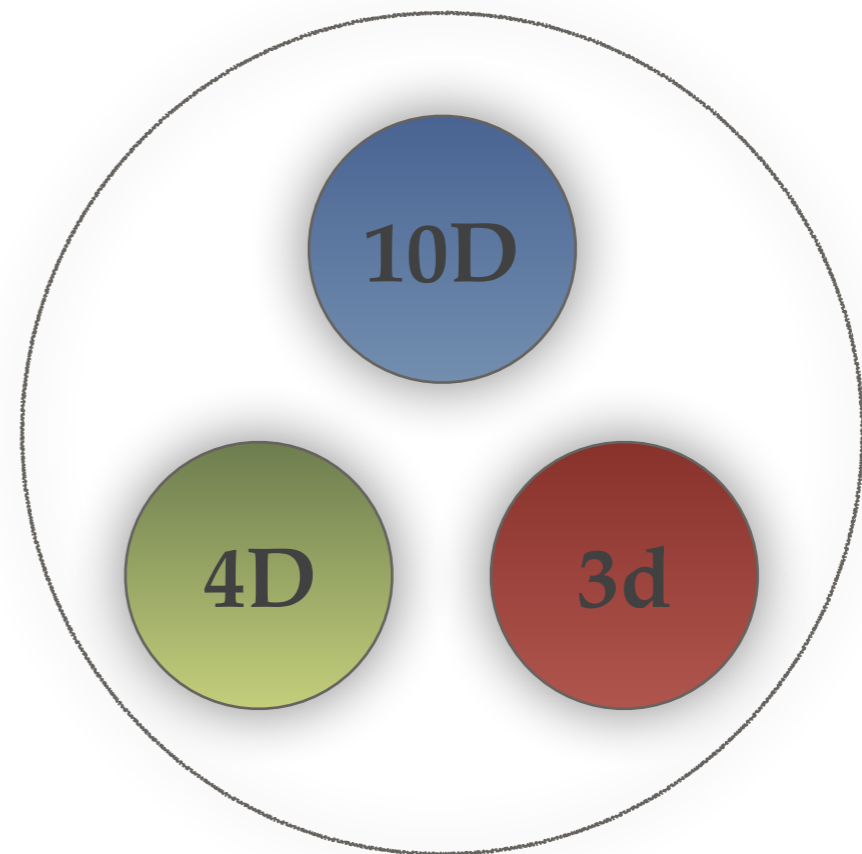


Deformed $N=8$ supergravity from IIA strings and its Chern-Simons duals

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November 19th , Brussels



Nikhef



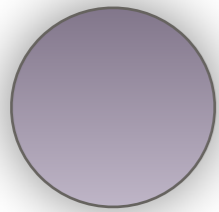
With Daniel Jafferis and Oscar Varela :

[arXiv:1504.08009](https://arxiv.org/abs/1504.08009) , [arXiv:1508.04432](https://arxiv.org/abs/1508.04432) , [arXiv:1509.02526](https://arxiv.org/abs/1509.02526)

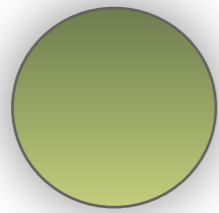
Outlook



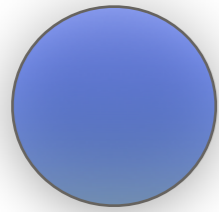
Motivation



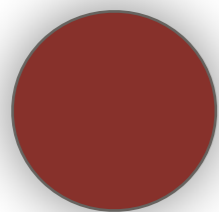
Deformed $SO(8)$ -gauged supergravity



Deformed $ISO(7)$ -gauged supergravity



Massive type IIA strings on S^6



New AdS_4 $N=2$ solution in massive IIA and its CFT_3 dual



Motivation

electric-magnetic deformations

- The uniqueness of the maximal ($N=8$) supergravities is historically inherited from their connection to sphere reductions

$$\text{AdS}_5 \times S^5 \text{ (D3-brane)} \quad , \quad \text{AdS}_4 \times S^7 \text{ (M2-brane)} \quad , \quad \text{AdS}_7 \times S^4 \text{ (M5-brane)}$$

- $N=8$ supergravity in 4D admits a **deformation parameter** c yielding **inequivalent** theories. It is an **electric/magnetic** deformation

$$D = \partial - g (A^{\text{elec}} - c \tilde{A}_{\text{mag}})$$

g = 4D gauge coupling
 c = **deformation param.**

- There are two generic situations :
 - 1) Family of $\text{SO}(8)_c$ theories : $c = [0, \sqrt{2} - 1]$ is a continuous param. [similar for $\text{SO}(p,q)_c$]
 - 2) Family of $\text{ISO}(7)_c$ theories : $c = 0 \text{ or } 1$ is an (on/off) param. [same for $\text{ISO}(p,q)_c$]

[Dall'Agata, Inverso, Trigiante '12]

[Dall'Agata, Inverso, Marrani '14]

The questions arise:

- Does such an electric / magnetic deformation of 4D maximal supergravity enjoy a string / M-theory origin, or is it just a 4D feature ?

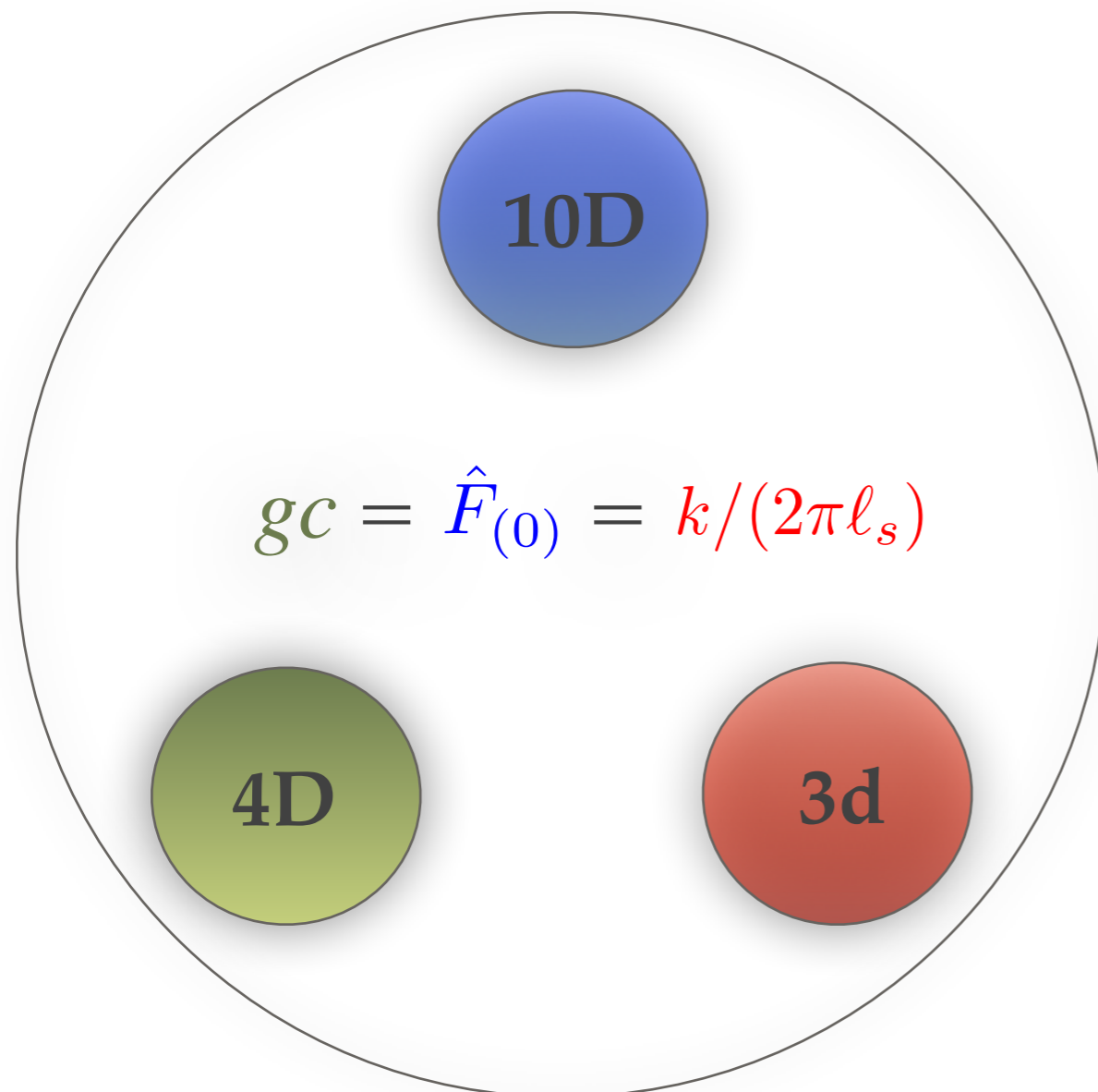
Obstruction for $SO(8)_c$, *cf.* [Lee, Strickland-Constable & Waldram '15]

[de Wit & Nicolai '13]

- For deformed 4D supergravities with supersymmetric AdS_4 vacua, are these AdS_4 / CFT_3 -dual to any identifiable 3d CFT ?

A new 10D / 4D / 3d correspondence

massive IIA on S^6 \ll ISO(7)_c-gauged sugra \gg SU(N)_k C-S-M theory



g_c = elec/mag deformation in 4D

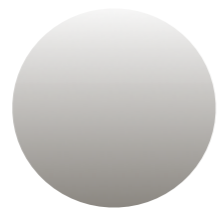
$\hat{F}_{(0)}$ = Romans mass in 10D

k = Chern-Simons level in 3d

[Schwarz '04]

[AG, Jafferis, Varela '15]

[AG, Varela '15]



Deformed $SO(8)$ -gauged supergravity

$N = 8$ supergravities in 4D

- SUGRA : metric + 8 gravitini + 28 vectors + 56 dilatini + 70 scalars
 (s = 2) (s = 3/2) (s = 1) (s = 1/2) (s = 0)

Ungauged (abelian) supergravity: Reduction of M-theory on a *torus* T^7
down to 4D produces $N = 8$ supergravity with $G = U(1)^{28}$ [Cremmer & Julia '79]

Gauged (non-abelian) supergravity: Reduction of M-theory on a *sphere* S^7
down to 4D produces $N = 8$ supergravity with $G = SO(8)$ [de Wit & Nicolai '82]

* $SO(8)$ -gauged supergravity believed to be **unique** for 30 years...

... but ... is this true?

Framework to study $N = 8$ supergravities in 4D

[de Wit, Samtleben & Trigiante '03 , '07]

Gauging procedure : Part of the **global E_7 symmetry** group is promoted to a local symmetry group G (gauging)

[$\alpha = 1, \dots, 133$]

Embedding tensor : It is a “selector” specifying which **generators of E_7** (there are 133!!) become the gauge symmetry G and, therefore, have associated gauge fields.

