO(4)_{v,s,c}$ related by Triality :

i) the vectorial embedding : $8_v = (1,1) + (3,1) + (1,1) + (1,3)$

$\omega \in [0, \frac{\pi}{4}]$



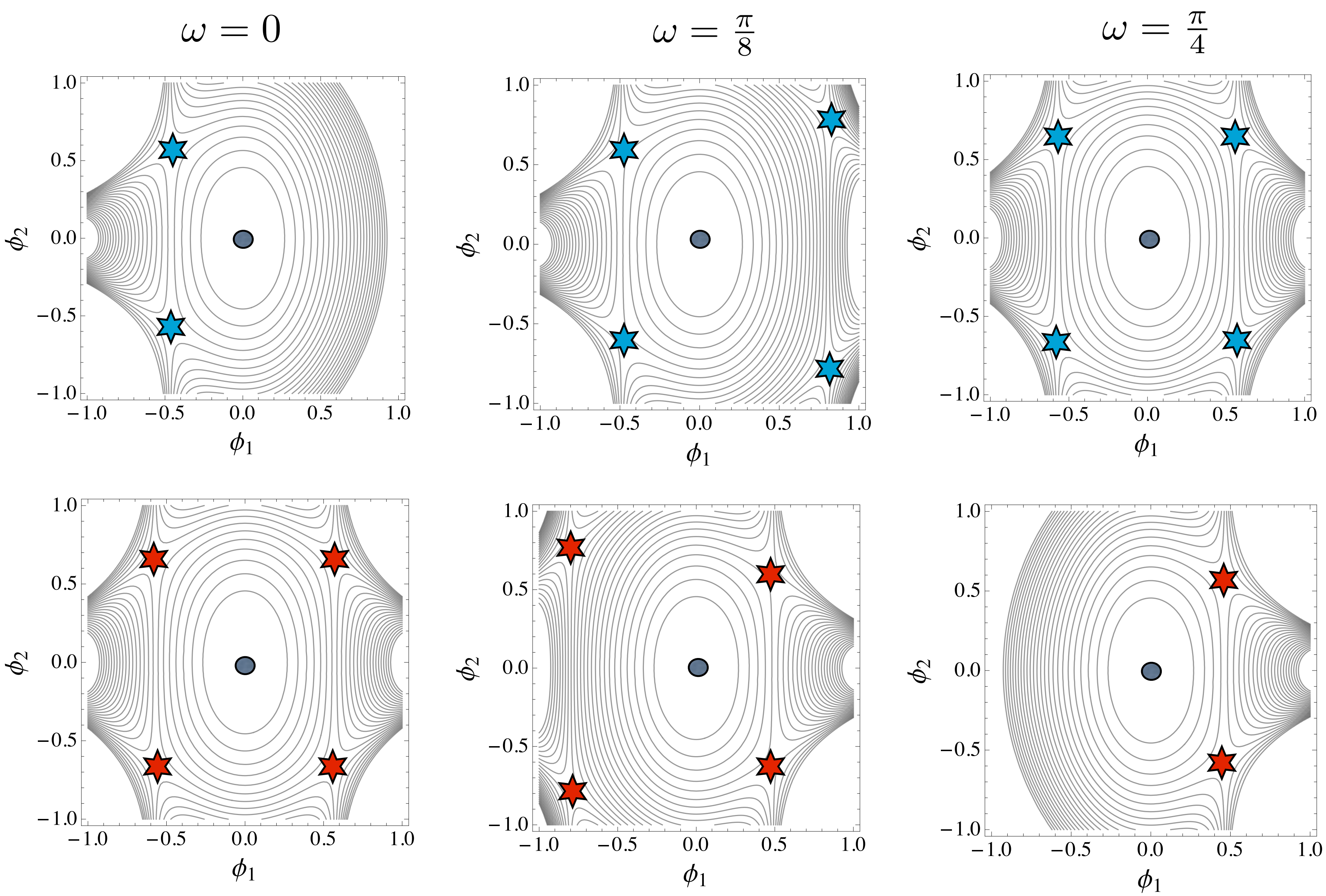
critical point	residual sym G_0	SUSY	Stability
●	$SO(8)$	$\mathcal{N} = 8$	✓
★	$SO(4)_v$	$\mathcal{N} = 0$	✓

> NO ω-dependence at all !!

[Warner '84]

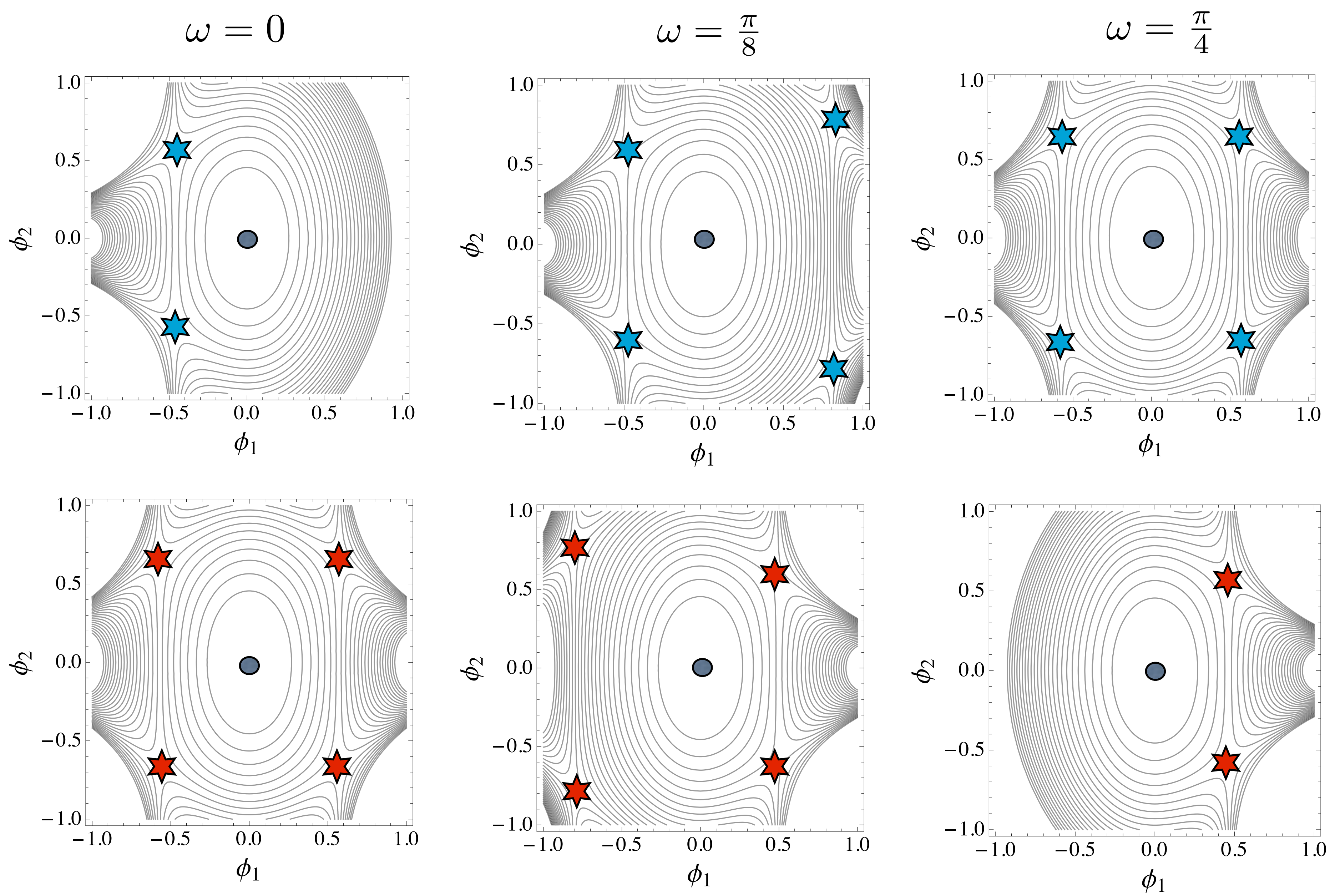
[Fischbacher, Pilch & Warner '10]

ii) spinorial (upper) & conjugate (lower) embeddings : $8_{s/c} = (1,1) + (3,1) + (1,1) + (1,3)$



critical point	residual sym G_0	SUSY	Stability
●	$SO(8)$	$\mathcal{N} = 8$	✓
★ (blue)	$SO(4)_s$	$\mathcal{N} = 0$	×
★ (red)	$SO(4)_c$	$\mathcal{N} = 0$	×

ii) spinorial (upper) & conjugate (lower) embeddings : $8_{s/c} = (1,1) + (3,1) + (1,1) + (1,3)$



> Mass spectra sensitive to ω

> $\frac{\pi}{4}$-periodicity restored by Triality

> Runaway of points at $\omega = n\,\frac{\pi}{4}$

critical point	residual sym G_0	SUSY	Stability
●	$SO(8)$	$\mathcal{N}=8$	✓
★	$SO(4)_s$	$\mathcal{N}=0$	×
★	$SO(4)_c$	$\mathcal{N}=0$	×

Runaway... but where to?

Runaway... but where to?

