Gaugings, fluxes and moduli fixing in (half-) maximal supergravities

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with G. Dibitetto and D. Roest : arXiv:1102.0239
arXiv:1104.3587
arXiv:11xx.xxxx *(...in progress)*

The footprint of extra dimensions

- 4d supergravities appear when compactifying string theory

- Fluctuations of the internal geometry translates into massless 4d scalar fields known as ``*moduli*"

$$\mathcal{L} = \frac{1}{2} R - \frac{1}{2} \partial_\mu \phi_i \, \partial^\mu \phi^i$$

Deviations from GR !!

massless scalars = long range interactions (precision tests of GR)

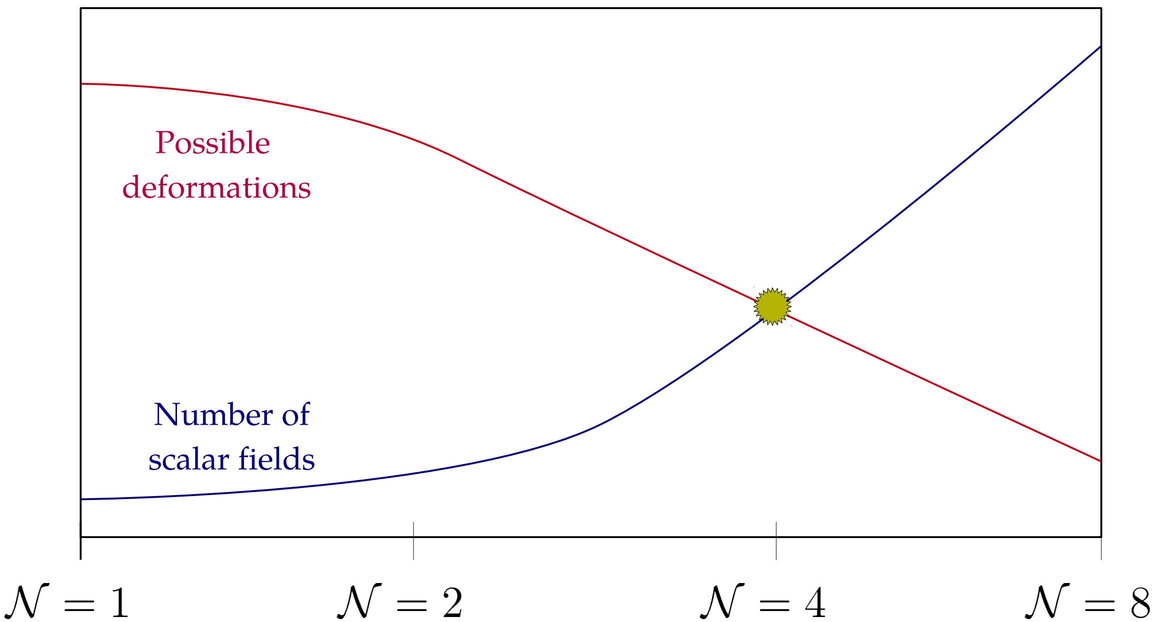
Linking strings to observations $\Rightarrow$ Mechanisms to stabilise moduli !!

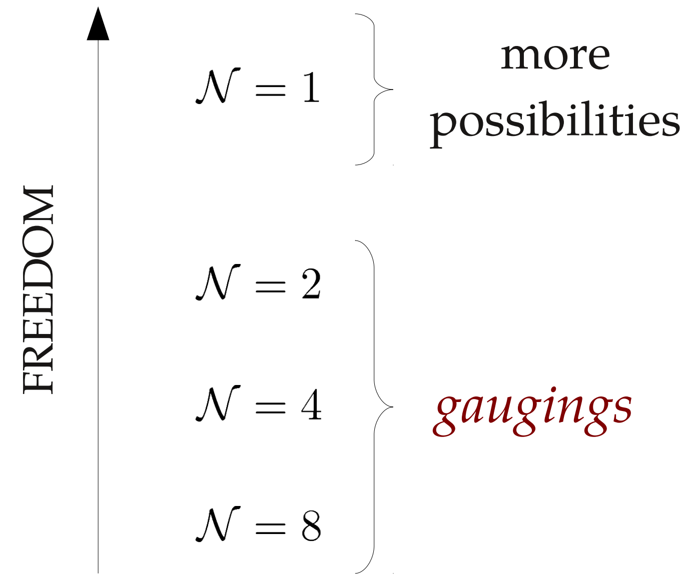
$$V(\phi) = m_{ij}^2 \, \phi^i \phi^j + \ldots$$

