How to get masses from Extended Field Theories

Adolfo Guarino

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II GRASS family and friends

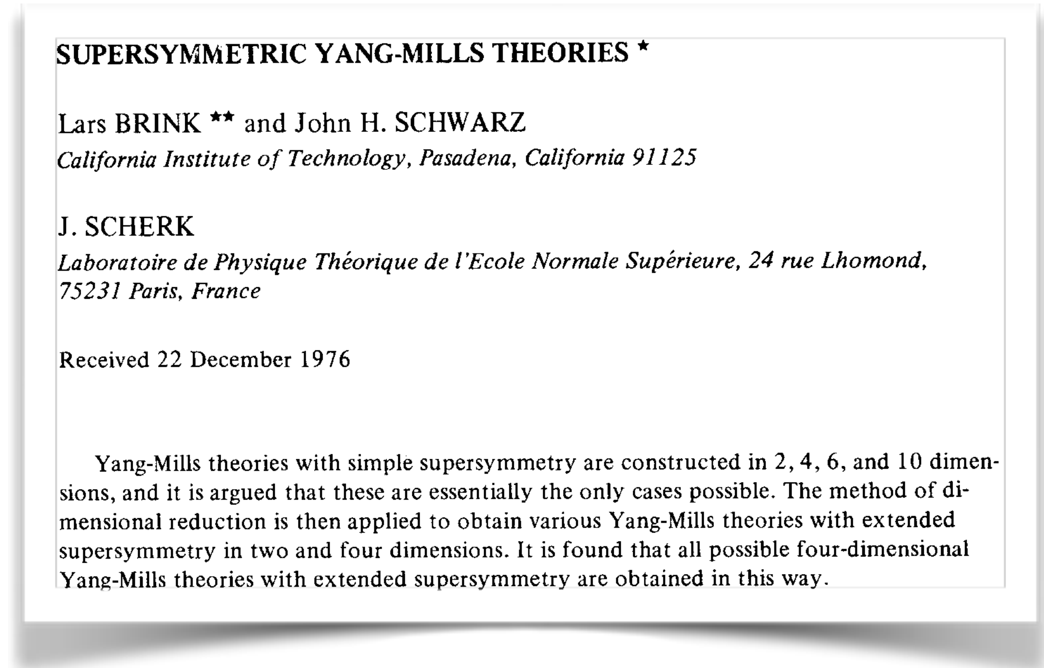
“almond tree" Meeting

40 years ago…

Reduction of SYM in 10D

- Masses in 4D from reduction of **non-abelian** SYM in 10D

  [***f*** = Lie algebra structure constants]



SUPERSYMMETRIC YANG-MILLS THEORIES *

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Yang-Mills theories with simple supersymmetry are constructed in 2, 4, 6, and 10 dimensions, and it is argued that these are essentially the only cases possible. The method of dimensional reduction is then applied to obtain various Yang-Mills theories with extended supersymmetry in two and four dimensions. It is found that all possible four-dimensional Yang-Mills theories with extended supersymmetry are obtained in this way.

[Brink, Scherk & Schwarz '76]

KK reduction

$$L_{10\text{D}} = -\tfrac{1}{4}F^2 + \tfrac{i}{2}\bar{\lambda}\not{D}\lambda \quad \Rightarrow \quad L_{4\text{D}} = -\tfrac{1}{4}F^2 + i\bar{\lambda}\not{D}\lambda - D_\mu M D^\mu M^{-1}$$

$$- \tfrac{i}{2} f\, M\, \bar{\lambda}\,\lambda + \text{c.c}$$

$$- \tfrac{1}{4} f\, f\, M^{-1}\, M^{-1}\, M\, M$$

Lower-dimensional masses from higher-dimensional deformations

Reduction of gravity theories in 10D/11D

- Masses in 4D from reduction of gravity theories in 10D/11D with **non-trivial** internal profiles

$$\Phi_m(x,y) = U(y)_m{}^n\,\Phi_n(x)$$

$$f = U^{-1}\,U^{-1}\partial U = cte$$

[***f*** = Lie algebra structure constants]