Limitation : If looking at the scalar potential, then the critical points are prisoners of the theory (gauging)

Runaway... but where to?

Limitation : If looking at the scalar potential, then the critical points are prisoners of the theory (gauging)

Hope : "Sit on top" of a critical point and travel with it to see what happens

Runaway... but where to?

Limitation : If looking at the scalar potential, then the critical points are prisoners of the theory (gauging)

Hope : "Sit on top" of a critical point and travel with it to see what happens...

Answer : It wants to migrate to a different theory (gauging) $\Longrightarrow$ the fermion masses can be used to monitoring its story !!

[Borghese, A.G & Roest '12, '13]

Tracking solutions using fermion masses

Going to the origin : If a critical point is found at $\phi = \phi_0$ with a residual symmetry G_0 , it can always be brought to $\phi_0 = 0$ via an E_7-transformation

[Dibitetto, A.G & Roest'11]
[Dall'Agata & Inverso '11]
[Kodama & Nozawa '12]

Applicability : After going to the origin, the quantities in the theory, e.g. fermi masses, will adopt a form compatible with G_0

[Borghese, A.G & Roest '12, '13]

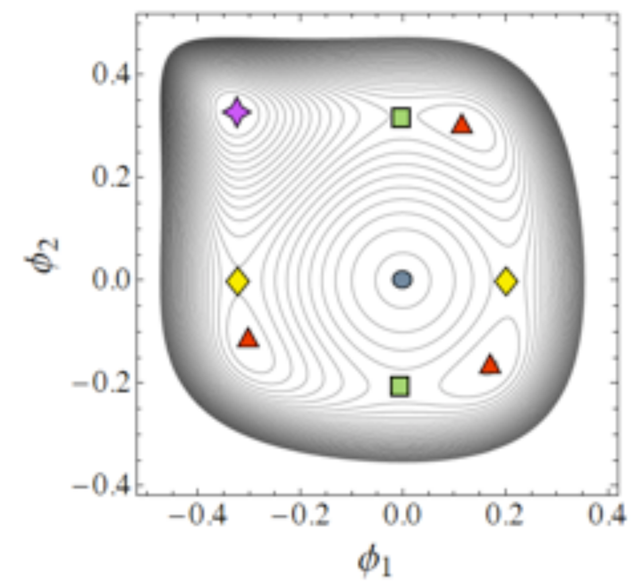
$$\mathcal{L}_{\text{fermi}} = \frac{\sqrt{2}}{2} \, \mathcal{A}_{\mathcal{I}\mathcal{J}} \, \overline{\psi}_\mu{}^{\mathcal{I}} \, \gamma^{\mu\nu} \, \psi_\nu{}^{\mathcal{J}} + \frac{1}{6} \, \mathcal{A}_{\mathcal{I}}{}^{\mathcal{J}\mathcal{K}\mathcal{L}} \, \overline{\psi}_\mu{}^{\mathcal{I}} \, \gamma^\mu \, \chi_{\mathcal{J}\mathcal{K}\mathcal{L}} + \mathcal{A}^{\mathcal{I}\mathcal{J}\mathcal{K},\mathcal{L}\mathcal{M}\mathcal{N}} \, \overline{\chi}_{\mathcal{I}\mathcal{J}\mathcal{K}} \, \chi_{\mathcal{L}\mathcal{M}\mathcal{N}}$$

gravitino-gravitino mass

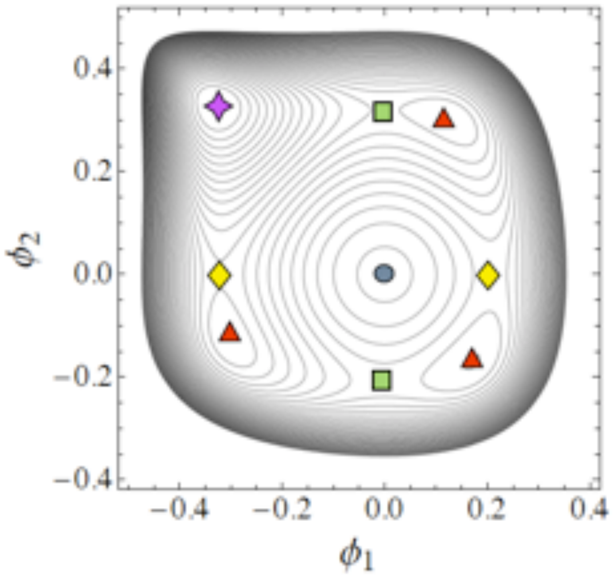
gravitino-dilatino mass

dilatino-dilatino mass (dependent)

Example 1 : Critical point preserving $G_0 = G_2$ (✦)



Example 1 : Critical point preserving $G_0 = G_2$ (✦)



- Pattern of fermi masses :

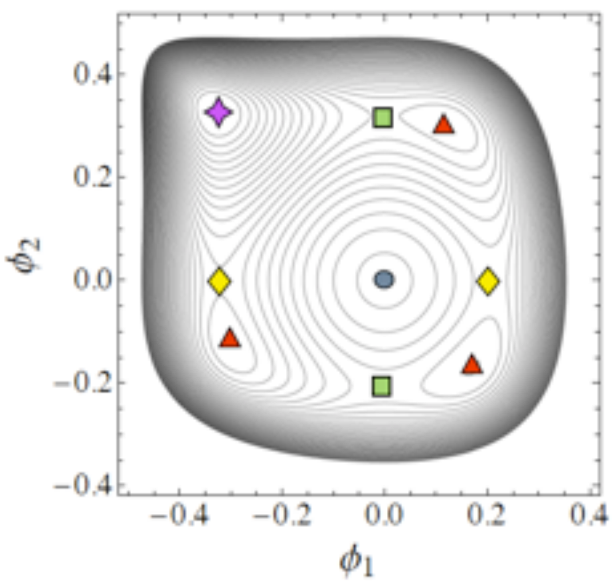
$$I = 1 \oplus m$$

i) gravitino-gravitino mass $\mathcal{A}^{IJ}(\phi_0)$ ⇒ $\mathcal{A}^{11} = \alpha_1 \;,\; \mathcal{A}^{mn} = \alpha_2\, \delta^{mn}$

ii) gravitino-dilatino mass ${\mathcal{A}_I}^{JKL}(\phi_0)$ ⇒ ${\mathcal{A}_1}^{mnp} = \beta_1\, \kappa^{mnp} \;,\; {\mathcal{A}_m}^{1np} = \beta_2\, {\kappa_m}^{np}$

> Four parameters $\alpha_1 , \alpha_2 , \beta_1 , \beta_2 \in \mathbb{C}$

Example 1 : Critical point preserving $G_0 = G_2$ (✦)



- Pattern of fermi masses :

$$I = 1 \oplus m$$

i) gravitino-gravitino mass $\mathcal{A}^{IJ}(\phi_0)$ ⇨ $\mathcal{A}^{11} = \alpha_1 \;,\; \mathcal{A}^{mn} = \alpha_2\, \delta^{mn}$

ii) gravitino-dilatino mass $\mathcal{A}_I{}^{JKL}(\phi_0)$ ⇨ $\mathcal{A}_1{}^{mnp} = \beta_1\, \kappa^{mnp} \;,\; \mathcal{A}_m{}^{1np} = \beta_2\, \kappa_m{}^{np}$

