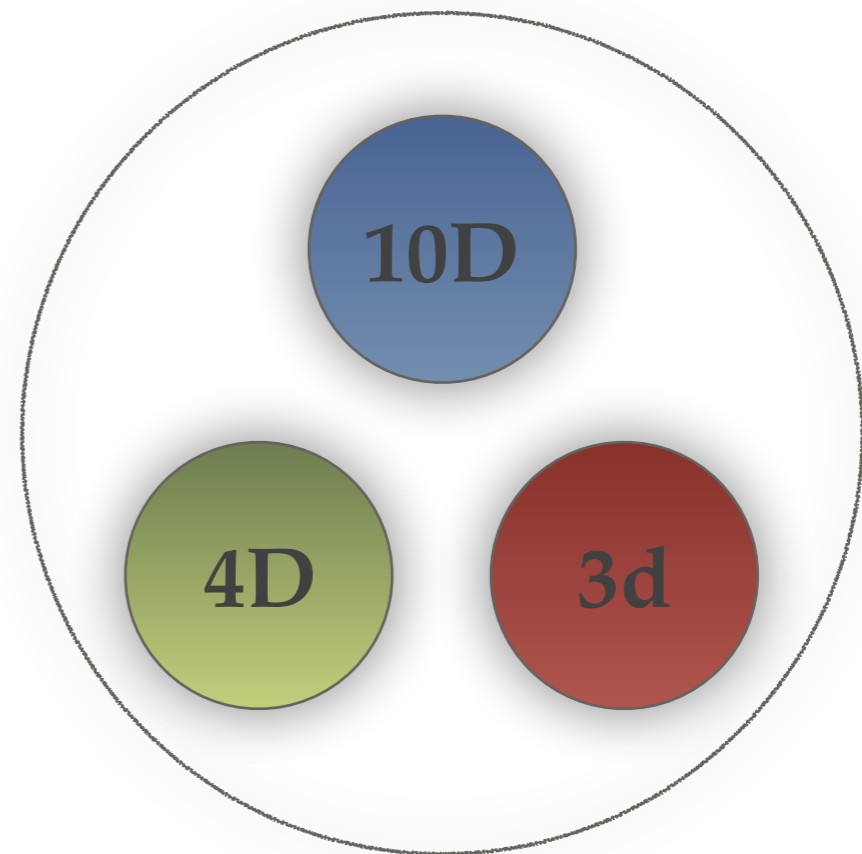


# Holographic RG flows from massive IIA on $S^6$

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August 8th , C.E.R.N.



With D. Jafferis , J. Tarrío and O. Varela :

[arXiv:1504.08009](https://arxiv.org/abs/1504.08009) , [arXiv:1508.04432](https://arxiv.org/abs/1508.04432) , [arXiv:1509.02526](https://arxiv.org/abs/1509.02526)

[arXiv:1605.09254](https://arxiv.org/abs/1605.09254) , [arXiv:1703.10833](https://arxiv.org/abs/1703.10833) , [arXiv:1706.01823](https://arxiv.org/abs/1706.01823)



# Outlook



Motivation



Deformed  $SO(8)$ -gauged supergravity



Deformed  $ISO(7)$ -gauged supergravity



Massive IIA on  $S^6$  / SYM-CS duality



Holographic RG flows: domain-walls and black holes



Motivation

# electric-magnetic deformations

- The uniqueness of the maximal (N=8) supergravities is historically inherited from their connection to sphere reductions [ Cvetič, Lu, Pope '00 ]

$$\text{AdS}_5 \times S^5 \text{ (D3-brane)} \quad , \quad \text{AdS}_4 \times S^7 \text{ (M2-brane)} \quad , \quad \text{AdS}_7 \times S^4 \text{ (M5-brane)}$$

- N=8 supergravity in 4D admits a **deformation parameter**  $c$  yielding **inequivalent** theories. It is an **electric/magnetic** deformation

$$D = \partial - g (A^{\text{elec}} - c \tilde{A}_{\text{mag}})$$

$g$  = 4D gauge coupling  
 $c$  = **deformation param.**

- There are two generic situations :
  - 1) Family of  $\text{SO}(8)_c$  theories :  $c = [0, \sqrt{2} - 1]$  is a continuous param [ similar for  $\text{SO}(p,q)_c$  ]
  - 2) Family of  $\text{ISO}(7)_c$  theories :  $c = 0 \text{ or } 1$  is an (on/off) param [ same for  $\text{ISO}(p,q)_c$  ]

[ Dall'Agata, Inverso, Marrani '14 ]  
 [ Dall'Agata, Inverso, Trigiante '12 ]



The questions arise:

- Does such an electric / magnetic deformation of 4D maximal supergravity enjoy a string / M-theory origin, or is it just a 4D feature ?

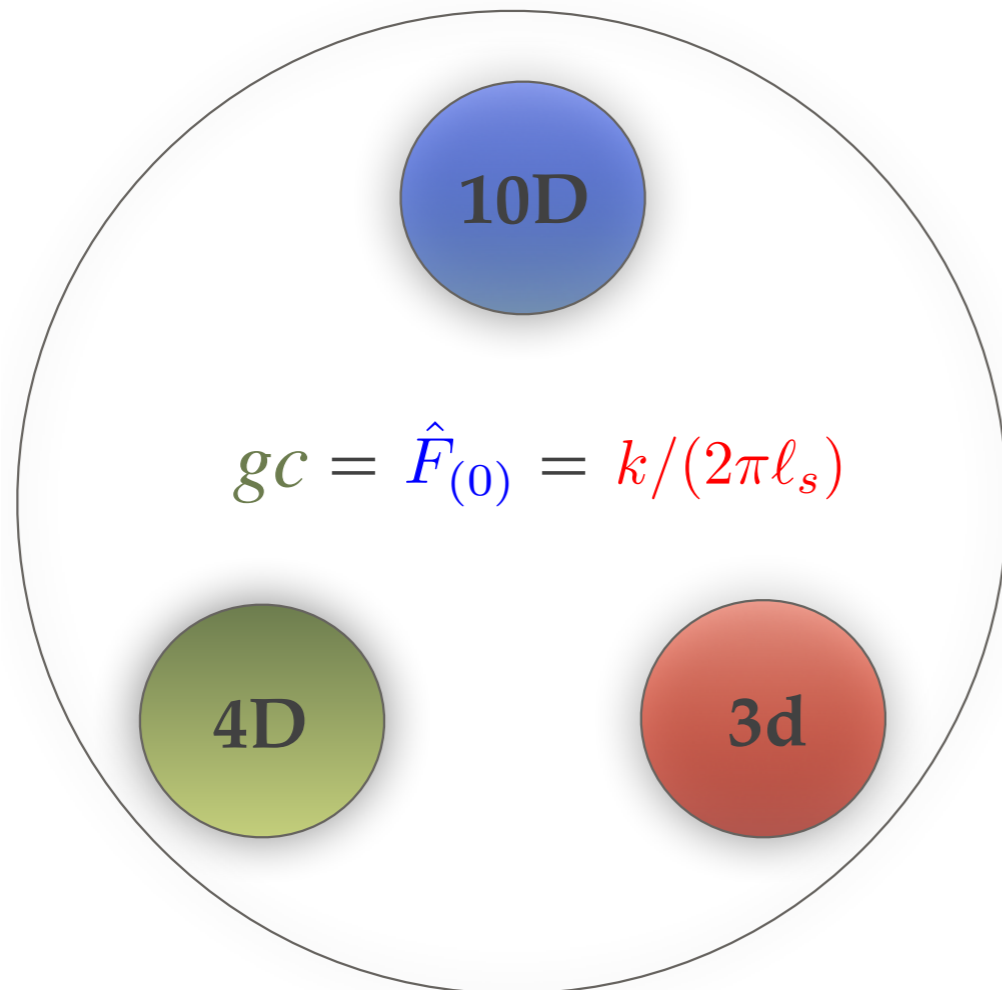
Obstruction for  $SO(8)_c$  , *cf.* [ Lee, Strickland-Constable, Waldram '15 ]

[ de Wit, Nicolai '13 ]

- For deformed 4D supergravities with supersymmetric  $AdS_4$  vacua, are these  $AdS_4$  /  $CFT_3$ -dual to any identifiable 3d CFT ?

