Holographic RG flows from massive IIA on S^6

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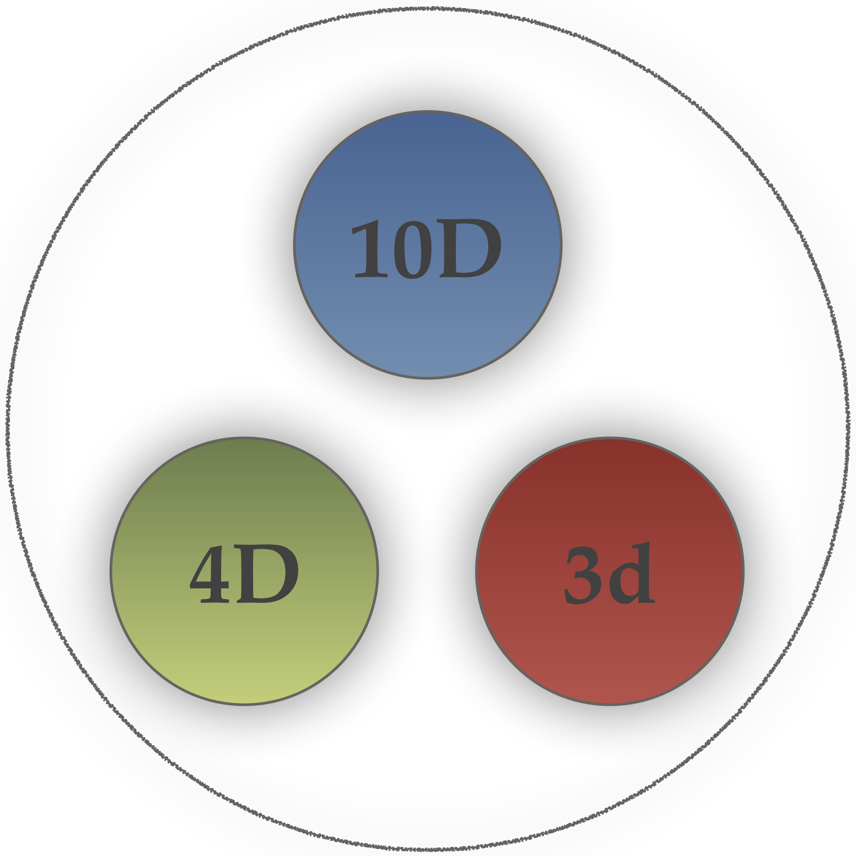


With D. Jafferis , J. Tarrío and O. Varela :

arXiv:1504.08009 , arXiv:1508.04432 , arXiv:1509.02526

arXiv:1605.09254 , arXiv:1703.10833

Outlook

- Motivation
- Deformed SO(8)-gauged supergravity
- Deformed ISO(7)-gauged supergravity
- Massive IIA on S^6 / SYM-CS duality
- Holographic RG flows: domain-walls and black holes

Motivation

electric-magnetic deformations

• The uniqueness of the maximal (N=8) supergravities is historically inherited from their connection to sphere reductions

$AdS_5 \times S^5$ (D3-brane) , $AdS_4 \times S^7$ (M2-brane) , $AdS_7 \times S^4$ (M5-brane)

• N=8 supergravity in 4D admits a deformation parameter c yielding inequivalent theories. It is an electric/magnetic deformation

$$D = \partial - g\,(A^{\mathrm{elec}} - c\,\tilde{A}_{\mathrm{mag}})$$

g = 4D gauge coupling
c = deformation param.

• There are two generic situations :

1) Family of $SO(8)_c$ theories : $c = [0, \sqrt{2} - 1]$ is a continuous param [similar for $SO(p,q)_c$]

2) Family of $ISO(7)_c$ theories : $c = 0$ or 1 is an (on/off) param [same for $ISO(p,q)_c$]

[Dall'Agata, Inverso, Trigiante '12]
[Dall'Agata, Inverso, Marrani '14]

The questions arise:

- Does such an electric/magnetic deformation of 4D maximal supergravity enjoy a string/M-theory origin, or is it just a 4D feature ?

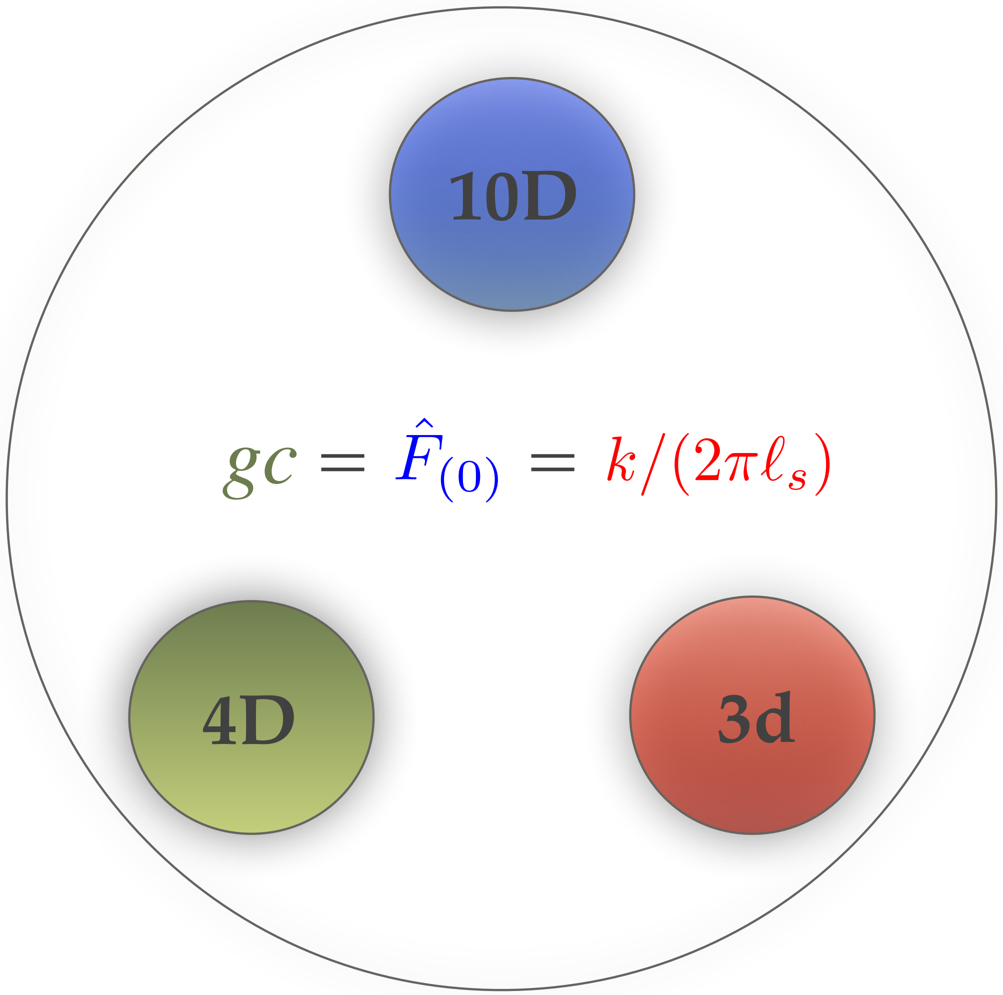
Obstruction for $SO(8)_c$, *cf*. [Lee, Strickland-Constable, Waldram '15]

[de Wit, Nicolai '13]

- For deformed 4D supergravities with supersymmetric AdS_4 vacua, are these AdS_4/CFT_3-dual to any identifiable 3d CFT ?

A new 10D/4D/3d correspondence

massive IIA on S^6 « ISO(7)$_c$-gauged sugra » SU(N)$_k$ CS-SYM theory



$$gc = \hat{F}_{(0)} = k/(2\pi\ell_s)$$

gc = elec/mag deformation in 4D

$\hat{F}_{(0)}$ = Romans mass in 10D

k = Chern-Simons level in 3d

[AG, Jafferis, Varela '15]
[AG, Varela '15]

Well-established and independent dualities :

Type IIB on S^5/N=4 SYM — M-theory on S^7/ABJM — mIIA on S^6/SYM-CS

Deformed SO(8)-gauged supergravity

N = 8 supergravities in 4D

- SUGRA : metric + 8 gravitini + 28 vectors + 56 dilatini + 70 scalars

metric	8 gravitini	28 vectors	56 dilatini	70 scalars
$(s = 2)$	$(s = 3/2)$	$(s = 1)$	$(s = 1/2)$	$(s = 0)$

Ungauged (abelian) supergravity: Reduction of M-theory on a *torus* T^7 down to 4D produces $N = 8$ supergravity with $G = U(1)^{28}$ [Cremmer, Julia '79]

Gauged (non-abelian) supergravity: Reduction of M-theory on a *sphere* S^7 down to 4D produces $N = 8$ supergravity with $G = SO(8)$ [de Wit, Nicolai '82]

✱ SO(8)-gauged supergravity believed to be unique for 30 years…

… but … is this true?

Framework to study N = 8 supergravities in 4D

[de Wit, Samtleben, Trigiante '03 , '07]

Gauging procedure : Part of the global E_7 symmetry group is promoted to a local symmetry group G (gauging)

$[\, \alpha = 1, \ldots, 133 \,]$

Embedding tensor : It is a "selector" specifying which generators of E_7 (there are 133!!) become the gauge symmetry G and, therefore, have associated gauge fields.

Formulation in terms of 56 vectors A_μ^M , though… $M = 1 , \ldots , 56 = 28$ (elec) + 28 (mag)

Sp(56) *Elec/Mag group*

$$A_\mu = A_\mu^M \, \Theta_M{}^\alpha \, t_\alpha$$

Redundancy!!

$$X_M = \Theta_M{}^\alpha \, t_\alpha \quad \Rightarrow \quad [\, X_M \, , \, X_N \,] = X_{MN}{}^P \, X_P \quad \text{with} \quad X_{MN}{}^P = \Theta_M{}^\alpha \, [t_\alpha]_N{}^P$$

✱ Closure of the gauge algebra : $\Omega^{MN} \, \Theta_M{}^\alpha \, \Theta_N{}^\beta = 0$

Only 28 physical l.c. of vectors!!

A family of G = SO(8) supergravities in 4D

- Choose G = SO(8)

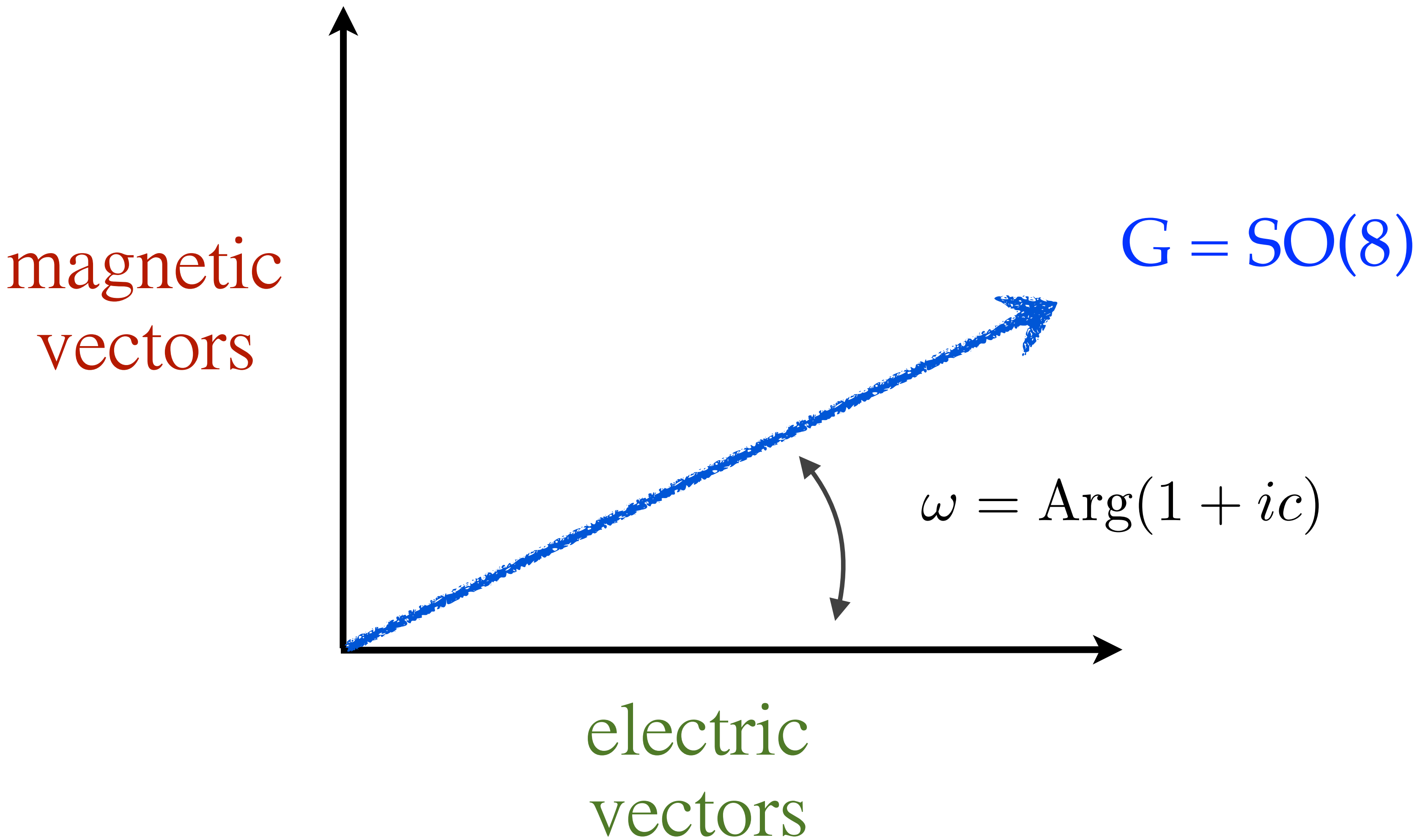
- Solve $\Omega^{MN}\,\Theta_M{}^{\alpha}\,\Theta_N{}^{\beta} = 0$ ⇒ One-parameter (c) family of $SO(8)_c$ theories !!