HOW TO GET MASSES FROM EXTRA DIMENSIONS

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Received 19 February 1979

A generalized method of dimensional reduction, applicable to theories in curved space, is described. As in previous works by other authors, the extra dimensions are related to the manifold of a Lie group. The new feature of this work is to define and study a class of Lie groups, called "flat groups", for which the resulting theory has no cosmological constant, a well-behaved potential, and a number of arbitrary mass parameters. In particular, when the analysis is applied to the reduction of 11-dimensional supergravity to four dimensions it becomes possible to incorporate three arbitrary mass parameters in the resulting $N = 8$ theory. This shows that extended supersymmetry theories allow more possibilities for spontaneous symmetry breaking than was previously believed to be the case.

[Scherk & Schwarz '79]

SS reduction

$$L_{10\text{D}/11\text{D}} = e\,R \quad \Rightarrow \quad L_{4\text{D}} = e\,R - \tfrac{1}{4}\,M\,F^2 - D_\mu M\,D^\mu M^{-1} - 2\,f\,f\,M^{-1} - f\,f\,M\,M^{-1}\,M^{-1}$$

Lower-dimensional masses from non-trivial internal dependence

After 40 years, a new framework where to jointly describe gauge and gravitational aspects of 10D/11D supergravities (string/M-theory) has been constructed based on the idea of **dualities**…

Dualities and Extended Field Theories (ExFT)

- Different strings related by dualities: IIA/IIB T-duality, IIB S-duality, …

- *String dualities* realised as *non-linear global symmetries* in SUGRA

lower-dimensional phenomenon vs **higher-dimensional** phenomenon

gaugings, embedding tensor, … non-geometry, ß-supergravity, ExFT …

Today's talk : **Extended Field Theories (ExFT)** [extend internal coords to transform *linearly* under duality]

- Exceptional Field Theory (EFT) ⇨ Exceptional groups $E_{d+1(d+1)}$ [max SUGRA (U-duality)]
- Double Field Theory (DFT) ⇨ Orthogonal groups O(d,d) [half-max SUGRA (T-duality)]

[Siegel '93] [Hull & Zwiebach (Hohm) '09 '10] [Hohm & Samtleben '13]

Dualities in SUGRA and Extended Field Theory

D	Maximal sugra / EFT	Half-maximal sugra	DFT
9	$\mathbb{R}^+ \times \mathrm{SL}(2)$	$\mathbb{R}^+ \times \mathrm{O}(1,1+n)$	$\mathbb{R}^+ \times \mathrm{O}(1,1+n)$
8	$\mathrm{SL}(2) \times \mathrm{SL}(3)$	$\mathbb{R}^+ \times \mathrm{O}(2,2+n)$	$\mathbb{R}^+ \times \mathrm{O}(2,2+n)$
7	$\mathrm{SL}(5)$	$\mathbb{R}^+ \times \mathrm{O}(3,3+n)$	$\mathbb{R}^+ \times \mathrm{O}(3,3+n)$
6	$\mathrm{SO}(5,5)$	$\mathbb{R}^+ \times \mathrm{O}(4,4+n)$	$\mathbb{R}^+ \times \mathrm{O}(4,4+n)$
5	$\mathrm{E}_{6(6)}$	$\mathbb{R}^+ \times \mathrm{O}(5,5+n)$	$\mathbb{R}^+ \times \mathrm{O}(5,5+n)$
4	$\mathrm{E}_{7(7)}$	$\mathrm{SL}(2) \times \mathrm{O}(6,6+n)$	$\mathbb{R}^+ \times \mathrm{O}(6,6+n)$
3	$\mathrm{E}_{8(8)}$	$\mathrm{O}(8,8+n)$	$\mathbb{R}^+ \times \mathrm{O}(7,7+n)$