- Moduli VEVs $\langle \phi \rangle = \phi_0$ determine 4d physics $\left\{ \begin{array}{l} \Lambda_{c.c} \equiv V(\phi_0) \\ g_s \text{ and } \mathrm{Vol}_{int} \\ \text{fermi masses} \end{array} \right.$

How to deform massless theories to have $V(\phi) \neq 0$?

- Supersymmetry dictates what deformations are allowed





gaugings = part of the global symmetry is promoted to local (*gauge*)

- Gauged supergravities can be systematically studied

embedding tensor formalism

[Nicolai, Samtleben '00]

[de Wit, Samtleben, Trigiante '02 '05 '07]

[Schon, Weidner '06]

undeformed = abelian = massless

- Reducing 10d supergravities down to 4d

global symmetry G = stringy features (*i.e.* U/T/S-duality)

- The physical scalar sector parameterises the coset space $\mathcal{M} = G/H$ where H is the maximal compact subgroup of G

$\mathcal{N} = 8$

$G = E_7 \qquad H = SU(8)$

133 - 63 = 70 scalars

$\mathcal{N} = 4$

$G = SL(2) \times SO(6,6)$

$H = SO(2) \times SO(6) \times SO(6)$

69 - 31 = 38 scalars

$$\boxed{V(\mathcal{M}) = 0}$$

- Abelian gauge fields $A_\mu^{\mathcal{F}}$ in the fundamental of G

deformed = non-abelian = massive

- *Gauging* : a non-abelian subgroup $G_0 \subset G$ is promoted to a gauge symmetry yielding a *gauged* supergravity

$$\nabla_\mu \longrightarrow \nabla_\mu - g\, A_\mu^{\mathcal{F}}\, \Theta_{\mathcal{F}}{}^{\mathcal{A}}\, t_{\mathcal{A}} \qquad \text{where} \quad \begin{cases} \mathcal{F} \equiv \text{fund} \\ \mathcal{A} \equiv \text{adj} \end{cases} \quad \text{reps of G}$$

embedding tensor

Consistent gauge group $\Longrightarrow$ Quadratic constraints on Θ

- Non-trivial scalar potential for the scalar fields $\mathcal{M} = G/H$

$$\boxed{V(\Theta, \mathcal{M}) \neq 0}$$

duality invariant = stringy !!

- The embedding tensor $\Theta_{\mathcal{F}}{}^{\mathcal{A}}$ encodes any possible deformation of the massless theory irrespective of its higher-dimensional origin

Higher-dimensional origin of gaugings

guiding principle = duality covariance

Example: T-duality = SO(d,d)

[Hull, Zwiebach '09]
[Hohm, Hull, Zwibach '10]
[Hohm, Kwak, Zwibach '11]
[Geissbühler '11]
[Aldazabal, Baron, Marques, Nunez '11]

- DFT / Doubled Geometry $\Longrightarrow$ $\{\, \partial_m \ , \ \partial^m \}$
  (fundamental = 1-form)

