

Cosmological and Holographical Applications of String Dualities

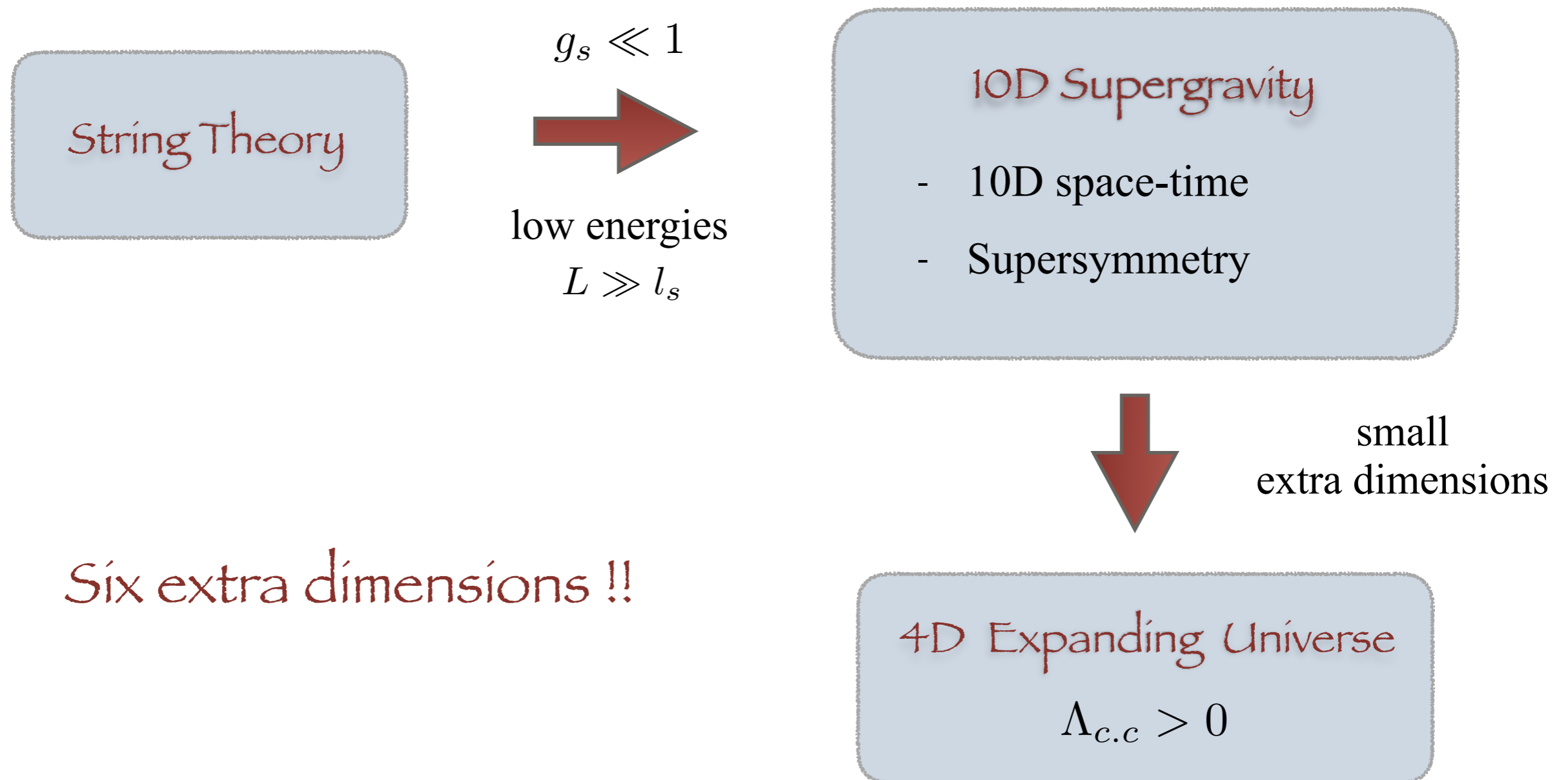


Based on various collaborations with

1 Borghese, Casas, Ciceri, Danielsson, de Carlos,
Derendinger, Dibitetto, F-Melgarejo, Font, Inverso,
Jafferis, Moreno, Roest, Tarrío, Varela & Weatherill

Linking strings to the real world

- ❖ String theory provides a framework where to describe General Relativity and Quantum Field Theory
- ❖ The fundamental building blocks are tiny vibrating strings with $l_s \sim 10^{-33}$ cm



The footprint of the extra dimensions

- ❖ Fluctuations of the extra dimensions (size and shape) translate into a set of **massless** 4D scalar fields known as “**moduli fields**”

$$\mathcal{L} = R - \frac{1}{2} \partial_\mu \phi_i \partial^\mu \phi^i$$



Deviations
from GR !!

massless scalars = long range interactions (precision tests of GR)

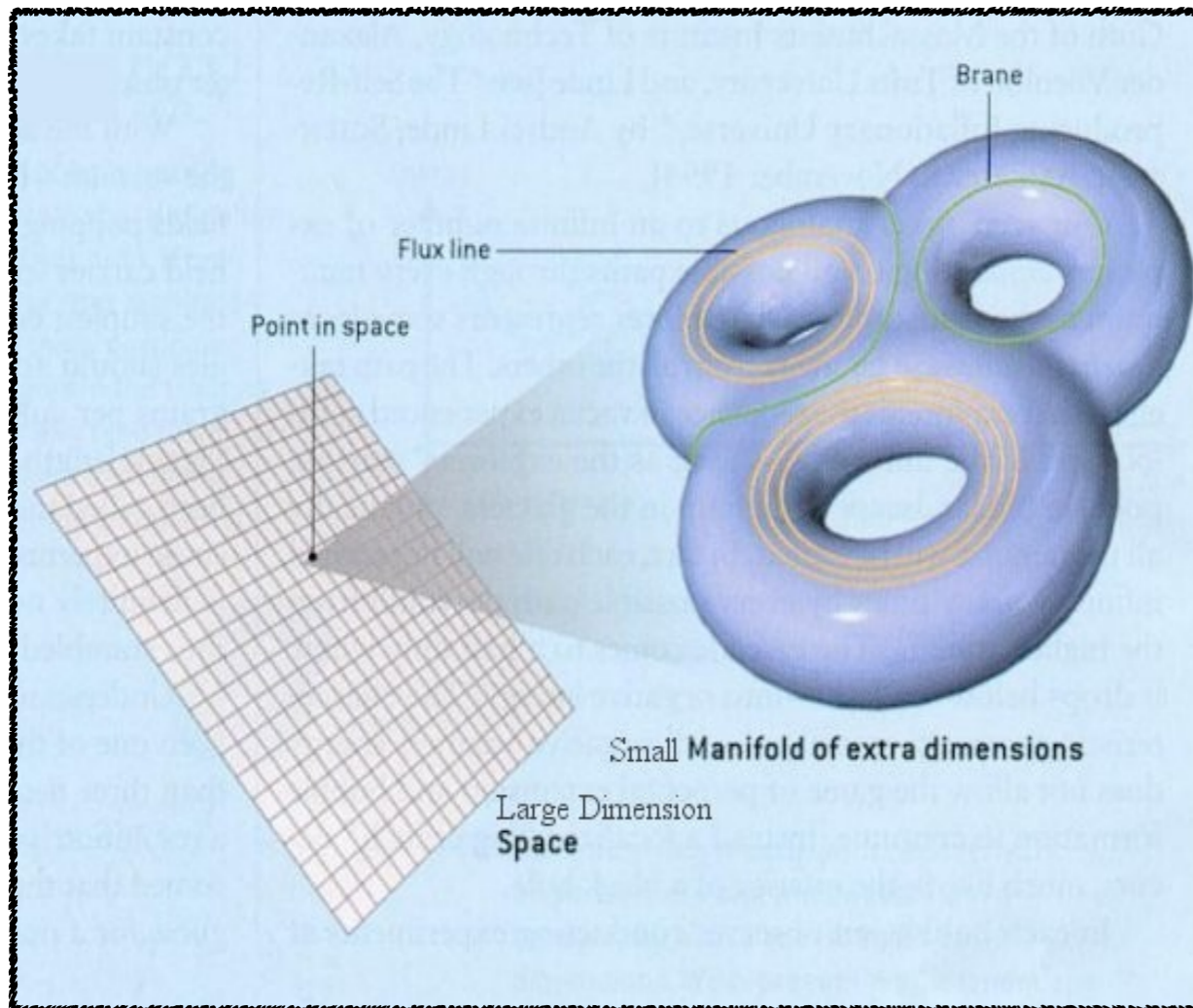
- ❖ String phenomenology \rightarrow Mechanisms for “**moduli stabilisation**”

$$V(\phi) = m_{ij}^2 \phi^i \phi^j + \dots$$

- ❖ The moduli VEVs $\langle \phi^i \rangle = \phi_0^i$ determine the **4D cosmological constant !!**

$$\Lambda_{c.c} \equiv V(\phi_0)$$

Extra dimensions...



