Mass deformations & SL(2) angles from exceptional field theory

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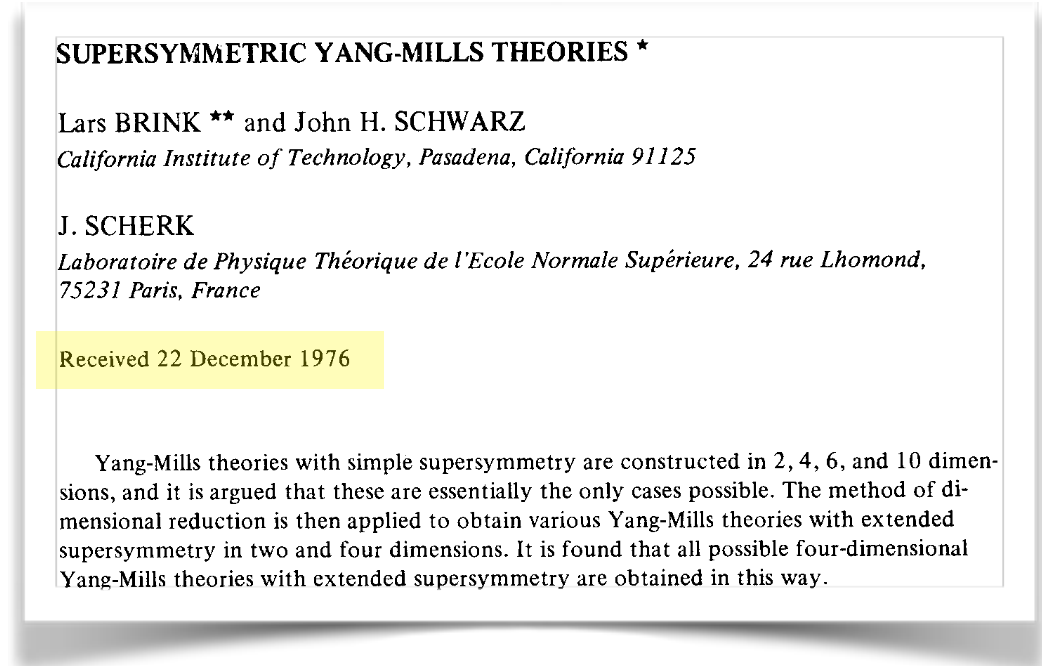
Supergravity at 40

40 years ago…

Reduction of SYM in 10D

- Masses in 4D from reduction of **non-abelian** SYM in 10D

[***f*** = Lie algebra structure constants]


SUPERSYMMETRIC YANG-MILLS THEORIES *

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Received 22 December 1976

Yang-Mills theories with simple supersymmetry are constructed in 2, 4, 6, and 10 dimensions, and it is argued that these are essentially the only cases possible. The method of dimensional reduction is then applied to obtain various Yang-Mills theories with extended supersymmetry in two and four dimensions. It is found that all possible four-dimensional Yang-Mills theories with extended supersymmetry are obtained in this way.

[Brink, Scherk & Schwarz **'76**]

KK reduction

$$L_{10\mathrm{D}} = -\tfrac{1}{4}F^2 + \tfrac{i}{2}\bar{\lambda}\not{D}\lambda$$

$$\begin{aligned} L_{4\mathrm{D}} = & -\tfrac{1}{4}F^2 + i\bar{\lambda}\not{D}\lambda - D_\mu M D^\mu M^{-1} \\ & - \tfrac{i}{2} f\, M\, \bar{\lambda}\,\lambda + \mathrm{c.c} \\ & - \tfrac{1}{4} f\, f\, M^{-1}\, M^{-1}\, M\, M \end{aligned}$$

Lower-dimensional masses from higher-dimensional deformations

Reduction of gravity theories in 10D/11D

- Masses in 4D from reduction of gravity theories in 10D/11D with **non-trivial** internal profiles

$$\Phi_m(x,y) = U(y)_m{}^n\, \Phi_n(x)$$

$$f = U^{-1}\, U^{-1} \partial U = cte$$

[***f*** = Lie algebra structure constants]