[Dall' Agata, Inverso, Trigiante '12]

- Immediate questions :

1) What?	(Yes, surprising but true)
2) Are these c-theories equivalent?	(No)
3) Are there new AdS_4 solutions?	(Yes)
4) Higher-dimensional origin?	(Good question…)
5) AdS_4/CFT_3 dual?	(Good question too… ABJ?)

Physical meaning in 4D : electric / magnetic deformation



$$D = \partial - g\,(A^{\mathrm{elec}} - c\,\tilde{A}_{\mathrm{mag}})$$

Physical meaning in 11D …

?

Holographic AdS_4/CFT_3 meaning …

?

[Hull '84 (electric)]

In this talk we are going to investigate the electric/magnetic deformation of a different N=8 supergravity closely related to the $G = SO(8)$ theory …

… the $G = ISO(7) = SO(7) \ltimes \mathbb{R}^7$ supergravity !!

electric/magnetic deformation	higher-dimensional origin	Holographic AdS_4/CFT_3 dual ?
✓	✓	✓

Deformed ISO(7)-gauged supergravity

A family of G = ISO(7) supergravities in 4D

- Choose G = ISO(7)

- Solve $\Omega^{MN}\,\Theta_M{}^{\alpha}\,\Theta_N{}^{\beta} = 0$ ⇨ One-parameter (c) family of $\mathrm{ISO}(7)_c$ theories !!

[Hull '84 (electric)]
[Dall'Agata, Inverso, Marrani '14]

- Immediate questions :

1) What? (Yes, and still surprising)

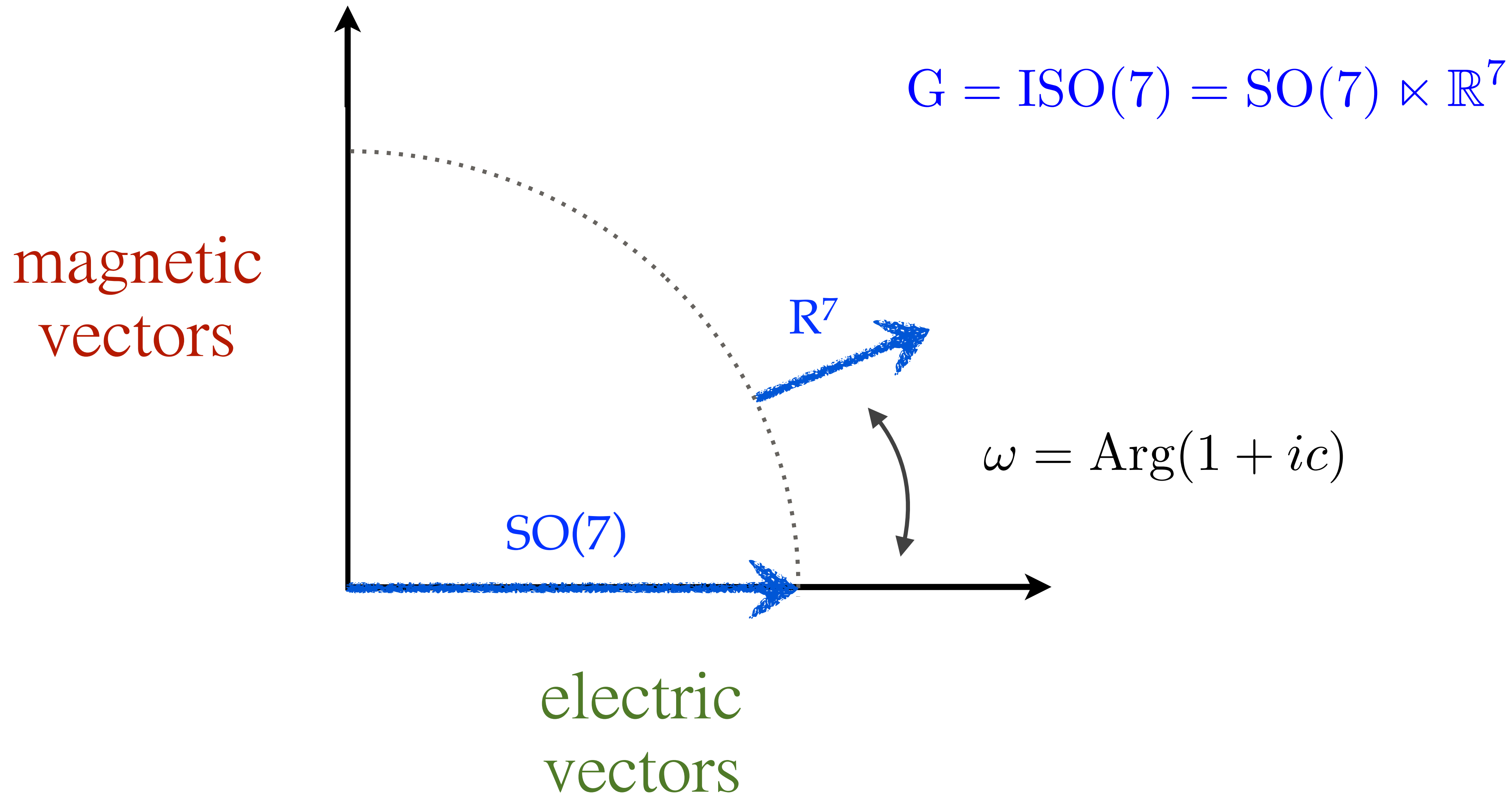
2) Are these c-theories equivalent? (No)

3) Are there new AdS_4 solutions? (Yes)

4) Higher-dimensional origin? (Yes)

5) AdS_4/CFT_3 dual? (Yes)

Physical meaning in 4D = electric / magnetic deformation



$$D = \partial - g\, A^{\mathrm{elec}}_{\mathrm{SO}(7)} - g\, (A^{\mathrm{elec}}_{\mathbb{R}^7} - c\, \tilde{A}_{\mathbb{R}^7\, \mathrm{mag}})$$

Deformed ISO(7)$_c$ Lagrangian $(m = gc)$

$\mathbb{M} = 1, \dots, 56$
$\Lambda = 1, \dots, 28$
$I = 1, \dots, 7$

$$\mathcal{L}_{\text{bos}} = (R - V)\,\text{vol}_4 - \tfrac{1}{48} D\mathcal{M}_{\mathbb{MN}} \wedge *D\mathcal{M}^{\mathbb{MN}} + \tfrac{1}{2}\mathcal{I}_{\Lambda\Sigma}\,\mathcal{H}^{\Lambda}_{(2)} \wedge *\mathcal{H}^{\Sigma}_{(2)} - \tfrac{1}{2}\mathcal{R}_{\Lambda\Sigma}\,\mathcal{H}^{\Lambda}_{(2)} \wedge \mathcal{H}^{\Sigma}_{(2)}$$
$$+\; m\,\Big[\mathcal{B}^I \wedge \big(\tilde{\mathcal{H}}_{(2)I} - \tfrac{g}{2}\delta_{IJ}\mathcal{B}^J\big) - \tfrac{1}{4}\tilde{\mathcal{A}}_I \wedge \tilde{\mathcal{A}}_J \wedge \big(d\mathcal{A}^{IJ} + \tfrac{g}{2}\,\delta_{KL}\,\mathcal{A}^{IK} \wedge \mathcal{A}^{JL}\big)\Big]$$

✦ Setting $m = 0$, all the magnetic pieces in the Lagrangian disappear.

✱ *Ingredients :*

- Electric vectors (21 + 7) : $\mathcal{A}^{IJ} = \mathcal{A}^{[IJ]}$ [SO(7)] and $\mathcal{A}^I$ [$\mathbb{R}^7$] with $\mathcal{H}^{\Lambda}_{(2)} = (\, \mathcal{H}^{IJ}_{(2)} \, , \, \mathcal{H}^{I}_{(2)} \,)$
- Auxiliary magnetic vectors (7) : $\tilde{\mathcal{A}}_I$ [$\mathbb{R}^7$] with $\tilde{\mathcal{H}}_{(2)I}$ field strength
- $E_7/SU(8)$ scalars : $\mathcal{M}_{\mathbb{MN}}$
- Auxiliary two-forms (7) : $\mathcal{B}^I$ [$\mathbb{R}^7$]
- Topological term : $m\,[\,\dots\,]$
- Scalar potential : $V(\mathcal{M}) = \dfrac{g^2}{672} X_{\mathbb{MN}}{}^{\mathbb{R}} X_{\mathbb{PQ}}{}^{\mathbb{S}} \mathcal{M}^{\mathbb{MP}} \big(\mathcal{M}^{\mathbb{NQ}} \mathcal{M}_{\mathbb{RS}} + 7\,\delta^{\mathbb{Q}}_{\mathbb{R}}\,\delta^{\mathbb{N}}_{\mathbb{S}}\big)$

A truncation : $G_0 = SU(3)$ invariant subsector

[Warner '83]

- Truncation : Retaining the fields and couplings which are invariant (singlets) under the action of a subgroup $G_0 \subset \mathrm{ISO}(7)$

- SU(8) R-symmetry branching : gravitini $\mathbf{8} \rightarrow \mathbf{1} + \mathbf{1} + \mathbf{3} + \bar{\mathbf{3}}$ ➡ N = 2 SUSY

- Scalars fields : $\mathbf{70} \rightarrow \mathbf{1}\ (\times 6) + \text{non-singlets}$ ➡ 6 real scalars $(\varphi\,,\,\chi\,;\,\phi\,,\,a\,,\,\zeta\,,\,\tilde{\zeta}\,)$

- Vector fields : $\mathbf{56} \rightarrow \mathbf{1}\ (\times 4) + \text{non-singlets}$ ➡ vectors $(\,A^0, A^1\,;\,\tilde{A}_0, \tilde{A}_1\,)$

- N = 2 gauged supergravity with $\mathrm{G} = \mathrm{U}(1) \times \mathbb{R}_c$ coupled to 1 vector & 1 hypermultiplet

$$\mathcal{M}_{\text{scalar}} = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(2,1)}{\mathrm{U}(2)}$$

The truncated Lagrangian

- The Lagrangian contains a non-dynamical tensor field B^0 :

$\Lambda = 0, 1$

$$\begin{aligned}\mathcal{L} &= (R-V)\,\text{vol}_4 + \tfrac{3}{2}\left[d\varphi \wedge * d\varphi + e^{2\varphi}\, d\chi \wedge * d\chi\right]\\ &+ 2\, d\phi \wedge * d\phi + \tfrac{1}{2}\, e^{2\phi}\left[D\zeta \wedge * D\zeta + D\tilde{\zeta} \wedge * D\tilde{\zeta}\right]\\ &+ \tfrac{1}{2}\, e^{4\phi}\left[Da + \tfrac{1}{2}(\zeta D\tilde{\zeta} - \tilde{\zeta} D\zeta)\right] \wedge * \left[Da + \tfrac{1}{2}(\zeta D\tilde{\zeta} - \tilde{\zeta} D\zeta)\right]\\ &+ \tfrac{1}{2}\,\mathcal{I}_{\Lambda\Sigma}\, H^{\Lambda}_{(2)} \wedge * H^{\Sigma}_{(2)} - \tfrac{1}{2}\,\mathcal{R}_{\Lambda\Sigma}\, H^{\Lambda}_{(2)} \wedge H^{\Sigma}_{(2)} + m\, B^0 \wedge d\tilde{A}_0 + \tfrac{1}{2}\, g\, m\, B^0 \wedge B^0\end{aligned}$$

with field strengths $H^1_{(2)} = dA^1$ and $H^0_{(2)} = dA^0 + m\, B^0$.

- Covariant derivatives :

$$Da = da + g\, A^0 - m\, \tilde{A}_0 \quad , \quad D\zeta = d\zeta - 3\, g\, A^1\, \tilde{\zeta} \quad , \quad D\tilde{\zeta} = d\tilde{\zeta} + 3\, g\, A^1\, \zeta$$

- Scalar potential :

$$\begin{aligned}V &= \tfrac{1}{2}\, g^2 \left[e^{4\phi-3\varphi}(1+e^{2\varphi}\chi^2)^3 - 12\, e^{2\phi-\varphi}(1+e^{2\varphi}\chi^2) - 12\, e^{2\phi+\varphi}\rho^2(1-3\, e^{2\varphi}\chi^2)\right.\\ &\left. - 24\, e^{\varphi} + 12\, e^{4\phi+\varphi}\,\chi^2\,\rho^2\,(1+e^{2\varphi}\chi^2) + 12\, e^{4\phi+\varphi}\rho^4\,(1+3\, e^{2\varphi}\chi^2)\right]\\ &- \tfrac{1}{2}\, g m\, \chi\, e^{4\phi+3\varphi}\,(12\,\rho^2 + 2\chi^2) + \tfrac{1}{2}\, m^2\, e^{4\phi+3\varphi} \ ,\end{aligned}$$

note : $\rho^2 \equiv \frac{1}{4}\,(\zeta^2 + \tilde{\zeta}^2)$

AdS critical points !!