Formulation in terms of **56** vectors A_μ^M , though... $M = 1, \dots, 56 = 28$ (elec) + **28** (mag)

Sp(56) Elec/Mag group

$$A_\mu = A_\mu^M \Theta_M^\alpha t_\alpha$$

Redundancy!!

$$X_M = \Theta_M^\alpha t_\alpha \quad \Rightarrow \quad [X_M, X_N] = X_{MN}^P X_P \quad \text{with} \quad X_{MN}^P = \Theta_M^\alpha [t_\alpha]_N^P$$

* Closure of the gauge algebra : $\Omega^{MN} \Theta_M^\alpha \Theta_N^\beta = 0$

Only 28 physical l.c. of vectors!!

A family of $G = \text{SO}(8)$ supergravities in 4D

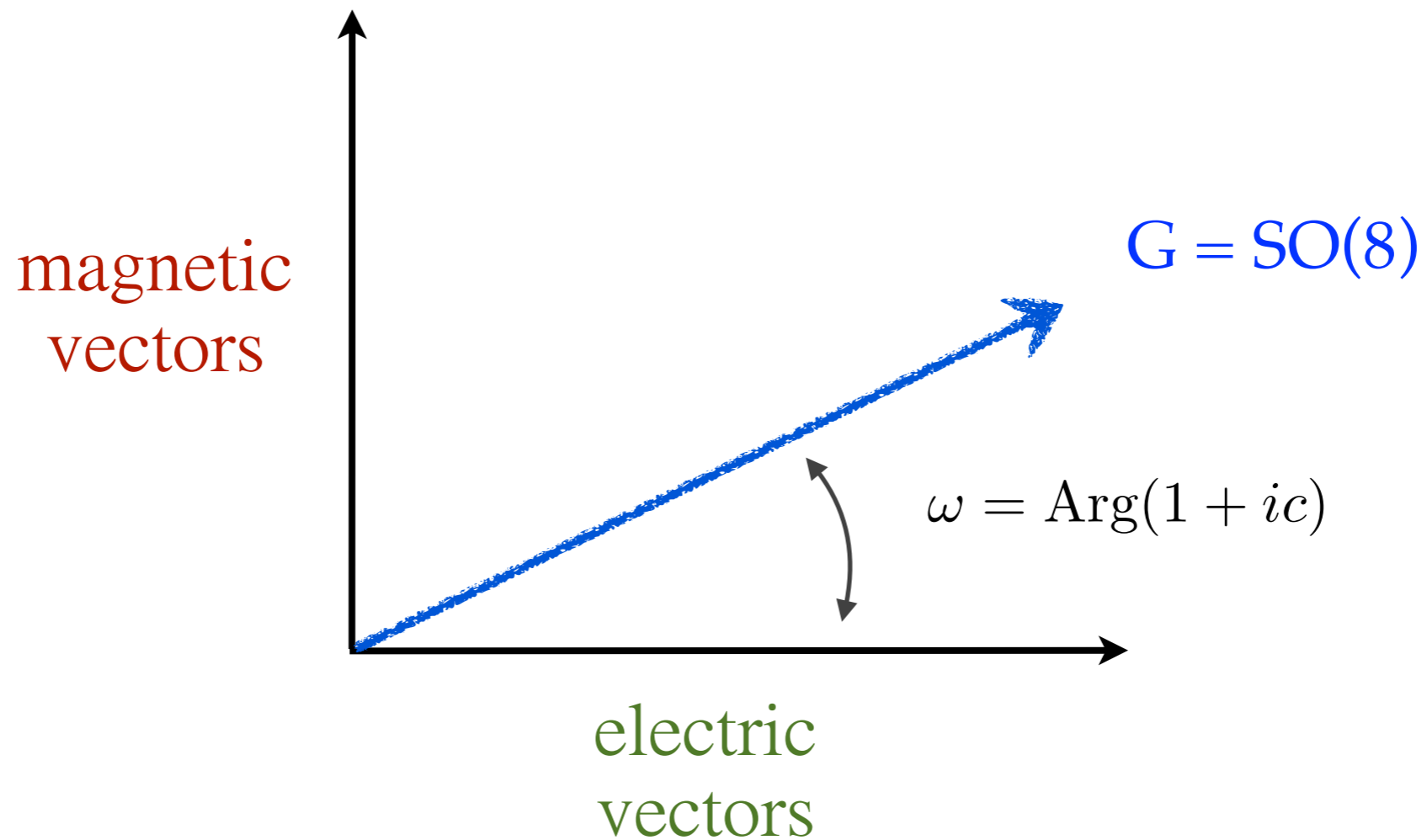
- Choose $G = \text{SO}(8)$
- Solve $\Omega^{MN} \Theta_M^\alpha \Theta_N^\beta = 0 \Rightarrow$ One-parameter (c) family of $\text{SO}(8)_c$ theories !!

[Dall'Agata, Inverso & Trigiante '12]

- Immediate questions :

- | | |
|--|------------------------------|
| 1) What? | (Yes, surprising but true) |
| 2) Are these c -theories equivalent? | (No) |
| 3) Are there new AdS_4 solutions? | (Yes) |
| 4) Higher-dimensional origin? | (Good question...) |
| 5) $\text{AdS}_4/\text{CFT}_3$ dual? | (Good question too... ABJ?) |

Physical meaning in 4D : electric/magnetic deformation



$$D = \partial - g (A^{\text{elec}} - c \tilde{A}_{\text{mag}})$$

Physical meaning 10D / 11D ...



Holographic $\text{AdS}_4/\text{CFT}_3$ meaning ...



In this talk we are going to investigate the electric/magnetic deformation of a different N=8 supergravity closely related to the $G = SO(8)$ theory ...

... the $G = ISO(7) = SO(7) \ltimes \mathbb{R}^7$ supergravity !!

electric / magnetic
deformation

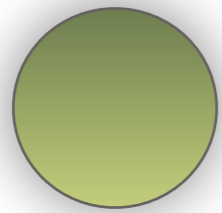


higher-dimensional
origin



Holographic
AdS₄/CFT₃ dual ?





Deformed ISO(7)-gauged supergravity

A family of $G = \text{ISO}(7)$ supergravities in 4D

- Choose $G = \text{ISO}(7)$
- Solve $\Omega^{MN} \Theta_M^\alpha \Theta_N^\beta = 0 \Rightarrow$ One-parameter (c) family of $\text{ISO}(7)_c$ theories !!

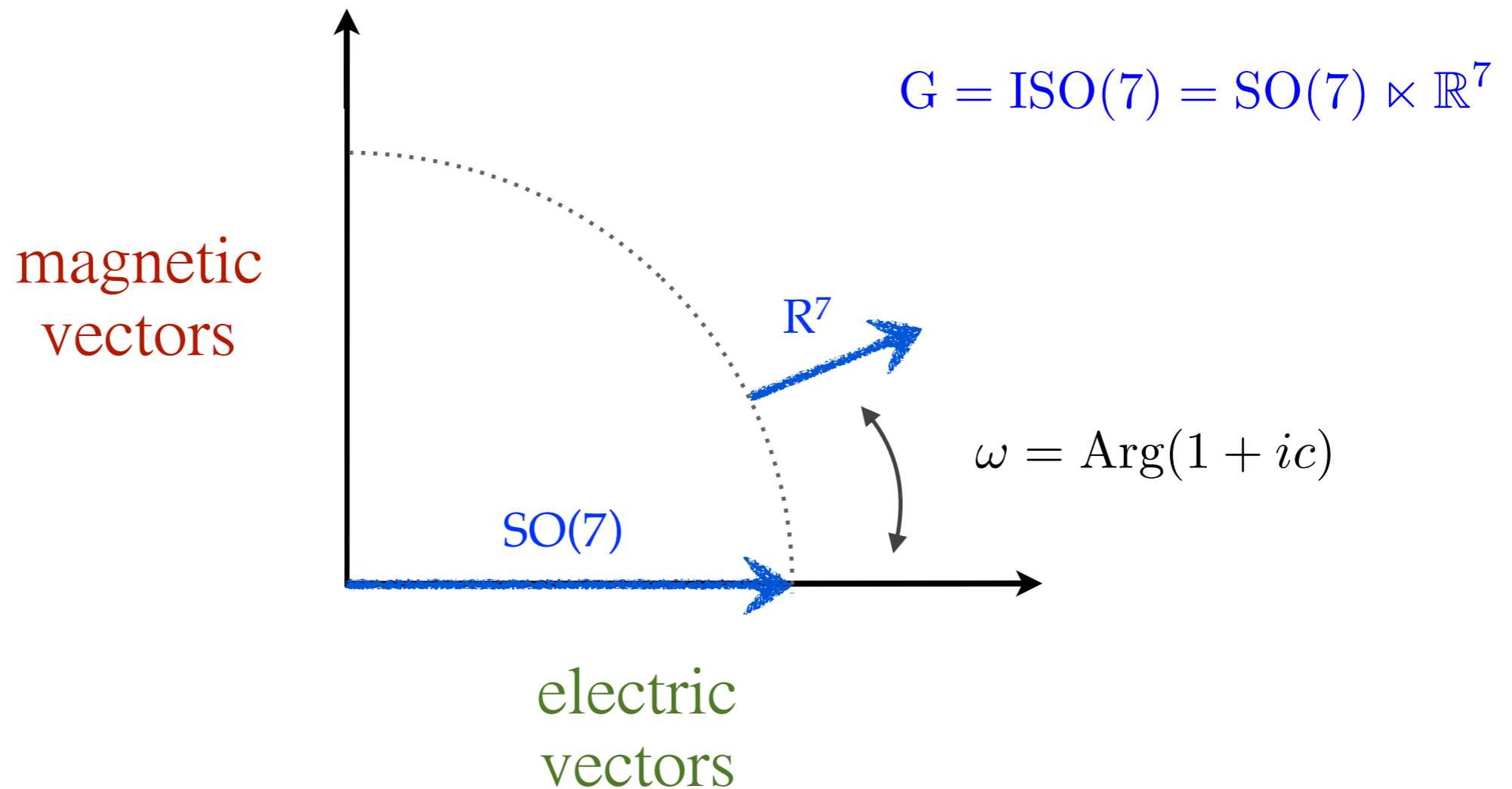
[Hull '84 (electric)]

[Dall'Agata, Inverso, Marrani '14]

- Immediate questions :

- | | |
|--|-----------------------------|
| 1) What? | (Yes, and still surprising) |
| 2) Are these c -theories equivalent? | (No) |
| 3) Are there new AdS_4 solutions? | (Yes) |
| 4) Higher-dimensional origin? | (Yes) |
| 5) $\text{AdS}_4/\text{CFT}_3$ dual? | (Yes) |

Physical meaning in 4D = electric/magnetic deformation



$$D = \partial - g A_{\text{SO}(7)}^{\text{elec}} - g (A_{\mathbb{R}^7}^{\text{elec}} - c \tilde{A}_{\mathbb{R}^7 \text{ mag}})$$

Deformed ISO(7)_c Lagrangian ($m = g\mathbf{c}$)

$$\begin{aligned} \mathbb{M} &= 1, \dots, 56 \\ \Lambda &= 1, \dots, 28 \\ I &= 1, \dots, 7 \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{bos}} &= (R - V) \text{vol}_4 - \frac{1}{48} D\mathcal{M}_{\text{MIN}} \wedge *D\mathcal{M}^{\text{MIN}} + \frac{1}{2} \mathcal{I}_{\Lambda\Sigma} \mathcal{H}_{(2)}^\Lambda \wedge *\mathcal{H}_{(2)}^\Sigma - \frac{1}{2} \mathcal{R}_{\Lambda\Sigma} \mathcal{H}_{(2)}^\Lambda \wedge \mathcal{H}_{(2)}^\Sigma \\ &+ m \left[\mathcal{B}^I \wedge (\tilde{\mathcal{H}}_{(2)I} - \frac{g}{2} \delta_{IJ} \mathcal{B}^J) - \frac{1}{4} \tilde{\mathcal{A}}_I \wedge \tilde{\mathcal{A}}_J \wedge (d\mathcal{A}^{IJ} + \frac{g}{2} \delta_{KL} \mathcal{A}^{IK} \wedge \mathcal{A}^{JL}) \right] \end{aligned}$$

◆ Setting $m = 0$, all the magnetic pieces in the Lagrangian disappear.