HOW TO GET MASSES FROM EXTRA DIMENSIONS

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Received 19 February 1979

A generalized method of dimensional reduction, applicable to theories in curved space, is described. As in previous works by other authors, the extra dimensions are related to the manifold of a Lie group. The new feature of this work is to define and study a class of Lie groups, called "flat groups", for which the resulting theory has no cosmological constant, a well-behaved potential, and a number of arbitrary mass parameters. In particular, when the analysis is applied to the reduction of 11-dimensional supergravity to four dimensions it becomes possible to incorporate three arbitrary mass parameters in the resulting $N = 8$ theory. This shows that extended supersymmetry theories allow more possibilities for spontaneous symmetry breaking than was previously believed to be the case.

[Scherk & Schwarz '79]

SS reduction

$$L_{10D/11D} = e\, R \quad \Rightarrow \quad L_{4D} = e\, R - \tfrac{1}{4}\, M\, F^2 - D_\mu M\, D^\mu M^{-1} - 2\, f\, f\, M^{-1} - f\, f\, M\, M^{-1}\, M^{-1}$$

Lower-dimensional masses from non-trivial internal dependence

After 40 years, a new framework where to jointly describe gravitational and gauge aspects of *massless* maximal supergravities in 10D/11D has been constructed…

Exceptional Field Theory (EFT)

[Hohm & Samtleben '13]

- Space-time : external (11-**n**) + **generalised internal** ($y^{\mathcal{M}}$ coordinates in $E_{n(n)}$ rep.)

[momentum, winding, ...]

Generalised diffs = *ordinary internal diffs* + internal *gauge transfos*

- Generalised Lie derivative built from an $E_{n(n)}$-invariant structure *Y*-tensor

[specific for each ***n***]

- Closure requires a **section constraint** : $Y^{\mathcal{P}\mathcal{Q}}{}_{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{P}} \otimes \partial_{\mathcal{Q}} = 0$

Two maximal solutions : M-theory (**n** dimensional) & Type IIB (**n-1** dimensional)

[**massless** theories]

Question : Is there an *exceptional* story behind **massive** IIA too ?

[Romans '86]

[Hohm & Kwak '11 (non-geometric DFT)]

How to get masses from EFT

Clues:

[Romans '86]

[Behrndt & Cvetic '04]
[Lüst & Tsimpis '04]
[Kashani-Poor '07]
[A.G. & Varela '15]

[Schwarz '04]
[Gaiotto & Tomasiello '09]
[A.G., Jafferis & Varela '15]

1. Deformations of EFT (à la SYM)

[Motivation = Massive IIA]

arXiv:1604.08602 with Ciceri and Inverso

2. Generalised Scherk-Schwarz reductions (à la gravity)

[Motivation = fluxes and moduli stabilisation]

work in progress

Overview

1. Deformations of EFT and massive IIA

2. Scherk-Schwarz reductions: DFT at SL(2) angles

3. What next ?

X deformation of EFT (XFT)

- Generalised Lie derivative

[no density term]

$$\mathbb{L}_\Lambda U^{\mathcal{M}} = \Lambda^{\mathcal{N}}\partial_{\mathcal{N}}U^{\mathcal{M}} - U^{\mathcal{N}}\partial_{\mathcal{N}}\Lambda^{\mathcal{M}} + Y^{\mathcal{M}\mathcal{N}}{}_{\mathcal{P}\mathcal{Q}}\,\partial_{\mathcal{N}}\Lambda^{\mathcal{P}}\,U^{\mathcal{Q}}$$

in terms of an $E_{n(n)}$-invariant structure Y-tensor. Closure requires **sec. constraint**