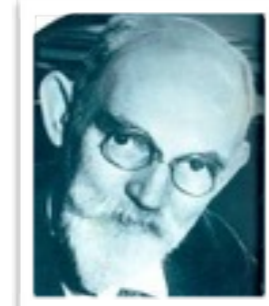
... will be non empty !!

- D-branes
- magnetic fluxes
- funny geometries

...

$$V(\phi) = V_{brane} + V_{flux} + V_{geom}$$

The problem = finding

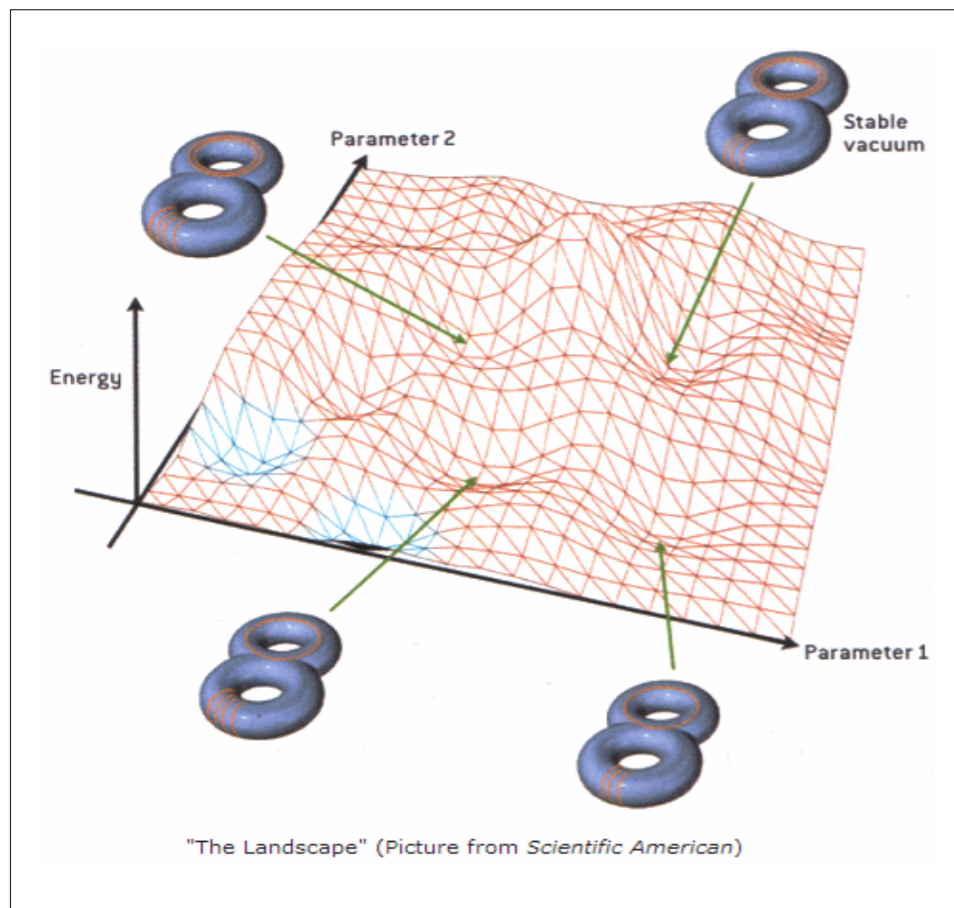


Willem de Sitter
(1872 – 1934)

❖ Model building :

branes + fluxes + geometries + ... = parameters

[Giddings, Kachru, Polchinski '01]



[Blumemhagen, Cámara, de Carlos, Dall'Agata, Derendinger, DeWolfe, Giryavets, Graña, Ibáñez, Kachru, Kounnas, Kors, Lüst, Minasian, Petrini, Petropoulos, Reffert, Schulz, Schulgin, Stieberger, Taylor, Tomasiello, Trigiante, Tripathy, Trivedi, Villadoro, Zwirner, ... 2002 - 2006]

$$\Lambda_{c.c} \equiv V(\phi_0) > 0$$

... but where is de Sitter

within the string landscape?

No-go theorems forbid perturbative dS vacua !!

[Hertzberg, Kachru, Taylor, Tegmark '07]



A way out:

To introduce *mysterious non-geometric* fluxes

[Dabholkar, Hull '05]

Motivation: what are non-geometric fluxes?

[Shelton, Taylor, Wecht '05, '06]

4D objects *conjectured* to exist based on string dualities (different regimes of strings)

[strong-weak coupling, winding-momentum states, ...]

[Hull, Townsend '94]

Questions to be addressed today :

- Are there stable dS vacua in these non-geometric 4D scenarios (useful at all)?
- Is there a 4D effective field theory where to describe such exotic fluxes systematically?
- Do they have a higher-dimensional (string) origin?

Challenges in string theory !!

From
non-geometric fluxes (2004-2006)

to

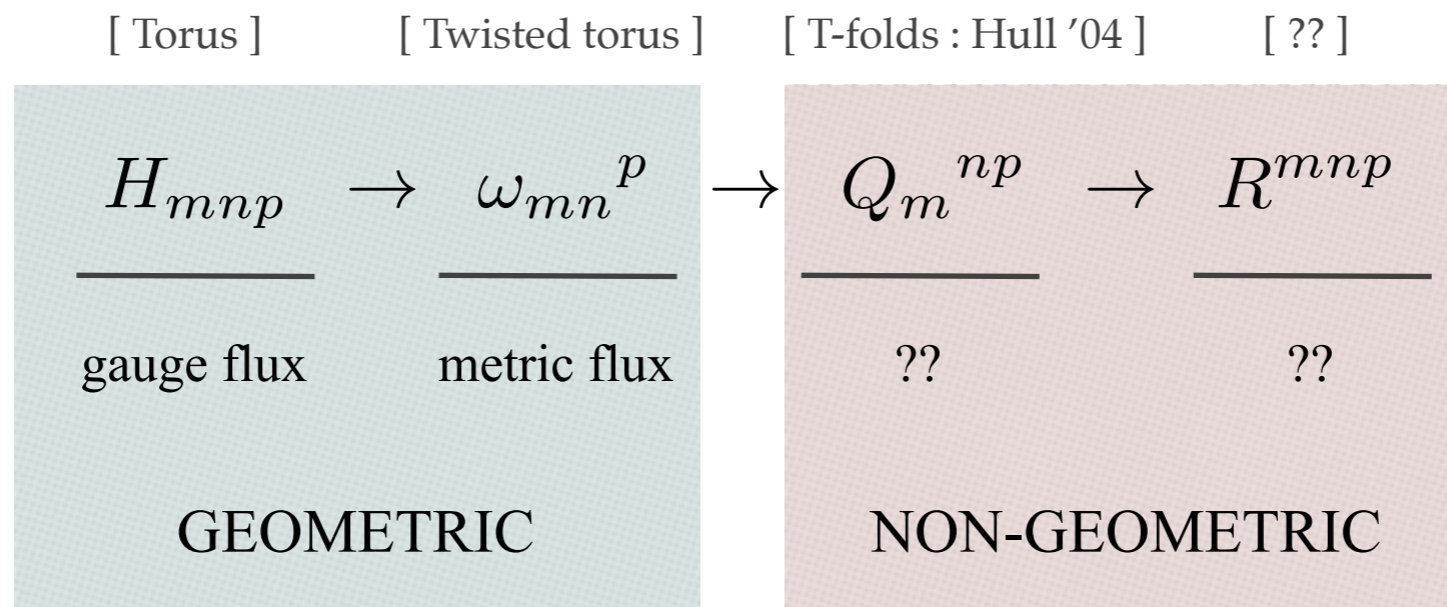
extended field theories (2014-2016)

(a personal trip)

(2004 - 2006)

[Dabholkar, Hull '05]
[Shelton, Taylor, Wecht '05, '06]

- ❖ Non-geometric fluxes were introduced for the first time in the context of effective N=1 supergravities in order to restore the *stringy* T-duality at the 4D level
- ❖ Starting from the field strength $H=dB$ of the B-field (2-form gauge potential) of string theory and applying a **chain of T-dualities**, one finds



[Kaloper, Myers '99]

- ❖ SO(6,6) T-duality symmetry of string theory becomes manifest at the 4D level !!
- ❖ Generalisation to other *stringy* SL(2) S-duality and E₇ U-duality : P-fluxes, etc...