* Ingredients :

- Electric vectors (21 + 7): $\mathcal{A}^{IJ} = \mathcal{A}^{[IJ]}$ [SO(7)] and \mathcal{A}^I [R⁷] with $\mathcal{H}_{(2)}^\Lambda = (\mathcal{H}_{(2)}^{IJ}, \mathcal{H}_{(2)}^I)$
- Auxiliary magnetic vectors (7): $\tilde{\mathcal{A}}_I$ [R⁷] with $\tilde{\mathcal{H}}_{(2)I}$
- E₇/SU(8) scalars : \mathcal{M}_{MIN}
- Auxiliary two-forms (7): \mathcal{B}^I [R⁷]
- Topological term : m [...]
- Scalar potential : $V(\mathcal{M}) = \frac{g^2}{672} X_{\text{MN}}^{\text{R}} X_{\text{PQ}}^{\text{S}} \mathcal{M}^{\text{MP}} (\mathcal{M}^{\text{NQ}} \mathcal{M}_{\text{RS}} + 7 \delta_{\text{R}}^{\text{Q}} \delta_{\text{S}}^{\text{N}})$

A truncation : $G_0 = \text{SU}(3)$ invariant sector

[Warner '83]

- **Truncation** : Retaining the fields and couplings which are invariant (singlets) under the action of a subgroup $G_0 \subset \text{ISO}(7)$

- **SU(8) R-symmetry branching** : **gravitini** $\mathbf{8} \rightarrow \mathbf{1} + \mathbf{1} + \mathbf{3} + \bar{\mathbf{3}} \Rightarrow \mathcal{N} = 2$ SUSY

- **Scalars fields** : $\mathbf{70} \rightarrow \mathbf{1} (\times 6) + \text{non-singlets} \Rightarrow 6$ real scalars $(\varphi, \chi; \phi, a, \zeta, \tilde{\zeta})$

- **Vector fields** : $\mathbf{56} \rightarrow \mathbf{1} (\times 4) + \text{non-singlets} \Rightarrow$ vectors $(A^0, A^1; \tilde{A}_0, \tilde{A}_1)$

- $N = 2$ gauged supergravity with $G = \text{U}(1) \times \text{SO}(1,1)_c$ coupled to **1 vector & 1 hyper**

$$\mathcal{M}_{\text{scalar}} = \frac{\text{SU}(1,1)}{\text{U}(1)} \times \frac{\text{SU}(2,1)}{\text{U}(2)}$$

- **Scalar potential** :
$$V = \frac{1}{2} g^2 \left[e^{4\phi-3\varphi} (1 + e^{2\varphi} \chi^2)^3 - 12 e^{2\phi-\varphi} (1 + e^{2\varphi} \chi^2) - 12 e^{2\phi+\varphi} \rho^2 (1 - 3 e^{2\varphi} \chi^2) \right. \\ \left. - 24 e^\varphi + 12 e^{4\phi+\varphi} \chi^2 \rho^2 (1 + e^{2\varphi} \chi^2) + 12 e^{4\phi+\varphi} \rho^4 (1 + 3 e^{2\varphi} \chi^2) \right] \\ - \frac{1}{2} g m \chi e^{4\phi+3\varphi} (12 \rho^2 + 2 \chi^2) + \frac{1}{2} m^2 e^{4\phi+3\varphi} ,$$

AdS critical points !!

AdS₄ solutions

\mathcal{N}	G_0	$c^{-1/3} \chi$	$c^{-1/3} e^{-\varphi}$	$c^{-1/3} \rho$	$c^{-1/3} e^{-\phi}$	$\frac{1}{4} g^{-2} c^{1/3} V_0$	$M^2 L^2$
$\mathcal{N} = 1$	G_2	$-\frac{1}{2^{7/3}}$	$\frac{5^{1/2} 3^{1/2}}{2^{7/3}}$	$-\frac{1}{2^{7/3}}$	$\frac{5^{1/2} 3^{1/2}}{2^{7/3}}$	$-\frac{2^{22/3} 3^{1/2}}{5^{5/2}}$	$4 \pm \sqrt{6}, -\frac{1}{6}(11 \pm \sqrt{6})$
$\mathcal{N} = 2$	$U(3)$	$-\frac{1}{2}$	$\frac{3^{1/2}}{2}$	0	$\frac{1}{2^{1/2}}$	$-3^{3/2}$	$3 \pm \sqrt{17}, 2, 2$
$\mathcal{N} = 1$	$SU(3)$	$\frac{1}{2^2}$	$\frac{3^{1/2} 5^{1/2}}{2^2}$	$-\frac{3^{1/2}}{2^2}$	$\frac{5^{1/2}}{2^2}$	$-\frac{2^6 3^{3/2}}{5^{5/2}}$	$4 \pm \sqrt{6}, 4 \pm \sqrt{6}$
$\mathcal{N} = 0$	$SO(6)_+$	0	$2^{1/6}$	0	$\frac{1}{2^{5/6}}$	$-3 2^{5/6}$	$6, 6, -\frac{3}{4}, 0$
$\mathcal{N} = 0$	$SO(7)_+$	0	$\frac{1}{5^{1/6}}$	0	$\frac{1}{5^{1/6}}$	$-\frac{3 5^{7/6}}{2^2}$	$6, -\frac{12}{5}, -\frac{6}{5}, -\frac{6}{5}$
$\mathcal{N} = 0$	G_2	$\frac{1}{2^{4/3}}$	$\frac{3^{1/2}}{2^{4/3}}$	$\frac{1}{2^{4/3}}$	$\frac{3^{1/2}}{2^{4/3}}$	$-\frac{2^{10/3}}{3^{1/2}}$	$6, 6, -1, -1$
$\mathcal{N} = 0$	$SU(3)$	0.455	0.838	0.335	0.601	-5.864	6.214, 5.925, 1.145, -1.284
$\mathcal{N} = 0$	$SU(3)$	0.270	0.733	0.491	0.662	-5.853	6.230, 5.905, 1.130, -1.264

◆ Relevant for holographic RG flows (in progress with J. Tarrio and O. Varela...)

The truncated Lagrangian (I)

- The Lagrangian still contains a **tensor field** B^0 :

$$\begin{aligned}
 \mathcal{L} = & (R - V) \text{vol}_4 + \frac{3}{2} [d\varphi \wedge *d\varphi + e^{2\varphi} d\chi \wedge *d\chi] \\
 & + 2 d\phi \wedge *d\phi + \frac{1}{2} e^{2\phi} [D\zeta \wedge *D\zeta + D\tilde{\zeta} \wedge *D\tilde{\zeta}] \\
 & + \frac{1}{2} e^{4\phi} [Da + \frac{1}{2}(\zeta D\tilde{\zeta} - \tilde{\zeta} D\zeta)] \wedge *[Da + \frac{1}{2}(\zeta D\tilde{\zeta} - \tilde{\zeta} D\zeta)] \\
 & + \frac{1}{2} \mathcal{I}_{\Lambda\Sigma} H_{(2)}^\Lambda \wedge *H_{(2)}^\Sigma - \frac{1}{2} \mathcal{R}_{\Lambda\Sigma} H_{(2)}^\Lambda \wedge H_{(2)}^\Sigma + m B^0 \wedge d\tilde{A}_0 + \frac{1}{2} g m B^0 \wedge B^0
 \end{aligned}$$

where

$$Da = da + g A^0 - m \tilde{A}_0 \quad , \quad D\zeta = d\zeta - 3g A^1 \tilde{\zeta} \quad , \quad D\tilde{\zeta} = d\tilde{\zeta} + 3g A^1 \zeta$$

and with

$$H_{(2)}^0 = dA^0 + m B^0 \quad , \quad H_{(2)}^1 = dA^1$$

- Vector kinetic & θ -term : $\mathcal{N}_{\Lambda\Sigma} = \mathcal{R}_{\Lambda\Sigma} + i\mathcal{I}_{\Lambda\Sigma} = \frac{1}{(2e^\varphi \chi + i)} \begin{pmatrix} -\frac{e^{3\varphi}}{(e^\varphi \chi - i)^2} & \frac{3e^{2\varphi} \chi}{(e^\varphi \chi - i)} \\ \frac{3e^{2\varphi} \chi}{(e^\varphi \chi - i)} & 3(e^\varphi \chi^2 + e^{-\varphi}) \end{pmatrix}$

◆ **Non-dynamical tensor** & magnetic vector via $Da + \text{topological term} !!$

The truncated Lagrangian (II) : dual formulation

- Solving the EOM for the magnetic vector, one finds

$$H_{(3)}^0 = e^{4\phi} * \left(Da + \frac{1}{2} (\zeta D\tilde{\zeta} - \tilde{\zeta} D\zeta) \right) \quad \Rightarrow \quad \text{scalar-tensor duality!!}$$

- The dual Lagrangian

$$\begin{aligned} \tilde{\mathcal{L}} = & (R - V) \text{vol}_4 + \frac{1}{2} e^{-4\phi} H_{(3)}^0 \wedge *H_{(3)}^0 + \frac{3}{2} [d\varphi \wedge *d\varphi + e^{2\varphi} d\chi \wedge *d\chi] \\ & + 2 d\phi \wedge *d\phi + \frac{1}{2} e^{2\phi} \left[D\zeta \wedge *D\zeta + D\tilde{\zeta} \wedge *D\tilde{\zeta} \right] \\ & + \frac{1}{2} \mathcal{I}_{\Lambda\Sigma} H_{(2)}^\Lambda \wedge *H_{(2)}^\Sigma - \frac{1}{2} \mathcal{R}_{\Lambda\Sigma} H_{(2)}^\Lambda \wedge H_{(2)}^\Sigma \\ & - H_{(3)}^0 \wedge \left[g A^0 + \frac{1}{2} (\zeta D\tilde{\zeta} - \tilde{\zeta} D\zeta) \right] + \frac{1}{2} gm B^0 \wedge B^0 . \end{aligned}$$

◆ Dynamical tensor $H_{(3)}^0 = dB^0$ & Da and \tilde{A}_0 have disappeared from the Lagrangian

◆ Natural duality frame to investigate possible higher-dimensional origin!!