- **Deformed** generalised Lie derivative

$$\widetilde{\mathbb{L}}_\Lambda U^{\mathcal{M}} = \Lambda^{\mathcal{N}}\partial_{\mathcal{N}}U^{\mathcal{M}} - U^{\mathcal{N}}\partial_{\mathcal{N}}\Lambda^{\mathcal{M}} + Y^{\mathcal{M}\mathcal{N}}{}_{\mathcal{P}\mathcal{Q}}\,\partial_{\mathcal{N}}\Lambda^{\mathcal{P}}\,U^{\mathcal{Q}} - \underbrace{X_{\mathcal{N}\mathcal{P}}{}^{\mathcal{M}}\,\Lambda^{\mathcal{N}}\,U^{\mathcal{P}}}_{\text{non-derivative}}$$

in terms of an X deformation which is $E_{n(n)}$-algebra valued

- Closure & triviality of the Jacobiator require (*together with* ***sec. constraint***)

$$X_{\mathcal{M}\mathcal{N}}{}^{\mathcal{P}}\,\partial_{\mathcal{P}} = 0$$

X constraint

$$X_{\mathcal{M}\mathcal{P}}{}^{\mathcal{Q}}\,X_{\mathcal{N}\mathcal{Q}}{}^{\mathcal{R}} - X_{\mathcal{N}\mathcal{P}}{}^{\mathcal{Q}}\,X_{\mathcal{M}\mathcal{Q}}{}^{\mathcal{R}} + X_{\mathcal{M}\mathcal{N}}{}^{\mathcal{Q}}\,X_{\mathcal{Q}\mathcal{P}}{}^{\mathcal{R}} = 0$$

Quadratic constraint (gauged max. supergravity)

[*X* deformation *vs* embedding tensor]

X deformation : background fluxes & Romans mass

$$Y^{\mathcal{P}\mathcal{Q}}{}_{\mathcal{M}\mathcal{N}}\,\partial_{\mathcal{P}} \otimes \partial_{\mathcal{Q}} = 0$$

section constraint

$$X_{\mathcal{M}\mathcal{N}}{}^{\mathcal{P}}\,\partial_{\mathcal{P}} = 0$$

X constraint

[algebraic system]

M-theory (**n** coords)

- SL(n) orbit
- Freund-Rubin param. (n = 4 and n = 7)
- *massless* IIA (subcase)

Type IIB (**n-1** coords)

- SL(n-1) orbit
- *p*-form fluxes compatible with SL(n-1)
- **SL(2)-triplet of 1-form flux** (includes compact SO(2))

[QC = flux-induced tadpoles]

X deformation : background fluxes & Romans mass

$$Y^{\mathcal{PQ}}{}_{\mathcal{MN}}\, \partial_{\mathcal{P}} \otimes \partial_{\mathcal{Q}} = 0$$

section constraint

$$X_{\mathcal{MN}}{}^{\mathcal{P}}\, \partial_{\mathcal{P}} = 0$$

X constraint

[algebraic system]

M-theory (**n** coords)

- SL(n) orbit
- Freund-Rubin param. (n = 4 and n = 7)
- *massless* IIA (subcase)

Type IIB (**n-1** coords)

- SL(n-1) orbit
- *p*-form fluxes compatible with SL(n-1)
- **SL(2)-triplet of 1-form flux** (includes compact SO(2))

+

New massive Type IIA (**n-1** coords)

- SL(n-1) orbit
- *p*-form fluxes compatible with SL(n-1)
- dilaton flux
- **Romans mass parameter** (*kills the M-theory coord*)

Massive Type IIA described in a purely geometric manner !!

[QC = flux-induced tadpoles]

EFT action (n=7 , D=4)

- $E_{7(7)}$-EFT action [$\mathcal{D}_\mu = \partial_\mu - \mathbb{L}_{A_\mu}$] [$y^{\mathcal{M}}$ coords in the **56** of $E_{7(7)}$]

$$S_{\text{EFT}} = \int d^4x\, d^{56}y\, e\, \big[\, \hat{R} + \tfrac{1}{48}\, g^{\mu\nu}\, \mathcal{D}_\mu \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \mathcal{D}_\nu \mathcal{M}_{\mathcal{M}\mathcal{N}} - \tfrac{1}{8}\, \mathcal{M}_{\mathcal{M}\mathcal{N}}\, \mathcal{F}^{\mu\nu\mathcal{M}} \mathcal{F}_{\mu\nu}{}^{\mathcal{N}} + e^{-1}\, \mathcal{L}_{\text{top}} - V_{\text{EFT}}(\mathcal{M}, g) \,\big]$$