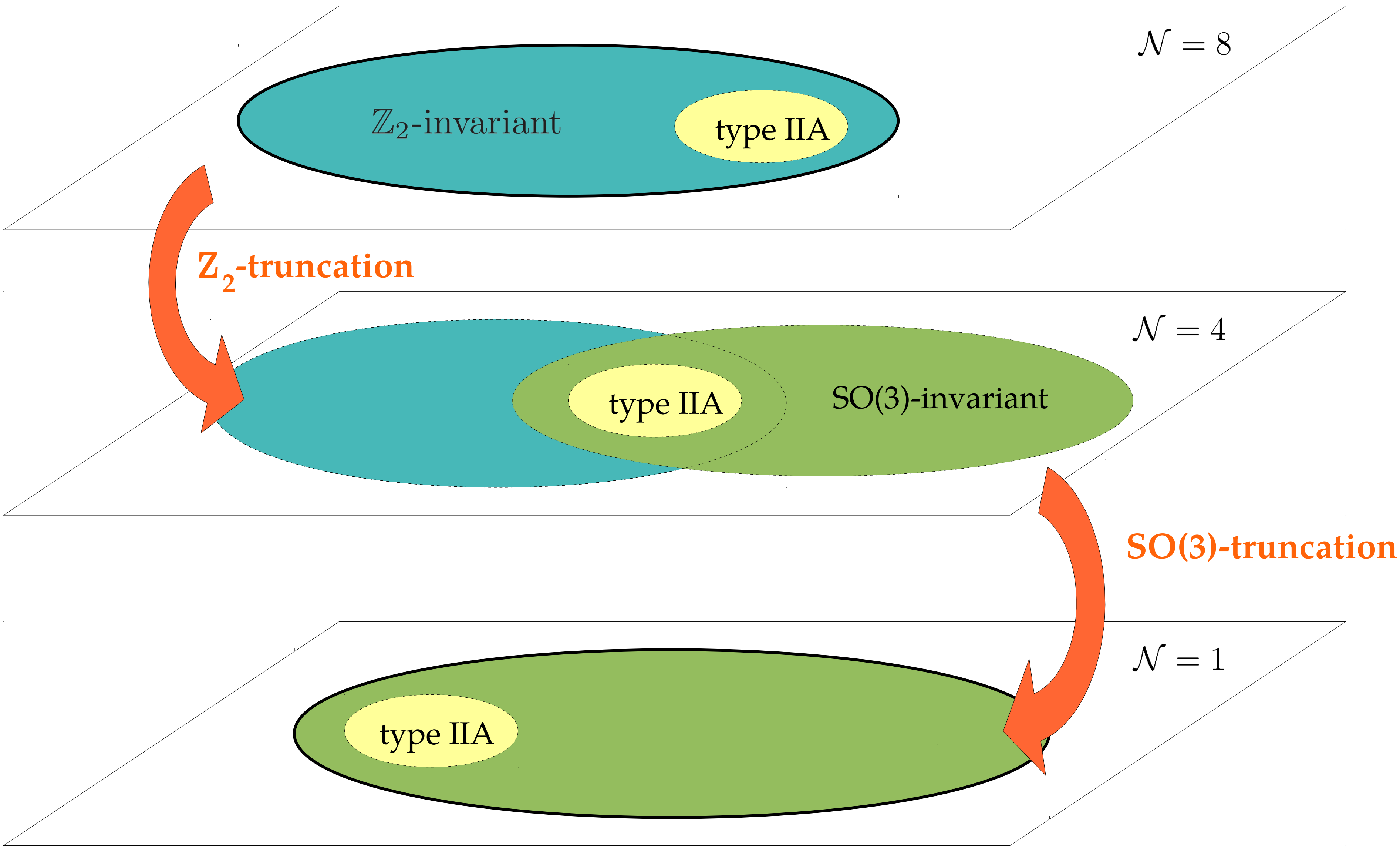
- Generalised Geometry $\Longrightarrow$ $\{\, G_{mn} \ , \ B_{mn} \ , \ \beta^{mn} \}$
  (adjoint = 2-form)

[Gualtieri '04]
[Grana, Minasian, Petrini, Tomasiello '04 '05]
[Grana, Minasian, Petrini, Waldram '08]
[Coimbra, Strickland-Constable, Waldram '11]

- Non-Geometry $\Longrightarrow$ $\{\, H_{mnp} \ , \ \omega_{np}^{m} \ , \ Q_p^{mn} \ , \ R^{mnp} \}$
  (3-form)

[Shelton, Taylor, Wecht '05 '06]
[Aldazabal, Cámara,Font, Ibañez '06]
[de Carlos, A.G. Moreno '09]
[Aldazabal, Andrés, Cámara, Grana '10]

A one minute summary



A chain of truncations in 4d

[Dibitetto, A.G, Roest in progress]

$$\begin{tikzcd}
G = E_7 \arrow[r, "SO(3)"] \arrow[d, "\mathbb{Z}_2"] & G = SL(2) \times G_2 \arrow[d, "\mathbb{Z}_2"] \\
G = SL(2) \times SO(6,6) \arrow[r, "SO(3)"] & G = SL(2) \times SL(2) \times SL(2)
\end{tikzcd}$$