AdS_4 solutions

[AG, Varela '15]

$\mathcal{N}$	G_0	$c^{-1/3}\,\chi$	$c^{-1/3}\,e^{-\varphi}$	$c^{-1/3}\,\rho$	$c^{-1/3}\,e^{-\phi}$	$\frac{1}{4}\,g^{-2}\,c^{1/3}\,V_0$	M^2L^2
$\mathcal{N}=1$	G_2	$-\frac{1}{2^{7/3}}$	$\frac{5^{1/2}\,3^{1/2}}{2^{7/3}}$	$-\frac{1}{2^{7/3}}$	$\frac{5^{1/2}\,3^{1/2}}{2^{7/3}}$	$-\frac{2^{22/3}\,3^{1/2}}{5^{5/2}}$	$4\pm\sqrt{6}\,,\ -\frac{1}{6}(11\pm\sqrt{6})$
$\mathcal{N}=2$	$U(3)$	$-\frac{1}{2}$	$\frac{3^{1/2}}{2}$	0	$\frac{1}{2^{1/2}}$	$-3^{3/2}$	$3\pm\sqrt{17}\,,\ 2\,,\ 2$
$\mathcal{N}=1$	$SU(3)$	$\frac{1}{2^2}$	$\frac{3^{1/2}\,5^{1/2}}{2^2}$	$-\frac{3^{1/2}}{2^2}$	$\frac{5^{1/2}}{2^2}$	$-\frac{2^6\,3^{3/2}}{5^{5/2}}$	$4\pm\sqrt{6}\,,\ 4\pm\sqrt{6}$
$\mathcal{N}=0$	$SO(6)_+$	0	$2^{1/6}$	0	$\frac{1}{2^{5/6}}$	$-3\,2^{5/6}$	$6\,,\ 6\,,\ -\frac{3}{4}\,,\ 0$
$\mathcal{N}=0$	$SO(7)_+$	0	$\frac{1}{5^{1/6}}$	0	$\frac{1}{5^{1/6}}$	$-\frac{3\,5^{7/6}}{2^2}$	$6\,,\ -\frac{12}{5}\,,\ -\frac{6}{5}\,,\ -\frac{6}{5}$
$\mathcal{N}=0$	G_2	$\frac{1}{2^{4/3}}$	$\frac{3^{1/2}}{2^{4/3}}$	$\frac{1}{2^{4/3}}$	$\frac{3^{1/2}}{2^{4/3}}$	$-\frac{2^{10/3}}{3^{1/2}}$	$6\,,\ 6\,,\ -1\,,\ -1$
$\mathcal{N}=0$	$SU(3)$	0.455	0.838	0.335	0.601	-5.864	$6.214\,,\ 5.925\,,\ 1.145\,,\ -1.284$
$\mathcal{N}=0$	$SU(3)$	0.270	0.733	0.491	0.662	-5.853	$6.230\,,\ 5.905\,,\ 1.130\,,\ -1.264$

- N = 2 solution very relevant for holography !!

Massive IIA on S^6 / SYM-CS duality

Collecting clues

- The deformed $\mathrm{ISO}(7)_c$ gauging has its SO(7) piece untouched by the deformation. This points towards an undeformed S^6 description in higher dimension.

- If the higher-dim geometry is not affected, it should then be the higher-dim theory the one changing. The massive IIA theory by Romans proves a natural candidate.

[Romans '86]

- The Romans mass parameter $\hat{F}_{(0)}$ is a discrete (on/off) deformation, exactly as the parameter c in the deformed $\mathrm{ISO}(7)_c$ theory.

Embedding of ISO(7)$_c$ into massive IIA supergravity

[AG, Varela '15]

$$
\begin{aligned}
d\hat{s}_{10}^2 &= \Delta^{-1}\, ds_4^2 \,+ g_{mn}\, Dy^m\, Dy^n \;, \\
\hat{A}_{(3)} &= \mu_I \mu_J \left(\mathcal{C}^{IJ} + \mathcal{A}^I \wedge \mathcal{B}^J + \tfrac{1}{6}\mathcal{A}^{IK} \wedge \mathcal{A}^{JL} \wedge \tilde{\mathcal{A}}_{KL} + \tfrac{1}{6}\mathcal{A}^I \wedge \mathcal{A}^{JK} \wedge \tilde{\mathcal{A}}_K\right) \\
&\quad + g^{-1}\left(\mathcal{B}_J{}^I + \tfrac{1}{2}\mathcal{A}^{IK} \wedge \tilde{\mathcal{A}}_{KJ} + \tfrac{1}{2}\mathcal{A}^I \wedge \tilde{\mathcal{A}}_J\right) \wedge \mu_I D\mu^J + \tfrac{1}{2} g^{-2}\, \tilde{\mathcal{A}}_{IJ} \wedge D\mu^I \wedge D\mu^J \\
&\quad - \tfrac{1}{2}\, \mu_I\, B_{mn}\, \mathcal{A}^I \wedge Dy^m \wedge Dy^n + \tfrac{1}{6} A_{mnp}\, Dy^m \wedge Dy^n \wedge Dy^p \;, \\
\hat{B}_{(2)} &= -\mu_I \left(\mathcal{B}^I + \tfrac{1}{2}\mathcal{A}^{IJ} \wedge \tilde{\mathcal{A}}_J\right) - g^{-1}\, \tilde{\mathcal{A}}_I \wedge D\mu^I + \tfrac{1}{2} B_{mn}\, Dy^m \wedge Dy^n \;, \\
\hat{A}_{(1)} &= -\mu_I\, \mathcal{A}^I + A_m\, Dy^m \;.
\end{aligned}
$$

where we have defined : $\quad Dy^m \equiv dy^m + \tfrac{1}{2}\, g\, K^m_{IJ}\, \mathcal{A}^{IJ} \quad , \quad D\mu^I \equiv d\mu^I - g\, \mathcal{A}^{IJ} \mu_J$

The scalars are embedded as

$$
\begin{aligned}
g^{mn} &= \tfrac{1}{4}\, g^2\, \Delta\, \mathcal{M}^{IJ\,KL}\, K^m_{IJ}\, K^n_{KL} \quad , \quad & B_{mn} &= -\tfrac{1}{2}\, \Delta\, g_{mp}\, K^p_{IJ}\, \partial_n \mu^K\, \mathcal{M}^{IJ}{}_{K8} \;, \\
A_m &= \tfrac{1}{2}\, g\, \Delta\, g_{mn}\, K^n_{IJ}\, \mu_K\, \mathcal{M}^{IJ\,K8} \quad , \quad & A_{mnp} &= \tfrac{1}{8}\, g\, \Delta\, g_{mq}\, K^q_{IJ}\, K^{KL}_{np}\, \mathcal{M}^{IJ}{}_{KL} + A_m B_{np} \;.
\end{aligned}
$$

A new N=2 solution of massive type IIA

- Using the uplifting formulae, it is a straightforward exercise to obtain the 10D embedding of a 4D critical point. An example is the N=2&U(3) AdS_4 point of the $ISO(7)_c$ theory

$$d\hat{s}^2_{10} = L^2 \frac{\left(3+\cos 2\alpha\right)^{\frac{1}{2}}}{\left(5+\cos 2\alpha\right)^{-\frac{1}{8}}} \left[ds^2(\mathrm{AdS}_4) + \frac{3}{2}\, d\alpha^2 + \frac{6\sin^2\alpha}{3+\cos 2\alpha}\, ds^2(\mathbb{CP}^2) + \frac{9\sin^2\alpha}{5+\cos 2\alpha}\, \boldsymbol{\eta}^2 \right] ,$$

$$e^{\hat{\phi}} = e^{\phi_0} \frac{\left(5+\cos 2\alpha\right)^{3/4}}{3+\cos 2\alpha} \quad , \qquad \hat{H}_{(3)} = 24\sqrt{2}\, L^2\, e^{\frac{1}{2}\phi_0} \frac{\sin^3\alpha}{\left(3+\cos 2\alpha\right)^2}\, \boldsymbol{J}\wedge d\alpha \ ,$$

$$L^{-1}\, e^{\frac{3}{4}\phi_0}\, \hat{F}_{(2)} = -4\sqrt{6}\, \frac{\sin^2\alpha\cos\alpha}{\left(3+\cos 2\alpha\right)\left(5+\cos 2\alpha\right)}\, \boldsymbol{J} - 3\sqrt{6}\, \frac{\left(3-\cos 2\alpha\right)}{\left(5+\cos 2\alpha\right)^2}\, \sin\alpha\ d\alpha\wedge\boldsymbol{\eta} \ ,$$

$$L^{-3}\, e^{\frac{1}{4}\phi_0}\, \hat{F}_{(4)} = 6\,\mathrm{vol}_4 + 12\sqrt{3}\, \frac{7+3\cos 2\alpha}{\left(3+\cos 2\alpha\right)^2}\, \sin^4\alpha\ \mathrm{vol}_{\mathbb{CP}^2} + 18\sqrt{3}\, \frac{\left(9+\cos 2\alpha\right)\sin^3\alpha\cos\alpha}{\left(3+\cos 2\alpha\right)\left(5+\cos 2\alpha\right)}\, \boldsymbol{J}\wedge d\alpha\wedge\boldsymbol{\eta} \ ,$$

where we have introduced the quantities $L^2 \equiv 2^{-\frac{5}{8}}\, 3^{-1}\, g^{-2}\, c^{\frac{1}{12}}$ and $e^{\phi_0} \equiv 2^{\frac{1}{4}}\, c^{-\frac{5}{6}}$

✦ The angle $0 \leq \alpha \leq \pi$ locally foliates S^6 with S^5 regarded as Hopf fibrations over $\mathbb{CP}^2$

CFT$_3$ candidate and matching of free energies

[Schwarz '04]
[Gaiotto & Tomassiello '09]

- We propose and N=2 Chern-Simons-matter theory with simple gauge group SU(N), level k and only adjoint matter, as the CFT of the N=2 massive IIA solution.

- The 3d free energy F = -Log(Z), where Z is the partition function on the CFT on a Euclidean S^3 can be computed via localisation over supersymmetric configurations

[Pestun '07] [Jafferis '10]
[Jafferis, Klebanov, Pufu, Safdi '11]

$$Z = \int \prod_{i=1}^{N} \frac{d\lambda_i}{2\pi} \prod_{i<j=1}^{N} \left(2\sinh^2\left(\frac{\lambda_i - \lambda_j}{2}\right)\right) \times \prod_{i,j=1}^{N} \left(\exp\left(\ell\left(\frac{1}{3} + \frac{i}{2\pi}(\lambda_i - \lambda_j)\right)\right)\right)^3 e^{\frac{ik}{4\pi}\sum \lambda_i^2}$$

where λ_i are the Coulomb branch parameters. In the $N \gg k$ limit, the result is given by

$$F = \frac{3^{13/6}\pi}{40}\left(\frac{32}{27}\right)^{2/3} k^{1/3} N^{5/3}$$

- The gravitational free energy can be computed from the warp factor in the N=2 massive IIA solution. Using the charge quantisation condition $N = -(2\pi\ell_s)^{-5}\int_{S^6} e^{\frac{1}{2}\phi}\hat{*}\hat{F}_{(4)} + \hat{B}_{(2)} \wedge d\hat{A}_{(3)} + \frac{1}{6}\hat{F}_{(0)}\hat{B}_{(2)}^3$ for the D2-brane, one finds

$$F = \frac{16\pi^3}{(2\pi\ell_s)^8}\int_{S^6} e^{8A}\mathrm{vol}_6 = \frac{\pi}{5} 2^{1/3} 3^{1/6} k^{1/3} N^{5/3}$$

provided

$$g\,c = \hat{F}_{(0)} = k/(2\pi\ell_s)$$

[Emparan, Johnson, Myers '99]

Holographic RG flows: domain-walls and black holes

Holographic description of RG flows

[Boonstra, Skenderis, Townsend '98]

- RG flows are described holographically as non-AdS_4 solutions in gravity

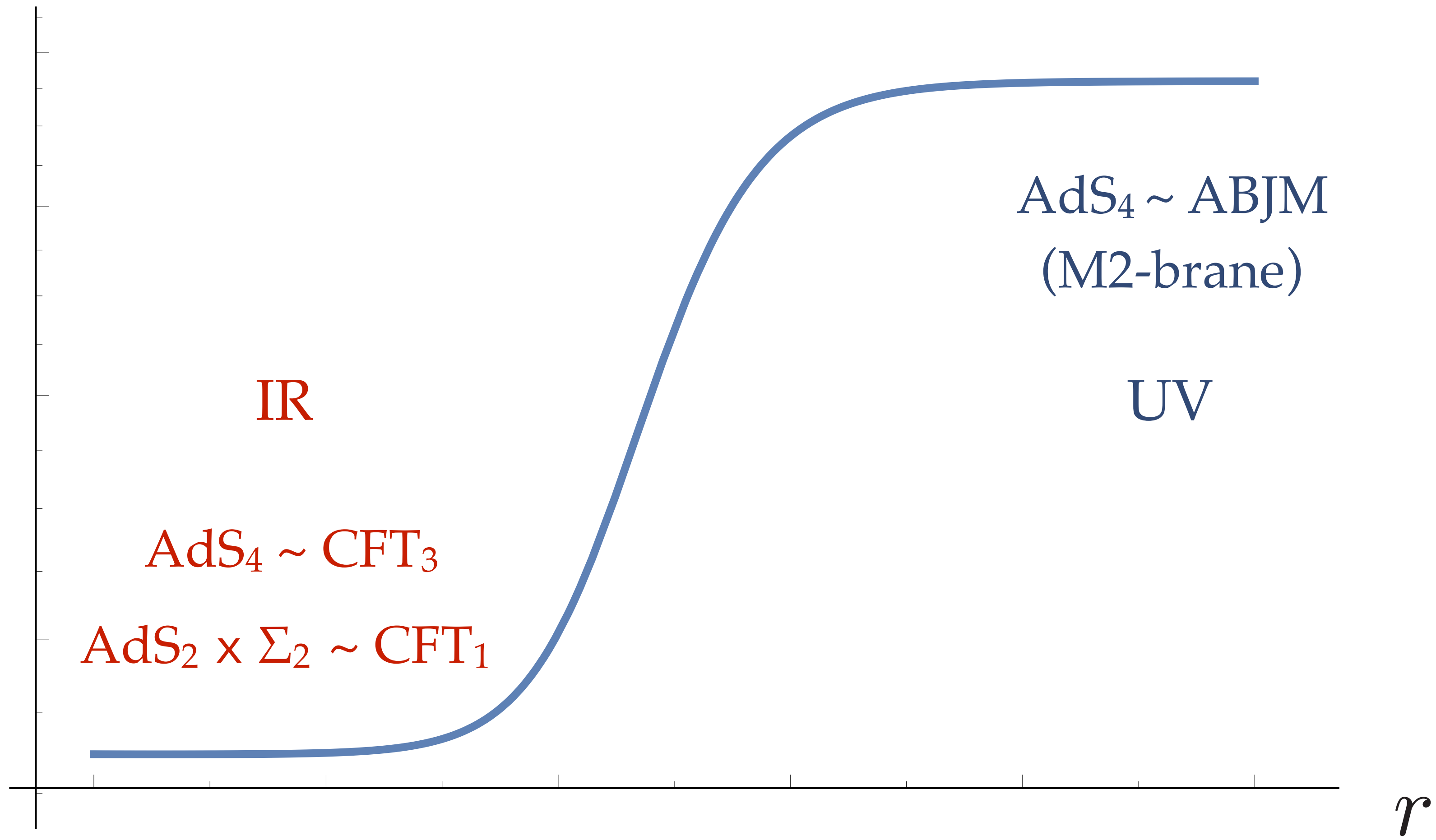
- RG flows on M2-brane : SO(8)-gauged sugra from M-theory on S^7