Duality groups of half-maximal SUGRA and DFT differ for **D<5**

* n = additional vector multiplets

Dualities in SUGRA and Extended Field Theory

D	Maximal sugra / EFT	Half-maximal sugra	DFT
9	$\mathbb{R}^+ \times SL(2)$	$\mathbb{R}^+ \times O(1,1+n)$	$\mathbb{R}^+ \times O(1,1+n)$
8	$SL(2) \times SL(3)$	$\mathbb{R}^+ \times O(2,2+n)$	$\mathbb{R}^+ \times O(2,2+n)$
7	$SL(5)$	$\mathbb{R}^+ \times O(3,3+n)$	$\mathbb{R}^+ \times O(3,3+n)$
6	$SO(5,5)$	$\mathbb{R}^+ \times O(4,4+n)$	$\mathbb{R}^+ \times O(4,4+n)$
5	$E_{6(6)}$	$\mathbb{R}^+ \times O(5,5+n)$	$\mathbb{R}^+ \times O(5,5+n)$
4	$E_{7(7)}$	$SL(2) \times O(6,6+n)$	$\mathbb{R}^+ \times O(6,6+n)$
3	$E_{8(8)}$	$O(8,8+n)$	$\mathbb{R}^+ \times O(7,7+n)$

Duality groups of half-maximal SUGRA and DFT differ for **D<5**

* n = additional vector multiplets

How to get masses from ExFT

1. Deformations à la SYM : $E_{7(7)}$ covariant ExFT

[Motivation = Massive IIA]

JHEP 08 (2016) 154 with F. Ciceri and G. Inverso

2. Deformations à la gravity : SL(2) x O(6,6) covariant ExFT

[Motivation = fluxes and moduli stabilisation]

JHEP 07 (2017) 028 with F. Ciceri and G. Inverso
+ G. Dibitetto and J.J. Fernández-Melgarejo

$E_{7(7)}$-EFT

[momentum, winding, …]

- Space-time : external (D=4) + **generalised internal** ($y^{\mathcal{M}}$ coordinates in **56** of $E_{7(7)}$)

Generalised diffs = *ordinary internal diffs* + internal *gauge transfos*

- Generalised Lie derivative built from an $E_{7(7)}$-invariant structure *Y*-tensor

$$\mathbb{L}_{\Lambda} U^{\mathcal{M}} = \Lambda^{\mathcal{N}} \partial_{\mathcal{N}} U^{\mathcal{M}} - U^{\mathcal{N}} \partial_{\mathcal{N}} \Lambda^{\mathcal{M}} + Y^{\mathcal{M}\mathcal{N}}{}_{\mathcal{P}\mathcal{Q}} \, \partial_{\mathcal{N}} \Lambda^{\mathcal{P}} \, U^{\mathcal{Q}}$$

[no density term]

Closure requires a **section constraint** : $Y^{\mathcal{P}\mathcal{Q}}{}_{\mathcal{M}\mathcal{N}} \, \partial_{\mathcal{P}} \otimes \partial_{\mathcal{Q}} = 0$

Two maximal solutions : M-theory (**7** dimensional) & Type IIB (**6** dimensional)

[**massless** theories]

Question: What about the massive IIA theory?

[Romans '86]

[Hohm & Kwak '11 (sec const violated)]

$E_{7(7)}$-EFT

- $E_{7(7)}$-EFT action $[\ \mathcal{D}_\mu = \partial_\mu - \mathbb{L}_{A_\mu}\ ]$

$$S_{\text{EFT}} = \int d^4x\, d^{56}y\, e\, \Big[\, \hat{R} + \tfrac{1}{48}\, g^{\mu\nu}\, \mathcal{D}_\mu \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \mathcal{D}_\nu \mathcal{M}_{\mathcal{M}\mathcal{N}} - \tfrac{1}{8}\, \mathcal{M}_{\mathcal{M}\mathcal{N}}\, \mathcal{F}^{\mu\nu\mathcal{M}} \mathcal{F}_{\mu\nu}{}^{\mathcal{N}} + e^{-1}\, \mathcal{L}_{\text{top}} - V_{\text{EFT}}(\mathcal{M}, g)\, \Big]$$