[N=2 Calabi-Yau and coset reds w/ fluxes]

[Polchinski & Strominger '95]

[Louis & Micu '02]

[Kashani-Poor '07]

[Kashani-Poor & Cassani '09]

Dual formulations seem to be crucial
to understand
higher-dimensional origins!!

... let's give up the Lagrangian.

SL(7)-covariant duality hierarchy

[de Wit, Nicolai & Samtleben '08]

[Bergshoeff, Hartong, Hohm, Huebscher & Ortin '09]

- Restricted SL(7)-covariant field content [index I]

	1	metric :	ds_4^2	
	21' + 7' + 21 + 7	coset representatives :	\mathcal{V}^{IJij} , \mathcal{V}^{I8ij} , $\tilde{\mathcal{V}}_{IJ}{}^{ij}$, $\tilde{\mathcal{V}}_{I8}{}^{ij}$,	
	21' + 7' + 21 + 7	vectors :	\mathcal{A}^{IJ} , \mathcal{A}^I , $\tilde{\mathcal{A}}_{IJ}$, $\tilde{\mathcal{A}}_I$,	
	48 + 7'	two-forms :	$\mathcal{B}_I{}^J$, \mathcal{B}^I ,	[out of 133]
	28'	three-forms :	\mathcal{C}^{IJ} ,	[out of 912]

- Two-form field strengths [**21' + 7' + 21 + 7**]

$$\mathcal{H}_{(2)}^{IJ} = d\mathcal{A}^{IJ} - g \delta_{KL} \mathcal{A}^{IK} \wedge \mathcal{A}^{LJ} ,$$

$$\mathcal{H}_{(2)}^I = d\mathcal{A}^I - g \delta_{JK} \mathcal{A}^{IJ} \wedge \mathcal{A}^K + \frac{1}{2} m \mathcal{A}^{IJ} \wedge \tilde{\mathcal{A}}_J + m \mathcal{B}^I ,$$

$$\tilde{\mathcal{H}}_{(2)IJ} = d\tilde{\mathcal{A}}_{IJ} + g \delta_{K[I} \mathcal{A}^{KL} \wedge \tilde{\mathcal{A}}_{J]L} + g \delta_{K[I} \mathcal{A}^K \wedge \tilde{\mathcal{A}}_{J]} - m \tilde{\mathcal{A}}_I \wedge \tilde{\mathcal{A}}_J + 2g \delta_{K[I} \mathcal{B}_{J]}{}^K ,$$

$$\tilde{\mathcal{H}}_{(2)I} = d\tilde{\mathcal{A}}_I - \frac{1}{2} g \delta_{IJ} \mathcal{A}^{JK} \wedge \tilde{\mathcal{A}}_K + g \delta_{IJ} \mathcal{B}^J ,$$

[i, j are SU(8) indices relevant to couple fermions]

- Three-form field strengths [**48 + 7'**]

$$\begin{aligned}
\mathcal{H}_{(3)I}{}^J &= DB_I{}^J + \frac{1}{2}\mathcal{A}^{JK} \wedge d\tilde{\mathcal{A}}_{IK} + \frac{1}{2}\mathcal{A}^J \wedge d\tilde{\mathcal{A}}_I + \frac{1}{2}\tilde{\mathcal{A}}_{IK} \wedge d\mathcal{A}^{JK} + \frac{1}{2}\tilde{\mathcal{A}}_I \wedge d\mathcal{A}^J \\
&\quad - \frac{1}{2}g\delta_{KL}\mathcal{A}^{JK} \wedge \mathcal{A}^{LM} \wedge \tilde{\mathcal{A}}_{IM} - \frac{1}{2}g\delta_{KL}\mathcal{A}^{JK} \wedge \mathcal{A}^L \wedge \tilde{\mathcal{A}}_I \\
&\quad + \frac{1}{6}g\delta_{IK}\mathcal{A}^{JL} \wedge \mathcal{A}^{KM} \wedge \tilde{\mathcal{A}}_{LM} - \frac{1}{3}g\delta_{IK}\mathcal{A}^{(J} \wedge \mathcal{A}^{K)L} \wedge \tilde{\mathcal{A}}_L \\
&\quad - \frac{1}{2}m\mathcal{A}^{JK} \wedge \tilde{\mathcal{A}}_I \wedge \tilde{\mathcal{A}}_K - 2g\delta_{IK}\mathcal{C}^{JK} - \frac{1}{7}\delta_I^J \text{ (trace) } ,
\end{aligned}$$

$$\mathcal{H}_{(3)}^I = DB^I - \frac{1}{2}\mathcal{A}^{IJ} \wedge d\tilde{\mathcal{A}}_J - \frac{1}{2}\tilde{\mathcal{A}}_J \wedge d\mathcal{A}^{IJ} + \frac{1}{2}g\delta_{JK}\mathcal{A}^{IJ} \wedge \mathcal{A}^{KL} \wedge \tilde{\mathcal{A}}_L ,$$

- Four-form field strengths [**28'**]

$$\begin{aligned}
\mathcal{H}_{(4)}^{IJ} &= DC^{IJ} - \mathcal{H}_{(2)}^{K(I} \wedge \mathcal{B}_K{}^{J)} + \mathcal{H}_{(2)}^{(I} \wedge \mathcal{B}^{J)} - \frac{1}{2}m\mathcal{B}^I \wedge \mathcal{B}^J - \frac{1}{6}\mathcal{A}^{K(I} \wedge \tilde{\mathcal{A}}_{KL} \wedge d\mathcal{A}^{J)L} \\
&\quad + \frac{1}{6}\mathcal{A}^{IK} \wedge \mathcal{A}^{JL} \wedge d\tilde{\mathcal{A}}_{KL} - \frac{1}{6}\mathcal{A}^{K(I} \wedge \tilde{\mathcal{A}}_K \wedge d\mathcal{A}^{J)} - \frac{1}{3}\mathcal{A}^{K(I} \wedge \mathcal{A}^{J)} \wedge d\tilde{\mathcal{A}}_K \\
&\quad - \frac{1}{6}\mathcal{A}^{(I} \wedge \tilde{\mathcal{A}}_K \wedge d\mathcal{A}^{J)K} - \frac{1}{6}g\delta_{KL}\mathcal{A}^{K(I} \wedge \mathcal{A}^{J)M} \wedge \mathcal{A}^{LN} \wedge \tilde{\mathcal{A}}_{MN} \\
&\quad + \frac{1}{6}g\delta_{KL}\mathcal{A}^{K(I} \wedge \mathcal{A}^{J)} \wedge \mathcal{A}^{LM} \wedge \tilde{\mathcal{A}}_M - \frac{1}{6}g\delta_{KL}\mathcal{A}^{K(I} \wedge \mathcal{A}^{J)M} \wedge \mathcal{A}^L \wedge \tilde{\mathcal{A}}_M \\
&\quad - \frac{1}{8}m\mathcal{A}^{IK} \wedge \mathcal{A}^{JL} \wedge \tilde{\mathcal{A}}_K \wedge \tilde{\mathcal{A}}_L .
\end{aligned}$$

Consistency checks

- Closed set of Bianchi identities

$$\begin{aligned}
 D\mathcal{H}_{(2)}^{IJ} &= 0 \quad , \quad D\mathcal{H}_{(2)}^I = m \mathcal{H}_{(3)}^I \quad , \quad D\tilde{\mathcal{H}}_{(2)IJ} = -2g \mathcal{H}_{(3)[I}{}^K \delta_{J]K} \quad , \quad D\tilde{\mathcal{H}}_{(2)I} = g \delta_{IJ} \mathcal{H}_{(3)}^J \quad , \\
 D\mathcal{H}_{(3)I}{}^J &= \mathcal{H}_{(2)}^{JK} \wedge \tilde{\mathcal{H}}_{(2)IK} + \mathcal{H}_{(2)}^J \wedge \tilde{\mathcal{H}}_{(2)I} - 2g \delta_{IK} \mathcal{H}_{(4)}^{JK} - \frac{1}{7} \delta_I^J \text{ (trace)} \quad , \\
 D\mathcal{H}_{(3)}^I &= -\mathcal{H}_{(2)}^{IJ} \wedge \tilde{\mathcal{H}}_{(2)J} \quad , \quad D\mathcal{H}_{(4)}^{IJ} \equiv 0 \quad .
 \end{aligned}$$

- Closed set of duality relations [right number of d.o.f] [short-hand notation]

$$\begin{aligned}
 \tilde{\mathcal{H}}_{(2)IJ} &= -\frac{1}{2} \mathcal{I}_{[IJ][KL]} * \mathcal{H}_{(2)}^{KL} - \mathcal{I}_{[IJ][K8]} * \mathcal{H}_{(2)}^K + \frac{1}{2} \mathcal{R}_{[IJ][KL]} \mathcal{H}_{(2)}^{KL} + \mathcal{R}_{[IJ][K8]} \mathcal{H}_{(2)}^K \quad , \\
 \tilde{\mathcal{H}}_{(2)I} &= -\frac{1}{2} \mathcal{I}_{[I8][KL]} * \mathcal{H}_{(2)}^{KL} - \mathcal{I}_{[I8][K8]} * \mathcal{H}_{(2)}^K + \frac{1}{2} \mathcal{R}_{[I8][KL]} \mathcal{H}_{(2)}^{KL} + \mathcal{R}_{[I8][K8]} \mathcal{H}_{(2)}^K \quad , \\
 \mathcal{H}_{(3)I}{}^J &= -\frac{1}{12} (t_I^J)_{\mathbb{M}^{\mathbb{P}}} \mathcal{M}_{\mathbb{NP}} * D\mathcal{M}^{\mathbb{MN}} - \frac{1}{7} \delta_I^J \text{ (trace)} \quad , \\
 \mathcal{H}_{(3)}^I &= -\frac{1}{12} (t_8^I)_{\mathbb{M}^{\mathbb{P}}} \mathcal{M}_{\mathbb{NP}} * D\mathcal{M}^{\mathbb{MN}} \quad , \\
 \mathcal{H}_{(4)}^{IJ} &= \frac{1}{84} X_{\mathbb{NQ}}^{\mathbb{S}} \left((t_K^{(I|})_{\mathbb{P}}^{\mathbb{R}} \mathcal{M}^{|J)KN} + (t_8^{(I|})_{\mathbb{P}}^{\mathbb{R}} \mathcal{M}^{|J)8N} \right) (\mathcal{M}^{\mathbb{PQ}} \mathcal{M}_{\mathbb{RS}} + 7 \delta_{\mathbb{S}}^{\mathbb{P}} \delta_{\mathbb{R}}^{\mathbb{Q}}) \text{vol}_4 \quad .
 \end{aligned}$$

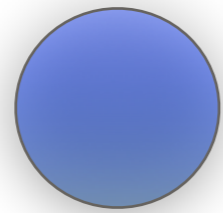
[t 's are $\text{SL}(7) \times \mathbb{R}^7$ generators]

- Closed set of SUSY transformations

Q : Why to bother with all these duality hierarchy issues?