# A new 10D/4D/3d correspondence

*massive IIA on  $S^6$*  «  $ISO(7)_c$ -gauged sugra »  $SU(N)_k$  CS-SYM theory



$gc$  = elec/mag deformation in 4D

$\hat{F}_{(0)}$  = Romans mass in 10D

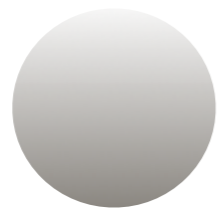
$k$  = Chern-Simons level in 3d

[ AG, Jafferis, Varela '15 ]

[ AG, Varela '15 ]

Well-established and independent dualities :

Type IIB on  $S^5$  /  $N=4$  SYM — M-theory on  $S^7$  / ABJM — mIIA on  $S^6$  / SYM-CS



# Deformed $SO(8)$ -gauged supergravity

# $N = 8$ supergravities in 4D

- SUGRA :    metric + 8 gravitini + 28 vectors + 56 dilatini + 70 scalars  
                  (s = 2)            (s = 3/2)            (s = 1)            (s = 1/2)            (s = 0)

Ungauged (abelian) supergravity: Reduction of M-theory on a *torus*  $T^7$   
down to 4D produces  $N = 8$  supergravity with  $G = U(1)^{28}$  [ Cremmer, Julia '79 ]

Gauged (non-abelian) supergravity: Reduction of M-theory on a *sphere*  $S^7$   
down to 4D produces  $N = 8$  supergravity with  $G = SO(8)$  [ de Wit, Nicolai '82 ]

\*  $SO(8)$ -gauged supergravity believed to be **unique** for 30 years...

... but ... is this true?

# Framework to study $N = 8$ supergravities in 4D

[ de Wit, Samtleben, Trigiante '03 , '07 ]

Gauging procedure : Part of the **global  $E_7$  symmetry** group is promoted to a local symmetry group  $G$  (gauging)

[  $\alpha = 1, \dots, 133$  ]

**Embedding tensor** : It is a “selector” specifying which **generators of  $E_7$**  (there are 133!!) become the gauge symmetry  $G$  and, therefore, have associated gauge fields.

Formulation in terms of **56** vectors  $A_\mu^M$ , though...  $M = 1, \dots, 56 = 28$  (elec) + **28** (mag)

*Sp(56) Elec/Mag group*

$$A_\mu = A_\mu^M \Theta_M^\alpha t_\alpha$$

*Redundancy!!*

$$X_M = \Theta_M^\alpha t_\alpha \quad \Rightarrow \quad [X_M, X_N] = X_{MN}^P X_P \quad \text{with} \quad X_{MN}^P = \Theta_M^\alpha [t_\alpha]_N^P$$

\* Closure of the gauge algebra :  $\Omega^{MN} \Theta_M^\alpha \Theta_N^\beta = 0$

*Only 28 physical l.c. of vectors!!*

# A family of $G = \text{SO}(8)$ supergravities in 4D

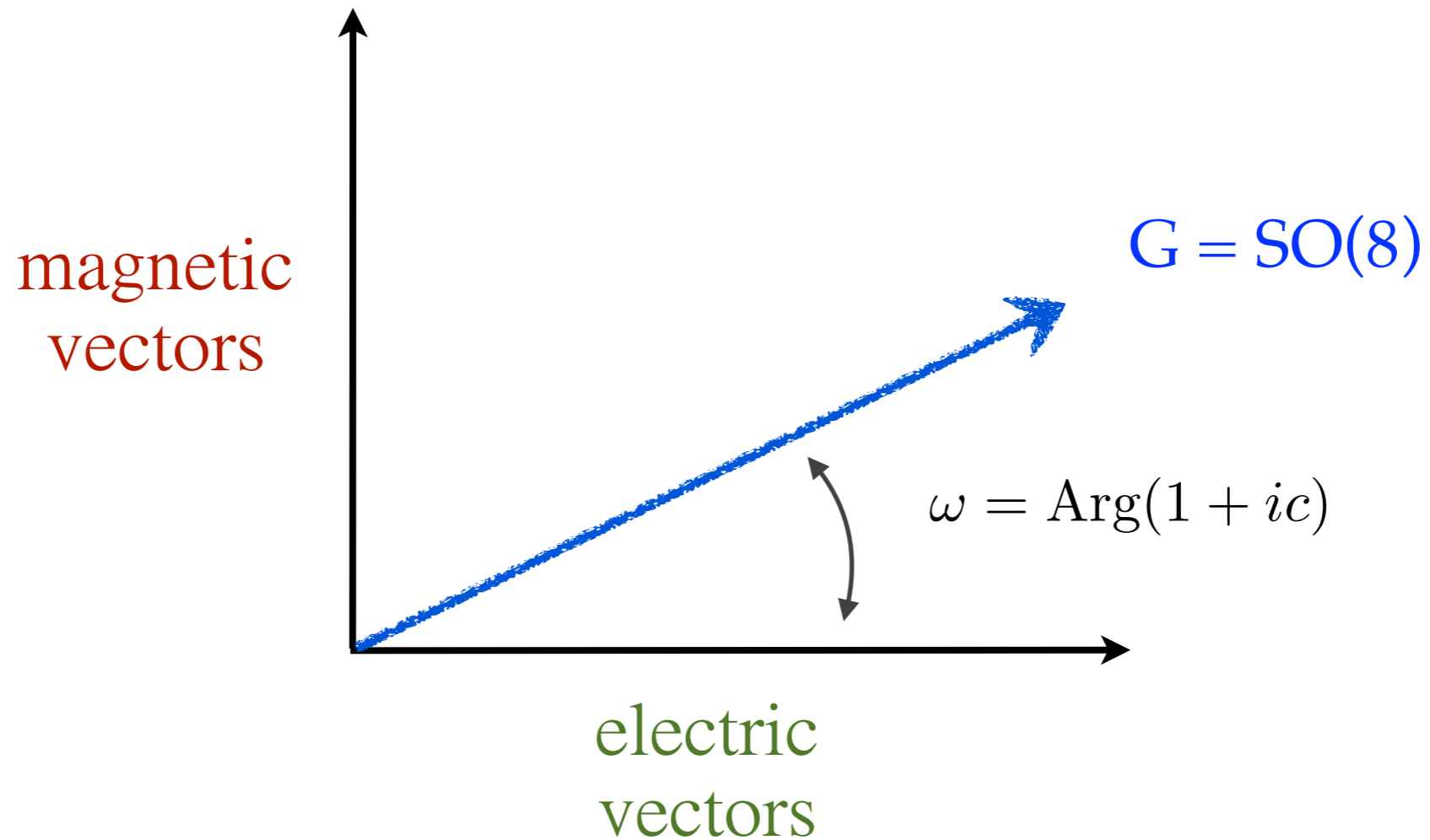
- Choose  $G = \text{SO}(8)$
- Solve  $\Omega^{MN} \Theta_M^\alpha \Theta_N^\beta = 0 \Rightarrow$  One-parameter ( $c$ ) family of  $\text{SO}(8)_c$  theories !!

[ Dall' Agata, Inverso, Trigiante '12 ]

- Immediate questions :

- |  |                              |
|--|------------------------------|
| 1) What?                                   | (Yes, surprising but true)   |
| 2) Are these $c$ -theories equivalent?     | (No)                         |
| 3) Are there new $\text{AdS}_4$ solutions? | (Yes)                        |
| 4) Higher-dimensional origin?              | (Good question... )          |
| 5) $\text{AdS}_4/\text{CFT}_3$ dual?       | (Good question too... ABJ? ) |

Physical meaning in 4D : electric/magnetic deformation



$$D = \partial - g (A^{\text{elec}} - c \tilde{A}_{\text{mag}})$$

Physical meaning in 11D ...