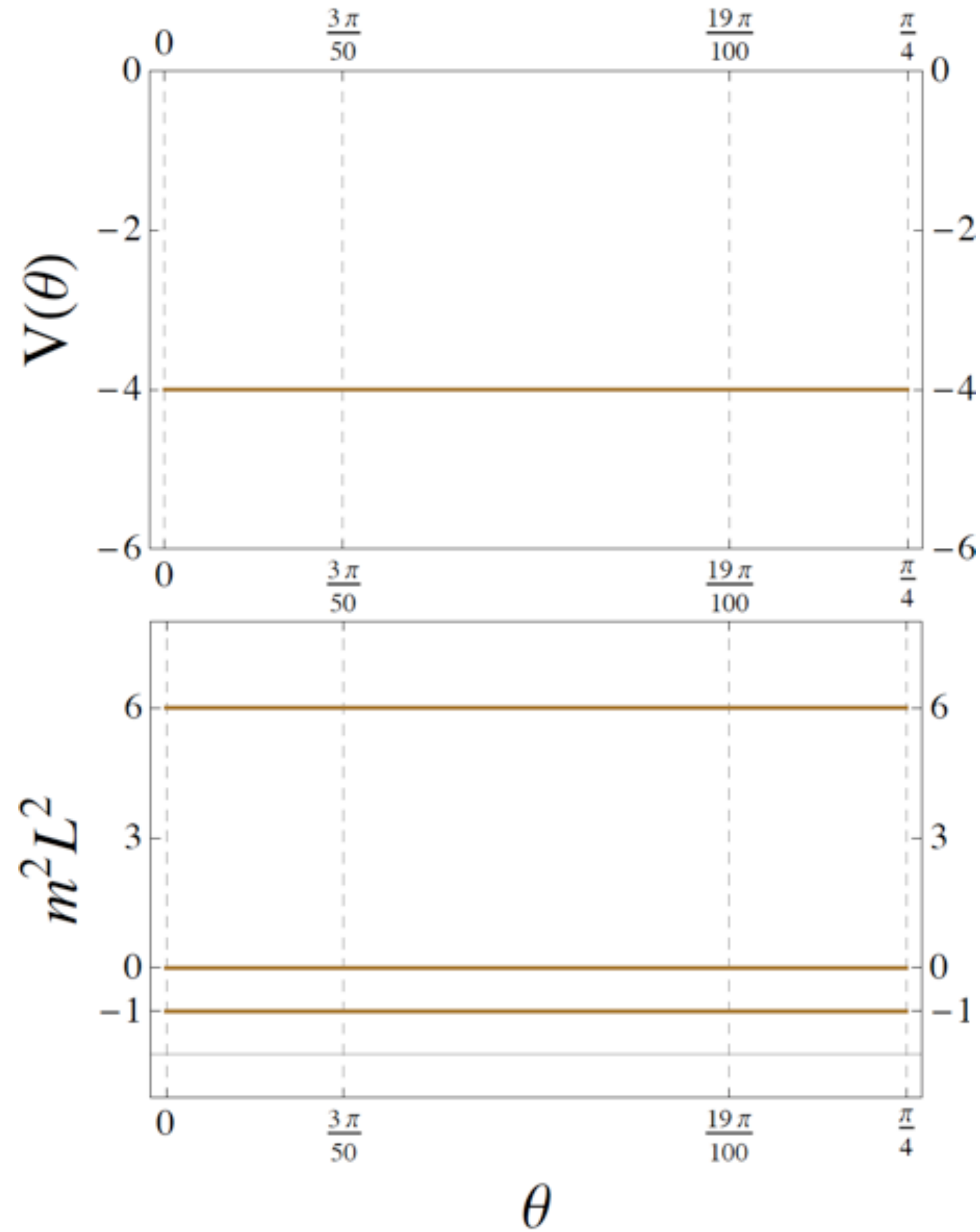
> Four parameters $\alpha_1 \,,\, \alpha_2 \,,\, \beta_1 \,,\, \beta_2 \in \mathbb{C}$

Solving QC & EOM : One-parameter family of theories compatible with $G_0 = G_2$

$$\alpha_1(\theta) \,,\, \alpha_2(\theta) \,,\, \beta_1(\theta) \,,\, \beta_2(\theta)$$

A boring excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = G_2$ can be tracked

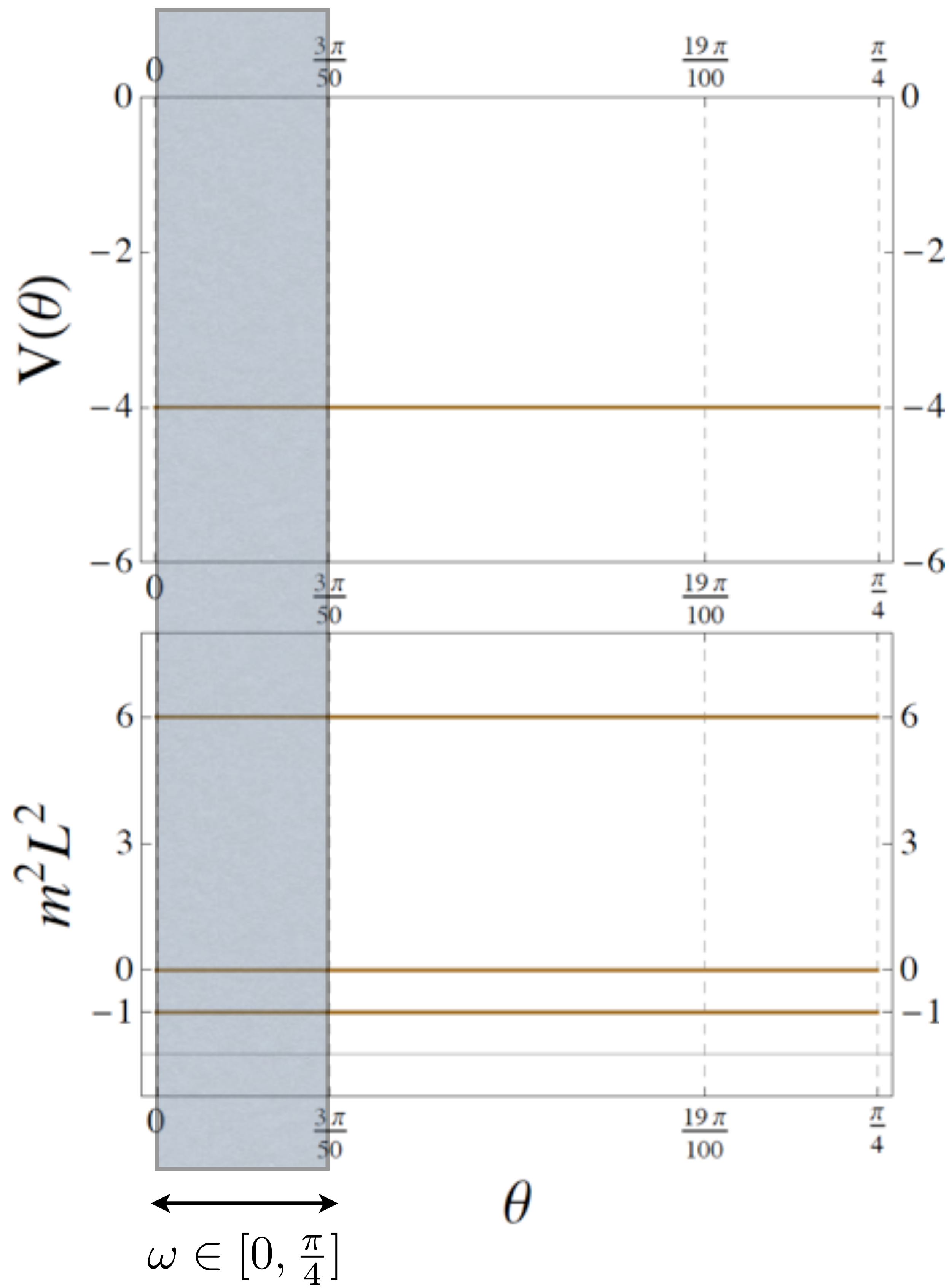


A boring excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = G_2$ can be tracked

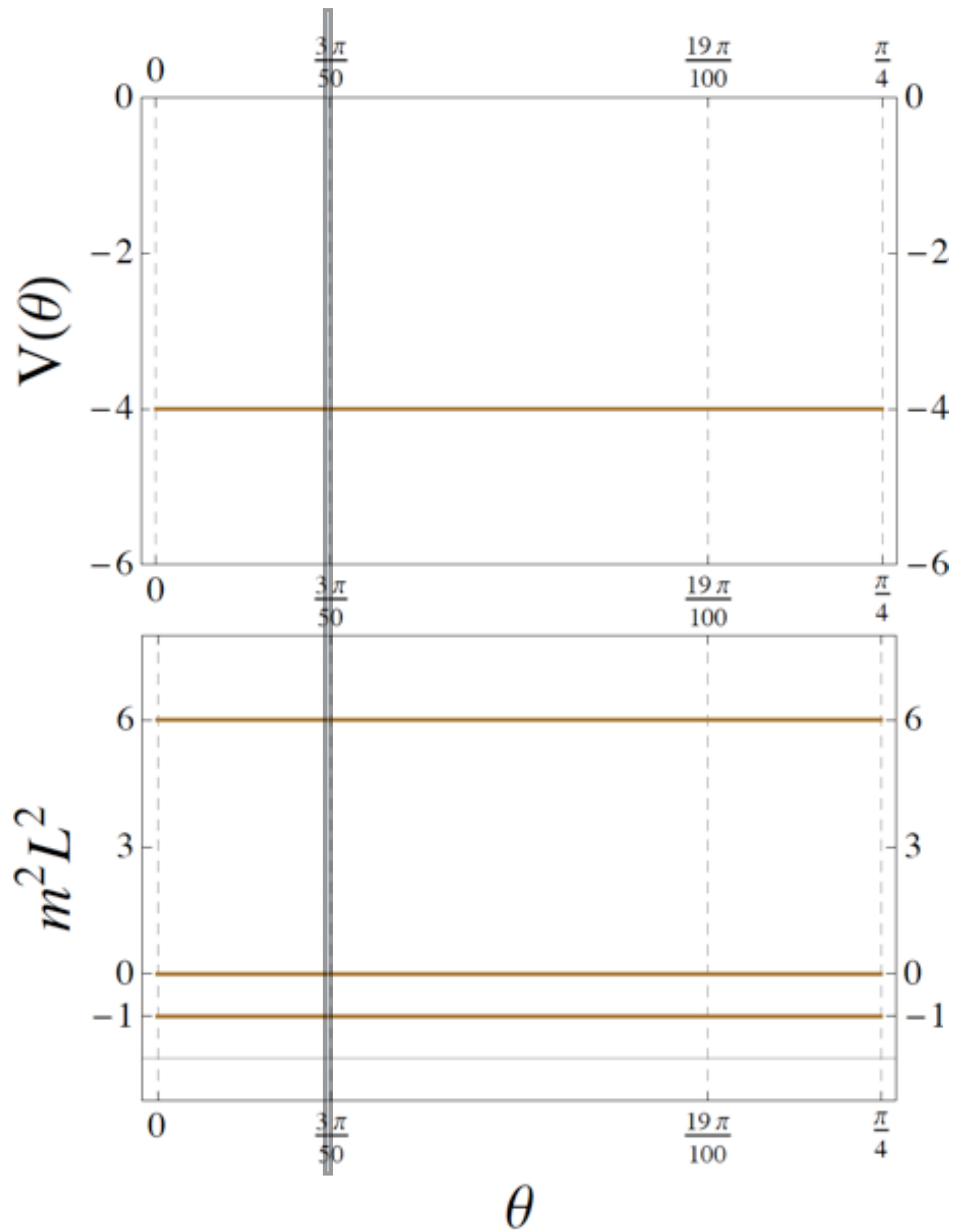
$i)\quad 0 \leq \theta < \frac{3\pi}{50} \;\rightarrow\; G = SO(8)$