A : Because the duality hierarchy allows us to derive simple embedding formulas to put the *4D dynamics* into a higher-dimensional one.

Remember : *No index, no clue.* Good luck trying to embed V into higher dimensions...



Massive type IIA strings on S^6

Collecting clues

- The deformed $\text{ISO}(7)_c$ gauging has its $\text{SO}(7)$ piece untouched by the deformation. This points towards an undeformed S^6 description in higher dimension.

- If the higher-dimensional geometry is not affected, it should then be the higher-dimensional theory the one changing. The massive IIA theory by Romans proves a natural candidate.

[Romans '86]

- The Romans mass parameter $\hat{F}_{(0)}$ is a discrete (on/off) deformation, exactly as the parameter c in the deformed $\text{ISO}(7)_c$ theory.

- The $\text{SU}(3)$ -invariant sector of the $\text{ISO}(7)_c$ theory connects to CY_3 reductions of massive IIA upon dualisation of some field. It is then natural to believe that the embedding of the full $\text{ISO}(7)_c$ theory would require an enlarged set of duality relations... duality hierarchy!!

Derivation of the IIA embedding [4-step process]

* Step 1 : 10D KK expansion that leaves 4D spacetime symmetry manifest

$A(x,y)$'s and $B(x,y)$'s fields

* Step 2 : Redefinitions of the A 's and B 's fields to conform 4D SUSY transformations

$C(x,y)$'s fields

* Step 3 : Connection to actual 4D fields by dressing up with S^6 geometrical data

$$C(x, y) = \text{geometry}(y) \times \mathcal{C}(x)$$

* Step 4 : Plug and play

* **Step 1** : 10D redefinitions (KK expansion) that leave 4D spacetime symmetry manifest

$$\text{SO}(1, 9) \rightarrow \text{SO}(1, 3) \times \text{SO}(6)$$

Then one has :

$$d\hat{s}_{10}^2 = \Delta^{-1} ds_4^2 + g_{mn} (dy^m + B^m) (dy^n + B^n) ,$$

$$\begin{aligned} \hat{A}_{(3)} &= \frac{1}{6} A_{\mu\nu\rho} dx^\mu \wedge dx^\nu \wedge dx^\rho + \frac{1}{2} A_{\mu\nu m} dx^\mu \wedge dx^\nu \wedge (dy^m + B^m) \\ &+ \frac{1}{2} A_{\mu mn} dx^\mu \wedge (dy^m + B^m) \wedge (dy^n + B^n) \\ &+ \frac{1}{6} A_{mnp} (dy^m + B^m) \wedge (dy^n + B^n) \wedge (dy^p + B^p) , \end{aligned}$$

$$\hat{B}_{(2)} = \frac{1}{2} B_{\mu\nu} dx^\mu \wedge dx^\nu + B_{\mu m} dx^\mu \wedge (dy^m + B^m) + \frac{1}{2} B_{mn} (dy^m + B^m) \wedge (dy^n + B^n) ,$$

$$\hat{A}_{(1)} = A_\mu dx^\mu + A_m (dy^m + B^m) ,$$

In terms of representations of $\text{SL}(6)$ [index m] :

	1	metric :	ds_4^2 ,
	21 + 6 + 1 + 20 + 15	scalars :	$g_{mn} , A_m , \hat{\phi} , A_{mnp} , B_{mn} ,$
	6' + 1 + 15 + 6	vectors :	$B_\mu^m , A_\mu , A_{\mu mn} , B_{\mu m} ,$
	6 + 1	two-forms :	$A_{\mu\nu m} , B_{\mu\nu} ,$
	1	three-form :	$A_{\mu\nu\rho} .$

$$\Delta^2 \equiv \frac{\det g_{mn}}{\det \hat{g}_{mn}}$$

* Step 2 : Non-linear field redefinitions to conform 4D SUSY transformations

- Vectors : $C_\mu^{m8} \equiv B_\mu^m$, $C_\mu^{78} \equiv A_\mu$, $\tilde{C}_{\mu mn} \equiv A_{\mu mn} - A_\mu B_{mn}$, $\tilde{C}_{\mu m7} \equiv B_{\mu m}$
- Two-forms : $C_{\mu\nu m}^8 \equiv -A_{\mu\nu m} + C_{[\mu}^{n8} \tilde{C}_{\nu]nm} + C_{[\mu}^{78} \tilde{C}_{\nu]m7}$, $C_{\mu\nu 7}^8 \equiv -B_{\mu\nu} + C_{[\mu}^{m8} \tilde{C}_{\nu]m7}$
- Three-form : $C_{\mu\nu\rho}^{88} \equiv A_{\mu\nu\rho} - C_{[\mu}^{m8} C_{\nu}^{n8} \tilde{C}_{\rho]mn} + C_{[\mu}^{m8} C_{\nu}^{78} \tilde{C}_{\rho]m7} + 3 C_{[\mu}^{78} C_{\nu\rho]7}^8$

These can be rearranged into representations of SL(7) [index I]

$$C_\mu^{I8} = (C_\mu^{m8}, C_\mu^{78}) \quad \tilde{C}_{\mu IJ} = (\tilde{C}_{\mu mn}, \tilde{C}_{\mu m7}) \quad C_{\mu\nu I}^8 = (C_{\mu\nu m}^8, C_{\mu\nu 7}^8) \quad C_{\mu\nu\rho}^{88}$$

with 10D SUSY transfs:

$$\delta C_\mu^{I8} = i V^{I8}{}_{AB} \left(\bar{\epsilon}^A \psi_\mu^B + \frac{1}{2\sqrt{2}} \bar{\epsilon}_C \gamma_\mu \chi^{ABC} \right) + \text{h.c.} ,$$

$$\delta \tilde{C}_{\mu IJ} = -i V_{IJ}{}_{AB} \left(\bar{\epsilon}^A \psi_\mu^B + \frac{1}{2\sqrt{2}} \bar{\epsilon}_C \gamma_\mu \chi^{ABC} \right) + \text{h.c.} ,$$

...

Mimicking the 4D tensor hierarchy !!

The result is then a set of $SL(7)$ -covariant 10D fields :

1	metric :	$ds_4^2(x, y)$,
7' + 21	generalised vielbeine :	$V^{I8}_{AB}(x, y)$, $\tilde{V}_{IJAB}(x, y)$,
7' + 21	vectors :	$C_\mu^{I8}(x, y)$, $\tilde{C}_{\mu IJ}(x, y)$,
7	two-forms :	$C_{\mu\nu I}^8(x, y)$,
1	three-form :	$C_{\mu\nu\rho}^{88}(x, y)$.

that is to be connected with the $SL(7)$ -covariant 4D fields of the tensor hierarchy :

1	metric :	$ds_4^2(x)$,
21' + 7' + 21 + 7	coset representatives :	$\mathcal{V}^{IJij}(x)$, $\mathcal{V}^{I8ij}(x)$, $\tilde{\mathcal{V}}_{IJij}(x)$, $\tilde{\mathcal{V}}_{I8ij}(x)$,
21' + 7' + 21 + 7	vectors :	$\mathcal{A}_\mu^{IJ}(x)$, $\mathcal{A}_\mu^I(x)$, $\tilde{\mathcal{A}}_{\mu IJ}(x)$, $\tilde{\mathcal{A}}_{\mu I}(x)$,
48 + 7'	two-forms :	$\mathcal{B}_{\mu\nu I}^J(x)$, $\mathcal{B}_{\mu\nu}^I(x)$,
28'	three-forms :	$\mathcal{C}_{\mu\nu\rho}^{IJ}(x)$,

> This connection is established by using geometrical data of the S^6 !!

* **Step 3** : Connecting 4D [SL(7)] and 10D [SL(6)] fields using the S^6 geometrical data in a “dressing up” process

[vectors]

$$C_{\mu}{}^{m8}(x, y) = \frac{1}{2} g K_{IJ}^m(y) \mathcal{A}_{\mu}{}^{IJ}(x) \quad , \quad C_{\mu}{}^{78}(x, y) = -\mu_I(y) \mathcal{A}_{\mu}{}^I(x) \quad ,$$

$$\tilde{C}_{\mu mn}(x, y) = \frac{1}{4} K_{mn}^{IJ}(y) \tilde{\mathcal{A}}_{\mu IJ}(x) \quad , \quad \tilde{C}_{\mu m7}(x, y) = -g^{-1} (\partial_m \mu^I)(y) \tilde{\mathcal{A}}_{\mu I}(x)$$

[two-forms]

$$C_{\mu\nu m}{}^8(x, y) = -g^{-1} (\mu_I \partial_m \mu^J)(y) \mathcal{B}_{\mu\nu J}{}^I(x)$$

$$C_{\mu\nu 7}{}^8(x, y) = \mu_I(y) \mathcal{B}_{\mu\nu}{}^I(x)$$

[three-form]

$$C_{\mu\nu\rho}{}^{88}(x, y) = (\mu_I \mu_J)(y) \mathcal{C}_{\mu\nu\rho}{}^{IJ}(x)$$

◆ S^6 geometrical data : embedding coordinates μ^I , Killing vectors K_{IJ}^m and tensors K_{IJ}^{mn}