[Aldazabal, Cámara, Font, Ibáñez '06]

(2007 - 2009)

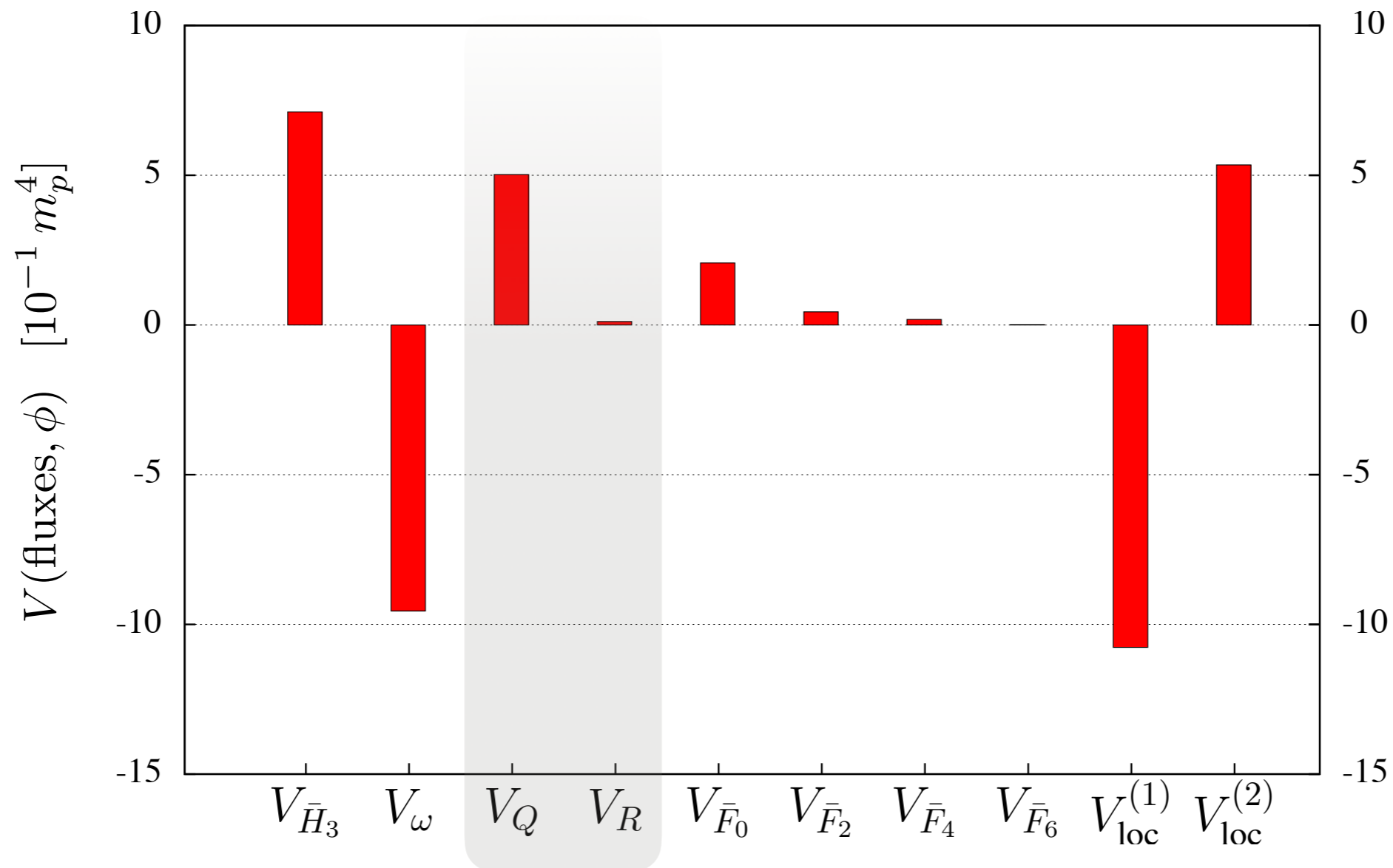
[Hull, Reid-Edwards '07] [Reid-Edwards '09]

[Dall'Agata, Prezas, Samtleben, Trigiante '07]

[Graña, Minasian, Petrini, Waldram '08]

[Font, AG, Moreno '08] [AG, Weatherill '08]

- ❖ Soon after their appearance, novel **algebraic structures** underlying the zoo of non-geometric fluxes started being uncovered. Making use of these algebraic structures, the first examples of *non-geometric stable dS vacua* were found!!

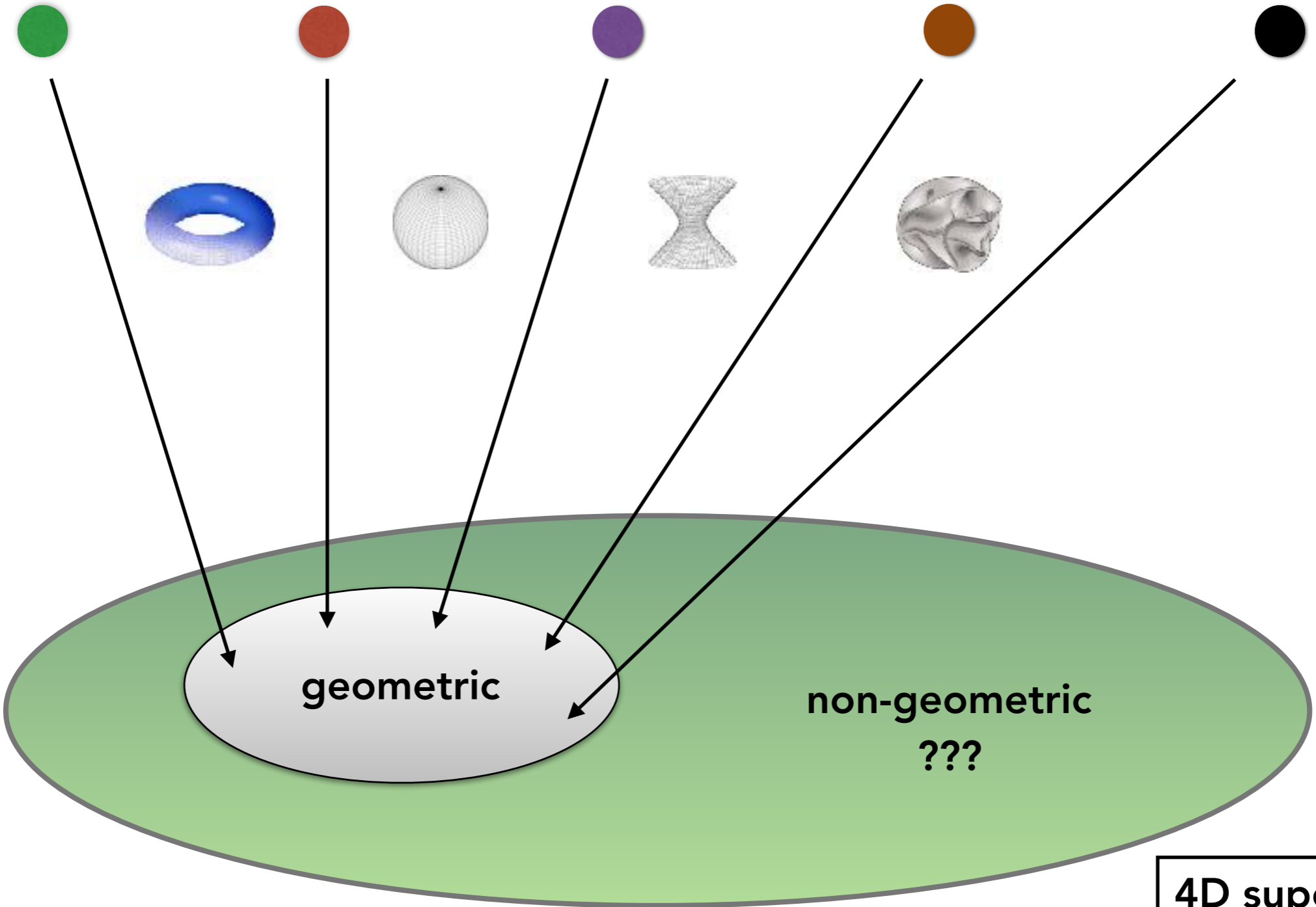


... although their higher-dimensional origin remained unclear

[de Carlos, AG, Moreno '09]

Higher-dimensional supergravities

Type IIA Type IIB 11D sugra | Type I Heterotic



4D supergravities
(ET framework)

Embedding Tensor framework

[3D : Nicolai, Samtleben '00 '01]
[de Wit, Samtleben, Trigiante '05 '07]
[Schön, Weidner '06]

- ❖ Parallel to these developments, an effective field theory framework where to describe *all* the N=8 and N=4 supergravities irrespective of their origin enters the scene :

the *embedding tensor* framework

- ❖ The ET formalism implements *all the stringy dualities in a 4D effective Lagrangian*, namely, the interactions are determined by the symmetries of string theory

couplings in the potential (fluxes) = embedding tensor (reps. of duality group)

Program : Precise *correspondence* between *embedding tensor* and *fluxes* (geometric and non-geometric) so that a systematic study of the latter can be achieved !!