with *field strengths* & *potential term* given by

$$\mathcal{F}_{\mu\nu}{}^{\mathcal{M}} = 2\, \partial_{[\mu} A_{\nu]}{}^{\mathcal{M}} - \big[A_\mu, A_\nu\big]_{\text{E}}^{\mathcal{M}} + \text{two-form terms}$$

(tensor hierarchy)

$$V_{\text{EFT}}(\mathcal{M}, g) = -\tfrac{1}{48}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{M}} \mathcal{M}^{\mathcal{K}\mathcal{L}}\, \partial_{\mathcal{N}} \mathcal{M}_{\mathcal{K}\mathcal{L}} + \tfrac{1}{2}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{M}} \mathcal{M}^{\mathcal{K}\mathcal{L}}\, \partial_{\mathcal{L}} \mathcal{M}_{\mathcal{N}\mathcal{K}} - \tfrac{1}{2}\, g^{-1} \partial_{\mathcal{M}} g\, \partial_{\mathcal{N}} \mathcal{M}^{\mathcal{M}\mathcal{N}} - \tfrac{1}{4}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, g^{-1} \partial_{\mathcal{M}} g\, g^{-1} \partial_{\mathcal{N}} g - \tfrac{1}{4}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{M}} g^{\mu\nu}\, \partial_{\mathcal{N}} g_{\mu\nu}$$

- Two-derivative potential : **ungauged** N=8 D=4 SUGRA when $\Phi(x,y) = \Phi(x)$

Deforming $E_{7(7)}$-EFT

- Generalised Lie derivative

$$\mathbb{L}_\Lambda U^{\mathcal{M}} \;=\; \Lambda^{\mathcal{N}}\partial_{\mathcal{N}}U^{\mathcal{M}} - U^{\mathcal{N}}\partial_{\mathcal{N}}\Lambda^{\mathcal{M}} + Y^{\mathcal{MN}}{}_{\mathcal{PQ}}\,\partial_{\mathcal{N}}\Lambda^{\mathcal{P}}\,U^{\mathcal{Q}}$$

- **Deformed** generalised Lie derivative

$$\widetilde{\mathbb{L}}_\Lambda U^{\mathcal{M}} \;=\; \Lambda^{\mathcal{N}}\partial_{\mathcal{N}}U^{\mathcal{M}} - U^{\mathcal{N}}\partial_{\mathcal{N}}\Lambda^{\mathcal{M}} + Y^{\mathcal{MN}}{}_{\mathcal{PQ}}\,\partial_{\mathcal{N}}\Lambda^{\mathcal{P}}\,U^{\mathcal{Q}} - \underbrace{X_{\mathcal{NP}}{}^{\mathcal{M}}\,\Lambda^{N}\,U^{\mathcal{P}}}_{\text{non-derivative}}$$

in terms of an *X* deformation which is $E_{n(n)}$-algebra valued

- Closure & triviality of the Jacobiator require (*together with **sec. constraint***)

$$X_{\mathcal{MN}}{}^{\mathcal{P}}\,\partial_{\mathcal{P}} = 0$$

X constraint

$$X_{\mathcal{MP}}{}^{\mathcal{Q}}\,X_{\mathcal{NQ}}{}^{\mathcal{R}} - X_{\mathcal{NP}}{}^{\mathcal{Q}}\,X_{\mathcal{MQ}}{}^{\mathcal{R}} + X_{\mathcal{MN}}{}^{\mathcal{Q}}\,X_{\mathcal{QP}}{}^{\mathcal{R}} = 0$$

Quadratic constraint (gauged max. supergravity)

[*X* deformation *vs* embedding tensor]