$G = E_7$

e.t comp = 912
vectors = 56
scalars = 133

$\mathcal{N} = 8$

$G = SL(2) \times G_2$

e.t comp = 72
vectors = 4
scalars = 17

$\mathcal{N} = 2$

$G = SL(2) \times SO(6,6)$

e.t comp = 464
vectors = 24
scalars = 69

$\mathcal{N} = 4$

$G = SL(2) \times SL(2) \times SL(2)$

e.t comp = 40
vectors = 0
scalars = 9

$\mathcal{N} = 1$

Half-maximal supergravities in 4d

[Schon, Weidner '06]

$$G = SL(2) \times SO(6,6)$$

$$\begin{aligned} \text{e.t comp} &= 464 \\ \text{vectors} &= 24 \\ \text{scalars} &= 69 \end{aligned}$$

$\mathcal{N} = 4$

* reps of G

Questions :

- Can the whole vacuum structure be charted in $\mathcal{N} = 4$ theories ?
- Are there connections in the landscape of vacua ?

Half-maximal : symmetry and fields

- Global symmetry (duality) group $\boxed{G = SL(2) \times SO(6,6)}$

- Field content = supergravity multiplet + six vector multiplets

- Vectors $A_\mu^{\alpha M}$ in the fundamental of G

$$\left.\begin{array}{l} \alpha = +,- \ \text{ is an electric-magnetic } SL(2) \text{ index} \\ M = 1, ..., 12 \ \text{ is an } SO(6,6) \text{ index} \end{array}\right\} \text{24 vectors}$$

- The physical scalars parameterise $\mathcal{M} = M_{\alpha\beta} \times M_{MN}$

$$\left.\begin{array}{l} \bullet\ 1 \text{ axion} + 1 \text{ dilaton in SL}(2) \\ \bullet\ 30 \text{ axions} + 6 \text{ dilatons in SO}(6,6) \end{array}\right\} \text{38 physical scalars}$$

Half-maximal : *gaugings* and scalar potential

- *Gaugings* are classified by two embedding tensor pieces

$$\xi_{\alpha M} \in (\mathbf{2}, \mathbf{12}) \qquad \text{and} \qquad f_{\alpha MNP} \in (\mathbf{2}, \mathbf{220})$$

* reps of SL(2) x SO(6,6)

- Supersymmetry + gauge invariance determine the scalar potential

$$V = \frac{1}{64} f_{\alpha MNP} f_{\beta QRS} M^{\alpha\beta} \left[\frac{1}{3} M^{MQ} M^{NR} M^{PS} + \left(\frac{2}{3}\eta^{MQ} - M^{MQ}\right)\eta^{NR}\eta^{PS}\right] - \\ - \frac{1}{144} f_{\alpha MNP} f_{\beta QRS} \epsilon^{\alpha\beta} M^{MNPQRS} + \frac{3}{64} \xi_\alpha^M \xi_\beta^N M^{\alpha\beta} M_{MN}$$

$\eta \equiv SO(6,6)$-metric

To keep in mind : V is quadratic in the emb. tens. parameters

- 464 e.t. components + 38 physical scalars = TOO MUCH !!

The SO(3) truncation

- Keeping only fields and embedding tensor components invariant under the action of a subgroup $SO(3) \subset SO(6,6)$

$G = SL(2) \times SO(6,6)$

e.t comp = 464
vectors = 24
physical scalars = 38

$\mathcal{N} = 4$

$\xrightarrow{SO(3)}$

$G = SL(2) \times SL(2) \times SL(2)$

e.t comp = 40
vectors = 0
physical scalars = 6

$\mathcal{N} = 1$

- R-symmetry group :

$$SU(4) \supset SO(3)$$
$$4 \longrightarrow 1 + 3$$

The SO(3) truncation : fields and *gaugings*

[Derendinger, Kounnas, Petropoulos, Zwirner '04]

- Symmetry and fields :

  - global symmetry $G = SL(2)_S \times SL(2)_T \times SL(2)_U$

  - $A_\mu^{\alpha M} = 0$ and $\xi_{\alpha M} = 0$

  - scalar coset = 3 complex scalars = STU – models !!