* Step 4 : Plug and play... so that the final embedding of $\text{ISO}(7)_c$ into type IIA is

$$d\hat{s}_{10}^2 = \Delta^{-1} ds_4^2 + g_{mn} Dy^m Dy^n ,$$

$$\begin{aligned} \hat{A}_{(3)} = & \mu_I \mu_J (C^{IJ} + \mathcal{A}^I \wedge \mathcal{B}^J + \frac{1}{6} \mathcal{A}^{IK} \wedge \mathcal{A}^{JL} \wedge \tilde{\mathcal{A}}_{KL} + \frac{1}{6} \mathcal{A}^I \wedge \mathcal{A}^{JK} \wedge \tilde{\mathcal{A}}_K) \\ & + g^{-1} (\mathcal{B}_J^I + \frac{1}{2} \mathcal{A}^{IK} \wedge \tilde{\mathcal{A}}_{KJ} + \frac{1}{2} \mathcal{A}^I \wedge \tilde{\mathcal{A}}_J) \wedge \mu_I D\mu^J + \frac{1}{2} g^{-2} \tilde{\mathcal{A}}_{IJ} \wedge D\mu^I \wedge D\mu^J \\ & - \frac{1}{2} \mu_I B_{mn} \mathcal{A}^I \wedge Dy^m \wedge Dy^n + \frac{1}{6} A_{mnp} Dy^m \wedge Dy^n \wedge Dy^p , \end{aligned}$$

$$\hat{B}_{(2)} = -\mu_I (\mathcal{B}^I + \frac{1}{2} \mathcal{A}^{IJ} \wedge \tilde{\mathcal{A}}_J) - g^{-1} \tilde{\mathcal{A}}_I \wedge D\mu^I + \frac{1}{2} B_{mn} Dy^m \wedge Dy^n ,$$

$$\hat{A}_{(1)} = -\mu_I \mathcal{A}^I + A_m Dy^m . \quad ($$

where we have defined : $Dy^m \equiv dy^m + \frac{1}{2} g K_{IJ}^m \mathcal{A}^{IJ}$, $D\mu^I \equiv d\mu^I - g \mathcal{A}^{IJ} \mu_J$

The scalars are embedded as

$$\begin{aligned} g^{mn} &= \frac{1}{4} g^2 \Delta \mathcal{M}^{IJKL} K_{IJ}^m K_{KL}^n , & B_{mn} &= -\frac{1}{2} \Delta g_{mp} K_{IJ}^p \partial_n \mu^K \mathcal{M}^{IJ}{}_{K8} , \\ A_m &= \frac{1}{2} g \Delta g_{mn} K_{IJ}^n \mu_K \mathcal{M}^{IJK8} , & A_{mnp} &= \frac{1}{8} g \Delta g_{mq} K_{IJ}^q K_{np}^{KL} \mathcal{M}^{IJ}{}_{KL} + A_m B_{np} . \end{aligned}$$

Remarks and consistency checks

- 10D (bosonic) SUSY transformations exactly reduce to those of the 4D tensor hierarchy.
- Computing the 10D field strengths $\hat{F}_{(2)} = d\hat{A}_{(1)} + m\hat{B}_{(2)}$, etc. one finds

$$\hat{F}_{(4)} = \mu_I \mu_J \mathcal{H}_{(4)}^{IJ} + g^{-1} \mathcal{H}_{(3)J}^I \wedge \mu_I D\mu^J + \frac{1}{2} g^{-2} \tilde{\mathcal{H}}_{(2)IJ} \wedge D\mu^I \wedge D\mu^J + \dots, \quad [\text{FR parameter}]$$

$$\hat{H}_{(3)} = -\mu_I \mathcal{H}_{(3)}^I - g^{-1} \tilde{\mathcal{H}}_{(2)I} \wedge D\mu^I + \dots,$$

$$\hat{F}_{(2)} = -\mu_I \mathcal{H}_{(2)}^I + g^{-1} (g \delta_{IJ} \mathcal{A}^J - m \tilde{\mathcal{A}}_I) \wedge D\mu^I + \dots,$$

which are expressed in terms of the 4D tensor hierarchy. The parameter $m=gc$ only appears through standard Romans' redefinitions of $\hat{F}_{(p)}$ in 10D (formulas also valid for massless IIA)

[Hull & Warner '88 (electric)]

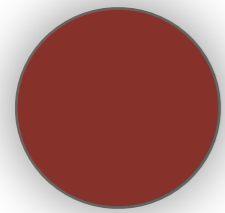
- The set of Bianchi identities of the above 10D field strengths reduces to

$$D\mathcal{H}_{(2)}^{IJ} = 0, \quad D\mathcal{H}_{(2)}^I = m \mathcal{H}_{(3)}^I, \quad D\tilde{\mathcal{H}}_{(2)IJ} = -2g \mathcal{H}_{(3)[I}^K \delta_{J]K}, \quad D\tilde{\mathcal{H}}_{(2)I} = g \delta_{IJ} \mathcal{H}_{(3)}^J,$$

$$D\mathcal{H}_{(3)I}^J = \mathcal{H}_{(2)}^{JK} \wedge \tilde{\mathcal{H}}_{(2)IK} + \mathcal{H}_{(2)}^J \wedge \tilde{\mathcal{H}}_{(2)I} - 2g \delta_{IK} \mathcal{H}_{(4)}^{JK} - \frac{1}{7} \delta_I^J (\text{trace}),$$

$$D\mathcal{H}_{(3)}^I = -\mathcal{H}_{(2)}^{IJ} \wedge \tilde{\mathcal{H}}_{(2)J}, \quad D\mathcal{H}_{(4)}^{IJ} \equiv 0.$$

which exactly matches the one of the 4D tensor hierarchy.



New AdS_4 $N=2$ solution in massive IIA and its CFT_3 dual

$N=2$ solution of massive type IIA

[Luest & Tsimpis '09]
[see also Petrini & Zaffaroni '09]

- Using the uplifting formulae, it is a straightforward exercise to obtain the 10D embedding of a 4D critical point. An example is the $N=2&U(3)$ AdS_4 point of the $ISO(7)_c$ theory

$$d\hat{s}_{10}^2 = L^2 \frac{(3 + \cos 2\alpha)^{\frac{1}{2}}}{(5 + \cos 2\alpha)^{-\frac{1}{8}}} \left[ds^2(AdS_4) + \frac{3}{2} d\alpha^2 + \frac{6 \sin^2 \alpha}{3 + \cos 2\alpha} ds^2(\mathbb{CP}^2) + \frac{9 \sin^2 \alpha}{5 + \cos 2\alpha} \boldsymbol{\eta}^2 \right],$$

$$e^{\hat{\phi}} = e^{\phi_0} \frac{(5 + \cos 2\alpha)^{3/4}}{3 + \cos 2\alpha}, \quad \hat{H}_{(3)} = 24\sqrt{2} L^2 e^{\frac{1}{2}\phi_0} \frac{\sin^3 \alpha}{(3 + \cos 2\alpha)^2} \mathbf{J} \wedge d\alpha,$$

$$L^{-1} e^{\frac{3}{4}\phi_0} \hat{F}_{(2)} = -4\sqrt{6} \frac{\sin^2 \alpha \cos \alpha}{(3 + \cos 2\alpha)(5 + \cos 2\alpha)} \mathbf{J} - 3\sqrt{6} \frac{(3 - \cos 2\alpha)}{(5 + \cos 2\alpha)^2} \sin \alpha d\alpha \wedge \boldsymbol{\eta},$$

$$L^{-3} e^{\frac{1}{4}\phi_0} \hat{F}_{(4)} = 6 \text{vol}_4 + 12\sqrt{3} \frac{7 + 3 \cos 2\alpha}{(3 + \cos 2\alpha)^2} \sin^4 \alpha \text{vol}_{\mathbb{CP}^2} + 18\sqrt{3} \frac{(9 + \cos 2\alpha) \sin^3 \alpha \cos \alpha}{(3 + \cos 2\alpha)(5 + \cos 2\alpha)} \mathbf{J} \wedge d\alpha \wedge \boldsymbol{\eta},$$

where we have introduced the quantities $L^2 \equiv 2^{-\frac{5}{8}} 3^{-1} g^{-2} c^{\frac{1}{12}}$ and $e^{\phi_0} \equiv 2^{\frac{1}{4}} c^{-\frac{5}{6}}$

- ◆ The angle $0 \leq \alpha \leq \pi$ locally foliates S^6 with S^5 regarded as Hopf fibrations over \mathbb{CP}^2

CFT₃ candidate and matching of free energies

[Schwarz '04]

[Gaiotto & Tomasiello '09]

- We propose an N=2 Chern-Simons-matter theory with simple gauge group SU(N), level k and only adjoint matter, as the CFT of the N=2 massive IIA solution.

- The 3d free energy $F = -\text{Log}(Z)$, where Z is the partition function on the CFT on a Euclidean S³ can be computed via localisation over supersymmetric configurations

[Pestun '07] [Jafferis '10]

[Jafferis, Klebanov, Pufu & Safdi '11]

$$Z = \int \prod_{i=1}^N \frac{d\lambda_i}{2\pi} \prod_{i<j=1}^N \left(2 \sinh^2 \left(\frac{\lambda_i - \lambda_j}{2} \right) \right) \times \prod_{i,j=1}^N \left(\exp \left(\ell \left(\frac{1}{3} + \frac{i}{2\pi} (\lambda_i - \lambda_j) \right) \right) \right)^3 e^{\frac{ik}{4\pi} \sum \lambda_i^2}$$

where λ_i are the Coulomb branch parameters. In the $N \gg k$ limit, the result is given by

$$F = \frac{3^{13/6} \pi}{40} \left(\frac{32}{27} \right)^{2/3} k^{1/3} N^{5/3}$$

- The gravitational free energy can be computed from the warp factor in the N=2 massive IIA solution. Using the charge quantisation condition $N = -(2\pi\ell_s)^{-5} \int_{S^6} e^{\frac{1}{2}\phi} \hat{F}_{(4)} + \hat{B}_{(2)} \wedge d\hat{A}_{(3)} + \frac{1}{6} \hat{F}_{(0)} \hat{B}_{(2)}^3$ for the D2-brane, one finds

$$F = \frac{16\pi^3}{(2\pi\ell_s)^8} \int_{S^6} e^{8A} \text{vol}_6 = \frac{\pi}{5} 2^{1/3} 3^{1/6} k^{1/3} N^{5/3} \quad \text{provided}$$

$$g c = \hat{F}_{(0)} = k / (2\pi\ell_s)$$

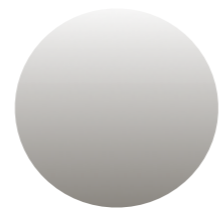
Summary

- 1) We propose a new method to embed 4D theories into 10D that makes extensive use of the 4D duality hierarchy. Using this method, we have connected the electric-magnetic deformed N=8 supergravity with $ISO(7)_c$ gauging to massive IIA reductions on S^6 .
- 2) Any 4D configuration is embedded into 10D via the uplifting formulas. As an example, we have found a new $AdS_4 \times S^6$ solution of massive IIA based on an N=2&U(3) AdS_4 vacua in 4D.
- 3) We propose a CFT_3 dual for the N=2 $AdS_4 \times S^6$ solution of massive IIA based on the D2-brane field theory. In the massive IIA case, there is a CS-term and a superpotential that make the theory flowing to a conformal phase (IR). This translates into the appearance of supersymmetric AdS_4 vacua in the deformed $ISO(7)_c$ supergravity theory.
- 4) For the new N=2 massive IIA solution, the gravitational and FT free energies do match provided

$$g c = \hat{F}_{(0)} = k/(2\pi\ell_s)$$

- 5) Extension to other dimensions? EGG / EFT for massive IIA? Insights into non-geometry?

Thanks a lot !!