Holographic  $\text{AdS}_4/\text{CFT}_3$  meaning ...



In this talk we are going to investigate the electric/magnetic deformation of a different N=8 supergravity closely related to the  $G = SO(8)$  theory ...

... the  $G = ISO(7) = SO(7) \ltimes \mathbb{R}^7$  supergravity !!

electric / magnetic  
deformation

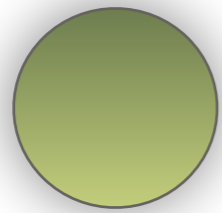


higher-dimensional  
origin



Holographic  
AdS<sub>4</sub>/CFT<sub>3</sub> dual ?





## Deformed ISO(7)-gauged supergravity

# A family of $G = \text{ISO}(7)$ supergravities in 4D

- Choose  $G = \text{ISO}(7)$
- Solve  $\Omega^{MN} \Theta_M^\alpha \Theta_N^\beta = 0 \Rightarrow$  One-parameter ( $c$ ) family of  $\text{ISO}(7)_c$  theories !!

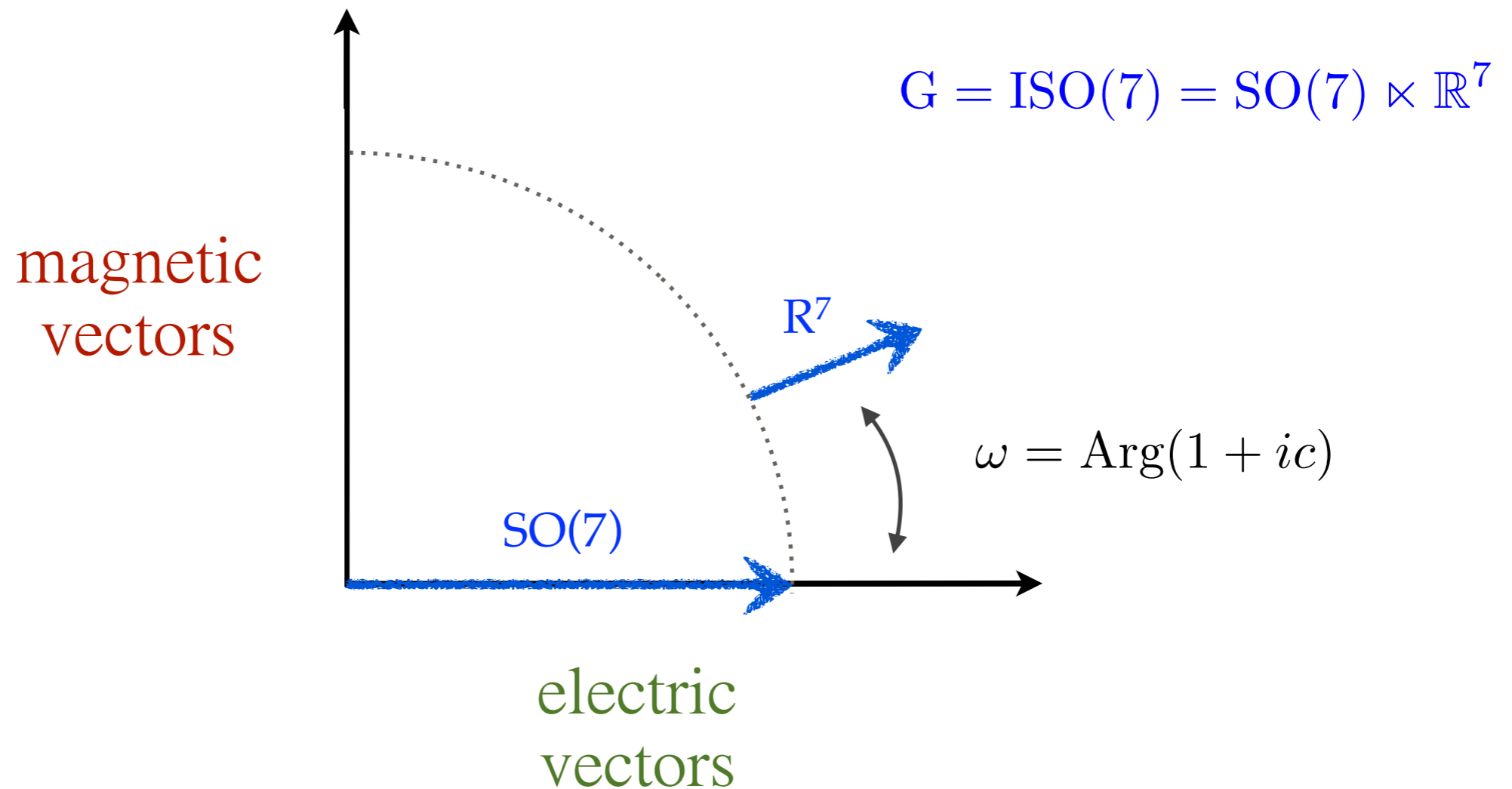
[ Hull '84 (electric) ]

[ Dall'Agata, Inverso, Marrani '14 ]

- Immediate questions :

- |  |                             |
|--|-----------------------------|
| 1) What?                                   | (Yes, and still surprising) |
| 2) Are these $c$ -theories equivalent?     | (No)                        |
| 3) Are there new $\text{AdS}_4$ solutions? | (Yes)                       |
| 4) Higher-dimensional origin?              | (Yes)                       |
| 5) $\text{AdS}_4/\text{CFT}_3$ dual?       | (Yes)                       |

# Physical meaning in 4D = electric/magnetic deformation



$$D = \partial - g A_{\text{SO}(7)}^{\text{elec}} - g (A_{\mathbb{R}^7}^{\text{elec}} - c \tilde{A}_{\mathbb{R}^7 \text{ mag}})$$

# Deformed ISO(7)<sub>c</sub> Lagrangian ( $m = g\mathbf{c}$ )

$$\begin{aligned} M &= 1, \dots, 56 \\ \Lambda &= 1, \dots, 28 \\ I &= 1, \dots, 7 \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{bos}} &= (R - V) \text{vol}_4 - \frac{1}{48} D\mathcal{M}_{\text{MIN}} \wedge *D\mathcal{M}^{\text{MIN}} + \frac{1}{2} \mathcal{I}_{\Lambda\Sigma} \mathcal{H}_{(2)}^\Lambda \wedge *\mathcal{H}_{(2)}^\Sigma - \frac{1}{2} \mathcal{R}_{\Lambda\Sigma} \mathcal{H}_{(2)}^\Lambda \wedge \mathcal{H}_{(2)}^\Sigma \\ &+ m \left[ \mathcal{B}^I \wedge (\tilde{\mathcal{H}}_{(2)I} - \frac{g}{2} \delta_{IJ} \mathcal{B}^J) - \frac{1}{4} \tilde{\mathcal{A}}_I \wedge \tilde{\mathcal{A}}_J \wedge (d\mathcal{A}^{IJ} + \frac{g}{2} \delta_{KL} \mathcal{A}^{IK} \wedge \mathcal{A}^{JL}) \right] \end{aligned}$$

◆ Setting  $m = 0$ , all the magnetic pieces in the Lagrangian disappear.