$$\underbrace{S \equiv \chi + i\, e^{-\phi}}_{SL(2)_S} \quad , \quad \underbrace{T \equiv \chi_1 + i\, e^{-\varphi_1} \quad \text{and} \quad U \equiv \chi_2 + i\, e^{-\varphi_2}}_{SL(2)_T \,\times\, SL(2)_U}$$

- The *gaugings* $G_0 \subset G$ and the scalar potential $V(S,T,U)$ are specified by the embedding tensor

$$\boxed{f_{\alpha MNP} = 40 \text{ components}}$$

Gaugings from fluxes

- String embedding as flux compactification

$$f_{\alpha MNP} \;=\; \text{generalised fluxes}$$

Example: SO(3) truncation $\Longleftrightarrow$ type II orientifolds of $\mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

- Type IIB fluxes and embedding tensor

$$f_{+mnp} = \tilde{F}'_{mnp} \quad , \quad f_{+mn}{}^{p} = Q'_{mn}{}^{p} \quad , \quad f_{+}{}^{mn}{}_{p} = Q^{mn}{}_{p} \quad , \quad f_{+}{}^{mnp} = \tilde{F}^{mnp} \quad ,$$
$$f_{-mnp} = \tilde{H}'_{mnp} \quad , \quad f_{-mn}{}^{p} = P'_{mn}{}^{p} \quad , \quad f_{-}{}^{mn}{}_{p} = P^{mn}{}_{p} \quad , \quad f_{-}{}^{mnp} = \tilde{H}^{mnp} \quad ,$$

[Dibitetto, Linares, Roest '10]

* index splitting $M = ({}_{m}\ ,\ {}^{m})$

- Perfect matching with flux-induced superpotential (up to Q.C)

$$W_{flux} = (P_F - P_H\, S) + 3\,T\,(P_Q - P_P\, S) + 3\,T^2(P_{Q'} - P_{P'}\, S) + T^3\,(P_{F'} - P_{H'}\, S)$$

Gaugings and sources

- Consistency of the gauge group = quadratic constraints on $f_{\alpha MNP}$

gaugings

$$A_\mu = A_\mu^{\alpha M}\, T_{\alpha M}$$

$$[T_{\alpha M}, T_{\beta N}] = f_{\alpha\, MN}{}^{P}\, T_{\beta P}$$

➡

Quadratic Constraints

$$\epsilon^{\alpha\beta}\, f_{\alpha MNR}\, f_{\beta PQ}{}^{R} = 0$$

$$f_{\alpha R[MN}\, f_{\beta PQ]}{}^{R} = 0$$

- String theory :

quadratic constraints = Absence of sources breaking $\mathcal{N}=4$

Example : Type IIB orientifolds with O3/O7-planes

- $(H\,,\,F\,)$ fluxes : Unconstrained D3-brane flux-induced tadpole
- $(H\,,\,F\,,\,Q\,)$ fluxes : Vanishing of D7-brane flux-induced tadpole
- (H , F , Q , P) fluxes : Vanishing of D7 , NS7 and I7 flux-induced tadpoles
- ??

We would like to . . .

1) build all the consistent SO(3)-invariant gaugings specified by $f_{\alpha MNP}$ by solving the quadratic constraints

$$\epsilon\, f\, f = 0 \qquad \text{and} \qquad f\, f = 0$$

2) compute all the SO(3)-invariant extrema of the f-induced scalar potential $V(f\,,\,\Phi)$ by solving the extremisation conditions

$$\left.\frac{\partial V}{\partial \Phi}\right|_{\Phi_0} = 0 \qquad \text{with} \qquad \Phi \equiv (\,S\,,\,T\,,\,U\,)$$