[Ahn, Paeng '00] [Ahn, Itoh '01]
[Bobev, Halmagyi, Pilch, Warner '09]

[Cacciatori, Klemm '09]
[Halmagyi, Petrino, Zaffaroni '13]
[Chimento, Klemm, Petri '15]
[Benini, Hristov, Zaffaroni '15 '16]

AdS_4 in IR : domain-wall

$AdS_2 \times \Sigma_2$ in IR : black hole



- RG flows on D3-brane : SO(6)-gauged sugra from type IIB on S^5 and N=4 SYM in 4D

[Freedman, Gubser, Pilch, Warner '99]
[Pilch, Warner '00] [Benini, Bobev '12,'13]

Holographic RG flows on the D2-brane of massive IIA

- D2-brane :

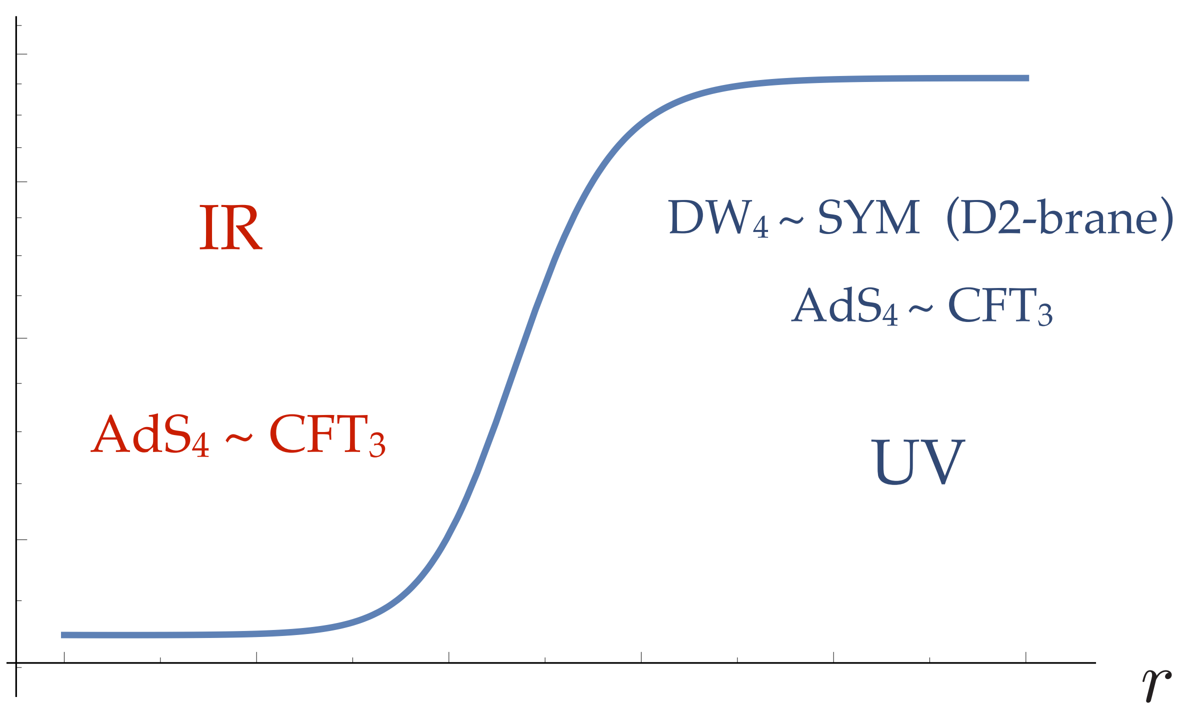
$$d\hat{s}_{10}^2 = e^{\frac{3}{4}\phi}\left(-e^{2U}dt^2 + e^{-2U}dr^2 + e^{2(\psi-U)}ds^2_{\mathrm{H}^2}\right) + g^{-2}e^{-\frac{1}{4}\phi}ds^2_{\mathrm{S}^6}$$

$$e^{\hat{\Phi}} = e^{\frac{5}{2}\phi}$$

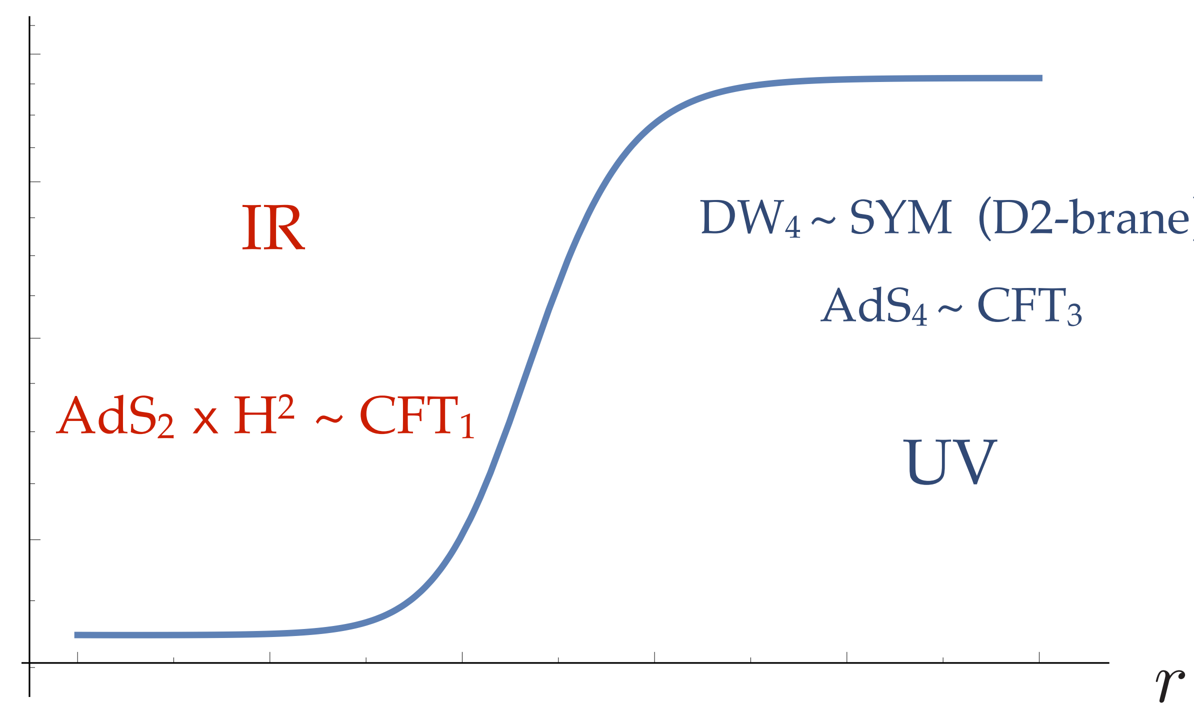
$$\hat{F}_{(4)} = 5\,g\,e^{\phi}\,e^{2(\psi-U)}\,\sinh\theta\,dt\wedge dr\wedge d\theta\wedge d\phi$$

with $e^{2U} \sim r^{\frac{7}{4}}$, $e^{2(\psi-U)} \sim r^{\frac{7}{4}}$ and $e^{\varphi} = e^{\phi} \sim r^{-\frac{1}{4}}$ ⟹ DW_4 domain-wall

- RG flows on D2-brane : ISO(7)-gauged sugra from type IIA on S^6



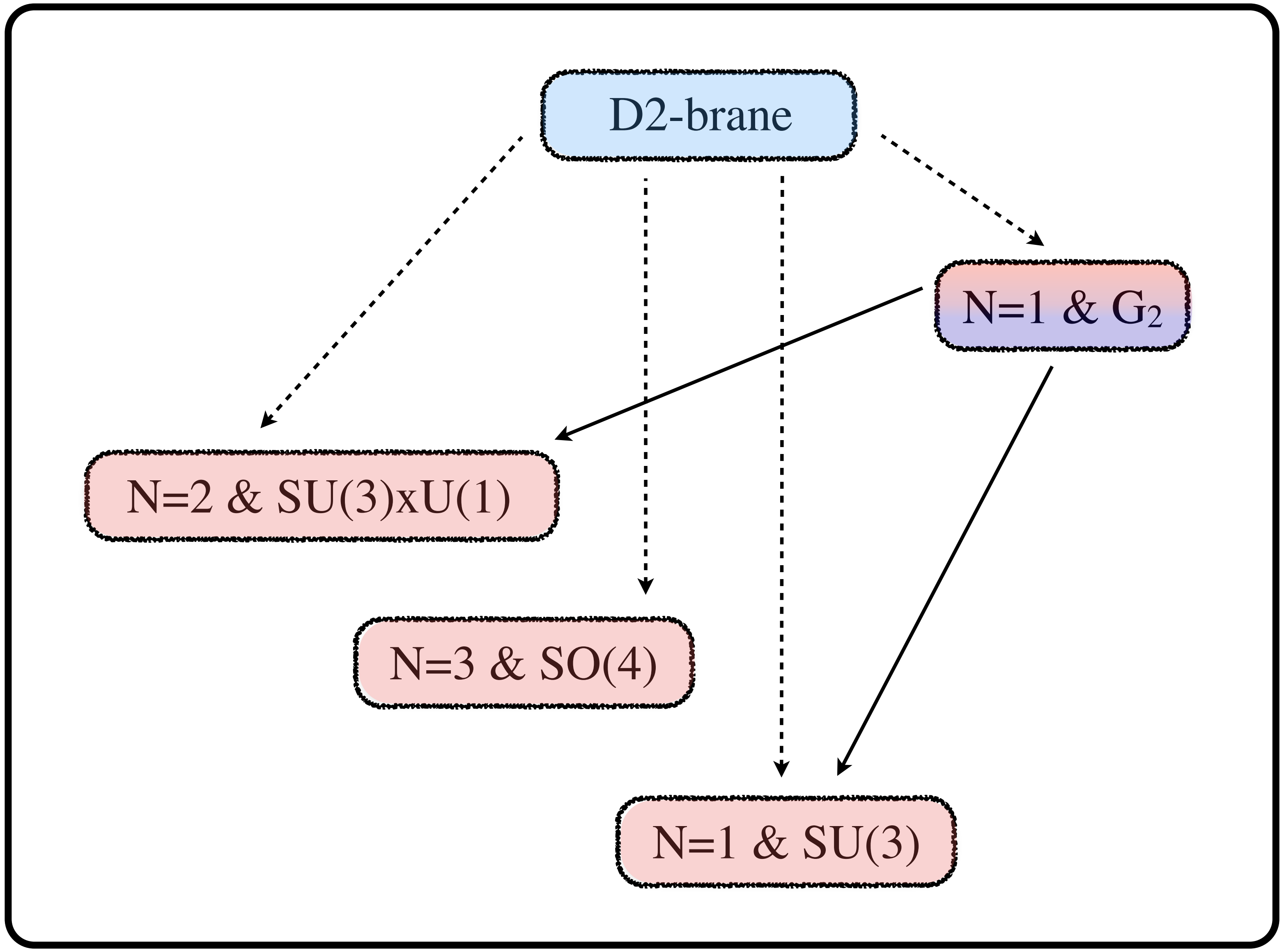
domain-walls



Black holes

Holographic RG flows: domain-walls

[AG, Tarrío, Varela '16]



- RG flows from SYM (dotted lines) and between CFT's (solid lines) dual to BPS domain-wall solutions of the dyonic ISO(7)-gauged supergravity

Holographic RG flows: black hole solutions (I)

$\Lambda = 0, 1$

- Black hole Anstaz :

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}dr^2 + e^{2(\psi(r)-U(r))}\left(d\theta^2 + \left(\frac{\sin\sqrt{\kappa}\,\theta}{\sqrt{\kappa}}\right)^2 d\phi^2\right)$$

$$\mathcal{A}^\Lambda = \mathcal{A}_t{}^\Lambda(r)\,dt - p^\Lambda\,\frac{\cos\sqrt{\kappa}\,\theta}{\kappa}\,d\phi$$

$$\tilde{\mathcal{A}}_\Lambda = \tilde{\mathcal{A}}_{t\,\Lambda}(r)dt - e_\Lambda\,\frac{\cos\sqrt{\kappa}\,\theta}{\kappa}\,d\phi$$

$$\mathcal{B}^0 = b_0(r)\,\frac{\sin\sqrt{\kappa}\,\theta}{\sqrt{\kappa}}\,d\theta\wedge d\phi$$

- Attractor equations :

$$\mathcal{Q} = \kappa\,L^2_{\Sigma_2}\,\Omega\,\mathcal{M}\,\mathcal{Q}^x\,\mathcal{P}^x - 4\,\mathrm{Im}(\bar{\mathcal{Z}}\,\mathcal{V})\ ,$$

$$\frac{L^2_{\Sigma_2}}{L_{\mathrm{AdS}_2}} = -2\,\mathcal{Z}\,e^{-i\beta}\ ,$$

$$\langle\mathcal{K}^u,\mathcal{V}\rangle = 0\ ,$$

[Klemm, Petri, Rabbiosi '16]

- **Unique solution** :

(N=2 & U(3) AdS$_4$ vev's)

$$e^{\varphi_h} = \frac{2}{\sqrt{3}}\left(\frac{g}{m}\right)^{\frac{1}{3}}\ ,\quad \chi_h = -\frac{1}{2}\left(\frac{g}{m}\right)^{-\frac{1}{3}}\ ,\quad e^{\phi_h} = \sqrt{2}\left(\frac{g}{m}\right)^{\frac{1}{3}}\ ,\quad a_h = \zeta_h = \tilde{\zeta}_h = 0\ ,$$

$$p^0 + \frac{1}{2}\,m\,b_0^h = \pm\,\frac{1}{6}\,m^{\frac{2}{3}}\,g^{-\frac{5}{3}}\ ,\quad e_0 + \frac{1}{2}\,g\,b_0^h = \pm\,\frac{1}{6}\,m^{-\frac{1}{3}}\,g^{-\frac{2}{3}}\ ,$$