## \* Ingredients :

- Electric vectors (21 + 7):  $\mathcal{A}^{IJ} = \mathcal{A}^{[IJ]}$  [SO(7)] and  $\mathcal{A}^I$  [R<sup>7</sup>] with  $\mathcal{H}_{(2)}^\Lambda = (\mathcal{H}_{(2)}^{IJ}, \mathcal{H}_{(2)}^I)$
- Auxiliary magnetic vectors (7):  $\tilde{\mathcal{A}}_I$  [R<sup>7</sup>] with  $\tilde{\mathcal{H}}_{(2)I}$  field strength
- E<sub>7</sub>/SU(8) scalars :  $\mathcal{M}_{\text{MIN}}$
- Auxiliary two-forms (7):  $\mathcal{B}^I$  [R<sup>7</sup>]
- Topological term :  $m$  [ ... ]
- Scalar potential :  $V(\mathcal{M}) = \frac{g^2}{672} X_{\text{MN}}^{\text{R}} X_{\text{PQ}}^{\text{S}} \mathcal{M}^{\text{MP}} (\mathcal{M}^{\text{NQ}} \mathcal{M}_{\text{RS}} + 7 \delta_{\text{R}}^{\text{Q}} \delta_{\text{S}}^{\text{N}})$

# A truncation : $G_0 = \text{SU}(3)$ invariant subsector

[ Warner '83 ]

- **Truncation** : Retaining the fields and couplings which are invariant (singlets) under the action of a subgroup  $G_0 \subset \text{ISO}(7)$ 
  - **SU(8) R-symmetry branching** : **gravitini**  $\mathbf{8} \rightarrow \mathbf{1} + \mathbf{1} + \mathbf{3} + \bar{\mathbf{3}} \Rightarrow \text{N} = 2 \text{ SUSY}$
  - **Scalars fields** :  $\mathbf{70} \rightarrow \mathbf{1} (\times 6) + \text{non-singlets} \Rightarrow 6 \text{ real scalars } (\varphi, \chi; \phi, a, \zeta, \tilde{\zeta})$
  - **Vector fields** :  $\mathbf{56} \rightarrow \mathbf{1} (\times 4) + \text{non-singlets} \Rightarrow \text{vectors } (A^0, A^1; \tilde{A}_0, \tilde{A}_1)$
- **N = 2 gauged supergravity with  $G = \text{U}(1) \times \mathbb{R}_c$  coupled to 1 vector & 1 hypermultiplet**

$$\mathcal{M}_{\text{scalar}} = \frac{\text{SU}(1, 1)}{\text{U}(1)} \times \frac{\text{SU}(2, 1)}{\text{U}(2)}$$

# The truncated Lagrangian

- The Lagrangian contains a **non-dynamical tensor field**  $B^0$  :

$$\Lambda = 0, 1$$

$$\begin{aligned} \mathcal{L} = & (R - V) \text{vol}_4 + \frac{3}{2} [d\varphi \wedge *d\varphi + e^{2\varphi} d\chi \wedge *d\chi] \\ & + 2 d\phi \wedge *d\phi + \frac{1}{2} e^{2\phi} [D\zeta \wedge *D\zeta + D\tilde{\zeta} \wedge *D\tilde{\zeta}] \\ & + \frac{1}{2} e^{4\phi} [Da + \frac{1}{2}(\zeta D\tilde{\zeta} - \tilde{\zeta} D\zeta)] \wedge *[Da + \frac{1}{2}(\zeta D\tilde{\zeta} - \tilde{\zeta} D\zeta)] \\ & + \frac{1}{2} \mathcal{I}_{\Lambda\Sigma} H_{(2)}^\Lambda \wedge *H_{(2)}^\Sigma - \frac{1}{2} \mathcal{R}_{\Lambda\Sigma} H_{(2)}^\Lambda \wedge H_{(2)}^\Sigma + m B^0 \wedge d\tilde{A}_0 + \frac{1}{2} g m B^0 \wedge B^0 \end{aligned}$$

with field strengths  $H_{(2)}^1 = dA^1$  and  $H_{(2)}^0 = dA^0 + m B^0$  .

- Covariant derivatives :

$$Da = da + g A^0 - m \tilde{A}_0 \quad , \quad D\zeta = d\zeta - 3g A^1 \tilde{\zeta} \quad , \quad D\tilde{\zeta} = d\tilde{\zeta} + 3g A^1 \zeta$$

- Scalar potential : 
$$\begin{aligned} V = & \frac{1}{2} g^2 [e^{4\phi-3\varphi} (1 + e^{2\varphi} \chi^2)^3 - 12 e^{2\phi-\varphi} (1 + e^{2\varphi} \chi^2) - 12 e^{2\phi+\varphi} \rho^2 (1 - 3 e^{2\varphi} \chi^2) \\ & - 24 e^\varphi + 12 e^{4\phi+\varphi} \chi^2 \rho^2 (1 + e^{2\varphi} \chi^2) + 12 e^{4\phi+\varphi} \rho^4 (1 + 3 e^{2\varphi} \chi^2)] \\ & - \frac{1}{2} g m \chi e^{4\phi+3\varphi} (12 \rho^2 + 2\chi^2) + \frac{1}{2} m^2 e^{4\phi+3\varphi} \quad , \end{aligned}$$

note :  $\rho^2 \equiv \frac{1}{4} (\zeta^2 + \tilde{\zeta}^2)$

*AdS critical points !!*

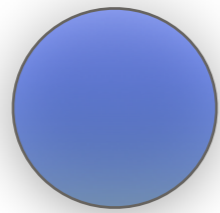


# AdS<sub>4</sub> solutions

[ AG, Varela '15 ]

$\mathcal{N}$	$G_0$	$c^{-1/3} \chi$	$c^{-1/3} e^{-\varphi}$	$c^{-1/3} \rho$	$c^{-1/3} e^{-\phi}$	$\frac{1}{4} g^{-2} c^{1/3} V_0$	$M^2 L^2$
$\mathcal{N} = 1$	$G_2$	$-\frac{1}{2^{7/3}}$	$\frac{5^{1/2} 3^{1/2}}{2^{7/3}}$	$-\frac{1}{2^{7/3}}$	$\frac{5^{1/2} 3^{1/2}}{2^{7/3}}$	$-\frac{2^{22/3} 3^{1/2}}{5^{5/2}}$	$4 \pm \sqrt{6}, -\frac{1}{6}(11 \pm \sqrt{6})$
$\mathcal{N} = 2$	$U(3)$	$-\frac{1}{2}$	$\frac{3^{1/2}}{2}$	0	$\frac{1}{2^{1/2}}$	$-3^{3/2}$	$3 \pm \sqrt{17}, 2, 2$
$\mathcal{N} = 1$	$SU(3)$	$\frac{1}{2^2}$	$\frac{3^{1/2} 5^{1/2}}{2^2}$	$-\frac{3^{1/2}}{2^2}$	$\frac{5^{1/2}}{2^2}$	$-\frac{2^6 3^{3/2}}{5^{5/2}}$	$4 \pm \sqrt{6}, 4 \pm \sqrt{6}$
$\mathcal{N} = 0$	$SO(6)_+$	0	$2^{1/6}$	0	$\frac{1}{2^{5/6}}$	$-3 2^{5/6}$	$6, 6, -\frac{3}{4}, 0$
$\mathcal{N} = 0$	$SO(7)_+$	0	$\frac{1}{5^{1/6}}$	0	$\frac{1}{5^{1/6}}$	$-\frac{3 5^{7/6}}{2^2}$	$6, -\frac{12}{5}, -\frac{6}{5}, -\frac{6}{5}$
$\mathcal{N} = 0$	$G_2$	$\frac{1}{2^{4/3}}$	$\frac{3^{1/2}}{2^{4/3}}$	$\frac{1}{2^{4/3}}$	$\frac{3^{1/2}}{2^{4/3}}$	$-\frac{2^{10/3}}{3^{1/2}}$	$6, 6, -1, -1$
$\mathcal{N} = 0$	$SU(3)$	0.455	0.838	0.335	0.601	-5.864	6.214, 5.925, 1.145, -1.284
$\mathcal{N} = 0$	$SU(3)$	0.270	0.733	0.491	0.662	-5.853	6.230, 5.905, 1.130, -1.264

◆  $\mathcal{N} = 2$  solution will play a central role in holography !!



## Massive IIA on $S^6$ / SYM-CS duality

# Collecting clues

- The deformed  $\text{ISO}(7)_c$  gauging has its  $\text{SO}(7)$  piece untouched by the deformation. This points towards an undeformed  $S^6$  description in higher dimension.
- If the higher-dim geometry is not affected, it should then be the higher-dim theory the one changing. The massive IIA theory by Romans proves a natural candidate.