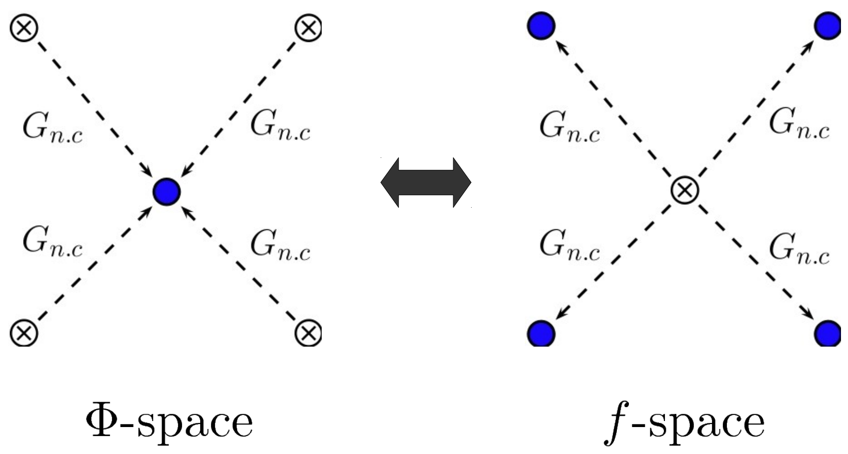
3) check stability of these extrema with respect to fluctuations of all the 38 scalars of half-maximal supergravity

4) identify the gauge group G_0 underlying all the different solutions

. . . but is this doable ?

Strategy and tools

- Idea : use the global symmetry group (non-compact part) to bring any field solution back to the origin !!



- At the origin everything is simply quadratic in the $f_{\alpha MNP}$ parameters

$$V(\Phi) = \sum_{\text{terms}} f\, f\ \Phi^{\text{high degree}}$$

so then,

computing the vacua structure	=	multivariate polynomial system $I = \langle\, \partial_\Phi V\|_{\Phi_0}\ ,\ \epsilon\, f\, f\ ,\ f\, f\,\rangle$

⇨ Algebraic Geometry techniques !!

[Gray, He, Lukas '06]

Basics of Algebraic Geometry

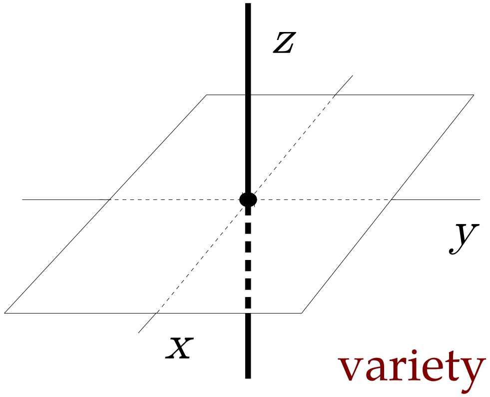
- Algebraic Geometry studies multivariate polynomial system and their link to geometry

$$I = \langle P_1\ ,\ P_2 \rangle$$

$$\begin{aligned} P_1(x,y,z) &= x\,z \\ P_2(x,y,z) &= y\,z \end{aligned}$$

algebraic system

$\Longleftrightarrow$



variety

- GTZ prime decomposition (analogous to integers dec. 30 = 2 x 3 x 5)

$$I = J_1 \cap J_2 \quad \text{where} \quad \begin{cases} J_1 = \langle z \rangle \longrightarrow xy\text{-plane} \\ J_2 = \langle x\ ,\ y \rangle \longrightarrow z\text{-axis} \end{cases}$$

$$\Rightarrow \quad \boxed{J_1 \cap J_2 \quad \longleftrightarrow \quad V(J_1) \cup V(J_2)}$$

algebra-geometry dictionary

- Applying the above procedure to our problem involving fluxes

$$I = \langle\ \partial_\Phi V|_{\Phi_0}\ \ ,\ \ \epsilon\, f\, f\ \ ,\ \ f\, f\ \rangle$$

$$I = J_1 \cap J_2 \cap \ldots \cap J_n$$

$\Rightarrow$ Splitting of the landscape into n disconnected pieces !!

[*Singular* project, 97]

An example : type IIA with metric fluxes

- Testing the method with type IIA orientifold models including gauge fluxes and a metric flux

[Dall'Agatta, Villadoro, Zwirner '09]

$$\boxed{\left(F_{p=0,2,4,6} \quad , \quad H_3 \right) \quad + \quad \omega \quad \subset \quad f_{\alpha MNP}}$$

- Subset of embedding tensor components closed under $G_{n.c}$
  - Fields can still be set at the origin without lost of generality
  - Stability with respect to fluctuations around the origin can be computed

[Borghese, Roest '10]