[Hohm & Samtleben '13]

with *field strengths* & *potential* given by

$$\mathcal{F}_{\mu\nu}{}^{\mathcal{M}} = 2\, \partial_{[\mu} A_{\nu]}{}^{\mathcal{M}} - \big[A_\mu, A_\nu\big]_{\text{E}}^{\mathcal{M}} + \text{two-form terms}$$

(tensor hierarchy)

$$V_{\text{EFT}}(\mathcal{M}, g) = -\tfrac{1}{48}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{M}} \mathcal{M}^{\mathcal{K}\mathcal{L}}\, \partial_{\mathcal{N}} \mathcal{M}_{\mathcal{K}\mathcal{L}} + \tfrac{1}{2}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{M}} \mathcal{M}^{\mathcal{K}\mathcal{L}}\, \partial_{\mathcal{L}} \mathcal{M}_{\mathcal{N}\mathcal{K}} - \tfrac{1}{2}\, g^{-1} \partial_{\mathcal{M}} g\, \partial_{\mathcal{N}} \mathcal{M}^{\mathcal{M}\mathcal{N}} - \tfrac{1}{4}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, g^{-1} \partial_{\mathcal{M}} g\, g^{-1} \partial_{\mathcal{N}} g - \tfrac{1}{4}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{M}} g^{\mu\nu}\, \partial_{\mathcal{N}} g_{\mu\nu}$$

- **Two**-derivative potential : **ungauged** 4D max. sugra when $\Phi(x,y) = \Phi(x)$

XFT action (n=7 , D=4)

- $E_{7(7)}$-XFT action [$\mathcal{D}_\mu = \partial_\mu - \widetilde{\mathbb{L}}_{A_\mu}$] [$y^{\mathcal{M}}$ coords in the **56** of $E_{7(7)}$]

$$S_{\text{XFT}} = \int d^4x\, d^{56}y\, e\, \Big[\, \hat{R} + \tfrac{1}{48}\, g^{\mu\nu}\, \mathcal{D}_\mu \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \mathcal{D}_\nu \mathcal{M}_{\mathcal{M}\mathcal{N}} - \tfrac{1}{8}\, \mathcal{M}_{\mathcal{M}\mathcal{N}}\, \mathcal{F}^{\mu\nu\mathcal{M}} \mathcal{F}_{\mu\nu}{}^{\mathcal{N}} + e^{-1}\, \mathcal{L}_{\text{top}} - V_{\text{XFT}}(\mathcal{M}, g) \,\Big]$$

with *field strengths* & *potential* given by

(deformed tensor hierarchy)

$$\mathcal{F}_{\mu\nu}{}^{\mathcal{M}} = 2\, \partial_{[\mu} A_{\nu]}{}^{\mathcal{M}} + X_{[\mathcal{P}\mathcal{Q}]}{}^{\mathcal{M}}\, A_\mu{}^{\mathcal{P}} A_\nu{}^{\mathcal{Q}} - \big[A_\mu, A_\nu\big]_{\text{E}}^{\mathcal{M}} + \text{two-form terms}$$

$$V_{\text{XFT}}(\mathcal{M}, g, X) = V_{\text{EFT}}(\mathcal{M}, g) + \underbrace{\tfrac{1}{12}\, \mathcal{M}^{MN} \mathcal{M}^{KL} X_{MK}{}^{P}\, \partial_N \mathcal{M}_{PL}}_{\text{cross term}} + \underbrace{V_{\text{SUGRA}}(\mathcal{M}, X)}_{\text{gauged max. sugra}}$$

- Two-One-**Zero**-derivative potential : **gauged** 4D max. sugra when $\Phi(x,y) = \Phi(x)$

Application : massive IIA on S^{n-1} $(2 < n < 8)$

Massless IIA reductions on S^{n-1} to gauged maximal sugra : EFT framework

Massive IIA reductions on S^{n-1} to gauged maximal sugra : XFT framework

Question : when is a consistent reduction Ansatz for *massless* IIA also consistent for *massive* IIA ?