[ Romans '86 ]

- The Romans mass parameter  $\hat{F}_{(0)}$  is a discrete (on/off) deformation, exactly as the parameter  $c$  in the deformed  $\text{ISO}(7)_c$  theory.

# Embedding of $ISO(7)_c$ into massive IIA supergravity

[ AG, Varela '15 ]

$$d\hat{s}_{10}^2 = \Delta^{-1} ds_4^2 + g_{mn} Dy^m Dy^n ,$$

$$\begin{aligned} \hat{A}_{(3)} = & \mu_I \mu_J (C^{IJ} + \mathcal{A}^I \wedge \mathcal{B}^J + \frac{1}{6} \mathcal{A}^{IK} \wedge \mathcal{A}^{JL} \wedge \tilde{\mathcal{A}}_{KL} + \frac{1}{6} \mathcal{A}^I \wedge \mathcal{A}^{JK} \wedge \tilde{\mathcal{A}}_K) \\ & + g^{-1} (\mathcal{B}_J^I + \frac{1}{2} \mathcal{A}^{IK} \wedge \tilde{\mathcal{A}}_{KJ} + \frac{1}{2} \mathcal{A}^I \wedge \tilde{\mathcal{A}}_J) \wedge \mu_I D\mu^J + \frac{1}{2} g^{-2} \tilde{\mathcal{A}}_{IJ} \wedge D\mu^I \wedge D\mu^J \\ & - \frac{1}{2} \mu_I B_{mn} \mathcal{A}^I \wedge Dy^m \wedge Dy^n + \frac{1}{6} A_{mnp} Dy^m \wedge Dy^n \wedge Dy^p , \end{aligned}$$

$$\hat{B}_{(2)} = -\mu_I (\mathcal{B}^I + \frac{1}{2} \mathcal{A}^{IJ} \wedge \tilde{\mathcal{A}}_J) - g^{-1} \tilde{\mathcal{A}}_I \wedge D\mu^I + \frac{1}{2} B_{mn} Dy^m \wedge Dy^n ,$$

$$\hat{A}_{(1)} = -\mu_I \mathcal{A}^I + A_m Dy^m .$$

where we have defined :  $Dy^m \equiv dy^m + \frac{1}{2} g K_{IJ}^m \mathcal{A}^{IJ}$  ,  $D\mu^I \equiv d\mu^I - g \mathcal{A}^{IJ} \mu_J$

The scalars are embedded as

$$\begin{aligned} g^{mn} &= \frac{1}{4} g^2 \Delta \mathcal{M}^{IJKL} K_{IJ}^m K_{KL}^n , & B_{mn} &= -\frac{1}{2} \Delta g_{mp} K_{IJ}^p \partial_n \mu^K \mathcal{M}^{IJ}_{K8} , \\ A_m &= \frac{1}{2} g \Delta g_{mn} K_{IJ}^n \mu_K \mathcal{M}^{IJK8} , & A_{mnp} &= \frac{1}{8} g \Delta g_{mq} K_{IJ}^q K_{np}^{KL} \mathcal{M}^{IJ}_{KL} + A_m B_{np} . \end{aligned}$$

# A new N=2 solution of massive type IIA

- Using the uplifting formulae, it is a straightforward exercise to obtain the 10D embedding of a 4D critical point. An example is the N=2&U(3) AdS<sub>4</sub> point of the ISO(7)<sub>c</sub> theory

$$d\hat{s}_{10}^2 = L^2 \frac{(3 + \cos 2\alpha)^{\frac{1}{2}}}{(5 + \cos 2\alpha)^{-\frac{1}{8}}} \left[ ds^2(\text{AdS}_4) + \frac{3}{2} d\alpha^2 + \frac{6 \sin^2 \alpha}{3 + \cos 2\alpha} ds^2(\mathbb{CP}^2) + \frac{9 \sin^2 \alpha}{5 + \cos 2\alpha} \boldsymbol{\eta}^2 \right],$$

$$e^{\hat{\phi}} = e^{\phi_0} \frac{(5 + \cos 2\alpha)^{3/4}}{3 + \cos 2\alpha}, \quad \hat{H}_{(3)} = 24\sqrt{2} L^2 e^{\frac{1}{2}\phi_0} \frac{\sin^3 \alpha}{(3 + \cos 2\alpha)^2} \mathbf{J} \wedge d\alpha,$$

$$L^{-1} e^{\frac{3}{4}\phi_0} \hat{F}_{(2)} = -4\sqrt{6} \frac{\sin^2 \alpha \cos \alpha}{(3 + \cos 2\alpha)(5 + \cos 2\alpha)} \mathbf{J} - 3\sqrt{6} \frac{(3 - \cos 2\alpha)}{(5 + \cos 2\alpha)^2} \sin \alpha d\alpha \wedge \boldsymbol{\eta},$$

$$L^{-3} e^{\frac{1}{4}\phi_0} \hat{F}_{(4)} = 6 \text{vol}_4 + 12\sqrt{3} \frac{7 + 3 \cos 2\alpha}{(3 + \cos 2\alpha)^2} \sin^4 \alpha \text{vol}_{\mathbb{CP}^2} + 18\sqrt{3} \frac{(9 + \cos 2\alpha) \sin^3 \alpha \cos \alpha}{(3 + \cos 2\alpha)(5 + \cos 2\alpha)} \mathbf{J} \wedge d\alpha \wedge \boldsymbol{\eta},$$

where we have introduced the quantities  $L^2 \equiv 2^{-\frac{5}{8}} 3^{-1} g^{-2} c^{\frac{1}{12}}$  and  $e^{\phi_0} \equiv 2^{\frac{1}{4}} c^{-\frac{5}{6}}$

- ◆ The angle  $0 \leq \alpha \leq \pi$  locally foliates S<sup>6</sup> with S<sup>5</sup> regarded as Hopf fibrations over  $\mathbb{CP}^2$

# CFT<sub>3</sub> candidate and matching of free energies

[ Schwarz '04 ]

[ Gaiotto, Tomasiello '09 ]

- We propose an N=2 Chern-Simons-matter theory with simple gauge group SU(N), level  $k$ , three adjoint matter and cubic superpotential, as the CFT dual of the N=2 massive IIA solution.
- The 3d free energy  $F = -\text{Log}(Z)$ , where  $Z$  is the partition function of the CFT on a Euclidean  $S^3$ , can be computed via localisation over supersymmetric configurations

$$Z = \int \prod_{i=1}^N \frac{d\lambda_i}{2\pi} \prod_{i<j=1}^N \left( 2 \sinh^2 \left( \frac{\lambda_i - \lambda_j}{2} \right) \right) \times \prod_{i,j=1}^N \left( \exp \left( \ell \left( \frac{1}{3} + \frac{i}{2\pi} (\lambda_i - \lambda_j) \right) \right) \right)^3 e^{\frac{ik}{4\pi} \sum \lambda_i^2}$$

[ Pestun '07 ] [ Kapustin, Willett, Yaakov '09 ]

[ Jafferis '10 ] [ Jafferis, Klebanov, Pufu, Safdi '11 ]

[ Closset, Dumitrescu, Festuccia, Komargodski '12 '13 ]

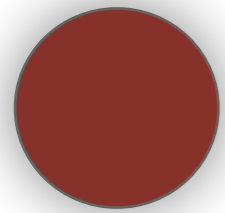
where  $\lambda_i$  are the Coulomb branch parameters. In the  $N \gg k$  limit, the result is given by

$$F = \frac{3^{13/6} \pi}{40} \left( \frac{32}{27} \right)^{2/3} k^{1/3} N^{5/3}$$