Extra material

More AdS₄ critical points

[here] = arXiv:1508.04432

SUSY	bos. sym.	$M^2 L^2$	stability	ref.
$\mathcal{N} = 3$	SO(4)	$3(1 \pm \sqrt{3})^{(1)}$, $(1 \pm \sqrt{3})^{(6)}$, $-\frac{9}{4}^{(4)}$, $-2^{(18)}$, $-\frac{5}{4}^{(12)}$, $0^{(22)}$ $(3 \pm \sqrt{3})^{(3)}$, $\frac{15}{4}^{(4)}$, $\frac{3}{4}^{(12)}$, $0^{(6)}$	yes	[30]
$\mathcal{N} = 2$	U(3)	$(3 \pm \sqrt{17})^{(1)}$, $-\frac{20}{9}^{(12)}$, $-2^{(16)}$, $-\frac{14}{9}^{(18)}$, $2^{(3)}$, $0^{(19)}$ $4^{(1)}$, $\frac{28}{9}^{(6)}$, $\frac{4}{9}^{(12)}$, $0^{(9)}$	yes	[15], [here]
$\mathcal{N} = 1$	G ₂	$(4 \pm \sqrt{6})^{(1)}$, $-\frac{1}{6}(11 \pm \sqrt{6})^{(27)}$, $0^{(14)}$ $\frac{1}{2}(3 \pm \sqrt{6})^{(7)}$, $0^{(14)}$	yes	[4]
$\mathcal{N} = 1$	SU(3)	$(4 \pm \sqrt{6})^{(2)}$, $-\frac{20}{9}^{(12)}$, $-2^{(8)}$, $-\frac{8}{9}^{(12)}$, $\frac{7}{9}^{(6)}$, $0^{(28)}$ $6^{(1)}$, $\frac{28}{9}^{(6)}$, $\frac{25}{9}^{(6)}$, $2^{(1)}$, $\frac{4}{9}^{(6)}$, $0^{(8)}$	yes	[here]
$\mathcal{N} = 0$	SO(7) ₊	$6^{(1)}$, $-\frac{12}{5}^{(27)}$, $-\frac{6}{5}^{(35)}$, $0^{(7)}$ $\frac{12}{5}^{(7)}$, $0^{(21)}$	no	[3]
$\mathcal{N} = 0$	SO(6) ₊	$6^{(2)}$, $-3^{(20)}$, $-\frac{3}{4}^{(20)}$, $0^{(28)}$ $6^{(1)}$, $\frac{9}{4}^{(12)}$, $0^{(15)}$	no	[3]
$\mathcal{N} = 0$	G ₂	$6^{(2)}$, $-1^{(54)}$, $0^{(14)}$ $3^{(14)}$, $0^{(14)}$	yes	[4]
$\mathcal{N} = 0$	SU(3)	see (3.44) see (3.45)	yes	[here]
$\mathcal{N} = 0$	SU(3)	see (3.46) see (3.47)	yes	[here]
$\mathcal{N} = 0$	SO(4)	see (5.12) see (5.13)	yes	[here]

SUSY transformations of the tensor hierarchy

[vielbein and scalars]

$$\begin{aligned}\delta e_\mu^\alpha &= \frac{1}{4} \bar{\epsilon}_i \gamma^\alpha \psi_\mu^i + \frac{1}{4} \bar{\epsilon}^i \gamma^\alpha \psi_{\mu i} \\ \delta \mathcal{V}_M^{ij} &= \frac{1}{\sqrt{2}} \mathcal{V}_M^{kl} \left(\bar{\epsilon}^{[i} \chi^{jkl]} + \frac{1}{4!} \varepsilon^{ijklmnpq} \bar{\epsilon}_m \chi_{npq} \right)\end{aligned}$$

[vectors]

$$\begin{aligned}\delta \mathcal{A}_\mu^{IJ} &= i \mathcal{V}^{IJ}{}_{ij} \left(\bar{\epsilon}^i \psi_\mu^j + \frac{1}{2\sqrt{2}} \bar{\epsilon}_k \gamma_\mu \chi^{ijk} \right) + \text{h.c.} \\ \delta \mathcal{A}_\mu^I &= i \mathcal{V}^{I8}{}_{ij} \left(\bar{\epsilon}^i \psi_\mu^j + \frac{1}{2\sqrt{2}} \bar{\epsilon}_k \gamma_\mu \chi^{ijk} \right) + \text{h.c.} \\ \delta \tilde{\mathcal{A}}_{\mu IJ} &= -i \tilde{\mathcal{V}}_{IJ}{}_{ij} \left(\bar{\epsilon}^i \psi_\mu^j + \frac{1}{2\sqrt{2}} \bar{\epsilon}_k \gamma_\mu \chi^{ijk} \right) + \text{h.c.} \\ \delta \tilde{\mathcal{A}}_{\mu I} &= -i \tilde{\mathcal{V}}_{I8}{}_{ij} \left(\bar{\epsilon}^i \psi_\mu^j + \frac{1}{2\sqrt{2}} \bar{\epsilon}_k \gamma_\mu \chi^{ijk} \right) + \text{h.c.}\end{aligned}$$

[two-forms]

$$\begin{aligned}\delta \mathcal{B}_{\mu\nu J}^I &= \left[-\frac{2}{3} (\mathcal{V}^{IK}{}_{jk} \tilde{\mathcal{V}}_{JK}{}^{ik} + \mathcal{V}^{I8}{}_{jk} \tilde{\mathcal{V}}_{J8}{}^{ik} + \tilde{\mathcal{V}}_{JK}{}_{jk} \mathcal{V}^{IK}{}^{ik} + \tilde{\mathcal{V}}_{J8}{}_{jk} \mathcal{V}^{I8}{}^{ik}) \bar{\epsilon}_i \gamma_{[\mu} \psi_{\nu]}^j \right. \\ &\quad \left. - \frac{\sqrt{2}}{3} (\mathcal{V}^{IK}{}_{ij} \tilde{\mathcal{V}}_{JK}{}^{kl} + \mathcal{V}^{I8}{}_{ij} \tilde{\mathcal{V}}_{J8}{}^{kl}) \bar{\epsilon}^{[i} \gamma_{\mu\nu} \chi^{jkl]} + \text{h.c.} \right] \\ &\quad + (\mathcal{A}_{[\mu}^{IK} \delta \tilde{\mathcal{A}}_{\nu]JK} + \mathcal{A}_{[\mu}^I \delta \tilde{\mathcal{A}}_{\nu]J} + \tilde{\mathcal{A}}_{[\mu|JK} \delta \mathcal{A}_{|\nu]}^{IK} + \tilde{\mathcal{A}}_{[\mu|J} \delta \mathcal{A}_{|\nu]}^I) - \frac{1}{7} \delta_J^I (\text{trace}), \\ \delta \mathcal{B}_{\mu\nu}^I &= \left[\frac{2}{3} (\mathcal{V}^{IJ}{}_{jk} \tilde{\mathcal{V}}_{J8}{}^{ik} + \tilde{\mathcal{V}}_{J8}{}_{jk} \mathcal{V}^{IJ}{}^{ik}) \bar{\epsilon}_i \gamma_{[\mu} \psi_{\nu]}^j + \frac{\sqrt{2}}{3} \mathcal{V}^{IJ}{}_{ij} \tilde{\mathcal{V}}_{J8}{}^{kl} \bar{\epsilon}^{[i} \gamma_{\mu\nu} \chi^{jkl]} + \text{h.c.} \right] \\ &\quad - (\mathcal{A}_{[\mu}^{IJ} \delta \tilde{\mathcal{A}}_{\nu]J} + \tilde{\mathcal{A}}_{[\mu|J} \delta \mathcal{A}_{|\nu]}^{IJ}).\end{aligned}$$

[three-forms]

$$\begin{aligned}
\delta\mathcal{C}_{\mu\nu\rho}{}^{IJ} = & \left[-\frac{4i}{7} \left(\mathcal{V}^{K(I}{}_{jl}(\mathcal{V}^{J)Llk} \tilde{\mathcal{V}}_{KLik} + \tilde{\mathcal{V}}_{KL}{}^{lk} \mathcal{V}^{J)L}{}_{ik} \right) \right. \\
& + \mathcal{V}^{K(I}{}_{jl}(\mathcal{V}^{J)8lk} \tilde{\mathcal{V}}_{K8ik} + \tilde{\mathcal{V}}_{K8}{}^{lk} \mathcal{V}^{J)8}{}_{ik} \\
& + \mathcal{V}^{(I|8}{}_{jl}(\mathcal{V}^{|J)Klk} \tilde{\mathcal{V}}_{K8ik} + \tilde{\mathcal{V}}_{K8}{}^{lk} \mathcal{V}^{|J)K}{}_{ik} \left. \right) \bar{\epsilon}^i \gamma_{[\mu\nu} \psi_{\rho]}^j \\
& + i\frac{\sqrt{2}}{3} \left(\mathcal{V}^{K(I|hi} \mathcal{V}^{|J)L}{}_{[ij]} \tilde{\mathcal{V}}_{KL|kl} + \mathcal{V}^{K(Ihi} \mathcal{V}^{J)8}{}_{[ij]} \tilde{\mathcal{V}}_{K8|kl} \right. \\
& \left. + \mathcal{V}^{(I|8hi} \mathcal{V}^{|J)K}{}_{[ij]} \tilde{\mathcal{V}}_{K8|kl} \right) \bar{\epsilon}_h \gamma_{\mu\nu\rho} \chi^{jkl} + \text{h.c.} \left. \right] \\
& - 3 \left(\mathcal{B}_{[\mu\nu|K}{}^{(I} \delta\mathcal{A}_{|\rho]}^{J)K} + \mathcal{B}_{[\mu\nu}{}^{(I} \delta\mathcal{A}_{\rho]}^{J)} \right) \\
& + \mathcal{A}_{[\mu}^{K(I} (\mathcal{A}_{\nu}^{J)L} \delta\tilde{\mathcal{A}}_{\rho]KL} + \tilde{\mathcal{A}}_{\nu KL} \delta\mathcal{A}_{\rho]}^{J)L}) + \mathcal{A}_{[\mu}^{K(I} (\mathcal{A}_{\nu}^{J)} \delta\tilde{\mathcal{A}}_{\rho]K} + \tilde{\mathcal{A}}_{\nu K} \delta\mathcal{A}_{\rho]}^{J)}) \\
& + \mathcal{A}_{[\mu}^{(I} (\mathcal{A}_{\nu}^{J)K} \delta\tilde{\mathcal{A}}_{\rho]K} + \tilde{\mathcal{A}}_{\nu K} \delta\mathcal{A}_{\rho]}^{J)K}) .
\end{aligned}$$

... all scalars, vectors and the fermions should be kept !!