[stable AdS_4 solutions]



A boring excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = G_2$ can be tracked



$ii)\quad \theta = \frac{3\pi}{50} \;\rightarrow\; G = ISO(7)$

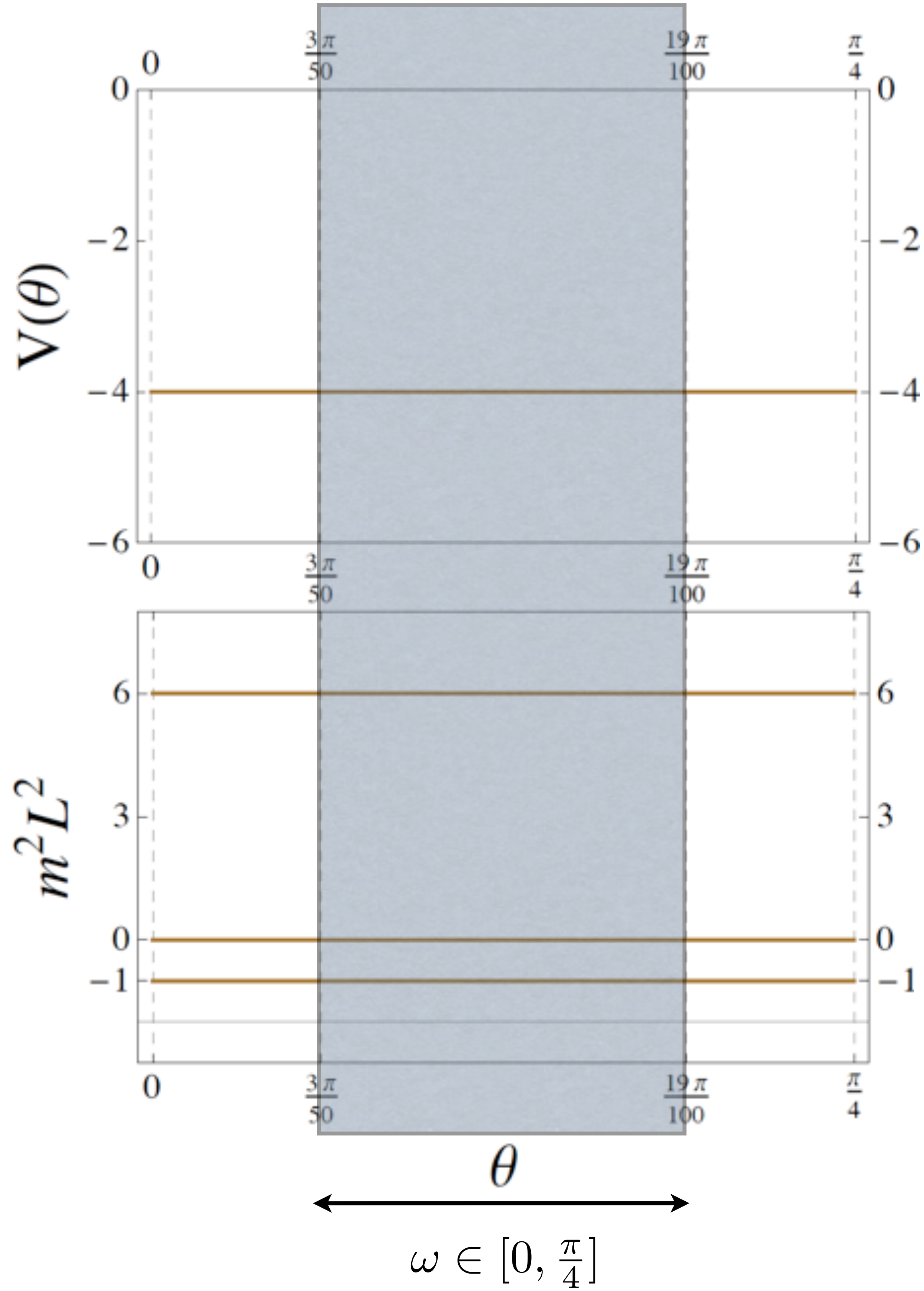
[stable AdS_4 solutions]

A boring excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = G_2$ can be tracked

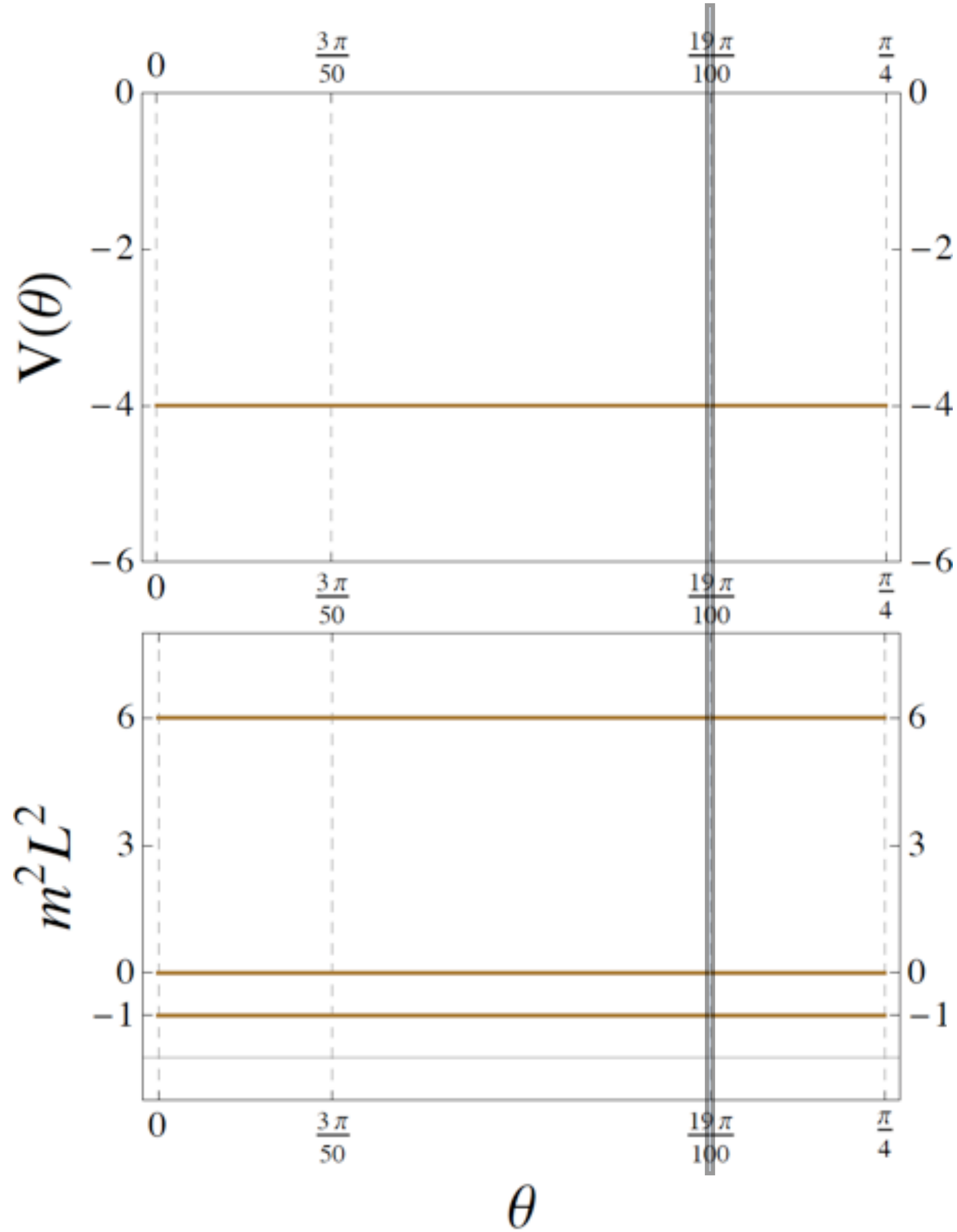
$iii)\quad \frac{3\pi}{50} < \theta < \frac{19\pi}{100} \;\rightarrow\; G = SO(7,1)$