- The gravitational free energy can be computed from the warp factor in the N=2 massive IIA solution. Using the charge quantisation condition  $N = -(2\pi\ell_s)^{-5} \int_{S^6} e^{\frac{1}{2}\phi} \hat{F}_{(4)} + \hat{B}_{(2)} \wedge d\hat{A}_{(3)} + \frac{1}{6} \hat{F}_{(0)} \hat{B}_{(2)}^3$  for the D2-brane, one finds

$$F = \frac{16\pi^3}{(2\pi\ell_s)^8} \int_{S^6} e^{8A} \text{vol}_6 = \frac{\pi}{5} 2^{1/3} 3^{1/6} k^{1/3} N^{5/3} \quad \text{provided}$$

$$gc = \hat{F}_{(0)} = k/(2\pi\ell_s)$$



## Holographic RG flows: domain-walls and black holes

# Holographic description of RG flows

[ Boonstra, Skenderis, Townsend '98 ]

- RG flows are described holographically as non-AdS<sub>4</sub> solutions in gravity
- RG flows on M2-brane : SO(8)-gauged sugra from M-theory on S<sup>7</sup>

[ Ahn, Paeng '00 ] [ Ahn, Itoh '01 ]

[ Bobev, Halmagyi, Pilch, Warner '09 ]

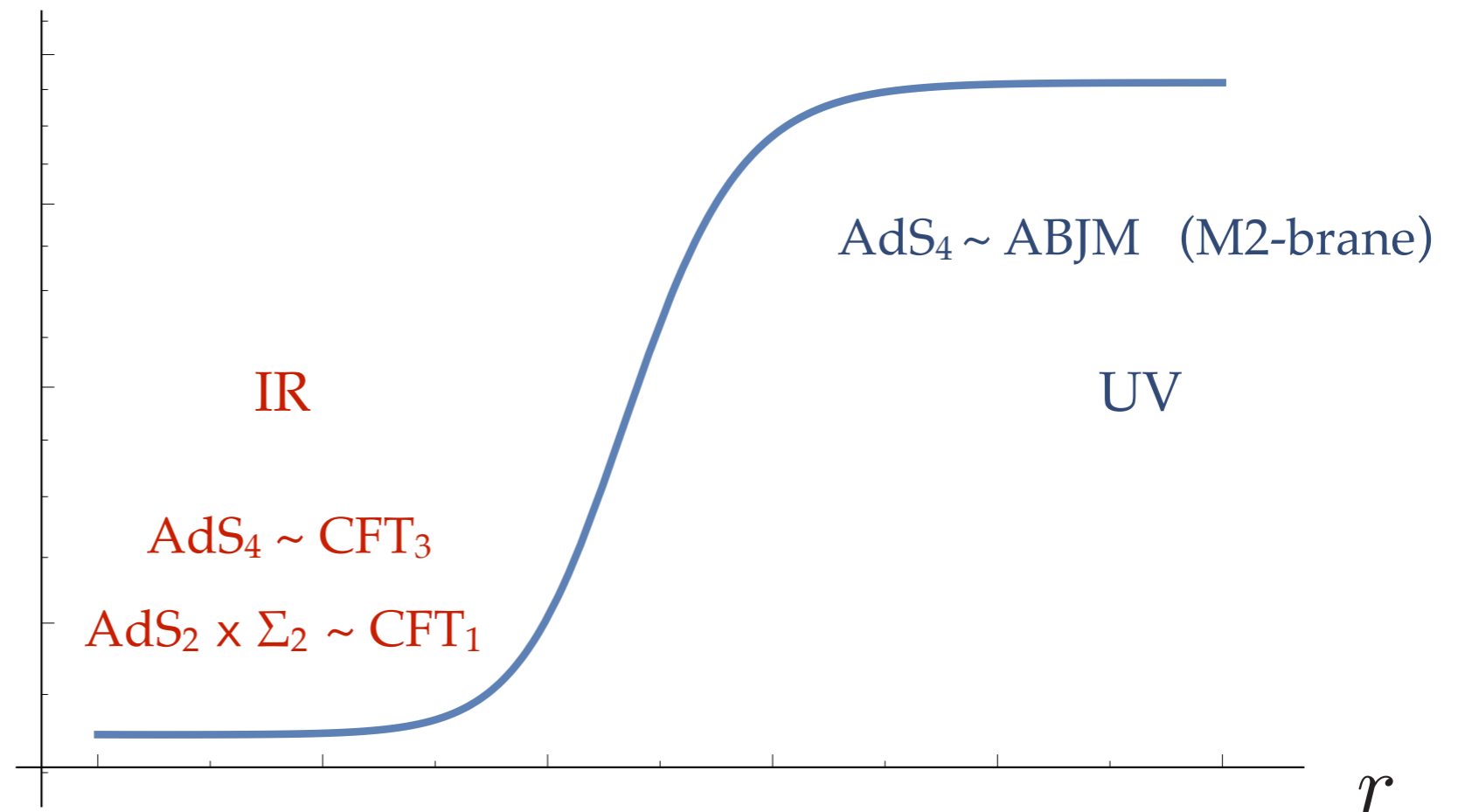
[ Cacciatori, Klemm '09 ]

[ Halmagyi, Petrini, Zaffaroni '13 ]

[ Chimento, Klemm, Petri '15 ]

[ Benini, Hristov, Zaffaroni '15 '16 ]

AdS<sub>4</sub> in IR : domain-wall  
AdS<sub>2</sub> × Σ<sub>2</sub> in IR : black hole



- RG flows on D3-brane : SO(6)-gauged sugra from type IIB on S<sup>5</sup> and N=4 SYM in 4D

[ Freedman, Gubser, Pilch, Warner '99 ]

[ Pilch, Warner '00 ] [ Benini, Bobev '12, '13 ]



# Holographic RG flows on the D2-brane of massive IIA

- D2-brane :

$$d\hat{s}_{10}^2 = e^{\frac{3}{4}\phi} \left( -e^{2U} dt^2 + e^{-2U} dr^2 + e^{2(\psi-U)} ds_{\mathbb{H}^2}^2 \right) + g^{-2} e^{-\frac{1}{4}\phi} ds_{S^6}^2$$

$$e^{\hat{\Phi}} = e^{\frac{5}{2}\phi}$$

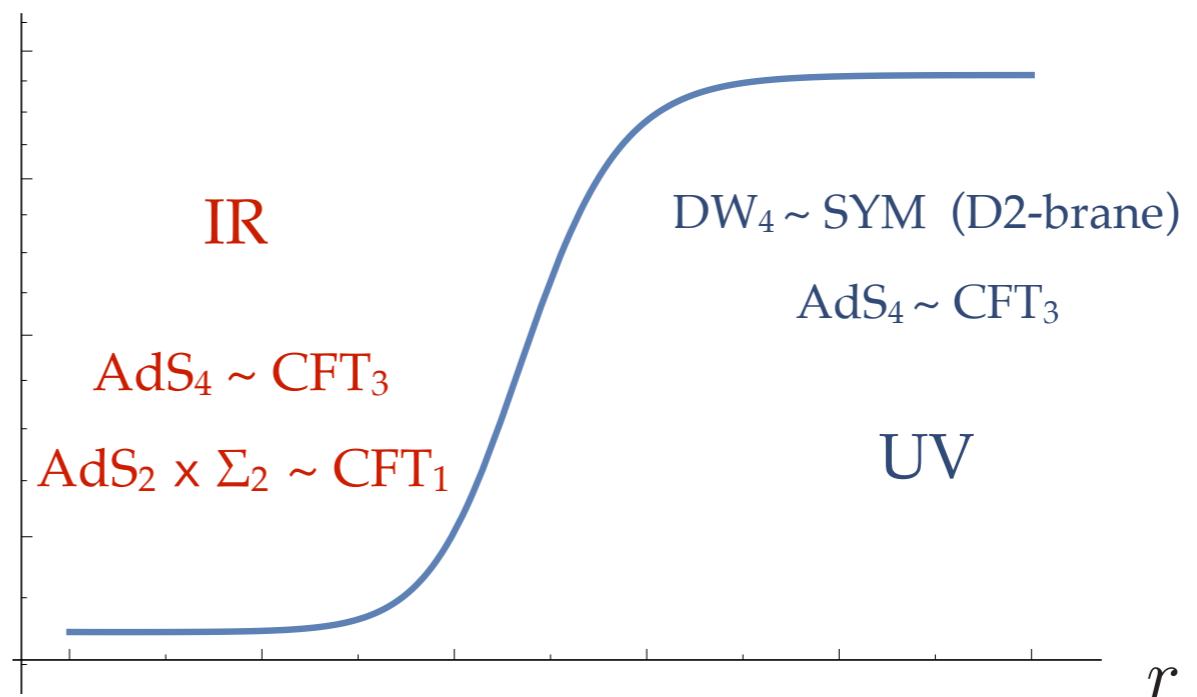
$$\hat{F}_{(4)} = 5 g e^{\phi} e^{2(\psi-U)} \sinh \theta dt \wedge dr \wedge d\theta \wedge d\phi$$