Previous results :

[de Wit, Samtleben, Trigiante '03] [Angelantonj, Ferrara, Trigiante '03 '04]

[Derendinger, Kounnas, Petropoulos & Zwirner '04 '05 '06]

[Villadoro, Zwirner '05] [Aldazabal, Cámara, Font, Ibáñez '06]

Embedding Tensor / Flux correspondence

[Dall'Agata, Villadoro, Zwirner '09]

[Dibitetto, AG, Roest '11 '12 '14]

couplings	SO(6, 6)
1	$-f_{+a\bar{b}\bar{c}}$
U	$f_{+a\bar{b}\bar{k}}$
U^2	$-f_{+a\bar{j}\bar{k}}$
U^3	$f_{+i\bar{j}\bar{k}}$
S	$-f_{-a\bar{b}\bar{c}}$
SU	$f_{-a\bar{b}\bar{k}}$
SU^2	$-f_{-a\bar{j}\bar{k}}$
SU^3	$f_{-i\bar{j}\bar{k}}$
T	$f_{+a\bar{b}\bar{k}}$
TU	$f_{+a\bar{j}\bar{k}} = f_{+i\bar{b}\bar{k}} , f_{+a\bar{b}\bar{c}}$
TU^2	$f_{+i\bar{b}\bar{c}} = f_{+a\bar{j}\bar{c}} , f_{+i\bar{j}\bar{k}}$
TU^3	$f_{+i\bar{j}\bar{c}}$
ST	$f_{-a\bar{b}\bar{k}}$
STU	$f_{-a\bar{j}\bar{k}} = f_{-i\bar{b}\bar{k}} , f_{-a\bar{b}\bar{c}}$
STU^2	$f_{-i\bar{b}\bar{c}} = f_{-a\bar{j}\bar{c}} , f_{-i\bar{j}\bar{k}}$
STU^3	$f_{-i\bar{j}\bar{c}}$



Type IIB	Type IIA
F_{ijk}	F_{aibjck}
F_{ijc}	F_{aibj}
F_{ibc}	F_{ai}
F_{abc}	F_0
H_{ijk}	H_{ijk}
H_{ijc}	ω_{ij}^c
H_{ibc}	Q_i^{bc}
H_{abc}	R^{abc}
Q_k^{ab}	H_{abk}
$Q_k^{aj} = Q_k^{ib} , Q_a^{bc}$	$\omega_{ka}^j = \omega_{bk}^i , \omega_{bc}^a$
$Q_c^{ib} = Q_c^{aj} , Q_k^{ij}$	$Q_b^{ci} = Q_a^{jc} , Q_k^{ij}$
Q_c^{ij}	R^{ijc}
P_k^{ab}	
$P_k^{aj} = P_k^{ib} , P_a^{bc}$	
$P_c^{ib} = P_c^{aj} , P_k^{ij}$	
P_c^{ij}	

(2009 - 2016)

- New **AdS** and **dS** vacua (systematically) found using the ET / flux correspondence
- ET framework relevant for **holography** and **cosmology** [A biased choice of applications]

AdS vacua: applications in holography

- Dyonic gaugings and dual CFT₃
[Dall'Agata, Inverso, Trigiante '12] [AG, Jafferis, Varela '15]
- New holographic RG flows from (M2)D2-branes and AdS₄/CFT₃
[AG & Tarrío, Varela '13] [Pang, Rong '15] [AG, Tarrío, Varela '16]
- Consistent reductions of *massive* IIA /M-theory
[Dall'Agata, Villadoro, Zwirner '09] [Godazgar's, Nicolai '13]
[Derendinger, AG '14] [Danielsson, Dibitetto, AG '14]
[Baron, Dall'Agata '14] [AG, Varela '15]
- Non-geometric superpotentials and BPS solutions
[AG, '13 '15] [Danielsson, Dibitetto '15]

dS vacua: applications in cosmology

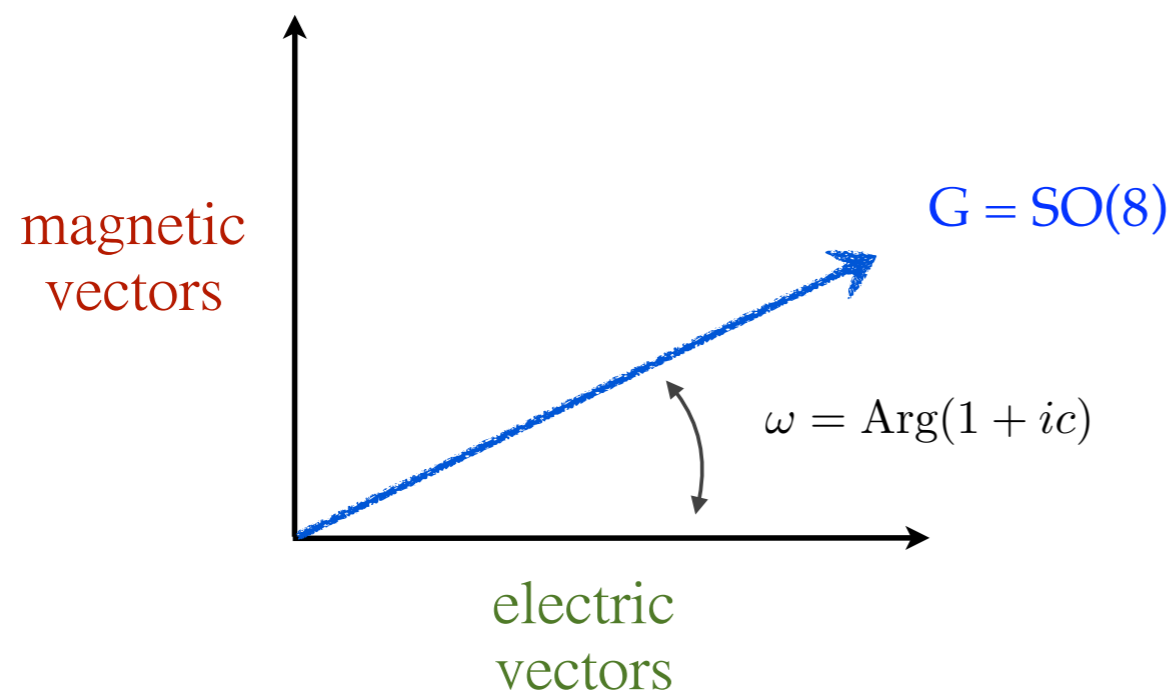
- dS in extended supergravity
[Roest, Rosseel '09] [Borghese, Roest '10]
[Dibitetto, AG, Roest '12]
- charting the landscape of flux vacua
[Dall'Agata, Inverso '11] [Damian, Loaiza-Brito '13]
[Danielsson, Haque, Shiu, van Riet & Koerber '10 '11]
[Aldazabal, Marques, Nuñez, Rosabal '11]
[Kodama, Nozawa '12] [Dibitetto, AG, Roest '11 '12 '14]
- inflationary models (slow-roll)
[de Carlos, AG, Moreno '09] [Hassler, Lüst, Massi '14]
[Kodama, Nozawa '15] [Blumenhagen et al '15]
- open/closed string interplay (alternative to anti-branes)
[Blåbæk, Roest, Zavala & Danielsson, Dibitetto '13]
[Kallosh, Linde, Vercnocke, Wrase '14] [AG, Inverso '15]

Holography : a new AdS_4/CFT_3 duality

- New *dyonic* maximal supergravities constructed using the ET framework

[Dall'Agata, Inverso, Trigiante & Marrani '12, '14] [Borghese, Dibitetto, AG, Roest, Varela '12] [Borghese, AG, Roest '12, '13]

Example : One-parameter family of $SO(8)$ -gauged supergravities !!



[Dall'Agata, Inverso, Trigiante '12]

$$D = \partial - g (A^{\text{elec}} - c \tilde{A}_{\text{mag}})$$

[de Wit, Nicolai '82]

- $AdS_4 \times S^7$ backgrounds of 11D supergravity (M2-brane) ?
- AdS_4/CFT_3 duality (ABJM or ABJ) ?