$E_{7(7)}$-XFT

- $E_{7(7)}$-XFT action [$\mathcal{D}_\mu = \partial_\mu - \widetilde{\mathbb{L}}_{A_\mu}$] [$y^{\mathcal{M}}$ coords in the **56** of $E_{7(7)}$]

$$S_{\text{XFT}} = \int d^4x\, d^{56}y\, e\,\Big[\,\hat{R} + \tfrac{1}{48}\, g^{\mu\nu}\,\mathcal{D}_\mu \mathcal{M}^{\mathcal{M}\mathcal{N}}\,\mathcal{D}_\nu \mathcal{M}_{\mathcal{M}\mathcal{N}} - \tfrac{1}{8}\,\mathcal{M}_{\mathcal{M}\mathcal{N}}\,\mathcal{F}^{\mu\nu\mathcal{M}}\mathcal{F}_{\mu\nu}{}^{\mathcal{N}} + e^{-1}\,\mathcal{L}_{\text{top}} - V_{\text{XFT}}(\mathcal{M}, g)\,\Big]$$

with *field strengths* & *potential* given by

(deformed tensor hierarchy)

$$\mathcal{F}_{\mu\nu}{}^{\mathcal{M}} = 2\,\partial_{[\mu}A_{\nu]}{}^{\mathcal{M}} + X_{[\mathcal{P}\mathcal{Q}]}{}^{\mathcal{M}}\,A_\mu{}^{\mathcal{P}}A_\nu{}^{\mathcal{Q}} - \big[A_\mu, A_\nu\big]_{\text{E}}^{\mathcal{M}} + \text{two-form terms}$$

$$V_{\text{XFT}}(\mathcal{M}, g, X) = V_{\text{EFT}}(\mathcal{M}, g) + \underbrace{\tfrac{1}{12}\,\mathcal{M}^{MN}\mathcal{M}^{KL}X_{MK}{}^{P}\,\partial_N \mathcal{M}_{PL}}_{\text{cross term}} + \underbrace{V_{\text{SUGRA}}(\mathcal{M}, X)}_{\text{gauged max. sugra}}$$

- Two-One-**Zero**-derivative potential : **gauged** 4D max. sugra when $\Phi(x,y) = \Phi(x)$

X deformation : background fluxes & Romans mass

$$Y^{\mathcal{PQ}}{}_{\mathcal{MN}}\, \partial_{\mathcal{P}} \otimes \partial_{\mathcal{Q}} = 0$$

section constraint

$$X_{\mathcal{MN}}{}^{\mathcal{P}}\, \partial_{\mathcal{P}} = 0$$

X constraint

[algebraic system]

→

M-theory (**7** coords)

- SL(7) orbit
- Freund-Rubin param.
- *massless* IIA (subcase)

Type IIB (**6** coords)

- SL(6) orbit
- *p*-form fluxes compatible with SL(6)
- **SL(2)-triplet of 1-form flux** (includes compact SO(2))

[QC = flux-induced tadpoles]

X deformation : background fluxes & Romans mass

$$Y^{\mathcal{P}\mathcal{Q}}{}_{\mathcal{M}\mathcal{N}}\, \partial_{\mathcal{P}} \otimes \partial_{\mathcal{Q}} = 0$$

section constraint

$$X_{\mathcal{M}\mathcal{N}}{}^{\mathcal{P}}\, \partial_{\mathcal{P}} = 0$$

X constraint

[algebraic system]

M-theory (**7** coords)

- SL(7) orbit
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- *massless* IIA (subcase)

Type IIB (**6** coords)

- SL(6) orbit
- *p*-form fluxes compatible with SL(6)
- **SL(2)-triplet of 1-form flux** (includes compact SO(2))

+

New massive Type IIA (**6** coords)

- SL(6) orbit
- *p*-form fluxes compatible with SL(6)
- dilaton flux
- **Romans mass parameter** *(kills the M-theory coord)*

Massive Type IIA described in a purely geometric manner !!