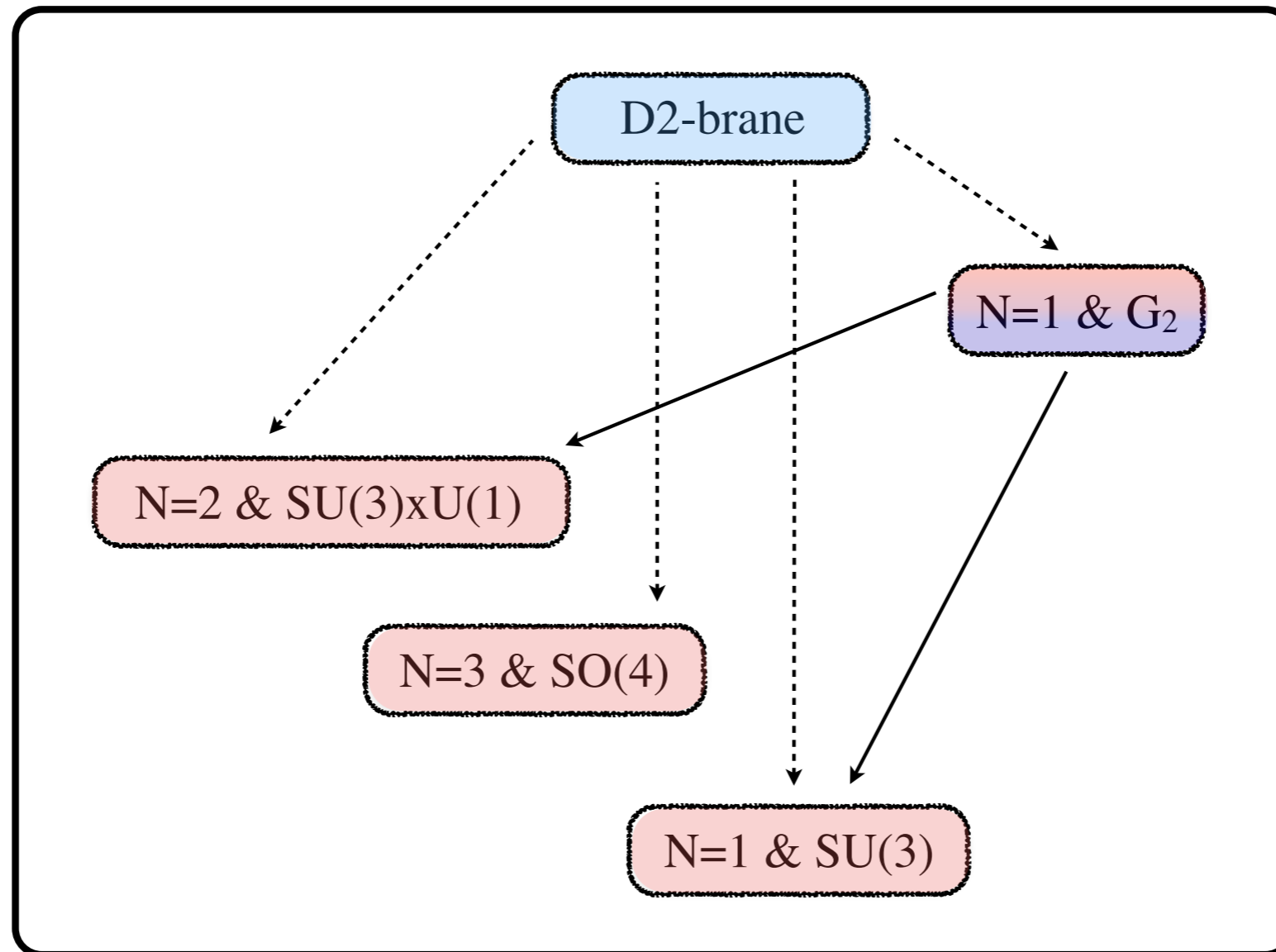
with  $e^{2U} \sim r^{\frac{7}{4}}$ ,  $e^{2(\psi-U)} \sim r^{\frac{7}{4}}$  and  $e^{\varphi} = e^{\phi} \sim r^{-\frac{1}{4}}$   $\rightarrow$

DW<sub>4</sub>  
domain-wall

- RG flows on D2-brane : ISO(7)-gauged sugra from type IIA on S<sup>6</sup>

AdS<sub>4</sub> in IR : domain-wall  
AdS<sub>2</sub> × Σ<sub>2</sub> in IR : black hole





- RG flows from **SYM** (dotted lines) and between **CFT's** (solid lines) dual to BPS domain-wall solutions of the dyonic ISO(7)-gauged supergravity

# Holographic RG flows: black hole solutions (I)

$$\Lambda = 0, 1$$

- Black hole Ansatz :

$$ds^2 = -e^{2U(r)} dt^2 + e^{-2U(r)} dr^2 + e^{2(\psi(r)-U(r))} \left( d\theta^2 + \left( \frac{\sin \sqrt{\kappa} \theta}{\sqrt{\kappa}} \right)^2 d\phi^2 \right)$$

$$\mathcal{A}^\Lambda = \mathcal{A}_t^\Lambda(r) dt - p^\Lambda \frac{\cos \sqrt{\kappa} \theta}{\kappa} d\phi$$

$$\tilde{\mathcal{A}}_\Lambda = \tilde{\mathcal{A}}_{t\Lambda}(r) dt - e_\Lambda \frac{\cos \sqrt{\kappa} \theta}{\kappa} d\phi$$

$$\mathcal{B}^0 = b_0(r) \frac{\sin \sqrt{\kappa} \theta}{\sqrt{\kappa}} d\theta \wedge d\phi$$

- Attractor equations :

$$Q = \kappa L_{\Sigma_2}^2 \Omega \mathcal{M} Q^x \mathcal{P}^x - 4 \text{Im}(\bar{\mathcal{Z}} \mathcal{V}) ,$$

$$\frac{L_{\Sigma_2}^2}{L_{\text{AdS}_2}} = -2 \mathcal{Z} e^{-i\beta} ,$$

$$\langle \mathcal{K}^u, \mathcal{V} \rangle = 0 ,$$

[ Dall'Agata, Guecchi '10 ]  
[ Klemm, Petri, Rabbiosi '16 ]

- Unique  $\text{AdS}_2 \times \text{H}^2$  :

( N=2 & U(3)  $\text{AdS}_4$  vev's )

$$e^{\varphi_h} = \frac{2}{\sqrt{3}} \left( \frac{g}{m} \right)^{\frac{1}{3}} , \quad \chi_h = -\frac{1}{2} \left( \frac{g}{m} \right)^{-\frac{1}{3}} , \quad e^{\phi_h} = \sqrt{2} \left( \frac{g}{m} \right)^{\frac{1}{3}} , \quad a_h = \zeta_h = \tilde{\zeta}_h = 0 ,$$

$$p^0 + \frac{1}{2} m b_0^h = \pm \frac{1}{6} m^{\frac{2}{3}} g^{-\frac{5}{3}} , \quad e_0 + \frac{1}{2} g b_0^h = \pm \frac{1}{6} m^{-\frac{1}{3}} g^{-\frac{2}{3}} ,$$