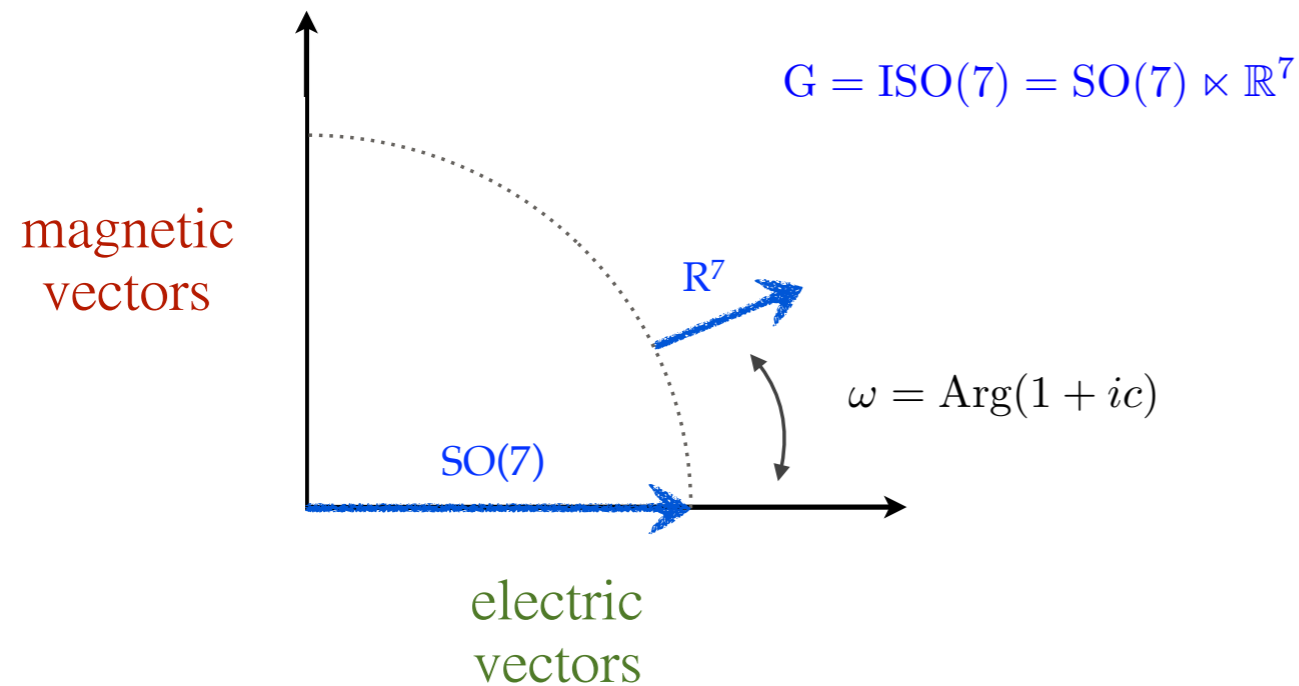
... no conclusive answer yet

[Aharony, Bergman, Jafferis, Maldacena '08]

[de Wit, Nicolai '13] [Lee, Strickland-Constable, Waldram '15]

Example : One-parameter family of ISO(7)-gauged supergravities

[Dall'Agata, Inverso, Marrani '14]



$$D = \partial - g A_{\text{SO}(7)}^{\text{elec}} - g (A_{\mathbb{R}^7}^{\text{elec}} - c \tilde{A}_{\mathbb{R}^7 \text{ mag}})$$

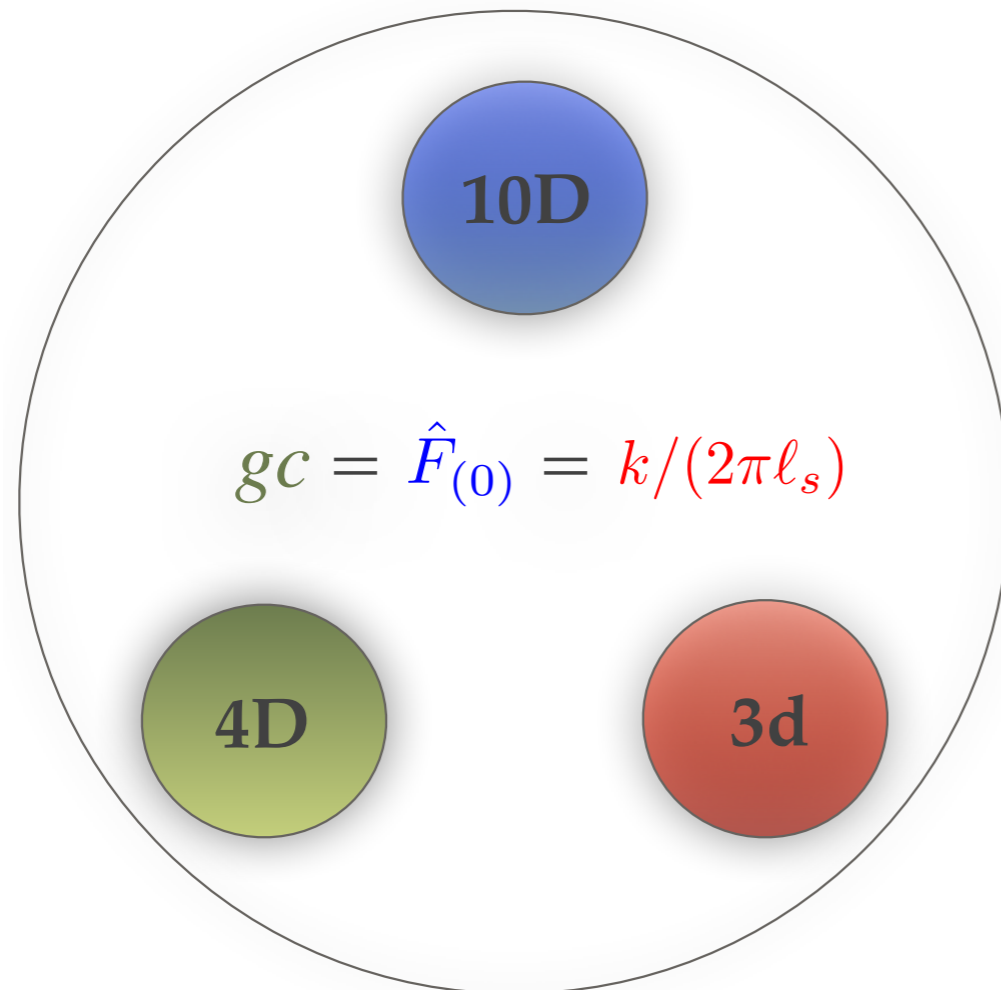
[AG, Jafferis, Varela '15]

[AG, Varela '15]

- $\text{AdS}_4 \times S^6$ background of **massive** IIA supergravity (D2-brane)
- $\text{AdS}_4/\text{CFT}_3$ duality : Chern-Simons-SYM theories with simple gauge group $\text{SU}(N)$ and level k given by Romans mass
[Schwarz '04]
- Matching of the free energies in supergravity and field theory sides (localisation)
- Further checks of the duality have been performed afterwards [Araujo, Nastase '16]

A new 10D/4D/3d correspondence

massive IIA on S^6 \ll $ISO(7)_c$ -gauged sugra \gg $SU(N)_k$ CS-SYM theory



gc = elec/mag deformation in 4D

$\hat{F}_{(0)}$ = Romans mass in 10D

k = Chern-Simons level in 3d

[AG, Jafferis, Varela '15]