- Vacua structure of these type IIA orientifolds

16 AdS_4 critical points + Mink of dim=1 → GKP after three T-dualities

[Giddings, Kachru, Polchinski '02]

- An AdS$_4$ landscape

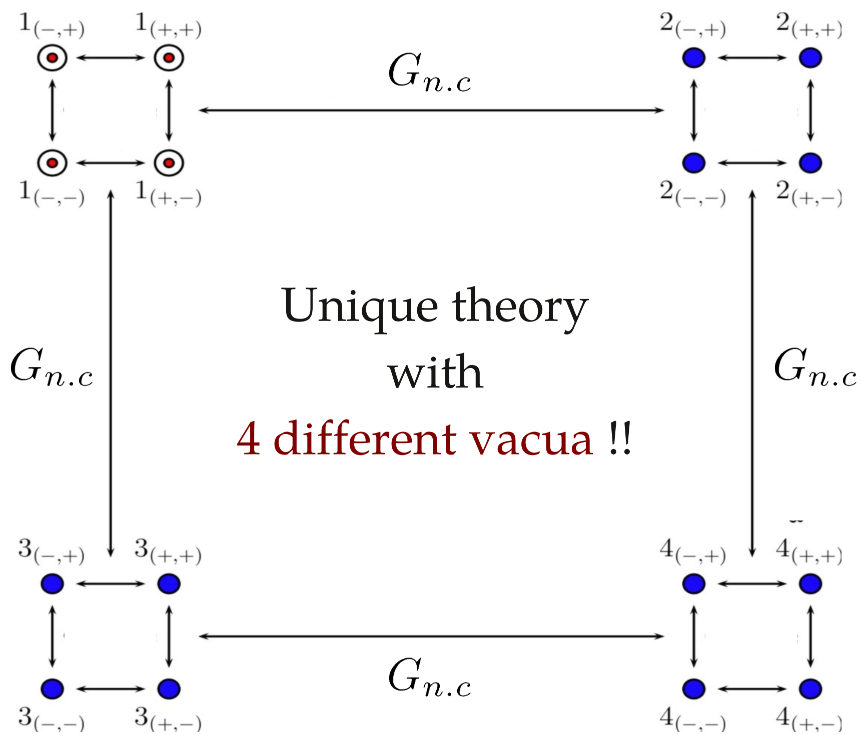
$$16 = 4 + 4 + 4 + 4$$

- Unique *gauging*

$$G_0 = \mathrm{ISO}(3) \ltimes \mathrm{U}(1)^6$$

$1_{(\pm,\pm)}$	$2_{(\pm,\pm)}$	$3_{(\pm,\pm)}$	$4_{(\pm,\pm)}$
$\mathcal{N}=1$ SUSY & ~~SUSY~~	~~SUSY~~	~~SUSY~~	~~SUSY~~
stable	unstable	stable	stable
m^2 = -2/3	m^2 = -4/5	$m^2 > 0$	$m^2 > 0$
V= -1	V= -32/27	V= -8/15	V= -32/27

$(*)\ m^2 \equiv$ lightest mode (B.F. bound $= -3/4$)



- All the solutions are connected and correspond to $\omega \equiv SU(2) \times SU(2)$

[Caviezel, Koerber, Körs, Lüst, Tsimpis/Wrase, Zagermann '08, '08]

NO SOURCES AT ALL !!

Lifting to maximal supergravity ?

$G = E_7$

e.t comp = 912
vectors = 56
scalars = 133
$\mathcal{N} = 8$

$\mathbb{Z}_2$

$G = SL(2) \times SO(6,6)$

e.t comp = (2,12) + (2,220)
vectors = (2,12)
scalars = (3,1) + (1,66)
$\mathcal{N} = 4$

* reps of G

Question :

- Is the absence of sources enough for the geometric IIA solutions to lift to a maximal supergravity theory?