[QC = flux-induced tadpoles]

How to get masses from ExFT

1. Deformations à la SYM : $E_{7(7)}$ covariant ExFT

[Motivation = Massive IIA]

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2. Deformations à la gravity : SL(2) x O(6,6) covariant ExFT

[Motivation = fluxes and moduli stabilisation]

JHEP 07 (2017) 028

From $E_{7(7)}$-EFT to SL(2)-DFT

- Halving $E_{7(7)}$-EFT to obtain SL(2)-DFT with SL(2) x SO(6,6) symmetry

$$\begin{array}{ccc} \mathrm{E}_{7(7)} & \rightarrow & \mathrm{SL}(2) \times \mathrm{SO}(6,6) \\ \mathbf{56} & \rightarrow & (\mathbf{2},\mathbf{12}) + (\mathbf{1},\mathbf{32}) \\ y^{\mathcal{M}} & \rightarrow & y^{\alpha M} + \cancel{y^{A}} \\ \text{EFT} & & \text{SL(2)-DFT} \end{array}$$

α = (+ , -) vector index of SL(2)

M vector index of SO(6,6)

A M-W spinor index of SO(6,6)

[see Dibitetto, A.G. & Roest '11 for sugra]

via a **Z_2 truncation** (*vector* = +1 , *spinor* = -1) on coordinates, fields, etc.

[Hull & Zwiebach '09] [Hohm, Hull & Zwiebach '10]

[DFT corresponds to an α = + orientation]

- SL(2)-DFT generalised Lie derivative

$$\mathbb{L}_{\Lambda} U^{\alpha M} = \Lambda^{\beta N} \partial_{\beta N} U^{\alpha M} - U^{\beta N} \partial_{\beta N} \Lambda^{\alpha M} + \eta^{MN} \eta_{PQ} \, \partial_{\beta N} \Lambda^{\beta P} \, U^{\alpha Q} + 2 \, \epsilon^{\alpha\beta} \, \epsilon_{\gamma\delta} \, \partial_{\beta N} \Lambda^{\gamma[M} \, U^{|\delta|N]}$$

- SL(2)-DFT section constraints :

$$\eta^{MN} \, \partial_{\alpha M} \otimes \partial_{\beta N} = 0 \quad , \quad \epsilon^{\alpha\beta} \, \partial_{\alpha[M|} \otimes \partial_{\beta|N]} = 0$$

SL(2)-DFT

- SL(2)-DFT action $[\ \mathcal{D}_\mu = \partial_\mu - \mathbb{L}_{A_\mu}\ ]$

$$S_{\text{SL(2)-DFT}} = \int d^4x\, d^{24}y\, e\,\big[\,\hat{R} + \tfrac{1}{4}\, g^{\mu\nu}\, \mathcal{D}_\mu M^{\alpha\beta}\, \mathcal{D}_\nu M_{\alpha\beta} + \tfrac{1}{8}\, g^{\mu\nu}\, \mathcal{D}_\mu M^{MN}\, \mathcal{D}_\nu M_{MN} \\ -\tfrac{1}{8}\, M_{\alpha\beta}\, M_{MN}\, \mathcal{F}^{\mu\nu\,\alpha M} \mathcal{F}_{\mu\nu}{}^{\beta N} + e^{-1}\, \mathcal{L}_{\text{top}} - V_{\text{SL(2)-DFT}}(M,g)\,\big]$$

with *field strengths* & *potential term* given by

$$\mathcal{F}_{\mu\nu}{}^{\alpha M} \;=\; 2\,\partial_{[\mu} A_{\nu]}{}^{\alpha M} - \big[A_\mu, A_\nu\big]_{\text{S}}^{\alpha M} + \text{two-form terms}$$