[stable AdS_4 solutions]



A boring excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = G_2$ can be tracked

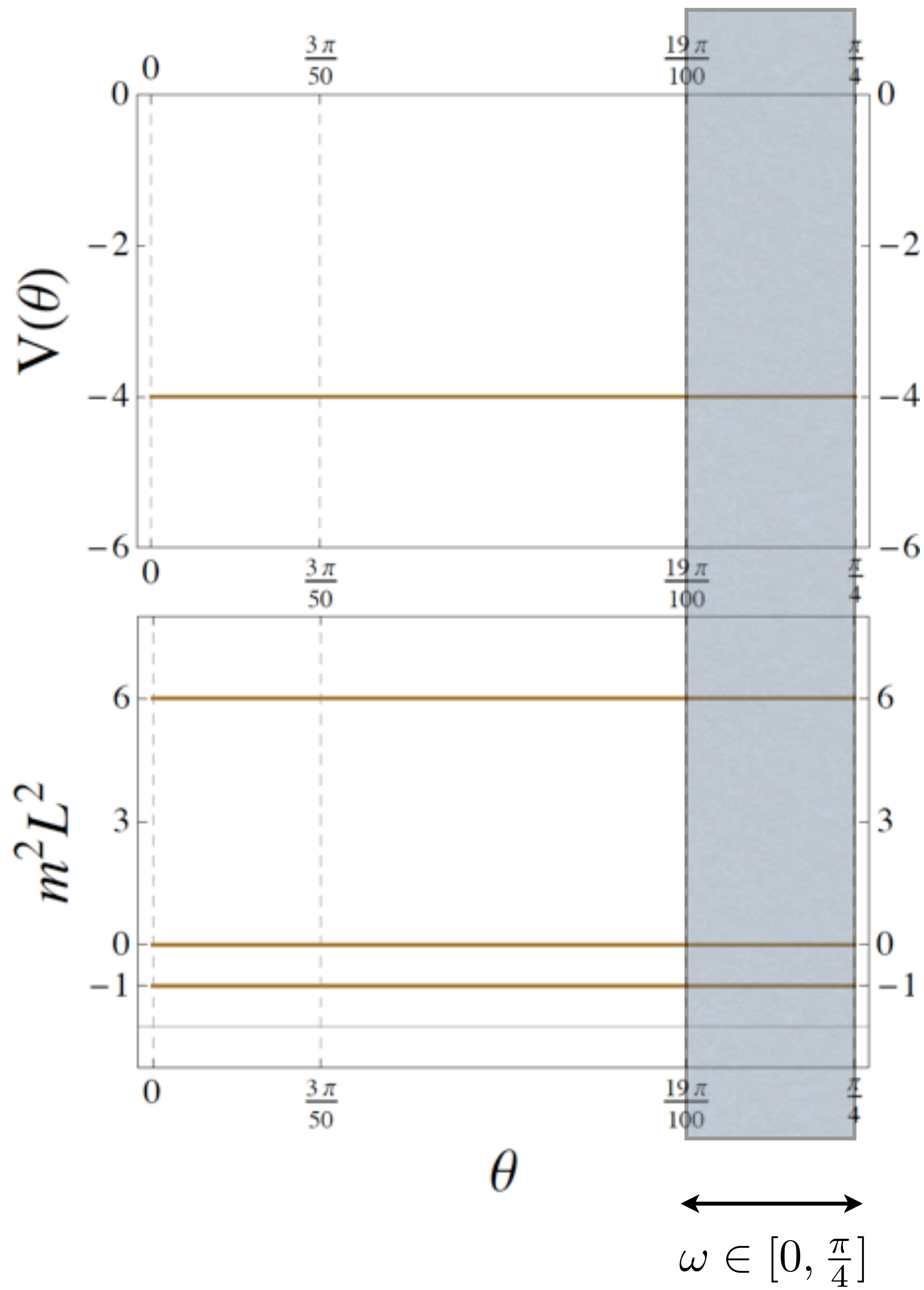


$iv) \quad \theta = \frac{19\pi}{100} \;\rightarrow\; G = ISO(7)$

[stable AdS_4 solutions]

A boring excursion through theories (gaugings)

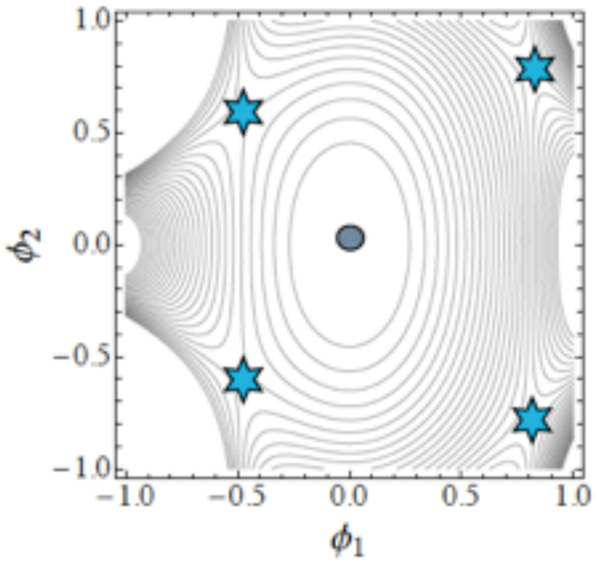
- The whole story of a solution preserving $G_0 = G_2$ can be tracked



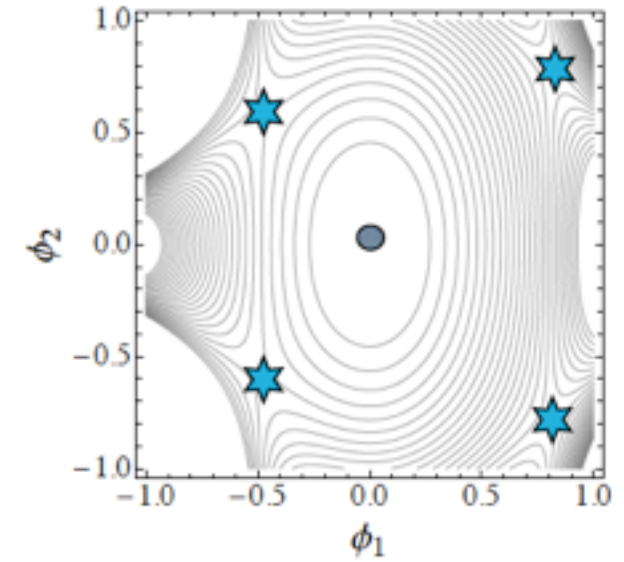
$v)\quad \frac{19\pi}{100} < \theta \leq \frac{\pi}{4} \;\rightarrow\; G = SO(8)$

[stable AdS_4 solutions]

Example 2 : Critical point preserving $G_0 = SO(4)_s$ (✡)



Example 2 : Critical point preserving $G_0 = SO(4)_s$ (★)



- Pattern of fermi masses :

$[\ I \rightarrow i \oplus \hat{i}\ ]$

i) gravitino-gravitino mass $\mathcal{A}^{IJ}(\phi_0)$ ⇨ $\mathcal{A}^{ij} = \alpha\ \delta^{ij} \quad , \quad \mathcal{A}^{\hat{i}\hat{j}} = \alpha\ \delta^{\hat{i}\hat{j}}$

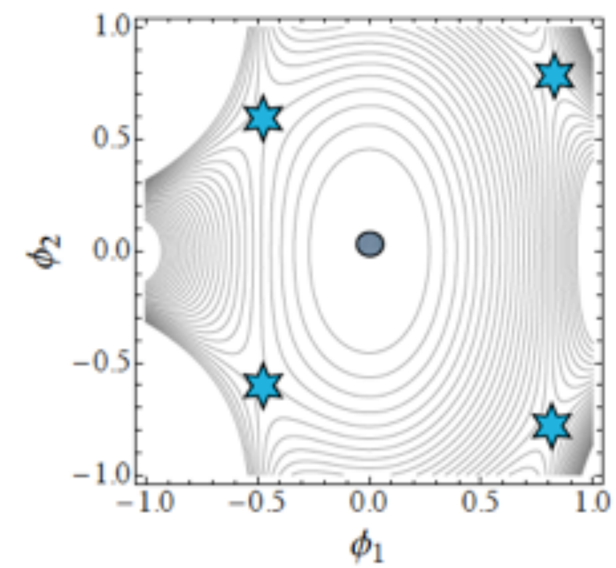
ii) gravitino-dilatino mass $\mathcal{A}_I{}^{JKL}(\phi_0)$ ⇨

$$\mathcal{A}_i{}^{jkl} = \beta\ \epsilon_i{}^{jkl} \quad , \quad \mathcal{A}_i{}^{\hat{j}\hat{k}l} = \delta\ \epsilon_i{}^{\hat{j}\hat{k}l} + \gamma\ \delta_i^{[\hat{j}}\delta^{\hat{k}]l}$$

$$\mathcal{A}_{\hat{i}}{}^{\hat{j}\hat{k}\hat{l}} = -\beta\ \epsilon_{\hat{i}}{}^{\hat{j}\hat{k}\hat{l}} \quad , \quad \mathcal{A}_{\hat{i}}{}^{jk\hat{l}} = -\delta\ \epsilon_{\hat{i}}{}^{jk\hat{l}} + \gamma\ \delta_{\hat{i}}^{[j}\delta^{k]\hat{l}}$$