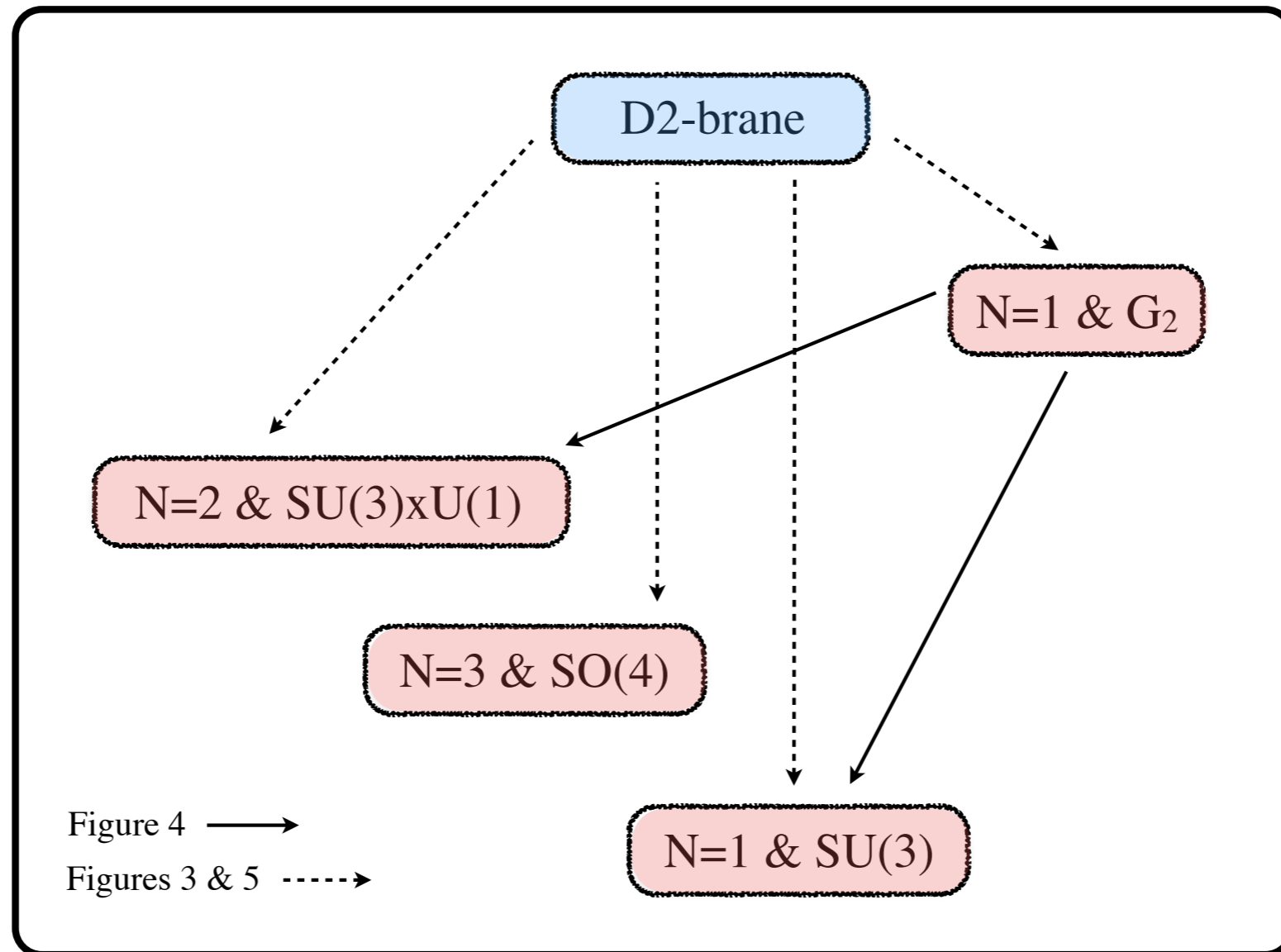
[AG, Varela '15]

Well-established and independent dualities :

Type IIB on S^5 / $N=4$ SYM — M-theory on S^7 / ABJM — mIIA on S^6 / CS-SYM

mIIA on S^6 /CS-SYM correspondence & holographic RG flows

[AG, Tarrío, Varela '16]



- RG flows from **SYM** (dotted lines) and between **CFT's** (solid lines) dual to BPS domain-wall solutions of the dyonic ISO(7)-gauged supergravity

Back to a long-standing question

What is the higher-dimensional origin of non-geometric fluxes and, more generally, of the various supergravities obtained using the lower-dimensional and duality covariant ET framework ?

(2013 - 2016) : string dualities in higher-dimensions

- Different strings related by dualities: IIA / IIB T-duality, IIB S-duality, ...

- *String dualities* realised as global symmetries in lower-dimensional SUGRA

lower-dimensional phenomenon

vs

higher-dimensional phenomenon

embedding tensor, non-geometric fluxes...

β -supergravity , γ -supergravity...

[Andriot, Betz '13] [Lee, Rey, Sakatani '16]

New approach : Extended Field Theories [extend internal coords to transform under duality]

- Double Field Theory (DFT) \Rightarrow Orthogonal groups $O(d,d)$ [half-max SUGRA (T-duality)]
- Exceptional Field Theory (EFT) \Rightarrow Exceptional groups $E_{d+1(d+1)}$ [max SUGRA (U-duality)]

[Siegel '93] [Hull, Zwiebach & Hohm '09 '10] [Hohm, Samtleben '13]

Double Field Theory (DFT)

[Siegel '93]

[Hull, Zwiebach & Hohm '09 '10]

- **T-duality** covariant framework to describe momentum + winding modes
- The theory lives in D (external) + $2d$ (internal) coordinates y^M , with $D + d = 10$, and has an $\mathbf{R}^+ \times \mathbf{O}(d, d)$ symmetry linearly realised

$$S_{\text{DFT}} = \int d^4x d^{12}y e e^{-2\phi} \left[\hat{R}(e) + 4 \mathcal{D}^\mu \phi \mathcal{D}_\mu \phi + \frac{1}{8} \mathcal{D}^\mu M^{MN} \mathcal{D}_\mu M_{MN} - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{1}{4} M_{MN} \mathcal{F}_{\mu\nu}^M \mathcal{F}^{\mu\nu N} - V_{\text{DFT}}(\phi, M_{MN}, g) \right]$$

Example : $D = 4$, $d = 6$ & $M = 1, \dots, 12$

- Consistency requires a **section constraint** : $\eta^{MN} \partial_M \otimes \partial_N = 0 \rightarrow$ **N=1 sugra in 10D**
- $M_{MN} \in \mathbf{O}(6, 6)/\mathbf{O}(6)^2$ generalised metric : metric + B-field
- **Generalised geometries (beyond Riemann) & non-geometry (T-folds)**

[Coimbra, Strickland-Constable, Waldram '11]

[asymmetric orbifolds]

Exceptional Field Theory (EFT)

[Hohm, Samtleben '13]

- **U-duality** covariant framework to describe momentum + winding + brane modes
- The theory lives in D (external) + $R[E_{d(d)}]$ (internal) coordinates $y^{\mathcal{M}}$, with $D + d = 11$, and has an $E_{d(d)}$ symmetry linearly realised

$$S_{E_{7(7)\text{-EFT}}} = \int d^4x d^{56}y e \left[\hat{R} + \frac{1}{48} g^{\mu\nu} \mathcal{D}_\mu M^{\mathcal{M}\mathcal{N}} \mathcal{D}_\nu M_{\mathcal{M}\mathcal{N}} - \frac{1}{8} M_{\mathcal{M}\mathcal{N}} \mathcal{F}^{\mu\nu\mathcal{M}} \mathcal{F}_{\mu\nu}{}^{\mathcal{N}} + e^{-1} \mathcal{L}_{\text{top}} - V_{E_{7(7)\text{-EFT}}}(M_{\mathcal{M}\mathcal{N}}, g) \right]$$

Example : $D = 4, d = 7$ & $\mathcal{M} = 1, \dots, 56$

[Massive IIA : Ciceri, AG, Inverso '16]

- Consistency requires a **section constraint** : $Y^{MN}{}_{PQ} \partial_M \otimes \partial_N = 0 \rightarrow$

**Type IIB
or
11D supergravity**

- $M_{\mathcal{M}\mathcal{N}} \in E_{7(7)}/SU(8)$ generalised metric : metric + type IIB or $A_{(3)}$ potentials
- **Exceptional generalised geometries (beyond Riemann) & non-geometry (U-folds)**

[Coimbra, Strickland-Constable, Waldram '11 '12]

Dualities in SUGRA and Extended Field Theory

D	Maximal sugra / EFT	Half-maximal sugra	DFT
9	$\mathbb{R}^+ \times \text{SL}(2)$	$\mathbb{R}^+ \times \text{O}(1, 1 + n)$	$\mathbb{R}^+ \times \text{O}(1, 1 + n)$
8	$\text{SL}(2) \times \text{SL}(3)$	$\mathbb{R}^+ \times \text{O}(2, 2 + n)$	$\mathbb{R}^+ \times \text{O}(2, 2 + n)$
7	$\text{SL}(5)$	$\mathbb{R}^+ \times \text{O}(3, 3 + n)$	$\mathbb{R}^+ \times \text{O}(3, 3 + n)$
6	$\text{SO}(5, 5)$	$\mathbb{R}^+ \times \text{O}(4, 4 + n)$	$\mathbb{R}^+ \times \text{O}(4, 4 + n)$
5	$\text{E}_{6(6)}$	$\mathbb{R}^+ \times \text{O}(5, 5 + n)$	$\mathbb{R}^+ \times \text{O}(5, 5 + n)$
4	$\text{E}_{7(7)}$	$\text{SL}(2) \times \text{O}(6, 6 + n)$	$\mathbb{R}^+ \times \text{O}(6, 6 + n)$
3	$\text{E}_{8(8)}$	$\text{O}(8, 8 + n)$	$\mathbb{R}^+ \times \text{O}(7, 7 + n)$

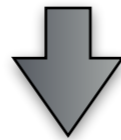
Duality groups of half-maximal SUGRA and DFT differ for $D \leq 4$

* n = additional vector multiplets

... so there is a theory in between EFT and DFT in $D = 4$

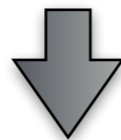
EFT with $E_{7(7)}$ duality group

[Hohm, Samtleben '13]