Scalar potential and three-form potentials

- How does the scalar potential potential V fit in the duality hierarchy ?

$$\Theta_{\mathbb{M}}^{\alpha} \mathcal{H}_{(4)\alpha}^{\mathbb{M}} = -2V \text{vol}_4$$

- In our deformed $\text{ISO}(7)_c$ theory, one has four-form field strengths

$$g \delta_{IJ} \mathcal{H}_{(4)}^{IJ} + m \tilde{\mathcal{H}}_{(4)} = -2V \text{vol}_4 \quad [\mathbf{28}' + \mathbf{1} \text{ of } \text{SL}(7)]$$

where we need the **SL(7)-singlet** four-form field strength $\tilde{\mathcal{H}}_{(4)}$ dual to the magnetic ET

- Consistency requires also the three-form field strength $\mathcal{H}_{(3)}$ rendering $\mathcal{H}_{(3)I}{}^J$ traceful

$$\mathcal{H}_{(3)} = \frac{1}{12} (t_8^8)_{\mathbb{M}}^{\mathbb{P}} \mathcal{M}_{\mathbb{NP}} * D\mathcal{M}^{\mathbb{MN}},$$

$$\tilde{\mathcal{H}}_{(4)} = \frac{1}{84} X_{\mathbb{NQ}}^{\mathbb{S}} (t_8^K)_{\mathbb{P}}^{\mathbb{R}} \mathcal{M}_{8K}^{\mathbb{N}} \mathcal{M}^{\mathbb{PQ}} \mathcal{M}_{\mathbb{RS}} \text{vol}_4$$

* Extended BI's :

$$D\mathcal{H}_{(3)} = \mathcal{H}_{(2)}^{IJ} \wedge \tilde{\mathcal{H}}_{(2)IJ} + \mathcal{H}_{(2)}^I \wedge \tilde{\mathcal{H}}_{(2)I} - 2g \delta_{IJ} \mathcal{H}_{(4)}^{IJ} - 14m \tilde{\mathcal{H}}_{(4)}$$

$$D\tilde{\mathcal{H}}_{(4)} \equiv 0,$$

Generalised vielbein

$$V^{I8}{}_{AB} = (V^{m8}{}_{AB}, V^{78}{}_{AB}) \quad , \quad \tilde{V}_{IJ}{}_{AB} = (\tilde{V}_{mn}{}_{AB}, \tilde{V}_{m7}{}_{AB}) :$$

with components

$$\begin{aligned} V^{m8}{}_{AB} &= -\frac{1}{4} \Delta^{-\frac{1}{2}} e_a{}^m (\Gamma^a C^{-1})^{AB} , \\ V^{78}{}_{AB} &= -\frac{1}{4} e^{-\frac{3}{4}\hat{\phi}} \Delta^{-\frac{1}{2}} (\Gamma_7 C^{-1})^{AB} - V^{m8}{}_{AB} A_m , \\ \tilde{V}_{m7}{}^{AB} &= \frac{1}{4} e^{\frac{1}{2}\hat{\phi}} \Delta^{-\frac{1}{2}} e_m{}^a (\Gamma_a \Gamma_7 C^{-1})^{AB} + V^{n8}{}_{AB} B_{nm} , \\ \tilde{V}_{mn}{}^{AB} &= \frac{1}{4} e^{-\frac{1}{4}\hat{\phi}} \Delta^{-\frac{1}{2}} e_m{}^a e_n{}^b (\Gamma_{ab} C^{-1})^{AB} + V^{p8}{}_{AB} (A_{pmn} - 2B_{p[m} A_{n]}) \\ &\quad + V^{78}{}_{AB} B_{mn} + 2\tilde{V}_{[m|7}{}^{AB} A_{|n]} \end{aligned}$$

and

$$\begin{aligned} V^{m8}{}_{AB} &= \frac{1}{4} \Delta^{-\frac{1}{2}} e_a{}^m (C\Gamma^a)_{AB} , \\ V^{78}{}_{AB} &= \frac{1}{4} e^{-\frac{3}{4}\hat{\phi}} \Delta^{-\frac{1}{2}} (C\Gamma_7)_{AB} - V^{m8}{}_{AB} A_m , \\ \tilde{V}_{m7}{}_{AB} &= \frac{1}{4} e^{\frac{1}{2}\hat{\phi}} \Delta^{-\frac{1}{2}} e_m{}^a (C\Gamma_a \Gamma_7)_{AB} + V^{n8}{}_{AB} B_{nm} , \\ \tilde{V}_{mn}{}_{AB} &= \frac{1}{4} e^{-\frac{1}{4}\hat{\phi}} \Delta^{-\frac{1}{2}} e_m{}^a e_n{}^b (C\Gamma_{ab})_{AB} + V^{p8}{}_{AB} (A_{pmn} - 2B_{p[m} A_{n]}) \\ &\quad + V^{78}{}_{AB} B_{mn} + 2\tilde{V}_{[m|7}{}_{AB} A_{|n]} \end{aligned}$$

“dressing up” process

[vielbein and scalars]

$$ds_4^2(x, y) = ds_4^2(x)$$

$$V^{m8 AB}(x, y) = \frac{1}{2} g K_{IJ}^m(y) \eta_i^A(y) \eta_j^B(y) \mathcal{V}^{IJ ij}(x) ,$$

$$V^{78 AB}(x, y) = -\mu_I(y) \eta_i^A(y) \eta_j^B(y) \mathcal{V}^{I8 ij}(x) ,$$

$$\tilde{V}_{mn}^{AB}(x, y) = \frac{1}{4} K_{mn}^{IJ}(y) \eta_i^A(y) \eta_j^B(y) \tilde{\mathcal{V}}_{IJ}^{ij}(x) ,$$

$$\tilde{V}_{m7}^{AB}(x, y) = -g^{-1} (\partial_m \mu^I)(y) \eta_i^A(y) \eta_j^B(y) \tilde{\mathcal{V}}_{I8}^{ij}(x) ,$$

[two-forms]

$$C_{\mu\nu m}{}^8(x, y) = -g^{-1} (\mu_I \partial_m \mu^J)(y) \mathcal{B}_{\mu\nu J}{}^I(x)$$

$$C_{\mu\nu 7}{}^8(x, y) = \mu_I(y) \mathcal{B}_{\mu\nu}{}^I(x)$$

[three-form]

$$C_{\mu\nu\rho}{}^{88}(x, y) = (\mu_I \mu_J)(y) \mathcal{C}_{\mu\nu\rho}{}^{IJ}(x)$$

[vectors]

$$C_\mu{}^{m8}(x, y) = \frac{1}{2} g K_{IJ}^m(y) \mathcal{A}_\mu{}^{IJ}(x) \quad , \quad C_\mu{}^{78}(x, y) = -\mu_I(y) \mathcal{A}_\mu{}^I(x) ,$$

$$\tilde{C}_{\mu mn}(x, y) = \frac{1}{4} K_{mn}^{IJ}(y) \tilde{\mathcal{A}}_{\mu IJ}(x) \quad , \quad \tilde{C}_{\mu m7}(x, y) = -g^{-1} (\partial_m \mu^I)(y) \tilde{\mathcal{A}}_{\mu I}(x)$$

◆ S^6 geometrical data : embedding coordinates μ^I , Killing vectors K_{IJ}^m and tensors K_{IJ}^{mn}

10D SUSY transformations

$$\delta C_\mu^{I8} = i V^{I8}{}_{AB} \left(\bar{\epsilon}^A \psi_\mu^B + \frac{1}{2\sqrt{2}} \bar{\epsilon}_C \gamma_\mu \chi^{ABC} \right) + \text{h.c.} ,$$

$$\delta \tilde{C}_{\mu IJ} = -i V_{IJ}{}_{AB} \left(\bar{\epsilon}^A \psi_\mu^B + \frac{1}{2\sqrt{2}} \bar{\epsilon}_C \gamma_\mu \chi^{ABC} \right) + \text{h.c.} ,$$

$$\begin{aligned} \delta C_{\mu\nu I}{}^8 &= \left[\frac{2}{3} (V^{J8}{}_{BC} \tilde{V}_{IJ}{}^{AC} + \tilde{V}_{IJ}{}_{BC} V^{J8AC}) \bar{\epsilon}_A \gamma_{[\mu} \psi_{\nu]}^B \right. \\ &\quad \left. + \frac{\sqrt{2}}{3} V^{J8}{}_{AB} \tilde{V}_{IJCD} \bar{\epsilon}^{[A} \gamma_{\mu\nu} \chi^{BCD]} + \text{h.c.} \right] - C_{[\mu}{}^{J8} \delta \tilde{C}_{\nu]IJ} - \tilde{C}_{[\mu|IJ} \delta C_{|\nu]}{}^{J8} , \end{aligned}$$

$$\begin{aligned} \delta C_{\mu\nu\rho}{}^{88} &= \left[\frac{4i}{7} V^{I8}{}_{BD} (V^{J8DC} \tilde{V}_{IJAC} + \tilde{V}_{IJ}{}^{DC} V^{J8AC}) \bar{\epsilon}^A \gamma_{[\mu\nu} \psi_{\rho]}^B \right. \\ &\quad \left. - i \frac{\sqrt{2}}{3} V^{I8AE} V^{J8}{}_{[EB|} \tilde{V}_{IJ|CD]} \bar{\epsilon}_A \gamma_{\mu\nu\rho} \chi^{BCD} + \text{h.c.} \right] \\ &\quad + 3 C_{[\mu\nu|I}{}^8 \delta C_{|\rho]}{}^{I8} - C_{[\mu}{}^{I8} (C_{\nu}{}^{J8} \delta \tilde{C}_{\rho]IJ} + \tilde{C}_{\nu|IJ} \delta C_{|\rho]}{}^{J8}) . \end{aligned}$$

Freund-Rubin term

[Freund & Rubin '80]

- By looking at the RR field strength $\hat{F}_{(4)} = \mathcal{H}_{(4)}^{IJ} \mu_I \mu_J + \dots$, one immediately identifies the Freund-Rubin term

$$\begin{aligned} \mathcal{H}_{(4)}^{IJ} \mu_I \mu_J = & -\frac{1}{3} g^{-1} V \text{vol}_4 + \frac{1}{84} g^{-1} \left(D\mathcal{H}_{(3)} - 7 \mathcal{H}_{(2)}^{IJ} \wedge \tilde{\mathcal{H}}_{(2)IJ} - 7 \mathcal{H}_{(2)}^I \wedge \tilde{\mathcal{H}}_{(2)I} \right) \\ & - \frac{1}{2} g^{-1} \left(D\mathcal{H}_{(3)I}{}^J - \mathcal{H}_{(2)}^{JK} \wedge \tilde{\mathcal{H}}_{(2)IK} - \mathcal{H}_{(2)}^J \wedge \tilde{\mathcal{H}}_{(2)I} \right) \mu^I \mu_J , \end{aligned}$$

NOTE: We have expressed the EOMs for the scalars as BI for the three-form field strengths of the tensor hierarchy.

- At a critical point of V one has $\hat{F}_{(4)} = -\frac{1}{3g} V \text{vol}_4 + \dots$, and the S^6 dependence drops out

[see also Godazgar, Godazgar, Krueger & Nicolai '15]