> Four parameters $\alpha, \beta, \gamma, \delta \in \mathbb{C}$

Example 2 : Critical point preserving $G_0 = SO(4)_s$ (★)



- Pattern of fermi masses :

$[\, I \rightarrow i \oplus \hat{i} \,]$

i) gravitino-gravitino mass $\mathcal{A}^{IJ}(\phi_0)$ ⇨ $\mathcal{A}^{ij} = \alpha\, \delta^{ij} \quad , \quad \mathcal{A}^{\hat{i}\hat{j}} = \alpha\, \delta^{\hat{i}\hat{j}}$

ii) gravitino-dilatino mass $\mathcal{A}_I{}^{JKL}(\phi_0)$ ⇨

$$\mathcal{A}_i{}^{jkl} = \beta\, \epsilon_i{}^{jkl} \quad , \quad \mathcal{A}_i{}^{\hat{j}\hat{k}l} = \delta\, \epsilon_i{}^{\hat{j}\hat{k}l} + \gamma\, \delta_i^{[\hat{j}} \delta^{\hat{k}]l}$$
$$\mathcal{A}_{\hat{i}}{}^{\hat{j}\hat{k}\hat{l}} = -\beta\, \epsilon_{\hat{i}}{}^{\hat{j}\hat{k}\hat{l}} \quad , \quad \mathcal{A}_{\hat{i}}{}^{jk\hat{l}} = -\delta\, \epsilon_{\hat{i}}{}^{jk\hat{l}} + \gamma\, \delta_{\hat{i}}^{[j} \delta^{k]\hat{l}}$$

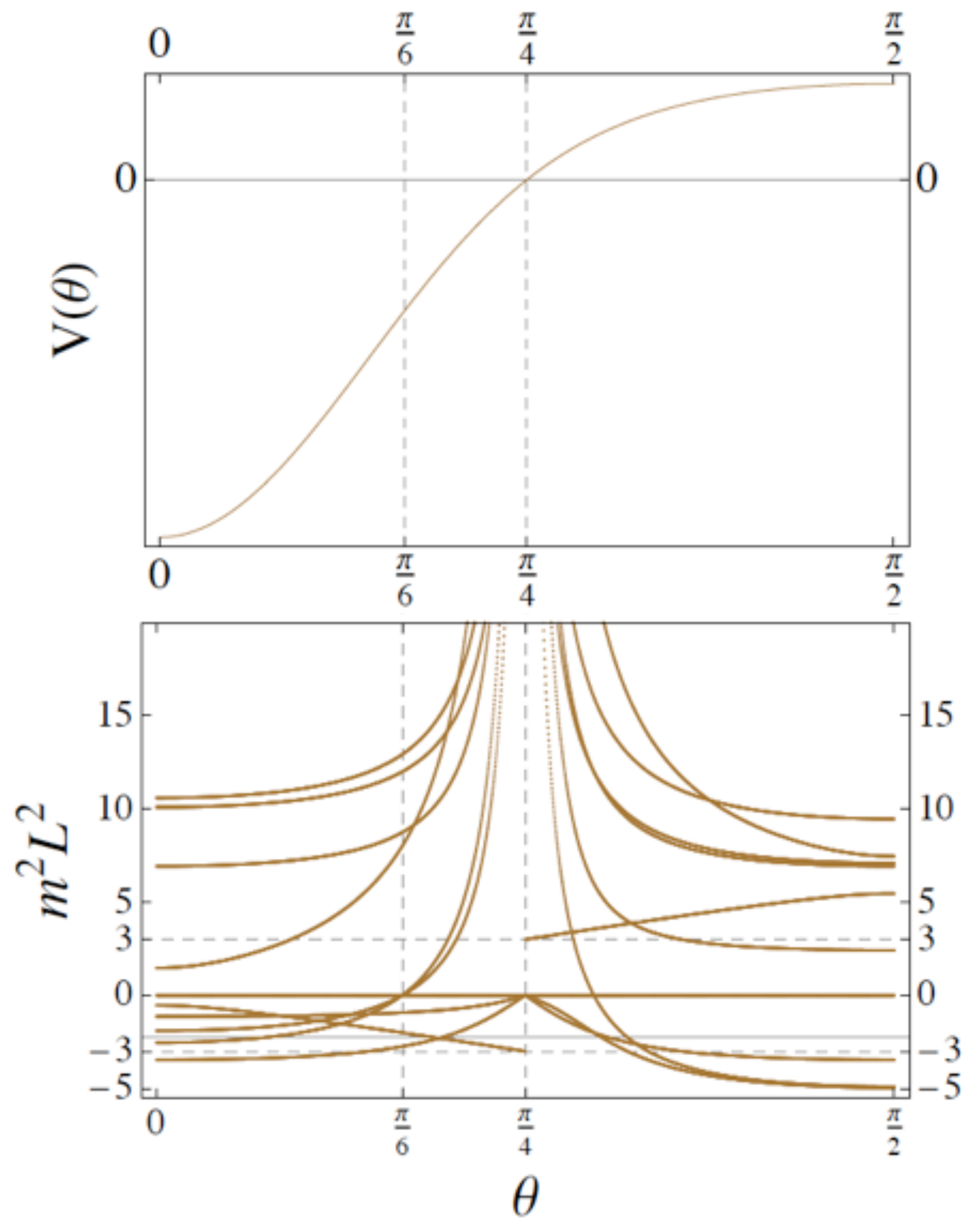
> Four parameters $\alpha, \beta, \gamma, \delta \in \mathbb{C}$

Solving QC & EOM : One-parameter family of theories compatible with $G_0 = SO(4)_s$

$$\alpha(\theta)\, ,\ \beta(\theta)\, ,\ \gamma(\theta)\, ,\ \delta(\theta)$$

A funny excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = \mathrm{SO}(4)_s$ can be tracked

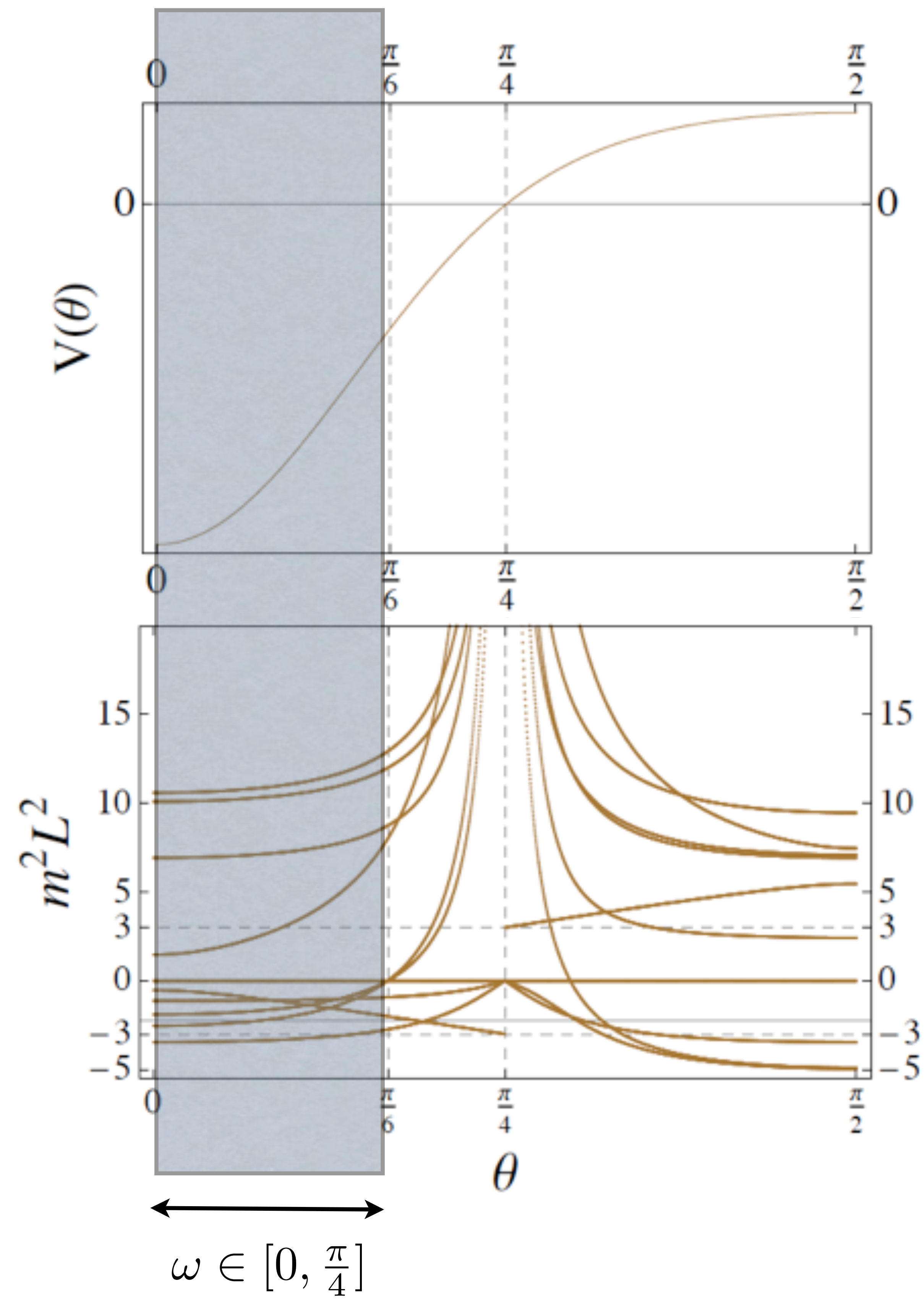


A funny excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = \mathrm{SO}(4)_s$ can be tracked