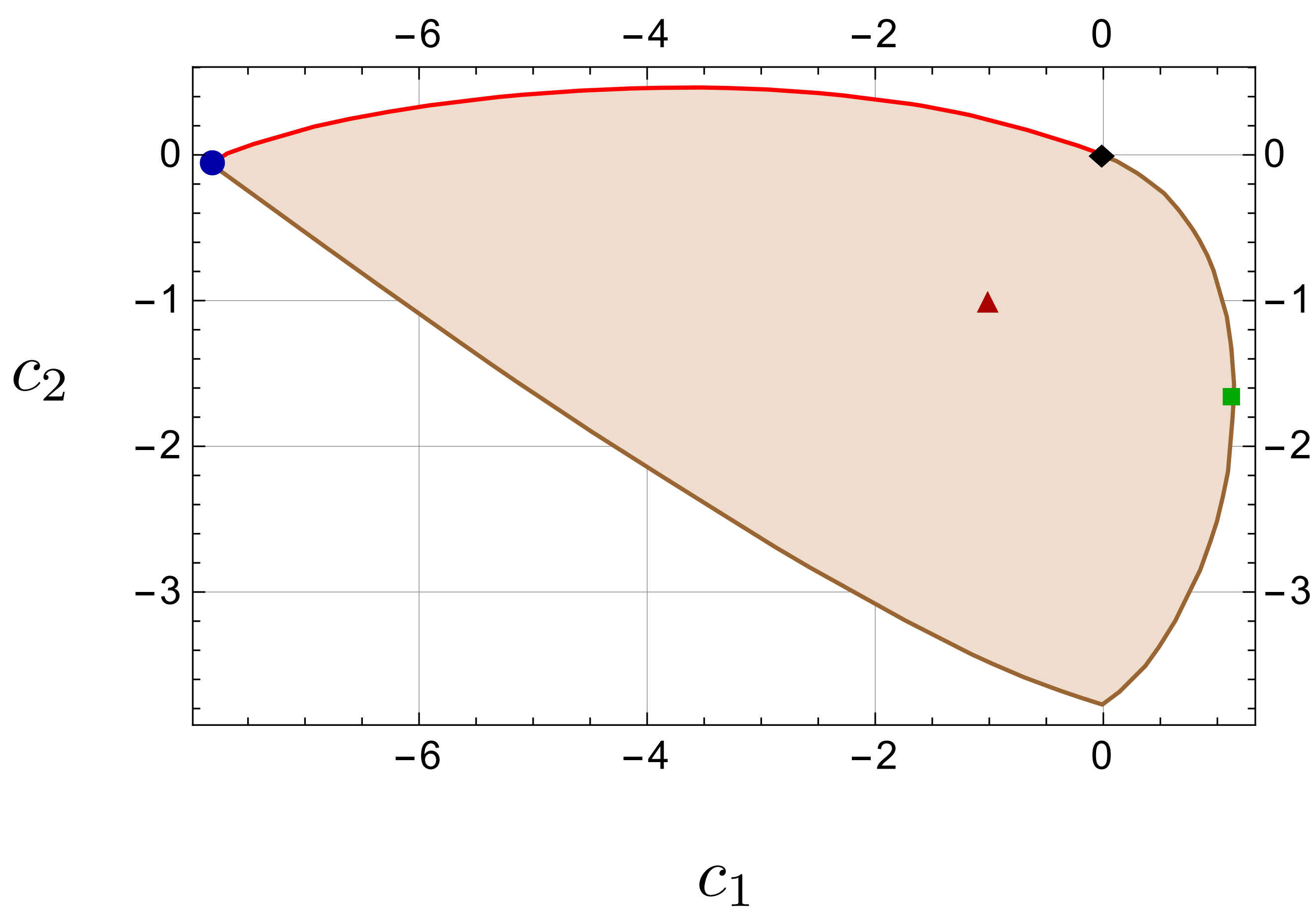
$$p^1 = \mp\,\frac{1}{3}\,g^{-1}\ ,\quad e_1 = \pm\,\frac{1}{2}\,m^{\frac{1}{3}}\,g^{-\frac{4}{3}}\ ,$$

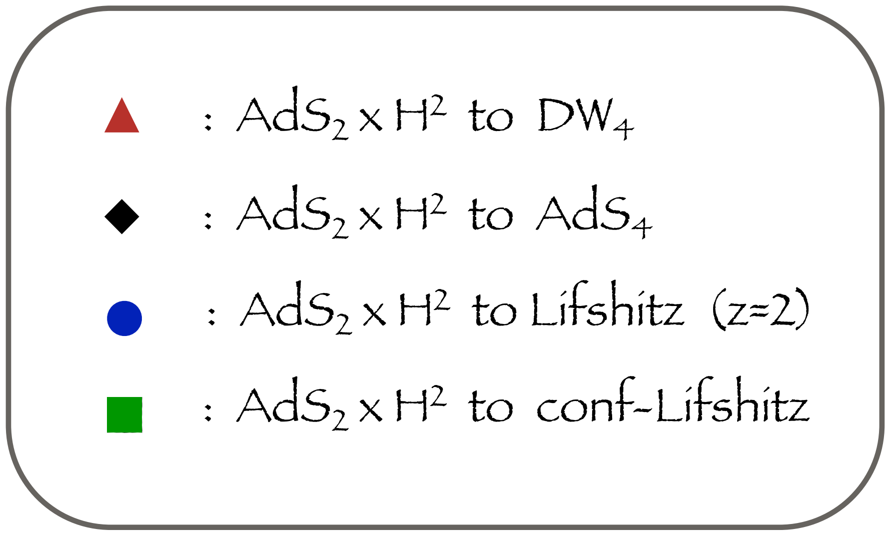
$$L^2_{\mathrm{AdS}_2} = \frac{1}{4\sqrt{3}}\,m^{\frac{1}{3}}\,g^{-\frac{7}{3}}\ ,\quad L^2_{\mathrm{H}^2} = \frac{1}{2\sqrt{3}}\,m^{\frac{1}{3}}\,g^{-\frac{7}{3}}\ .$$

Holographic RG flows: black hole solutions (II)

[AG, Tarrío '17]

- Two irrelevant modes (c_1, c_2) when perturbing around the $AdS_2 \times H^2$ solution in the IR





- RG flows across dimension from SYM or CFT_3 or non-relativistic to CFT_1 dual to BPS black hole solutions of the dyonic ISO(7)-gauged supergravity

Summary

- We connected the dyonic N=8 supergravity with $ISO(7)_c$ gauging to massive IIA reductions on S^6.

- Any 4D configuration (AdS, DW, BH) is embedded into 10D via the uplifting formulas. As an example, we found an $AdS_4 \times S^6$ solution of massive IIA based on an N=2&U(3) AdS_4 vacuum.

- We proposed a CFT_3 dual for the N=2 $AdS_4 \times S^6$ solution of massive IIA based on the D2-brane field theory (SYM-CS). The gravitational and field theory free energies perfectly match provided

$$g\,c \;=\; \hat{F}_{(0)} \;=\; k/(2\pi\ell_s)$$

- RG flows were studied holographically : DW solutions (CFT_3-CFT_3 & SYM_3-CFT_3)
  BH solutions (CFT_3-CFT_1 & SYM_3-CFT_1)

- Flows across dimensions : IR = **unique** solution of the attractor equations
  UV = non-conformal, conformal and non-relativistic behaviours

- Need for an understanding in terms of the field theory on the D2-brane of mIIA !!

[Hosseini, Zaffaroni '16]

Thanks !!

Extra material

More AdS_4 critical points

[here] = arXiv:1508.04432

SUSY	bos. sym.	M^2L^2	stability	ref.
$\mathcal{N}=3$	SO(4)	$3(1\pm\sqrt{3})^{(1)}\ ,\ (1\pm\sqrt{3})^{(6)}\ ,\ -\frac{9}{4}^{(4)}\ ,\ -2^{(18)}\ ,\ -\frac{5}{4}^{(12)}\ ,\ 0^{(22)}$ $(3\pm\sqrt{3})^{(3)}\ ,\ \frac{15}{4}^{(4)}\ ,\ \frac{3}{4}^{(12)}\ ,\ 0^{(6)}$	yes	[30]
$\mathcal{N}=2$	U(3)	$(3\pm\sqrt{17})^{(1)}\ ,\ -\frac{20}{9}^{(12)}\ ,\ -2^{(16)}\ ,\ -\frac{14}{9}^{(18)}\ ,\ 2^{(3)}\ ,\ 0^{(19)}$ $4^{(1)}\ ,\ \frac{28}{9}^{(6)}\ ,\ \frac{4}{9}^{(12)}\ ,\ 0^{(9)}$	yes	[15] , [here]
$\mathcal{N}=1$	G_2	$(4\pm\sqrt{6})^{(1)}\ ,\ -\frac{1}{6}(11\pm\sqrt{6})^{(27)}\ ,\ 0^{(14)}$ $\frac{1}{2}(3\pm\sqrt{6})^{(7)}\ ,\ 0^{(14)}$	yes	[4]
$\mathcal{N}=1$	SU(3)	$(4\pm\sqrt{6})^{(2)}\ ,\ -\frac{20}{9}^{(12)}\ ,\ -2^{(8)}\ ,\ -\frac{8}{9}^{(12)}\ ,\ \frac{7}{9}^{(6)}\ ,\ 0^{(28)}$ $6^{(1)}\ ,\ \frac{28}{9}^{(6)}\ ,\ \frac{25}{9}^{(6)}\ ,\ 2^{(1)}\ ,\ \frac{4}{9}^{(6)}\ ,\ 0^{(8)}$	yes	[here]
$\mathcal{N}=0$	$SO(7)_+$	$6^{(1)}\ ,\ -\frac{12}{5}^{(27)}\ ,\ -\frac{6}{5}^{(35)}\ ,\ 0^{(7)}$ $\frac{12}{5}^{(7)}\ ,\ 0^{(21)}$	no	[3]
$\mathcal{N}=0$	$SO(6)_+$	$6^{(2)}\ ,\ -3^{(20)}\ ,\ -\frac{3}{4}^{(20)}\ ,\ 0^{(28)}$ $6^{(1)}\ ,\ \frac{9}{4}^{(12)}\ ,\ 0^{(15)}$	no	[3]
$\mathcal{N}=0$	G_2	$6^{(2)}\ ,\ -1^{(54)}\ ,\ 0^{(14)}$ $3^{(14)}\ ,\ 0^{(14)}$	yes	[4]
$\mathcal{N}=0$	SU(3)	see (3.44) see (3.45)	yes	[here]
$\mathcal{N}=0$	SU(3)	see (3.46) see (3.47)	yes	[here]
$\mathcal{N}=0$	SO(4)	see (5.12) see (5.13)	yes	[here]

Tensor hierarchy

[de Wit, Nicolai & Samtleben '08]
[Bergshoeff, Hartong, Hohm, Huebscher & Ortin '09]

- Idea: To describe the dynamics of the full ISO(7) theory in terms of a set of p-form fields with $p = 0\,,1\,,2\,,3$ [no Lagrangian!]

- Restricted SL(7)-covariant field content [index I]

$\mathbf{21}' + \mathbf{7}' + \mathbf{21} + \mathbf{7}$	vectors :	$\mathcal{A}^{IJ}\,, \quad \mathcal{A}^{I}\,, \quad \tilde{\mathcal{A}}_{IJ}\,, \quad \tilde{\mathcal{A}}_{I}\,,$	
$\mathbf{48} + \mathbf{7}'$	two-forms :	$\mathcal{B}_I{}^J\,, \quad \mathcal{B}^I\,,$	[out of 133]
$\mathbf{28}'$	three-forms :	$\mathcal{C}^{IJ}\,,$	[out of 912]

- Two-form field strengths [**21′** + **7′** + **21** + **7**]

$$
\begin{aligned}
\mathcal{H}^{IJ}_{(2)} &= d\mathcal{A}^{IJ} - g\,\delta_{KL}\,\mathcal{A}^{IK}\wedge\mathcal{A}^{LJ}\,,\\
\mathcal{H}^{I}_{(2)} &= d\mathcal{A}^{I} - g\,\delta_{JK}\,\mathcal{A}^{IJ}\wedge\mathcal{A}^{K} + \tfrac{1}{2}m\,\mathcal{A}^{IJ}\wedge\tilde{\mathcal{A}}_{J} + m\,\mathcal{B}^{I}\,,\\
\tilde{\mathcal{H}}_{(2)IJ} &= d\tilde{\mathcal{A}}_{IJ} + g\,\delta_{K[I}\,\mathcal{A}^{KL}\wedge\tilde{\mathcal{A}}_{J]L} + g\,\delta_{K[I}\,\mathcal{A}^{K}\wedge\tilde{\mathcal{A}}_{J]} - m\,\tilde{\mathcal{A}}_{I}\wedge\tilde{\mathcal{A}}_{J} + 2g\,\delta_{K[I}\,\mathcal{B}_{J]}{}^{K}\,,\\
\tilde{\mathcal{H}}_{(2)I} &= d\tilde{\mathcal{A}}_{I} - \tfrac{1}{2}g\,\delta_{IJ}\,\mathcal{A}^{JK}\wedge\tilde{\mathcal{A}}_{K} + g\,\delta_{IJ}\,\mathcal{B}^{J}\,,
\end{aligned}
$$