Half-maximal inside maximal

[Dibitetto, A.G., Roest '11]

- Look at the maximal theory with the half-maximal *``glasses''*

$$\begin{array}{lccl} & E_7 & \supset & SL(2)\times SO(6,6) \\ \text{vectors}: & 56 & \longrightarrow & (2,12)+(1,32) \\ \text{scalars}: & 133 & \longrightarrow & (3,1)+(1,66)+(2,32') \\ \text{e.t:} & \underbrace{912}_{X_{56\,56}{}^{56}} & \longrightarrow & \underbrace{(2,12)}_{\xi_{\alpha M}}+\underbrace{(2,220)}_{f_{\alpha MNP}}+(1,352')+(3,32) \end{array}$$

$$\left.\begin{array}{lcccc} & & & & \mathbb{Z}_2 \\ rep & \equiv & bos\ (B) & \rightarrow & even \\ rep & \equiv & \underbrace{fermi\ (F)}_{SO(6,6)} & \rightarrow & odd \end{array}\right.$$

- Gauge algebra in the maximal theory

$$\left.\begin{array}{lcl} [A_B, A_B] & = & X_{BB}{}^{B}\ A_B + X_{BB}{}^{F}\ A_F \\ [A_B, A_F] & = & X_{BF}{}^{B}\ A_B + X_{BF}{}^{F}\ A_F \\ [A_F, A_F] & = & X_{FF}{}^{B}\ A_B + X_{FF}{}^{F}\ A_F \end{array}\right\} \quad \text{Jacobi identities} = QC_{\mathcal{N}=8}$$

- Components with an odd number of fermionic indices projected out !!

Extra conditions for the lifting

- For a half-maximal to be embeddable in a maximal theory

$$QC_{\mathcal{N}=8} \quad = \quad QC_{\mathcal{N}=4} \quad + \quad \text{extra conditions for the lifting}$$

- The extra conditions are

$$\underbrace{f_{\alpha MNP}\, f_{\beta}{}^{MNP} = 0}_{(3,1)} \qquad \text{and} \qquad \underbrace{\epsilon^{\alpha\beta}\; f_{\alpha[MNP}\, f_{\beta QRS]}\, \big|_{\text{SD}} = 0}_{(1,462')}$$

* reps of SL(2) x SO(6,6)

Example: type IIB dual sources ?

$$\underbrace{H_3 \wedge F_3}_{\substack{\text{D3-brane} \\ \text{flux-induced tadpole}}} \subset (1, 462') \quad \Longrightarrow \quad \text{which objects fill the rep ?}$$

An example : lifting of geometric type IIA

- All the 16 geom. IIA solutions lift to maximal supergravity

- Fake SUSY becomes SUSY

$$\underbrace{SU(8)}_{\begin{matrix}\mathcal{N}=8\\ \text{SUSY}\end{matrix}} \rightarrow \underbrace{SU(4)}_{\begin{matrix}\mathcal{N}=4\\ \text{SUSY}\end{matrix}} \times \underbrace{SU(4)}_{\begin{matrix}\mathcal{N}=4\\ \text{FAKE}\end{matrix}}$$

$1_{(\pm,\pm)}$	$2_{(\pm,\pm)}$	$3_{(\pm,\pm)}$	$4_{(\pm,\pm)}$
$\mathcal{N}=1$ SUSY	~~SUSY~~	~~SUSY~~	~~SUSY~~
stable	unstable	stable	unstable
$m^2 = -2/3$	$m^2 = -4/5$	$m^2 > 0$	$m^2 = -4/3$
$V = -1$	$V = -32/27$	$V = -8/15$	$V = -32/27$

$(*)\ m^2 \equiv$ lightest mode (B.F. bound $= -3/4$)

- Masses of the 70 physical scalars $\Longrightarrow$ ~~SUSY~~ and stable minimum !!

[Le Diffon, Samtleben, Trigiante '11]

- Underlying gaugings in maximal supergravity (28 vectors) ??

(…in progress)

Conclusions

- Some progress towards disentangling the landscape of extended supergravities can still be done without performing statistics of vacua

- The approach relies on the combined use of global symmetries and of algebraic geometry techniques

- As a warming-up, the complete vacua structure of simple type IIA orientifold theories can be worked out revealing some odd features :

  i) connections between vacua
  ii) $\mathcal{N}=8$ lifting of the entire vacua structure
  iii) stability without supersymmetry

- For the future :

  - Beyond the geometric limit : non-geometric backgrounds, dual branes . . .
  - de Sitter in extended supergravity (maybe $\mathcal{N}=2$) and links to Cosmology
  - Higher dimensional origin of gaugings : DFT, Generalised Geometry . . .

Thanks !!