  $i)$ $0 \leq \theta < \frac{\pi}{6} \rightarrow G = SO(8)$

  [unstable AdS_4 solutions]

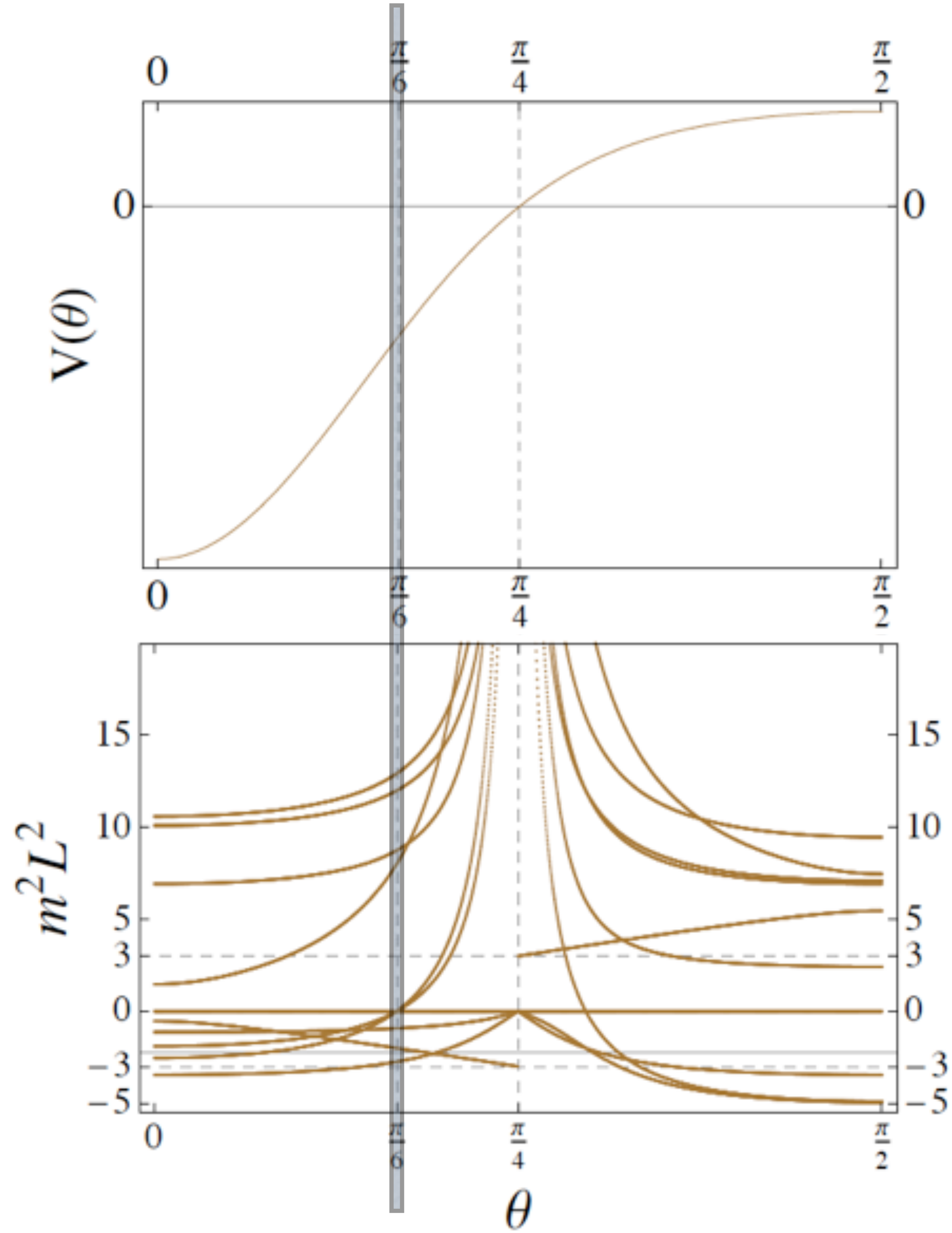


A funny excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = SO(4)_s$ can be tracked

$$ii) \quad \theta = \frac{\pi}{6} \quad \rightarrow \quad G = SO(2) \times SO(6) \ltimes T^{12}$$

[unstable AdS_4 solution]

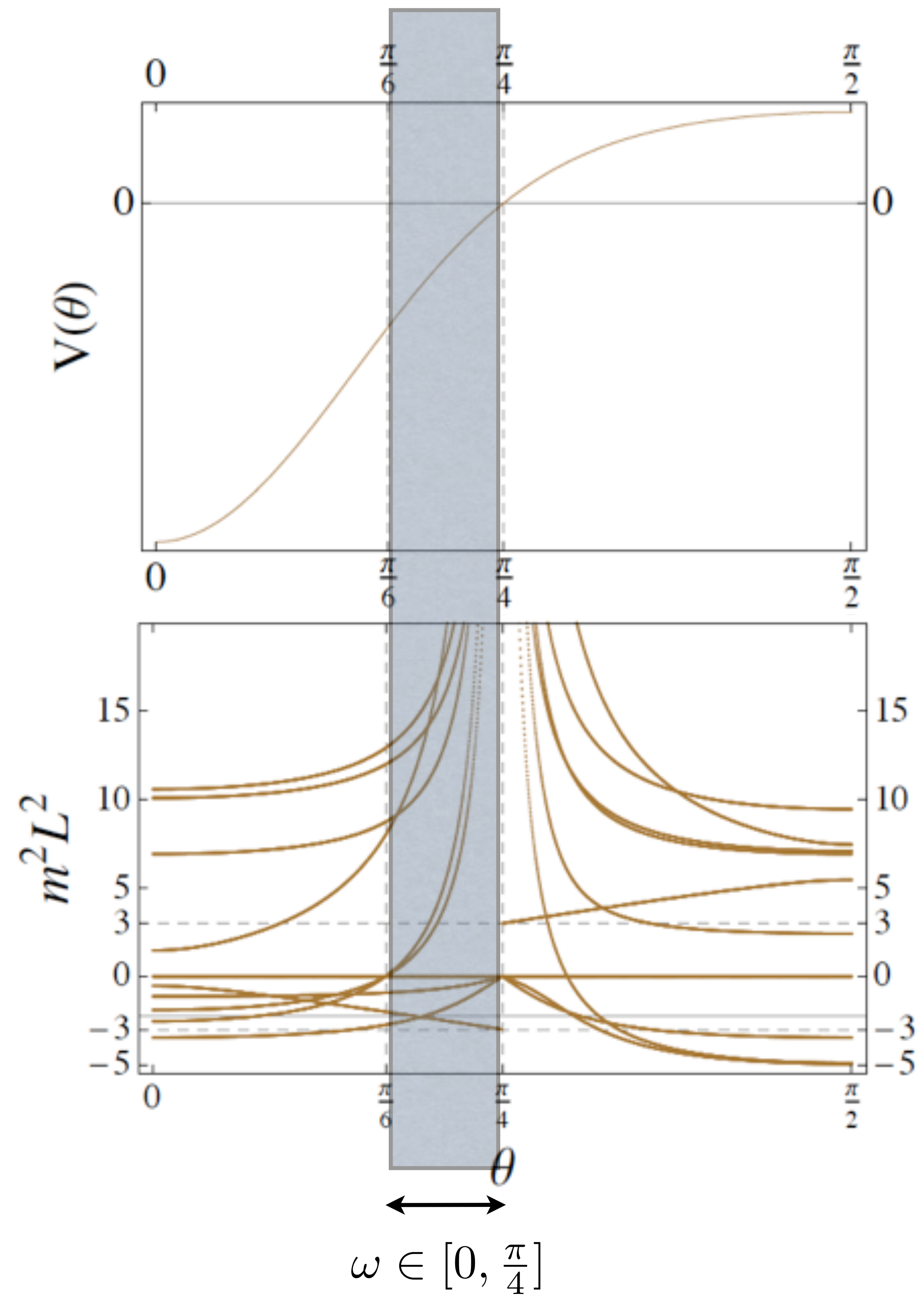


A funny excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = \mathrm{SO}(4)_s$ can be tracked