[Ciceri, Dibitetto, F-Melgarejo, AG, Inverso '16]

SL(2)-DFT with $SL(2) \times O(6,6+n)$ duality group



DFT with $R^+ \times O(6,6+n)$ duality group

[Siegel '93]

[Hull, Zwiebach '09]

[Hohm, Hull, Zwiebach '10]

[Hohm, Kwak '11]

SL(2) -Double Field Theory

- $SL(2) \times O(6, 6)$ covariant framework $[n = 0]$ [Ciceri, Dibitetto, F-Melgarejo, AG, Inverso '16]
- The theory lives in D (external) + 2×12 (internal) coordinates $y^{\alpha M} = (y^{+M}, y^{-M})$, and has an $SL(2) \times O(6, 6)$ symmetry linearly realised

$$S_{SL(2)\text{-DFT}} = \int d^4x d^{24}y e \left[\hat{R} + \frac{1}{4} g^{\mu\nu} \mathcal{D}_\mu M^{\alpha\beta} \mathcal{D}_\nu M_{\alpha\beta} + \frac{1}{8} g^{\mu\nu} \mathcal{D}_\mu M^{MN} \mathcal{D}_\nu M_{MN} - \frac{1}{8} M_{\alpha\beta} M_{MN} \mathcal{F}^{\mu\nu\alpha M} \mathcal{F}_{\mu\nu}{}^{\beta N} + e^{-1} \mathcal{L}_{\text{top}} - V_{SL(2)\text{-DFT}}(M, g) \right]$$

Example : $D = 4, d = 6$ & $\alpha = +, -, M = 1, \dots, 12$

- Consistency section constraints : $\eta^{MN} \partial_{\alpha M} \otimes \partial_{\beta N} = 0$
 $\epsilon^{\alpha\beta} \partial_{\alpha[M} \otimes \partial_{\beta N]} = 0$ \rightarrow
- $M_{\alpha\beta} \in SL(2)/SO(2)$ generalised dilaton : dilaton + axion
- $M_{MN} \in SO(6, 6)/SO(6)^2$ generalised metric : metric + B-field

N=1 sugra in 10D
or
(2,0) sugra in 6D

- DFT corresponds to $SL(2)_+$ -DFT

... but what for ?

SL(2) angles and moduli stabilisation

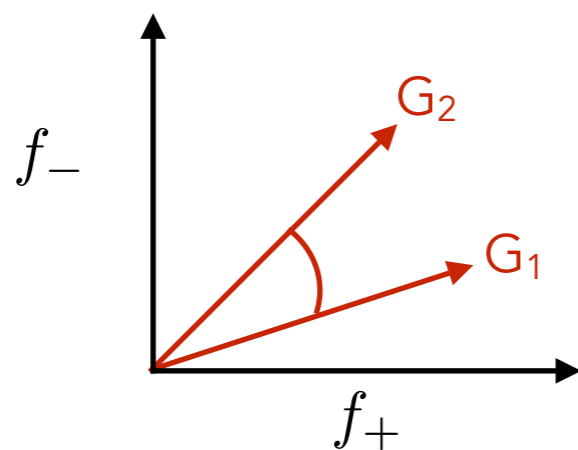
- Scherk-Schwarz (SS) reductions with $SL(2) \times O(6,6)$ twist matrices $U_{\alpha M}{}^{\beta N} = e^\lambda e_\alpha{}^\beta U_M{}^N$ yield **N = 4, D = 4 gaugings** [Schön, Weidner '06]

$$f_{\alpha MNP} = -3 e^{-\lambda} e_\alpha{}^\beta \eta_{Q[M} U_N{}^R U_P]{}^S \partial_{\beta R} U_S{}^Q$$

$$\xi_{\alpha M} = 2 U_M{}^N \partial_{\beta N} (e^{-\lambda} e_\alpha{}^\beta)$$

[de Roo, Wagemans '85]

- Moduli stabilisation **requires** gaugings $G = G_1 \times G_2$ at **relative SL(2) angles**



(sec. constraint **violated**)

$$\epsilon^{\alpha\beta} \partial_{\alpha[M} \otimes \partial_{\beta|N]} \neq 0$$

- Dependence on **both type + & type -** coordinates [**not possible in DFT**]

Example : $SO(4) \times SO(4)$ -gauged sugra and non-geometry

- SS with $U(y^{\alpha M}) \in O(6, 6)$: Half of the coords of **type +** & half of **type -**
- $SL(2)$ -superposition of **two chains** of **non-geometric** fluxes $(H, \omega, Q, R)_{\pm}$

$$f_+ \quad f_{+mnp} = H^+_{mnp} \quad , \quad f_{+mn\bar{p}} = \omega^+_{mn}{}^p \quad , \quad f_{+\bar{m}\bar{n}p} = Q^{+mn}{}_p \quad , \quad f_{+\bar{m}\bar{n}\bar{p}} = R^{+mnp}$$

$$f_- \quad f_{-mnp} = H^-_{mnp} \quad , \quad f_{-mn\bar{p}} = \omega^-_{mn}{}^p \quad , \quad f_{-\bar{m}\bar{n}p} = Q^{-mn}{}_p \quad , \quad f_{-\bar{m}\bar{n}\bar{p}} = R^{-mnp}$$

Most general family (8 params) of $SO(4) \times SO(4)$ -gauged N=4 sugra

- $SO(4) \times SO(4)$ SUGRA : **AdS₄** & **dS₄** vacua (sphere/hyperboloid reductions)

[de Roo, Westra, Panda, Trigiante '03] [Dibitetto, AG, Roest '12]

- ``Hybrid \pm '' sources to cancel flux-induced tadpoles : **SL(2)-dual NS-NS branes ??**

[exotic branes : de Boer, Shigemori '10]

Summary & Future directions (2017 - ...)

- Understanding the string dynamics beyond the standard 10D/11D supergravity regime is a key step towards linking strings to cosmological data, *e.g.* **de Sitter from strings**
- String dualities connect different regimes of the string dynamics, thus motivating the search for **duality covariant frameworks** :

- **Lower-dimensional** (Embed. Tensor)

Cosmology : non-geometry & de Sitter vacua
charting the string landscape

Holography : new AdS₄/CFT₃ duality
mIIA on S⁶/CS-SYM

- **Higher-dimensional** (Ext. Field Theories)

Unification of 10D/11D supergravities

Generalised geometries (T/S/U-folds)

Consistent reductions (AdS₅ × S⁵ , ...)

Non-geometry vs section constraint

...

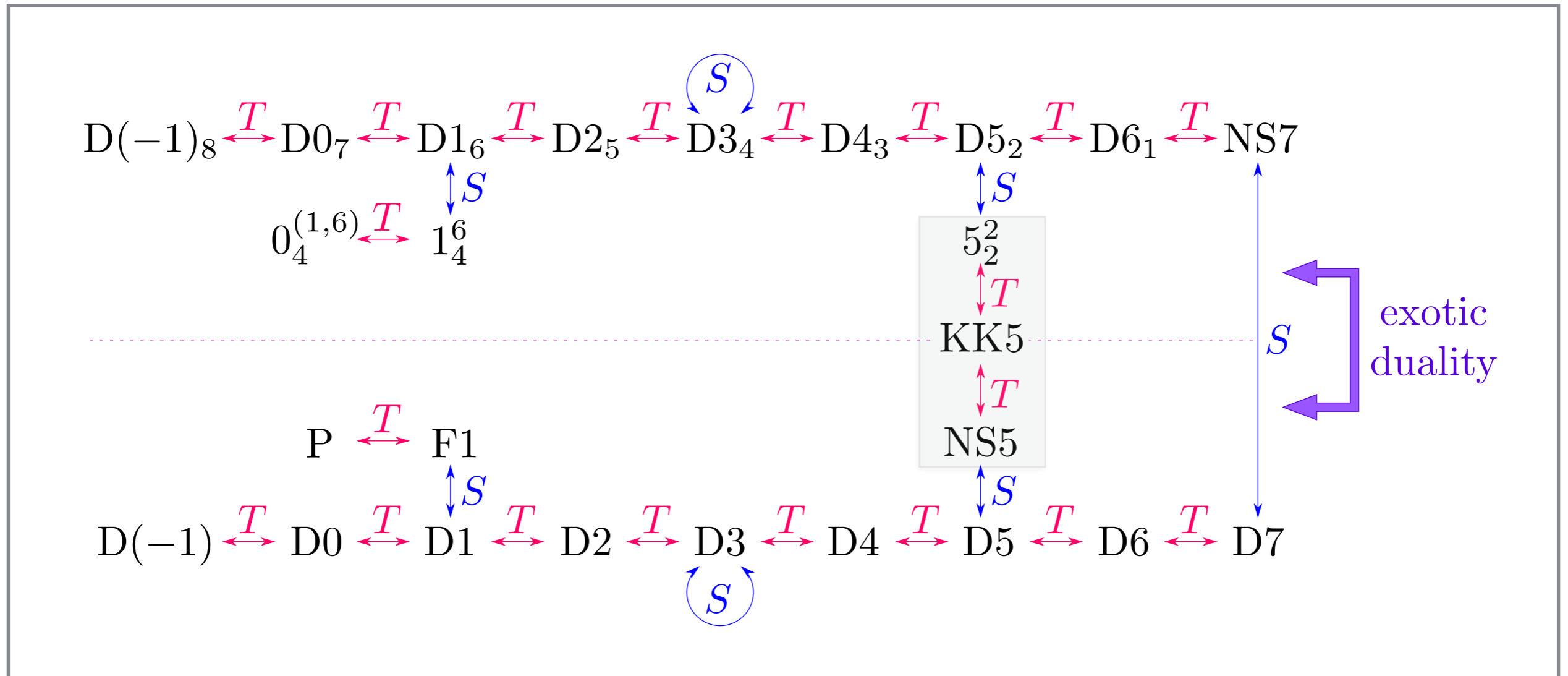
- Exotic branes & BH microstates counting