- Three-form field strengths [**48** + **7′**]

$$
\begin{aligned}
\mathcal{H}_{(3)I}{}^{J} \;=\;& D\mathcal{B}_I{}^J + \tfrac{1}{2}\mathcal{A}^{JK}\wedge d\tilde{\mathcal{A}}_{IK} + \tfrac{1}{2}\mathcal{A}^{J}\wedge d\tilde{\mathcal{A}}_{I} + \tfrac{1}{2}\tilde{\mathcal{A}}_{IK}\wedge d\mathcal{A}^{JK} + \tfrac{1}{2}\tilde{\mathcal{A}}_{I}\wedge d\mathcal{A}^{J}\\
&-\tfrac{1}{2}g\,\delta_{KL}\,\mathcal{A}^{JK}\wedge\mathcal{A}^{LM}\wedge\tilde{\mathcal{A}}_{IM} - \tfrac{1}{2}g\,\delta_{KL}\,\mathcal{A}^{JK}\wedge\mathcal{A}^{L}\wedge\tilde{\mathcal{A}}_{I}\\
&+\tfrac{1}{6}g\,\delta_{IK}\,\mathcal{A}^{JL}\wedge\mathcal{A}^{KM}\wedge\tilde{\mathcal{A}}_{LM} - \tfrac{1}{3}g\,\delta_{IK}\,\mathcal{A}^{(J}\wedge\mathcal{A}^{K)L}\wedge\tilde{\mathcal{A}}_{L}\\
&-\tfrac{1}{2}m\,\mathcal{A}^{JK}\wedge\tilde{\mathcal{A}}_{I}\wedge\tilde{\mathcal{A}}_{K} - 2g\,\delta_{IK}\,\mathcal{C}^{JK} - \tfrac{1}{7}\,\delta_I^J\,(\text{trace})\;,\\
\mathcal{H}^{I}_{(3)} \;=\;& D\mathcal{B}^{I} - \tfrac{1}{2}\mathcal{A}^{IJ}\wedge d\tilde{\mathcal{A}}_{J} - \tfrac{1}{2}\tilde{\mathcal{A}}_{J}\wedge d\mathcal{A}^{IJ} + \tfrac{1}{2}g\,\delta_{JK}\,\mathcal{A}^{IJ}\wedge\mathcal{A}^{KL}\wedge\tilde{\mathcal{A}}_{L}\;,
\end{aligned}
$$

- Four-form field strengths [**28′**]

$$
\begin{aligned}
\mathcal{H}^{IJ}_{(4)} \;=\;& D\mathcal{C}^{IJ} - \mathcal{H}^{K(I}_{(2)}\wedge\mathcal{B}_K{}^{J)} + \mathcal{H}^{(I}_{(2)}\wedge\mathcal{B}^{J)} - \tfrac{1}{2}m\,\mathcal{B}^{I}\wedge\mathcal{B}^{J} - \tfrac{1}{6}\mathcal{A}^{K(I}\wedge\tilde{\mathcal{A}}_{KL}\wedge d\mathcal{A}^{J)L}\\
&+\tfrac{1}{6}\mathcal{A}^{IK}\wedge\mathcal{A}^{JL}\wedge d\tilde{\mathcal{A}}_{KL} - \tfrac{1}{6}\mathcal{A}^{K(I}\wedge\tilde{\mathcal{A}}_{K}\wedge d\mathcal{A}^{J)} - \tfrac{1}{3}\mathcal{A}^{K(I}\wedge\mathcal{A}^{J)}\wedge d\tilde{\mathcal{A}}_{K}\\
&-\tfrac{1}{6}\mathcal{A}^{(I}\wedge\tilde{\mathcal{A}}_{K}\wedge d\mathcal{A}^{J)K} - \tfrac{1}{6}g\,\delta_{KL}\,\mathcal{A}^{K(I}\wedge\mathcal{A}^{J)M}\wedge\mathcal{A}^{LN}\wedge\tilde{\mathcal{A}}_{MN}\\
&+\tfrac{1}{6}g\,\delta_{KL}\,\mathcal{A}^{K(I}\wedge\mathcal{A}^{J)}\wedge\mathcal{A}^{LM}\wedge\tilde{\mathcal{A}}_{M} - \tfrac{1}{6}g\,\delta_{KL}\,\mathcal{A}^{K(I}\wedge\mathcal{A}^{J)M}\wedge\mathcal{A}^{L}\wedge\tilde{\mathcal{A}}_{M}\\
&-\tfrac{1}{8}m\,\mathcal{A}^{IK}\wedge\mathcal{A}^{JL}\wedge\tilde{\mathcal{A}}_{K}\wedge\tilde{\mathcal{A}}_{L}\;.
\end{aligned}
$$

Consistency checks

- Closed set of Bianchi identities

$$D\mathcal{H}^{IJ}_{(2)} = 0 \ , \ \ D\mathcal{H}^{I}_{(2)} = m\,\mathcal{H}^{I}_{(3)} \ , \ \ D\tilde{\mathcal{H}}_{(2)IJ} = -2\,g\,\mathcal{H}_{(3)[I}{}^{K}\,\delta_{J]K} \ , \ \ D\tilde{\mathcal{H}}_{(2)I} = g\,\delta_{IJ}\,\mathcal{H}^{J}_{(3)} \ ,$$

$$D\mathcal{H}_{(3)I}{}^{J} = \mathcal{H}^{JK}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)IK} + \mathcal{H}^{J}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)I} - 2g\,\delta_{IK}\,\mathcal{H}^{JK}_{(4)} - \tfrac{1}{7}\,\delta^{J}_{I}\,(\text{trace}) \ ,$$

$$D\mathcal{H}^{I}_{(3)} = -\mathcal{H}^{IJ}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)J} \quad , \qquad D\mathcal{H}^{IJ}_{(4)} \equiv 0 \ .$$

- Closed set of duality relations [right number of d.o.f] [short-hand notation]

$$\tilde{\mathcal{H}}_{(2)IJ} = -\tfrac{1}{2}\mathcal{I}_{[IJ][KL]} * \mathcal{H}^{KL}_{(2)} - \mathcal{I}_{[IJ][K8]} * \mathcal{H}^{K}_{(2)} + \tfrac{1}{2}\mathcal{R}_{[IJ][KL]}\,\mathcal{H}^{KL}_{(2)} + \mathcal{R}_{[IJ][K8]}\,\mathcal{H}^{K}_{(2)} \ ,$$

$$\tilde{\mathcal{H}}_{(2)I} = -\tfrac{1}{2}\mathcal{I}_{[I8][KL]} * \mathcal{H}^{KL}_{(2)} - \mathcal{I}_{[I8][K8]} * \mathcal{H}^{K}_{(2)} + \tfrac{1}{2}\mathcal{R}_{[I8][KL]}\,\mathcal{H}^{KL}_{(2)} + \mathcal{R}_{[I8][K8]}\,\mathcal{H}^{K}_{(2)} \ ,$$

$$\mathcal{H}_{(3)I}{}^{J} = -\tfrac{1}{12}(t_{I}{}^{J})_{\mathbb{M}}{}^{\mathbb{P}}\,\mathcal{M}_{\mathbb{NP}} * D\mathcal{M}^{\mathbb{MN}} - \tfrac{1}{7}\,\delta^{J}_{I}\,(\text{trace}) \ ,$$

$$\mathcal{H}_{(3)}{}^{I} = -\tfrac{1}{12}(t_{8}{}^{I})_{\mathbb{M}}{}^{\mathbb{P}}\,\mathcal{M}_{\mathbb{NP}} * D\mathcal{M}^{\mathbb{MN}} \ ,$$

$$\mathcal{H}^{IJ}_{(4)} = \tfrac{1}{84}X_{\mathbb{NQ}}{}^{\mathbb{S}}\big((t_{K}{}^{(I|})_{\mathbb{P}}{}^{\mathbb{R}}\mathcal{M}^{|J)K\,\mathbb{N}} + (t_{8}{}^{(I|})_{\mathbb{P}}{}^{\mathbb{R}}\mathcal{M}^{|J)8\,\mathbb{N}}\big)\big(\mathcal{M}^{\mathbb{PQ}}\mathcal{M}_{\mathbb{RS}} + 7\,\delta^{\mathbb{P}}_{\mathbb{S}}\,\delta^{\mathbb{Q}}_{\mathbb{R}}\big)\text{vol}_4 \ .$$

[t's are $SL(7) \times R^7$ generators]

- Closed set of SUSY transformations

SUSY transformations of the tensor hierarchy

[vielbein and scalars]

$$\delta e_\mu{}^\alpha = \tfrac{1}{4}\,\bar\epsilon_i\,\gamma^\alpha\,\psi_\mu{}^i + \tfrac{1}{4}\,\bar\epsilon^i\,\gamma^\alpha\,\psi_{\mu i}$$

$$\delta\mathcal{V}_{\mathbb{M}}{}^{ij} = \tfrac{1}{\sqrt{2}}\,\mathcal{V}_{\mathbb{M}\,kl}\,\left(\bar\epsilon^{[i}\,\chi^{jkl]} + \tfrac{1}{4!}\,\varepsilon^{ijklmnpq}\,\bar\epsilon_m\,\chi_{npq}\right)$$

[vectors]

$$\delta\mathcal{A}_\mu{}^{IJ} = i\,\mathcal{V}^{IJ}{}_{ij}\left(\bar\epsilon^i\psi_\mu{}^j + \tfrac{1}{2\sqrt{2}}\,\bar\epsilon_k\gamma_\mu\chi^{ijk}\right) + \text{h.c.}$$

$$\delta\mathcal{A}_\mu{}^{I} = i\,\mathcal{V}^{I8}{}_{ij}\left(\bar\epsilon^i\psi_\mu{}^j + \tfrac{1}{2\sqrt{2}}\,\bar\epsilon_k\gamma_\mu\chi^{ijk}\right) + \text{h.c.}$$

$$\delta\tilde{\mathcal{A}}_{\mu\,IJ} = -i\,\tilde{\mathcal{V}}_{IJ\,ij}\left(\bar\epsilon^i\psi_\mu{}^j + \tfrac{1}{2\sqrt{2}}\,\bar\epsilon_k\gamma_\mu\chi^{ijk}\right) + \text{h.c.}$$

$$\delta\tilde{\mathcal{A}}_{\mu\,I} = -i\,\tilde{\mathcal{V}}_{I8\,ij}\left(\bar\epsilon^i\psi_\mu{}^j + \tfrac{1}{2\sqrt{2}}\,\bar\epsilon_k\gamma_\mu\chi^{ijk}\right) + \text{h.c.}$$

[two-forms]

$$\begin{aligned}\delta\mathcal{B}_{\mu\nu\,J}{}^{I} = \Big[&-\tfrac{2}{3}\big(\mathcal{V}^{IK}{}_{jk}\,\tilde{\mathcal{V}}_{JK}{}^{ik} + \mathcal{V}^{I8}{}_{jk}\,\tilde{\mathcal{V}}_{J8}{}^{ik} + \tilde{\mathcal{V}}_{JK\,jk}\,\mathcal{V}^{IK\,ik} + \tilde{\mathcal{V}}_{J8\,jk}\,\mathcal{V}^{I8ik}\big)\bar\epsilon_i\gamma_{[\mu}\psi^j_{\nu]} \\ &-\tfrac{\sqrt{2}}{3}\,\big(\mathcal{V}^{IK}{}_{ij}\,\tilde{\mathcal{V}}_{JK\,kl} + \mathcal{V}^{I8}{}_{ij}\,\tilde{\mathcal{V}}_{J8\,kl}\big)\,\bar\epsilon^{[i}\gamma_{\mu\nu}\chi^{jkl]} + \text{h.c.}\Big] \\ &+\big(\mathcal{A}^{IK}_{[\mu}\,\delta\tilde{\mathcal{A}}_{\nu]JK} + \mathcal{A}^{I}_{[\mu}\,\delta\tilde{\mathcal{A}}_{\nu]J} + \tilde{\mathcal{A}}_{[\mu|\,JK}\,\delta\mathcal{A}_{|\nu]}{}^{IK} + \tilde{\mathcal{A}}_{[\mu|\,J}\,\delta\mathcal{A}_{|\nu]}{}^{I}\big) - \tfrac{1}{7}\,\delta^I_J\,(\text{trace})\,,\end{aligned}$$

$$\begin{aligned}\delta\mathcal{B}_{\mu\nu}{}^{I} = \Big[&\tfrac{2}{3}\big(\mathcal{V}^{IJ}{}_{jk}\,\tilde{\mathcal{V}}_{J8}{}^{ik} + \tilde{\mathcal{V}}_{J8\,jk}\,\mathcal{V}^{IJik}\big)\,\bar\epsilon_i\gamma_{[\mu}\psi^j_{\nu]} + \tfrac{\sqrt{2}}{3}\,\mathcal{V}^{IJ}{}_{ij}\,\tilde{\mathcal{V}}_{J8\,kl}\,\bar\epsilon^{[i}\gamma_{\mu\nu}\chi^{jkl]} + \text{h.c.}\Big] \\ &-\big(\mathcal{A}^{IJ}_{[\mu}\,\delta\tilde{\mathcal{A}}_{\nu]J} + \tilde{\mathcal{A}}_{[\mu|\,J}\,\delta\mathcal{A}_{|\nu]}{}^{IJ}\big)\,.\end{aligned}$$

[three-forms]

$$
\begin{aligned}
\delta\mathcal{C}_{\mu\nu\rho}{}^{IJ} = \Big[- \tfrac{4i}{7} \Big(& \mathcal{V}^{K(I}{}_{jl}\big(\mathcal{V}^{J)L\,lk}\,\tilde{\mathcal{V}}_{KL\,ik} + \tilde{\mathcal{V}}_{KL}{}^{lk}\,\mathcal{V}^{J)L}{}_{ik}\big) \\
& + \mathcal{V}^{K(I}{}_{jl}\big(\mathcal{V}^{J)8\,lk}\,\tilde{\mathcal{V}}_{K8\,ik} + \tilde{\mathcal{V}}_{K8}{}^{lk}\,\mathcal{V}^{J)8}{}_{ik}\big) \\
& + \mathcal{V}^{(I|8}{}_{jl}\big(\mathcal{V}^{|J)K\,lk}\,\tilde{\mathcal{V}}_{K8\,ik} + \tilde{\mathcal{V}}_{K8}{}^{lk}\,\mathcal{V}^{|J)K}{}_{ik}\big)\Big)\,\bar{\epsilon}^{i}\gamma_{[\mu\nu}\psi^{j}_{\rho]} \\
+ i\tfrac{\sqrt{2}}{3}\Big(& \mathcal{V}^{K(I|hi}\,\mathcal{V}^{|J)L}{}_{[ij|}\,\tilde{\mathcal{V}}_{KL|kl]} + \mathcal{V}^{K(Ihi}\,\mathcal{V}^{J)8}{}_{[ij|}\,\tilde{\mathcal{V}}_{K8|kl]} \\
& + \mathcal{V}^{(I|8hi}\,\mathcal{V}^{|J)K}{}_{[ij|}\,\tilde{\mathcal{V}}_{K8|kl]}\Big)\,\bar{\epsilon}_{h}\gamma_{\mu\nu\rho}\chi^{jkl} \;+\text{h.c.}\Big] \\
- 3\,\Big(& \mathcal{B}_{[\mu\nu|K}{}^{(I}\,\delta\mathcal{A}^{J)K}_{|\rho]} + \mathcal{B}_{[\mu\nu}{}^{(I}\,\delta\mathcal{A}^{J)}_{\rho]}\Big) \\
+ \mathcal{A}^{K(I}_{[\mu}\big(& \mathcal{A}^{J)L}_{\nu}\,\delta\tilde{\mathcal{A}}_{\rho]KL} + \tilde{\mathcal{A}}_{\nu KL}\,\delta\mathcal{A}^{J)L}_{\rho]}\big) + \mathcal{A}^{K(I}_{[\mu}\big(\mathcal{A}^{J)}_{\nu}\,\delta\tilde{\mathcal{A}}_{\rho]K} + \tilde{\mathcal{A}}_{\nu K}\,\delta\mathcal{A}^{J)}_{\rho]}\big) \\
& + \mathcal{A}^{(I}_{[\mu}\big(\mathcal{A}^{J)K}_{\nu}\,\delta\tilde{\mathcal{A}}_{\rho]K} + \tilde{\mathcal{A}}_{\nu K}\,\delta\mathcal{A}^{J)K}_{\rho]}\big)\ .
\end{aligned}
$$

… all scalars, vectors and the fermions should be kept !!