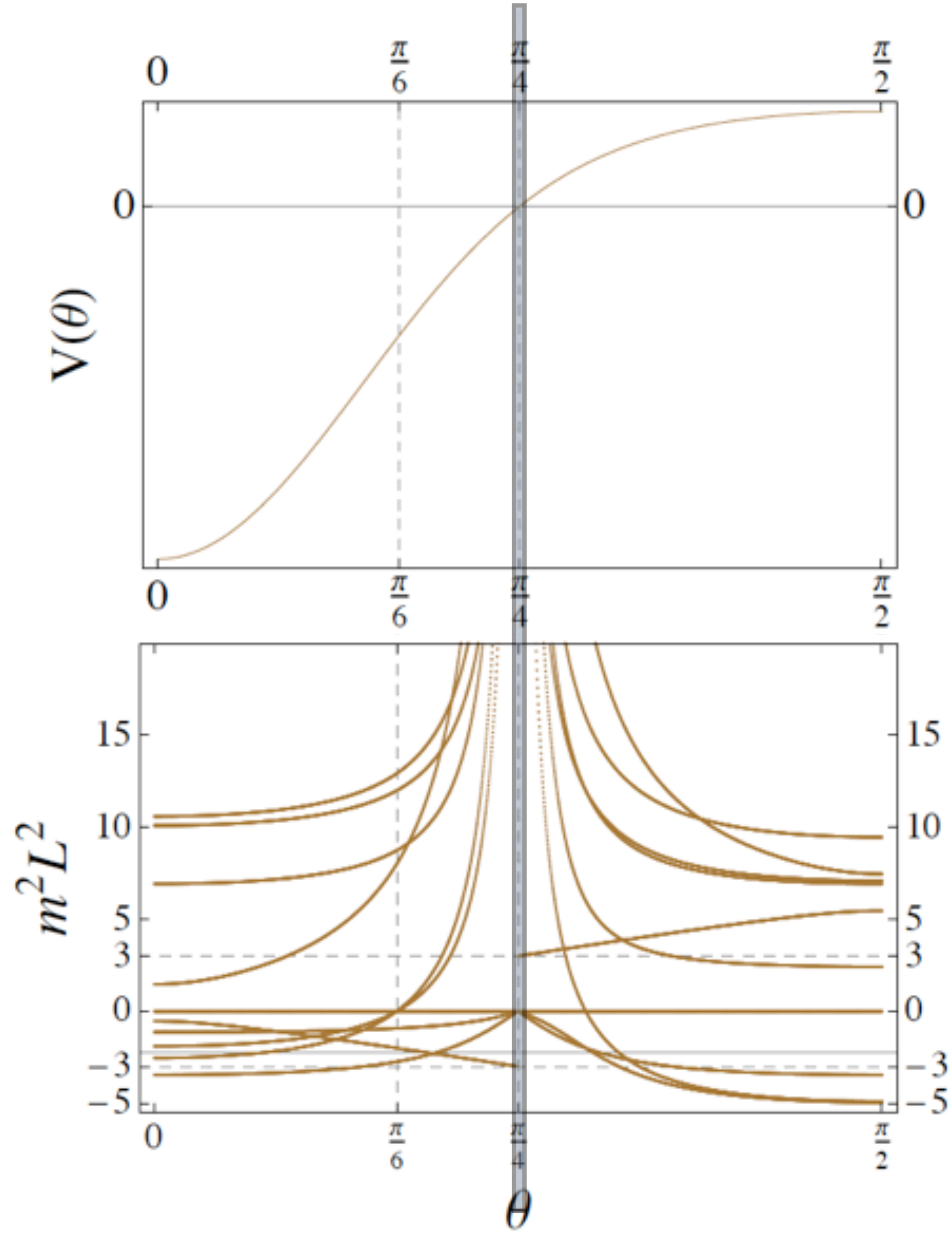
$iii) \quad \frac{\pi}{6} < \theta < \frac{\pi}{4} \quad \rightarrow \quad G = SO(6,2)$

[unstable AdS_4 solutions]



A funny excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = SO(4)_s$ can be tracked

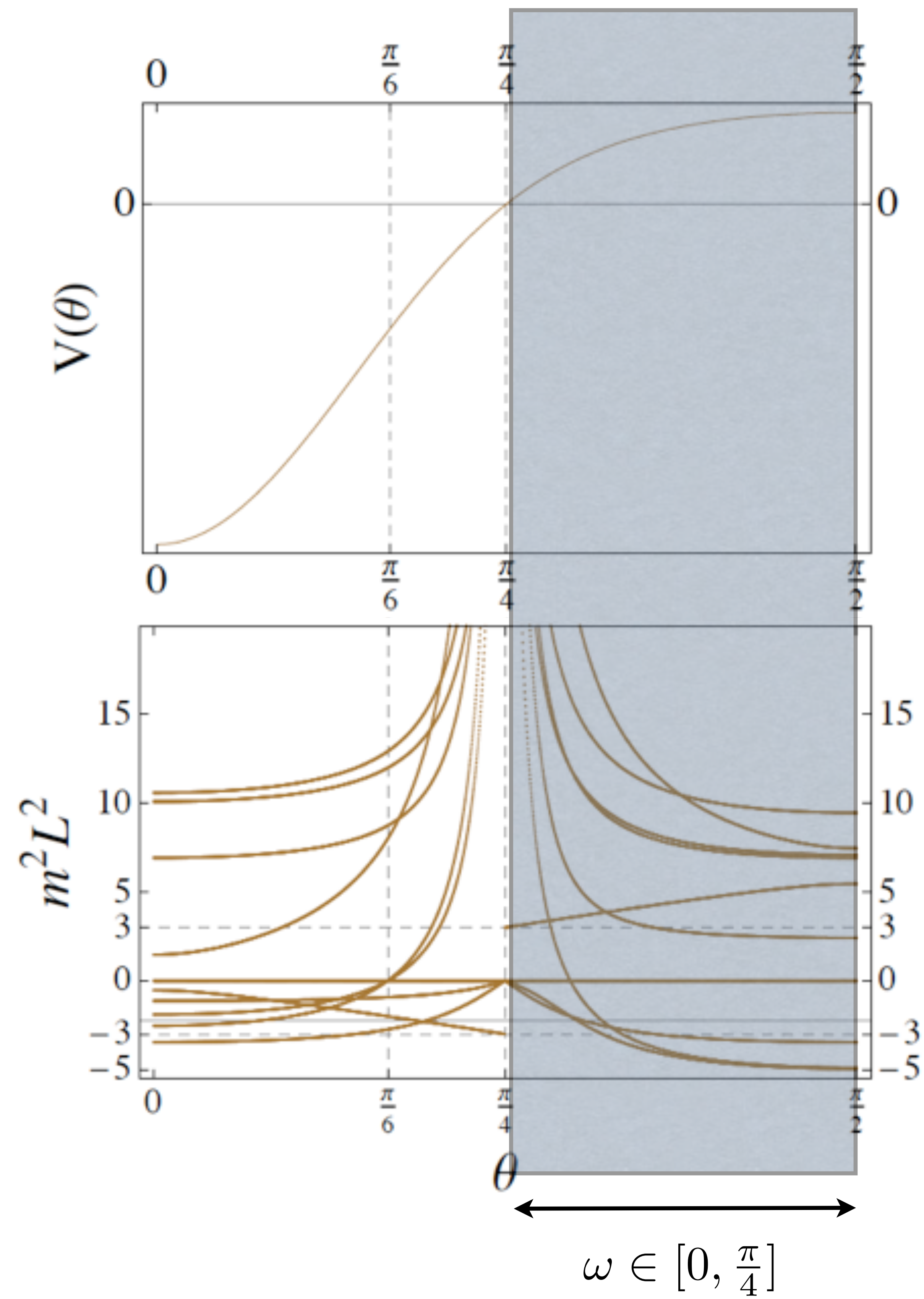


$iv)\quad \theta = \frac{\pi}{4} \quad \rightarrow \quad G = SO(3,1)^2 \ltimes T^{16}$

[Mkw solution with flat directions]

A funny excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = SO(4)_s$ can be tracked



$v) \quad \frac{\pi}{4} < \theta \leq \frac{\pi}{2} \quad \rightarrow \quad G = SO(4,4)$

[dS_4 solutions with tachyon dilution]

[Dall'Agata & Inverso '12]

Final remarks

- Electromagnetic U(1) rotations pick up a physically relevant direction in the space of the embedding tensor deformations and provide new vacua of $\mathcal{N}=8$ supergravity with interesting properties : increase of critical points, partial $\mathcal{N}=2 \rightarrow \mathcal{N}=1$ breaking, stability without SUSY, ...

- Small residual symmetry groups like SU(3) & SO(4) show ω-dependent mass spectra. Triality restores $\frac{\pi}{4}$-periodicity.

[Borghese, Dibitetto, A.G , Roest & Varela '13]

- Critical points running away at $\omega = n\,\frac{\pi}{4}$ in one theory, show up in another. The entire story of a solution can be tracked by computing fermi masses in the GTTO approach

- Tachyon dilution around AdS/Mkw/dS transitions.

- All these solutions can be obtained as $Z_2 x Z_2$ Type IIB orientifolds with non-geometric fluxes (both electric and magnetic). Lifting to M-theory including vectors from A_3 and A_6 ? And oxidation to DFT ?

[de Wit & Nicolai '13]

[Blumenhagen, Gao, Herschmann & Shukla '13]

Thanks for your attention !!