[Cvetic , Lü & Pope '00]
[Lee, Strickland-Constable & Waldram '14]
[Hohm & Samtleben '14]

Procedure :

a) *Massless* IIA : generalised twist matrices valued in SL(n)

b) Romans mass introduced as an X^R deformation

c) Consistency requires the *stabiliser* of X^R in $E_{n(n)}$ to contain SL(n)

Answer : Only **massive IIA on S^6** works (n=7) !!

[A.G. & Varela '15]

Overview

1. Deformations of EFT and massive IIA

2. Scherk-Schwarz reductions: DFT at SL(2) angles

3. What next ?

From EFT to SL(2)-DFT

- Halving EFT with $E_{7(7)}$ symmetry to obtain SL(2)-DFT with SL(2) x SO(6,6) symmetry

$$\begin{array}{ccc} \mathrm{E}_{7(7)} & \rightarrow & \mathrm{SL}(2) \times \mathrm{SO}(6,6) \\ \mathbf{56} & \rightarrow & (\mathbf{2},\mathbf{12}) + (\mathbf{1},\mathbf{32}) \\ y^{\mathcal{M}} & \rightarrow & y^{\alpha M} + \cancel{y}^{A} \end{array}$$

EFT — SL(2)-DFT

α = (+ , -) vector index of SL(2)

M vector index of SO(6,6)

A M-W spinor index of SO(6,6)

[see Dibitetto, A.G. & Roest '11 for sugra]

via a **Z_2 truncation** (*vector* = +1 , *spinor* = -1) on coordinates, fields, etc.

[Hull & Zwiebach '09] [Hohm, Hull & Zwiebach '10]

- SL(2)-DFT generalised Lie derivative

[DFT corresponds to an α = + orientation]

$$\mathbb{L}_{\Lambda} U^{\alpha M} = \Lambda^{\beta N} \partial_{\beta N} U^{\alpha M} - U^{\beta N} \partial_{\beta N} \Lambda^{\alpha M} + \eta^{MN} \eta_{PQ} \, \partial_{\beta N} \Lambda^{\beta P} \, U^{\alpha Q} + 2 \, \epsilon^{\alpha\beta} \, \epsilon_{\gamma\delta} \, \partial_{\beta N} \Lambda^{\gamma[M} \, U^{|\delta|N]}$$

- SL(2)-DFT section constraints :

$$\eta^{MN} \, \partial_{\alpha M} \otimes \partial_{\beta N} = 0 \quad , \quad \epsilon^{\alpha\beta} \, \partial_{\alpha[M|} \otimes \partial_{\beta|N]} = 0$$

- SL(2)-DFT action extendable to SL(2) x SO(6,6**+n_v**) and also X-deformable

[see Hohm & Kwak '11 for Het-DFT]

[additional deform. params]

Section constraint & SL(2) angles

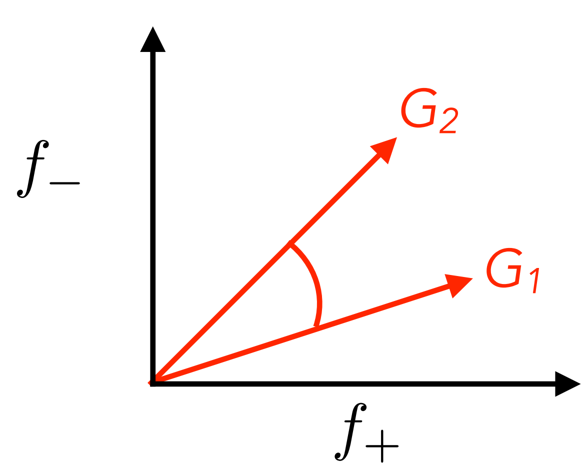
- One maximal solution of sec. constraint : it describes Type I/Heterotic [**as** in DFT]