New generalised *stringy* geometries, exotic dual branes , black hole microstates , inflationary models , new de sitter vacua , AdS_4/CFT_3 , holographic RG flows and much more to be explored in the near future!!

Thanks

Extra material

A family of exotic branes



[Sakatani '15]

Deformations of EFT (XFT)

- Generalised Lie derivative

[no density term]

$$\mathbb{L}_\Lambda U^\mathcal{M} = \Lambda^\mathcal{N} \partial_\mathcal{N} U^\mathcal{M} - U^\mathcal{N} \partial_\mathcal{N} \Lambda^\mathcal{M} + Y^{\mathcal{MN}}{}_{\mathcal{PQ}} \partial_\mathcal{N} \Lambda^\mathcal{P} U^\mathcal{Q}$$

in terms of an $E_{n(n)}$ -invariant **structure Y-tensor**. Closure requires **sec. constraint**

- **Deformed** generalised Lie derivative

$$\tilde{\mathbb{L}}_\Lambda U^\mathcal{M} = \Lambda^\mathcal{N} \partial_\mathcal{N} U^\mathcal{M} - U^\mathcal{N} \partial_\mathcal{N} \Lambda^\mathcal{M} + Y^{\mathcal{MN}}{}_{\mathcal{PQ}} \partial_\mathcal{N} \Lambda^\mathcal{P} U^\mathcal{Q} - \underbrace{X_{\mathcal{NP}}{}^\mathcal{M}}_{\text{non-derivative}} \Lambda^\mathcal{N} U^\mathcal{P}$$

in terms of an **X deformation** which is $E_{n(n)}$ -algebra valued

non-derivative

- Closure & triviality of the Jacobiator require (together with **sec. constraint**)

$$X_{\mathcal{MN}}{}^\mathcal{P} \partial_\mathcal{P} = 0$$

X constraint

$$X_{\mathcal{MP}}{}^\mathcal{Q} X_{\mathcal{NQ}}{}^\mathcal{R} - X_{\mathcal{NP}}{}^\mathcal{Q} X_{\mathcal{MQ}}{}^\mathcal{R} + X_{\mathcal{MN}}{}^\mathcal{Q} X_{\mathcal{QP}}{}^\mathcal{R} = 0$$

Quadratic constraint (gauged max. supergravity)

X deformation : background fluxes & Romans mass

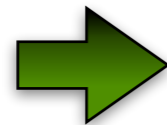
$$Y^{\mathcal{P}\mathcal{Q}}{}_{MN} \partial_{\mathcal{P}} \otimes \partial_{\mathcal{Q}} = 0$$

section constraint

$$X_{MN}{}^{\mathcal{P}} \partial_{\mathcal{P}} = 0$$

X constraint

[algebraic system]



M-theory (n coords)

- SL(n) orbit
- Freund-Rubin param. (n = 4 and n = 7)
- *massless* IIA (subcase)

Type IIB (n-1 coords)

- SL(n-1) orbit
- p-form fluxes compatible with SL(n-1)
- **SL(2)-triplet of 1-form flux** (includes compact SO(2))

+

New massive Type IIA (n-1 coords)

- SL(n-1) orbit
- p-form fluxes compatible with SL(n-1)
- dilaton flux
- **Romans mass parameter** (kills the M-theory coord)

Massive Type IIA described in a purely geometric manner !!

[QC = flux-induced tadpoles]

E₇₍₇₎-XFT action

- E₇₍₇₎-XFT action [$\mathcal{D}_\mu = \partial_\mu - \tilde{\mathbb{L}}_{A_\mu}$] [$y^{\mathcal{M}}$ coords in the **56** of E₇₍₇₎]

$$S_{\text{XFT}} = \int d^4x d^{56}y e \left[\hat{R} + \frac{1}{48} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{\mathcal{MN}} \mathcal{D}_\nu \mathcal{M}_{\mathcal{MN}} - \frac{1}{8} \mathcal{M}_{\mathcal{MN}} \mathcal{F}^{\mu\nu\mathcal{M}} \mathcal{F}_{\mu\nu}{}^{\mathcal{N}} + e^{-1} \mathcal{L}_{\text{top}} - V_{\text{XFT}}(\mathcal{M}, g) \right]$$

with field strengths & potential given by

(deformed tensor hierarchy)

$$\mathcal{F}_{\mu\nu}{}^{\mathcal{M}} = 2 \partial_{[\mu} A_{\nu]}{}^{\mathcal{M}} + X_{[\mathcal{P}\mathcal{Q}]}{}^{\mathcal{M}} A_\mu{}^{\mathcal{P}} A_\nu{}^{\mathcal{Q}} - [A_\mu, A_\nu]_{\text{E}}{}^{\mathcal{M}} + \text{two-form terms}$$

$$V_{\text{XFT}}(\mathcal{M}, g, X) = V_{\text{EFT}}(\mathcal{M}, g) + \underbrace{\frac{1}{12} \mathcal{M}^{\mathcal{MN}} \mathcal{M}^{\mathcal{KL}} X_{\mathcal{MK}}{}^{\mathcal{P}} \partial_{\mathcal{N}} \mathcal{M}_{\mathcal{PL}}}_{\text{cross term}} + \underbrace{V_{\text{SUGRA}}(\mathcal{M}, X)}_{\text{gauged max. sugra}}$$

- Two-One-**Zero**-derivative potential : **gauged** 4D max. sugra when $\Phi(x, y) = \Phi(x)$

Extended (super) Poincaré superalgebra

- Central charges (internal symmetries) $Z_{IJ} = (a_{IJ}^a) T^a$

- The algebra :

$$[P_\mu, P_\nu] = 0 \quad [M_{\mu\nu}, M_{\rho\sigma}] = i(\eta_{\nu\rho} M_{\mu\sigma} - \eta_{\nu\sigma} M_{\mu\rho} - \eta_{\mu\rho} M_{\nu\sigma} + \eta_{\mu\sigma} M_{\nu\rho})$$

$$[P_\mu, M_{\rho\sigma}] = i(\eta_{\mu\rho} P_\sigma - \eta_{\mu\sigma} P_\rho)$$

$$[T^a, T^b] = if_c^{ab} T^c \quad [T^a, P_\mu] = [T^a, M_{\mu\nu}] = 0$$

$$[Q'_\alpha, P_\mu] = [\bar{Q}'_{\dot{\alpha}}, P_\mu] = 0 \quad [Q'_\alpha, T^a] = (b_a)'_J Q'^J_\alpha \quad [\bar{Q}'_{\dot{\alpha}}, T^a] = -\bar{Q}'_{\dot{\alpha}} (b_a)_{J'}^J$$

$$[Q'_\alpha, M_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu})_\alpha^\beta Q'_\beta \quad [\bar{Q}'_{\dot{\alpha}}, M_{\mu\nu}] = -\frac{1}{2}\bar{Q}'_{\dot{\beta}} (\bar{\sigma}_{\mu\nu})^{\dot{\beta}}_{\dot{\alpha}}$$

$$\{\bar{Q}'_{\dot{\alpha}}, \bar{Q}'_{\dot{\beta}}\} = -2\epsilon_{\dot{\alpha}\dot{\beta}} Z^{IJ\dagger} \quad \{Q'_\alpha, Q'_\beta\} = 2\epsilon_{\alpha\beta} Z^{IJ} \quad \{Q'_\alpha, \bar{Q}'_{\dot{\beta}}\} = 2\delta^{IJ} (\sigma^\mu)_{\alpha\dot{\beta}} P_\mu$$