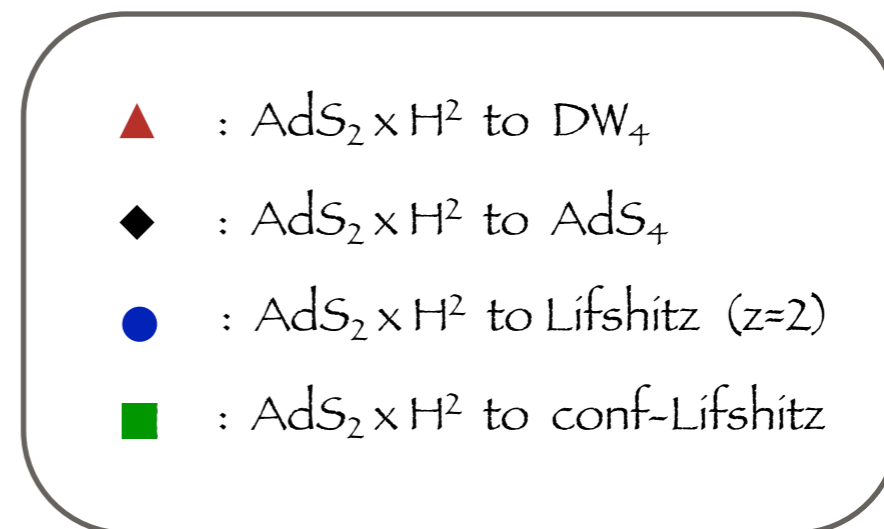
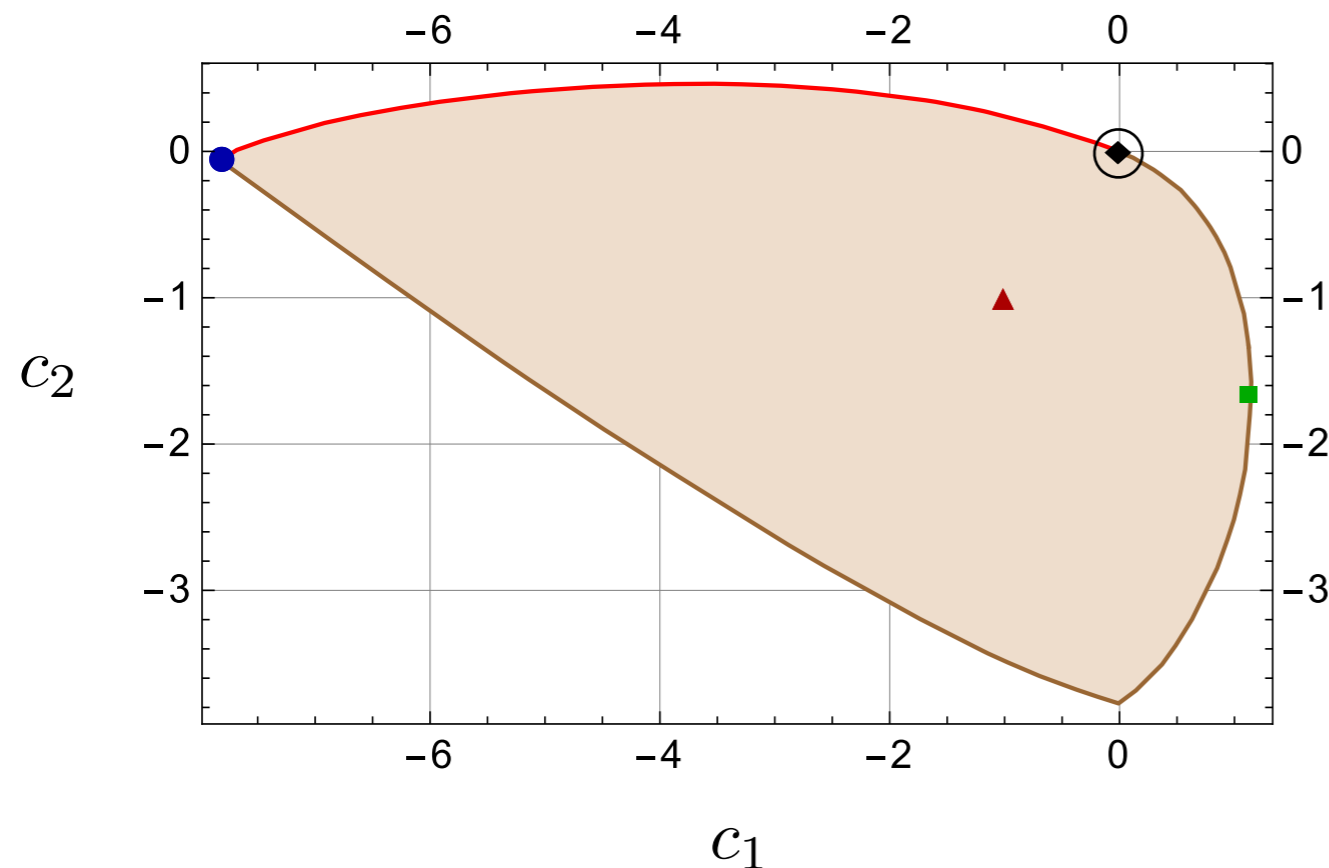
$$p^1 = \mp \frac{1}{3} g^{-1} , \quad e_1 = \pm \frac{1}{2} m^{\frac{1}{3}} g^{-\frac{4}{3}} ,$$

$$L_{\text{AdS}_2}^2 = \frac{1}{4\sqrt{3}} m^{\frac{1}{3}} g^{-\frac{7}{3}} , \quad L_{\text{H}^2}^2 = \frac{1}{2\sqrt{3}} m^{\frac{1}{3}} g^{-\frac{7}{3}} .$$

[ AG, Tarrío '17 ]

# Holographic RG flows: black hole solutions (II)

- Two irrelevant modes  $(c_1, c_2)$  when perturbing around the  $\text{AdS}_2 \times \text{H}^2$  solution in the IR



[ AG, Tarrío '17 ]

- RG flows across dimension from **SYM or  $\text{CFT}_3$  or non-relativistic** to  **$\text{CFT}_1$**
- Universal RG flow (◆)  **$\text{CFT}_3$**  to  **$\text{CFT}_1$**  [ Azzurli, Bobev, Cricigno, Min, Zaffaroni '17 ]
- $\text{AdS}_2 \times \Sigma_g$  horizon classification in the mIIA STU-model : 3 vector + 1 hyper [ AG '17 ]

# Summary

- We connected the dyonic  $N = 8$  supergravity with  $ISO(7)_c$  gauging to massive IIA reductions on  $S^6$ .
- Any 4D configuration (AdS, DW, BH) is embedded into 10D via the uplifting formulas. As an example, we found an  $AdS_4 \times S^6$  solution of massive IIA based on an  $N = 2$  &  $U(3)$   $AdS_4$  vacuum.
- We proposed a  $CFT_3$  dual for the  $N = 2$   $AdS_4 \times S^6$  solution of massive IIA based on the D2-brane field theory (SYM-CS). The gravitational and field theory free energies perfectly match!
- Holographic study of RG flows on D2-brane :  
DW solutions (  $CFT_3$ - $CFT_3$  &  $SYM_3$ - $CFT_3$  )  
BH solutions (  $CFT_3$ - $CFT_1$  &  $SYM_3$ - $CFT_1$  )
- Recent progress in the holographic counting of BH microstates  
[ Benini, Hristov, Zaffaroni '16 ]  
[ Azzurli, Bobev, Cricigno, Min, Zaffaroni '17 ]  
[ Hosseini, Hristov, Passias '17 ] [ Benini, Khachatryan, Milan '17 ]
- New holographic perspective on the one-parameter family of  $SO(8)$  theories?

Thanks !!