- Generalised SS reductions with **SL(2) x O(6,6)** twist matrices $U_{\alpha M}{}^{\beta N} = e^{\lambda}\, e_{\alpha}{}^{\beta}\, U_M{}^N$ yield N=4 gauging parameters

$$
\begin{aligned}
f_{\alpha MNP} &= -3\, e^{-\lambda}\, e_{\alpha}{}^{\beta}\, \eta_{Q[M}\, U_N{}^R\, U_{P]}{}^S\ \partial_{\beta R} U_S{}^Q \\
\xi_{\alpha M} &= 2\, U_M{}^N \partial_{\beta N}(e^{-\lambda} e_{\alpha}{}^{\beta})
\end{aligned}
$$

[de Roo & Wagemans '85]

- *Moduli stabilisation* **requires** gaugings $G = G_1 \times G_2$ at **relative** SL(2) angles



(sec. constraint violated)

$$\epsilon^{\alpha\beta}\, \partial_{\alpha[M|} \otimes \partial_{\beta|N]} \neq 0$$

[**not** possible in DFT]

Example : SO(4) x SO(4) gaugings and non-geometry

- SS with $U(y^{\alpha M}) \in \mathrm{O}(6,6)$: Half of the coords of type + & half of type -

- SL(2)-superposition of *two chains* of **non-geometric** fluxes $(\, H \, , \, \omega \, , \, Q \, , \, R \,)_{\pm}$

f_+	$f_{+abc} = H^{(+)}{}_{abc}$,	$f_{+ijk} = H^{(+)}{}_{ijk}$,	$f_{+ab\bar{c}} = \omega^{(+)}{}_{ab}{}^{c}$,	$f_{+ij\bar{k}} = \omega^{(+)}{}_{ij}{}^{k}$
	$f_{+\bar{a}\bar{b}c} = Q^{(+)ab}{}_{c}$,	$f_{+\bar{i}\bar{j}k} = Q^{(+)ij}{}_{k}$,	$f_{+\bar{a}\bar{b}\bar{c}} = R^{(+)abc}$,	$f_{+\bar{i}\bar{j}\bar{k}} = R^{(+)ijk}$

f_-	$f_{-ijk} = H^{(-)}{}_{ijk}$,	$f_{-abc} = H^{(-)}{}_{abc}$,	$f_{-ij\bar{k}} = \omega^{(-)}{}_{ij}{}^{k}$,	$f_{-ab\bar{c}} = \omega^{(-)}{}_{ab}{}^{c}$
	$f_{-\bar{i}\bar{j}k} = Q^{(-)ij}{}_{k}$,	$f_{-\bar{a}\bar{b}c} = Q^{(-)ab}{}_{c}$,	$f_{-\bar{i}\bar{j}\bar{k}} = R^{(-)ijk}$,	$f_{-\bar{a}\bar{b}\bar{c}} = R^{(-)abc}$

Most general family (8 params) of SO(4) x SO(4) gaugings of N=4 sugra

- SO(4) x SO(4) N=4 sugra : **AdS** & **dS** vacua (sphere/hyperboloid reductions)

[de Roo, Westra, Panda & Trigiante '03]
[Dibitetto, A.G. & Roest '12]

- \`\`Hybrid ±" sources to cancel flux-induced tadpoles (QC) : *dual branes ?*

[Bergshoeff, de Roo, Kerstan, Ortín & Riccioni '06]

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Future directions

- *X constraint* $X_{\mathcal{MN}}{}^{\mathcal{P}}\,\partial_{\mathcal{P}} = 0$ and mutually BPS states
  [Bossard & Kleinschmidt '15]
  [Bandos '15]

- Connection of *X* deformation to other approaches
  [du Bosque, Hassler, Lüst '15]

- The role of the maximal compact subgroups
  [Bandos & Ortín '15]
  [Aldazabal, Andres, Cámara & Graña '15]

- Flux formulation of SL(2)-DFT : sec. constraint, branes and non-geometry
  [Aldazabal, Graña, Marqués & Rosabal '13]

- SL(2)-DFT and massive IIA sphere reductions to *half-maximal* supergravity
  [Cassani, Felice, Petrini, Strickland-Constable & Waldram '16]

- Cosmological applications of SL(2)-DFT (moduli stab, de Sitter, inflation, …)
  [Hassler, Lüst & Massai '14]

Thank you !!