Scalar potential and three-form potentials

- How does the scalar potential potential V fit in the duality hierarchy ?

$$\Theta_{\mathbb{M}}{}^{\alpha}\,\mathcal{H}_{(4)\alpha}{}^{\mathbb{M}} = -2\,V\,\text{vol}_4$$

\- In our deformed ISO(7)$_c$ theory, one has four-form field strengths

$$g\,\delta_{IJ}\,\mathcal{H}^{IJ}_{(4)} + m\,\tilde{\mathcal{H}}_{(4)} = -2\,V\,\text{vol}_4 \qquad [\ \mathbf{28'} + \mathbf{1} \text{ of SL(7) }]$$

where we need the **SL(7)-*singlet*** four-form field strength $\tilde{\mathcal{H}}_{(4)}$ dual to the magnetic ET

\- Consistency requires also the three-form field strength $\mathcal{H}_{(3)}$ rendering $\mathcal{H}_{(3)I}{}^{J}$ traceful

$$\begin{aligned} \mathcal{H}_{(3)} &= \tfrac{1}{12}(t_8{}^8)_{\mathbb{M}}{}^{\mathbb{P}}\,\mathcal{M}_{\mathbb{N}\mathbb{P}} * D\mathcal{M}^{\mathbb{M}\mathbb{N}}\ , \\ \tilde{\mathcal{H}}_{(4)} &= \tfrac{1}{84}\,X_{\mathbb{N}\mathbb{Q}}{}^{\mathbb{S}}(t_8{}^K)_{\mathbb{P}}{}^{\mathbb{R}}\mathcal{M}_{8K}{}^{\mathbb{N}}\mathcal{M}^{\mathbb{P}\mathbb{Q}}\mathcal{M}_{\mathbb{R}\mathbb{S}}\,\text{vol}_4 \end{aligned}$$

* Extended BI's :

$$\begin{aligned} D\mathcal{H}_{(3)} &= \mathcal{H}^{IJ}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)IJ} + \mathcal{H}^{I}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)I} - 2g\,\delta_{IJ}\,\mathcal{H}^{IJ}_{(4)} - 14\,m\,\tilde{\mathcal{H}}_{(4)} \\ D\tilde{\mathcal{H}}_{(4)} &\equiv 0\ , \end{aligned}$$

Derivation of the IIA embedding [4-step process]

* Step 1 : 10D KK decomposition that leaves 4D spacetime symmetry manifest

$A(x,y)$'s and $B(x,y)$'s fields

* Step 2 : Redefinitions of the A's and B's fields to conform 4D SUSY transformations

$C(x,y)$'s fields

* Step 3 : Connection to actual 4D fields by dressing up with S^6 geometrical data

$$C(x,y) = \text{geometry}(y) \times \mathcal{C}(x)$$

* Step 4 : Plug and play

[Following prescription by de Wit & Nicolai '86, Godazgar, Godazgar & Nicolai '13, …]

∗ Step 1 : 10D redefinitions (KK decomp) that leave 4D spacetime symmetry manifest

$$\mathrm{SO}(1,9) \rightarrow \mathrm{SO}(1,3) \times \mathrm{SO}(6)$$

Then one has :

$$\begin{aligned}
d\hat{s}^2_{10} &= \Delta^{-1}\, ds^2_4 + g_{mn}\big(dy^m + B^m\big)\big(dy^n + B^n\big) \ , \\
\hat{A}_{(3)} &= \tfrac{1}{6} A_{\mu\nu\rho}\, dx^\mu \wedge dx^\nu \wedge dx^\rho + \tfrac{1}{2} A_{\mu\nu m}\, dx^\mu \wedge dx^\nu \wedge \big(dy^m + B^m\big) \\
&\quad + \tfrac{1}{2} A_{\mu mn}\, dx^\mu \wedge \big(dy^m + B^m\big) \wedge \big(dy^n + B^n\big) \\
&\quad + \tfrac{1}{6} A_{mnp}\big(dy^m + B^m\big) \wedge \big(dy^n + B^n\big) \wedge \big(dy^p + B^p\big) \ , \\
\hat{B}_{(2)} &= \tfrac{1}{2} B_{\mu\nu}\, dx^\mu \wedge dx^\nu + B_{\mu m}\, dx^\mu \wedge \big(dy^m + B^m\big) + \tfrac{1}{2} B_{mn}\big(dy^m + B^m\big) \wedge \big(dy^n + B^n\big) \ , \\
\hat{A}_{(1)} &= A_\mu\, dx^\mu + A_m\big(dy^m + B^m\big) \ ,
\end{aligned}$$

In terms of representations of SL(6) [index m] :

$\mathbf{1}$	metric :	ds^2_4 ,
$\mathbf{21} + \mathbf{6} + \mathbf{1} + \mathbf{20} + \mathbf{15}$	scalars :	g_{mn} , A_m , $\hat{\phi}$, A_{mnp} , B_{mn} ,
$\mathbf{6}' + \mathbf{1} + \mathbf{15} + \mathbf{6}$	vectors :	$B_\mu{}^m$, A_μ , $A_{\mu mn}$, $B_{\mu m}$,
$\mathbf{6} + \mathbf{1}$	two-forms :	$A_{\mu\nu m}$, $B_{\mu\nu}$,
$\mathbf{1}$	three-form :	$A_{\mu\nu\rho}$.

$$\Delta^2 \equiv \frac{\det\ g_{mn}}{\det\ \mathring{g}_{mn}}$$

∗ Step 2 : Non-linear field redefinitions to conform 4D SUSY transformations

- Vectors : $C_\mu{}^{m8} \equiv B_\mu{}^m \quad , \quad C_\mu{}^{78} \equiv A_\mu \quad , \quad \tilde{C}_{\mu\, mn} \equiv A_{\mu mn} - A_\mu B_{mn} \quad , \quad \tilde{C}_{\mu\, m7} \equiv B_{\mu m}$
- Two-forms : $C_{\mu\nu\, m}{}^8 \equiv -A_{\mu\nu m} + C_{[\mu}{}^{n8}\, \tilde{C}_{\nu]nm} + C_{[\mu}{}^{78}\, \tilde{C}_{\nu]m7} \quad , \quad C_{\mu\nu\, 7}{}^8 \equiv -B_{\mu\nu} + C_{[\mu}{}^{m8}\, \tilde{C}_{\nu]m7}$
- Three-form : $C_{\mu\nu\rho}{}^{88} \equiv A_{\mu\nu\rho} - C_{[\mu}{}^{m8}\, C_\nu{}^{n8}\, \tilde{C}_{\rho]mn} + C_{[\mu}{}^{m8}\, C_\nu{}^{78}\, \tilde{C}_{\rho]m7} + 3\, C_{[\mu}{}^{78} C_{\nu\rho]7}{}^8$

These can be rearranged into representations of SL(7) [index *I*]

$C_\mu{}^{I8} = (C_\mu{}^{m8}\, ,\, C_\mu{}^{78}) \qquad \tilde{C}_{\mu\, IJ} = (\tilde{C}_{\mu\, mn}\, ,\, \tilde{C}_{\mu\, m7}) \qquad C_{\mu\nu\, I}{}^8 = (C_{\mu\nu\, m}{}^8\, ,\, C_{\mu\nu\, 7}{}^8) \qquad C_{\mu\nu\rho}{}^{88}$

with 10D SUSY transfs:

$$\delta C_\mu{}^{I8} = i\, V^{I8}{}_{AB} \left(\bar{\epsilon}^A \psi_\mu{}^B + \tfrac{1}{2\sqrt{2}}\, \bar{\epsilon}_C \gamma_\mu \chi^{ABC} \right) + \text{h.c.}\, ,$$

$$\delta \tilde{C}_{\mu\, IJ} = -i\, V_{IJ\, AB} \left(\bar{\epsilon}^A \psi_\mu{}^B + \tfrac{1}{2\sqrt{2}}\, \bar{\epsilon}_C \gamma_\mu \chi^{ABC} \right) + \text{h.c.}\, ,$$

$\ldots$

Mimicking the 4D tensor hierarchy !!

The result is then a set of SL(7)-covariant 10D fields :

$$
\begin{array}{rcl}
\mathbf{1} & \text{metric :} & ds_4^2\,(x,y) \;, \\
\mathbf{7}' + \mathbf{21} & \text{generalised vielbeine :} & V^{I8}{}_{AB}(x,y) \;,\; \tilde{V}_{IJ\,AB}(x,y) \;, \\
\mathbf{7}' + \mathbf{21} & \text{vectors :} & C_{\mu}{}^{I8}(x,y) \;,\; \tilde{C}_{\mu\,IJ}(x,y) \;, \\
\mathbf{7} & \text{two-forms :} & C_{\mu\nu\,I}{}^{8}(x,y) \;, \\
\mathbf{1} & \text{three-form :} & C_{\mu\nu\rho}{}^{88}(x,y) \;.
\end{array}
$$

that is to be connected with the SL(7)-covariant 4D fields of the tensor hierarchy :

$$
\begin{array}{rcl}
\mathbf{1} & \text{metric :} & ds_4^2\,(x) \;, \\
\mathbf{21}' + \mathbf{7}' + \mathbf{21} + \mathbf{7} & \text{coset representatives :} & \mathcal{V}^{IJ\,ij}(x) \;,\; \mathcal{V}^{I8\,ij}(x) \;,\; \tilde{\mathcal{V}}_{IJ}{}^{ij}(x) \;,\; \tilde{\mathcal{V}}_{I8}{}^{ij}(x) \;, \\
\mathbf{21}' + \mathbf{7}' + \mathbf{21} + \mathbf{7} & \text{vectors :} & \mathcal{A}_{\mu}{}^{IJ}(x) \;,\; \mathcal{A}_{\mu}{}^{I}(x) \;,\; \tilde{\mathcal{A}}_{\mu\,IJ}(x) \;,\; \tilde{\mathcal{A}}_{\mu\,I}(x) \;, \\
\mathbf{48} + \mathbf{7}' & \text{two-forms :} & \mathcal{B}_{\mu\nu\,I}{}^{J}(x) \;,\; \mathcal{B}_{\mu\nu}{}^{I}(x) \;, \\
\mathbf{28}' & \text{three-forms :} & \mathcal{C}_{\mu\nu\rho}{}^{IJ}(x) \;,
\end{array}
$$

> This connection is established by using geometrical data of the S^6 !!

∗ Step 3 : Connecting 4D [SL(7)] and 10D [SL(6)] fields using the S^6 geometrical data in a "dressing up" process

[vectors]

$$C_\mu{}^{m8}(x,y) = \tfrac{1}{2}\, g\, K^m_{IJ}(y)\, \mathcal{A}_\mu{}^{IJ}(x) \quad , \quad C_\mu{}^{78}(x,y) = -\mu_I(y)\, \mathcal{A}_\mu{}^I(x) \ ,$$

$$\tilde{C}_{\mu\, mn}(x,y) = \tfrac{1}{4}\, K^{IJ}_{mn}(y)\, \tilde{\mathcal{A}}_{\mu\, IJ}(x) \quad , \quad \tilde{C}_{\mu\, m7}(x,y) = -g^{-1}\, (\partial_m \mu^I)(y)\, \tilde{\mathcal{A}}_{\mu\, I}(x)$$

[two-forms]

$$C_{\mu\nu\, m}{}^8(x,y) = -g^{-1}\, (\mu_I \partial_m \mu^J)(y)\, \mathcal{B}_{\mu\nu\, J}{}^I(x)$$

$$C_{\mu\nu\, 7}{}^8(x,y) = \mu_I(y)\, \mathcal{B}_{\mu\nu}{}^I(x)$$

[three-form]

$$C_{\mu\nu\rho}{}^{88}(x,y) = (\mu_I \mu_J)(y)\, \mathcal{C}_{\mu\nu\rho}{}^{IJ}(x)$$

✦ S^6 geometrical data : embedding coordinates μ^I , Killing vectors K^m_{IJ} and tensors K^{mn}_{IJ}

∗ Step 4 : Plug and play… so that the final embedding of $\text{ISO}(7)_c$ into type IIA is

$$
\begin{aligned}
d\hat{s}_{10}^2 &= \Delta^{-1}\, ds_4^2 + g_{mn}\, Dy^m\, Dy^n \ , \\
\hat{A}_{(3)} &= \mu_I \mu_J \left(\mathcal{C}^{IJ} + \mathcal{A}^I \wedge \mathcal{B}^J + \tfrac{1}{6}\mathcal{A}^{IK} \wedge \mathcal{A}^{JL} \wedge \tilde{\mathcal{A}}_{KL} + \tfrac{1}{6}\mathcal{A}^I \wedge \mathcal{A}^{JK} \wedge \tilde{\mathcal{A}}_K\right) \\
&\quad + g^{-1}\left(\mathcal{B}_J{}^I + \tfrac{1}{2}\mathcal{A}^{IK} \wedge \tilde{\mathcal{A}}_{KJ} + \tfrac{1}{2}\mathcal{A}^I \wedge \tilde{\mathcal{A}}_J\right) \wedge \mu_I D\mu^J + \tfrac{1}{2} g^{-2}\, \tilde{\mathcal{A}}_{IJ} \wedge D\mu^I \wedge D\mu^J \\
&\quad - \tfrac{1}{2}\, \mu_I\, B_{mn}\, \mathcal{A}^I \wedge Dy^m \wedge Dy^n + \tfrac{1}{6} A_{mnp}\, Dy^m \wedge Dy^n \wedge Dy^p \ , \\
\hat{B}_{(2)} &= -\mu_I \left(\mathcal{B}^I + \tfrac{1}{2}\mathcal{A}^{IJ} \wedge \tilde{\mathcal{A}}_J\right) - g^{-1}\, \tilde{\mathcal{A}}_I \wedge D\mu^I + \tfrac{1}{2} B_{mn}\, Dy^m \wedge Dy^n \ , \\
\hat{A}_{(1)} &= -\mu_I\, \mathcal{A}^I + A_m\, Dy^m \ . \qquad (
\end{aligned}
$$

where we have defined : $\quad Dy^m \equiv dy^m + \tfrac{1}{2}\, g\, K^m_{IJ}\, \mathcal{A}^{IJ} \quad , \quad D\mu^I \equiv d\mu^I - g\, \mathcal{A}^{IJ} \mu_J$

The scalars are embedded as

$$
\begin{aligned}
g^{mn} &= \tfrac{1}{4}\, g^2\, \Delta\, \mathcal{M}^{IJ\,KL}\, K^m_{IJ}\, K^n_{KL} \quad , \quad & B_{mn} &= -\tfrac{1}{2}\, \Delta\, g_{mp}\, K^p_{IJ}\, \partial_n \mu^K\, \mathcal{M}^{IJ}{}_{K8} \ , \\
A_m &= \tfrac{1}{2}\, g\, \Delta\, g_{mn}\, K^n_{IJ}\, \mu_K\, \mathcal{M}^{IJ\,K8} \quad , \quad & A_{mnp} &= \tfrac{1}{8}\, g\, \Delta\, g_{mq}\, K^q_{IJ}\, K^{KL}_{np}\, \mathcal{M}^{IJ}{}_{KL} + A_m B_{np} \ .
\end{aligned}
$$

Remarks and consistency checks

- 10D (bosonic) SUSY transformations exactly reduce to those of the 4D tensor hierarchy.

- Computing the 10D field strengths $\hat{F}_{(2)} = d\hat{A}_{(1)} + m\,\hat{B}_{(2)}$, etc. one finds

$$\hat{F}_{(4)} = \mu_I \mu_J\, \mathcal{H}^{IJ}_{(4)} + g^{-1}\, \mathcal{H}_{(3)\,J}{}^{I} \wedge \mu_I D\mu^J + \tfrac{1}{2}\, g^{-2}\, \tilde{\mathcal{H}}_{(2)IJ} \wedge D\mu^I \wedge D\mu^J + \dots \,, \qquad \text{[FR parameter]}$$

$$\hat{H}_{(3)} = -\mu_I\, \mathcal{H}^{I}_{(3)} - g^{-1}\, \tilde{\mathcal{H}}_{(2)I} \wedge D\mu^I + \dots \,,$$

$$\hat{F}_{(2)} = -\mu_I \mathcal{H}^{I}_{(2)} + g^{-1} \left(g\, \delta_{IJ}\, \mathcal{A}^J - m\, \tilde{\mathcal{A}}_I \right) \wedge D\mu^I + \dots \,,$$

which are expressed in terms of the 4D tensor hierarchy. The parameter $m{=}gc$ only appears through standard Romans' redefinitions of $\hat{F}_{(p)}$ in 10D (formulas also valid for massless IIA)

[Hull & Warner '88 (electric)]

- The set of Bianchi identities of the above 10D field strengths reduces to

$$D\mathcal{H}^{IJ}_{(2)} = 0 \;\;,\;\; D\mathcal{H}^{I}_{(2)} = m\, \mathcal{H}^{I}_{(3)} \;\;,\;\; D\tilde{\mathcal{H}}_{(2)IJ} = -2\, g\, \mathcal{H}_{(3)[I}{}^{K}\, \delta_{J]K} \;\;,\;\; D\tilde{\mathcal{H}}_{(2)I} = g\, \delta_{IJ}\, \mathcal{H}^{J}_{(3)} \;,$$

$$D\mathcal{H}_{(3)I}{}^{J} = \mathcal{H}^{JK}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)IK} + \mathcal{H}^{J}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)I} - 2g\, \delta_{IK}\, \mathcal{H}^{JK}_{(4)} - \tfrac{1}{7}\, \delta^{J}_{I}\, (\text{trace}) \;,$$

$$D\mathcal{H}^{I}_{(3)} = -\mathcal{H}^{IJ}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)J} \quad , \qquad D\mathcal{H}^{IJ}_{(4)} \equiv 0 \;.$$

which exactly matches the one of the 4D tensor hierarchy.

Generalised vielbein

$$V^{I8}{}_{AB} = (V^{m8}{}_{AB}\,,\, V^{78}{}_{AB}) \qquad , \qquad \tilde{V}_{IJ\,AB} = (\tilde{V}_{mn\,AB}\,,\, \tilde{V}_{m7\,AB}) \,.$$

with components

$$\begin{aligned}
V^{m8\,AB} &= -\tfrac{1}{4}\,\Delta^{-\frac{1}{2}}\,e_a{}^m(\Gamma^a C^{-1})^{AB}\,, \\
V^{78\,AB} &= -\tfrac{1}{4}\,e^{-\frac{3}{4}\hat{\phi}}\,\Delta^{-\frac{1}{2}}\,(\Gamma_7 C^{-1})^{AB} - V^{m8\,AB}A_m\,, \\
\tilde{V}_{m7}{}^{AB} &= \tfrac{1}{4}\,e^{\frac{1}{2}\hat{\phi}}\,\Delta^{-\frac{1}{2}}\,e_m{}^a(\Gamma_a\Gamma_7 C^{-1})^{AB} + V^{n8\,AB}B_{nm}\,, \\
\tilde{V}_{mn}{}^{AB} &= \tfrac{1}{4}\,e^{-\frac{1}{4}\hat{\phi}}\,\Delta^{-\frac{1}{2}}\,e_m{}^a e_n{}^b(\Gamma_{ab}C^{-1})^{AB} + V^{p8\,AB}(A_{pmn} - 2B_{p[m}A_{n]}) \\
&\quad + V^{78\,AB}B_{mn} + 2\,\tilde{V}_{[m|7}{}^{AB}\,A_{|n]}
\end{aligned}$$

and

$$\begin{aligned}
V^{m8}{}_{AB} &= \tfrac{1}{4}\,\Delta^{-\frac{1}{2}}\,e_a{}^m(C\Gamma^a)_{AB}\,, \\
V^{78}{}_{AB} &= \tfrac{1}{4}\,e^{-\frac{3}{4}\hat{\phi}}\,\Delta^{-\frac{1}{2}}\,(C\Gamma_7)_{AB} - V^{m8}{}_{AB}\,A_m\,, \\
\tilde{V}_{m7\,AB} &= \tfrac{1}{4}\,e^{\frac{1}{2}\hat{\phi}}\,\Delta^{-\frac{1}{2}}\,e_m{}^a(C\Gamma_a\Gamma_7)_{AB} + V^{n8}{}_{AB}\,B_{nm}\,, \\
\tilde{V}_{mn\,AB} &= \tfrac{1}{4}\,e^{-\frac{1}{4}\hat{\phi}}\,\Delta^{-\frac{1}{2}}\,e_m{}^a e_n{}^b(C\Gamma_{ab})_{AB} + V^{p8}{}_{AB}(A_{pmn} - 2B_{p[m}A_{n]}) \\
&\quad + V^{78}{}_{AB}\,B_{mn} + 2\,\tilde{V}_{[m|7\,AB}A_{|n]}
\end{aligned}$$

"dressing up" process

[vielbein and scalars]

$$ds_4^2(x,y) = ds_4^2(x)$$

$$\begin{aligned} V^{m8\,AB}(x,y) &= \tfrac{1}{2}\, g\, K^m_{IJ}(y)\, \eta_i^A(y)\, \eta_j^B(y)\, \mathcal{V}^{IJ\,ij}(x)\ , \\ V^{78\,AB}(x,y) &= -\mu_I(y)\, \eta_i^A(y)\, \eta_j^B(y)\, \mathcal{V}^{I8\,ij}(x)\ , \\ \tilde{V}_{mn}{}^{AB}(x,y) &= \tfrac{1}{4}\, K^{IJ}_{mn}(y)\, \eta_i^A(y)\, \eta_j^B(y)\, \tilde{\mathcal{V}}_{IJ}{}^{ij}(x)\ , \\ \tilde{V}_{m7}{}^{AB}(x,y) &= -g^{-1}\, (\partial_m \mu^I)(y)\, \eta_i^A(y)\, \eta_j^B(y)\, \tilde{\mathcal{V}}_{I8}{}^{ij}(x)\ , \end{aligned}$$

[two-forms]

$$C_{\mu\nu\, m}{}^8(x,y) = -g^{-1}\, (\mu_I \partial_m \mu^J)(y)\, \mathcal{B}_{\mu\nu\, J}{}^I(x)$$

$$C_{\mu\nu\, 7}{}^8(x,y) = \mu_I(y)\, \mathcal{B}_{\mu\nu}{}^I(x)$$

[three-form]

$$C_{\mu\nu\rho}{}^{88}(x,y) = (\mu_I \mu_J)(y)\, \mathcal{C}_{\mu\nu\rho}{}^{IJ}(x)$$

[vectors]

$$C_\mu{}^{m8}(x,y) = \tfrac{1}{2}\, g\, K^m_{IJ}(y)\, \mathcal{A}_\mu{}^{IJ}(x) \quad , \quad C_\mu{}^{78}(x,y) = -\mu_I(y)\, \mathcal{A}_\mu{}^I(x)\ ,$$

$$\tilde{C}_{\mu\, mn}(x,y) = \tfrac{1}{4}\, K^{IJ}_{mn}(y)\, \tilde{\mathcal{A}}_{\mu\, IJ}(x) \quad , \quad \tilde{C}_{\mu\, m7}(x,y) = -g^{-1}\, (\partial_m \mu^I)(y)\, \tilde{\mathcal{A}}_{\mu\, I}(x)$$

- S^6 geometrical data : embedding coordinates μ^I, Killing vectors K^m_{IJ} and tensors K^{mn}_{IJ}

10D SUSY transformations

$$\delta C_\mu{}^{I8} = i\, V^{I8}{}_{AB} \left(\bar\epsilon^A \psi_\mu{}^B + \tfrac{1}{2\sqrt{2}}\, \bar\epsilon_C \gamma_\mu \chi^{ABC} \right) + \text{h.c.}\ ,$$

$$\delta \tilde C_{\mu\, IJ} = -i\, V_{IJ\, AB} \left(\bar\epsilon^A \psi_\mu{}^B + \tfrac{1}{2\sqrt{2}}\, \bar\epsilon_C \gamma_\mu \chi^{ABC} \right) + \text{h.c.}\ ,$$

$$\begin{aligned} \delta C_{\mu\nu\, I}{}^8 = \Big[& \tfrac{2}{3} \big(V^{J8}{}_{BC}\, \tilde V_{IJ}{}^{AC} + \tilde V_{IJ\, BC}\, V^{J8\, AC} \big)\, \bar\epsilon_A \gamma_{[\mu} \psi^B_{\nu]} \\ & + \tfrac{\sqrt{2}}{3}\, V^{J8}{}_{AB}\, \tilde V_{IJ\, CD}\, \bar\epsilon^{[A} \gamma_{\mu\nu} \chi^{BCD]} + \text{h.c.} \Big] - C_{[\mu}{}^{J8}\, \delta \tilde C_{\nu] IJ} - \tilde C_{[\mu|\, IJ}\, \delta C_{|\nu]}{}^{J8}\ , \end{aligned}$$

$$\begin{aligned} \delta C_{\mu\nu\rho}{}^{88} = \Big[& \tfrac{4i}{7}\, V^{I8}{}_{BD} \big(V^{J8\, DC}\, \tilde V_{IJ\, AC} + \tilde V_{IJ}{}^{DC}\, V^{J8}{}_{AC} \big)\, \bar\epsilon^A \gamma_{[\mu\nu} \psi^B_{\rho]} \\ & - i \tfrac{\sqrt{2}}{3}\, V^{I8\, AE}\, V^{J8}{}_{[EB|}\, \tilde V_{IJ\, |CD]}\, \bar\epsilon_A \gamma_{\mu\nu\rho} \chi^{BCD} + \text{h.c.} \Big] \\ & + 3\, C_{[\mu\nu|\, I}{}^8\, \delta C_{|\rho]}{}^{I8} - C_{[\mu}{}^{I8} \big(C_\nu{}^{J8}\, \delta \tilde C_{\rho] IJ} + \tilde C_{\nu|\, IJ}\, \delta C_{|\rho]}{}^{J8} \big)\ . \end{aligned}$$

Freund-Rubin term

[Freund & Rubin '80]

- By looking at the RR field strength $\hat{F}_{(4)} = \mathcal{H}^{IJ}_{(4)}\,\mu_I\,\mu_J + \dots$, one immediately identifies the Freund-Rubin term

$$\begin{aligned}\mathcal{H}^{IJ}_{(4)}\,\mu_I\,\mu_J = &-\tfrac{1}{3}\,g^{-1}\,V\,\mathrm{vol}_4 + \tfrac{1}{84}\,g^{-1}\left(D\mathcal{H}_{(3)} - 7\,\mathcal{H}^{IJ}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)IJ} - 7\,\mathcal{H}^{I}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)I}\right)\\ &-\tfrac{1}{2}\,g^{-1}\left(D\mathcal{H}_{(3)I}{}^{J} - \mathcal{H}^{JK}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)IK} - \mathcal{H}^{J}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)I}\right)\mu^I\,\mu_J\ ,\end{aligned}$$

NOTE: We have expressed the EOMs for the scalars as BI for the three-form field strengths of the tensor hierarchy.

- At a critical point of V one has $\hat{F}_{(4)} = -\dfrac{1}{3g} V\,\mathrm{vol}_4 + \dots$, and the S^6 dependence drops out

[see also Godazgar, Godazgar, Krueger & Nicolai '15]