(tensor hierarchy)

$$\begin{aligned} V_{\text{SL(2)-DFT}}(M,g) \;=\;& M^{\alpha\beta} M^{MN}\big[-\tfrac{1}{4}\,(\partial_{\alpha M} M^{\gamma\delta})(\partial_{\beta N} M_{\gamma\delta}) - \tfrac{1}{8}\,(\partial_{\alpha M} M^{PQ})(\partial_{\beta N} M_{PQ}) \\ & + \tfrac{1}{2}\,(\partial_{\alpha M} M^{\gamma\delta})(\partial_{\delta N} M_{\beta\gamma}) + \tfrac{1}{2}(\partial_{\alpha M} M^{PQ})(\partial_{\beta Q} M_{NP})\big] \\ +\;& \tfrac{1}{2}\, M^{MN} M^{PQ} (\partial_{\alpha M} M^{\alpha\delta})(\partial_{\delta Q} M_{NP}) + \tfrac{1}{2}\, M^{\alpha\beta} M^{\gamma\delta} (\partial_{\alpha M} M^{MQ})(\partial_{\delta Q} M_{\beta\gamma}) \\ -\;& \tfrac{1}{4}\, M^{\alpha\beta}\, M^{MN}\, \big[g^{-1}(\partial_{\alpha M} g)\, g^{-1}(\partial_{\beta N} g) + (\partial_{\alpha M} g^{\mu\nu})\,(\partial_{\beta N} g_{\mu\nu})\big] \\ -\;& \tfrac{1}{2}\, g^{-1}\,(\partial_{\alpha M} g)\,\partial_{\beta N}(M^{\alpha\beta} M^{MN}) \end{aligned}$$

- Two-derivative potential : **ungauged** N=4 D=4 SUGRA when $\Phi(x,y) = \Phi(x)$

Section constraint & SL(2) angles

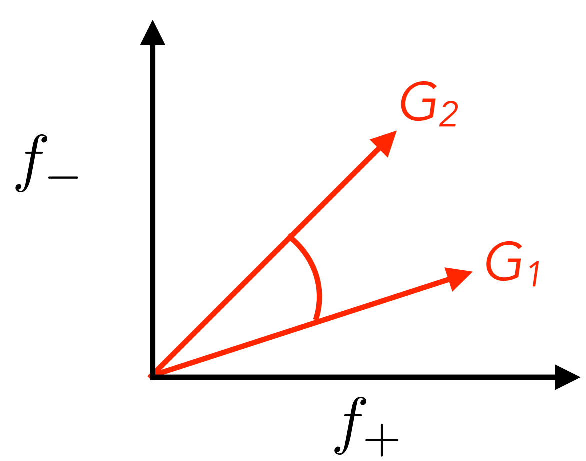
- One maximal solution of sec. constraint : it describes Type I/Heterotic [**as** in DFT]

- Generalised SS reductions with **SL(2) x O(6,6)** twist matrices $U_{\alpha M}{}^{\beta N} = e^{\lambda}\, e_{\alpha}{}^{\beta}\, U_M{}^N$ yield N=4 gauging parameters

$$
\begin{aligned}
f_{\alpha MNP} &= -3\, e^{-\lambda}\, e_{\alpha}{}^{\beta}\, \eta_{Q[M}\, U_N{}^R\, U_{P]}{}^S\ \partial_{\beta R} U_S{}^Q \\
\xi_{\alpha M} &= 2\, U_M{}^N \partial_{\beta N}(e^{-\lambda} e_{\alpha}{}^{\beta})
\end{aligned}
$$

[de Roo & Wagemans '85]

- *Moduli stabilisation* **requires** gaugings *G = G₁ x G₂* at **relative** SL(2) angles



(sec. constraint violated)

$$
\epsilon^{\alpha\beta}\, \partial_{\alpha[M|} \otimes \partial_{\beta|N]} \neq 0
$$

[**not** possible in DFT]

Example : SO(4) x SO(4) gaugings and non-geometry

- SS with $U(y^{\alpha M}) \in \mathrm{O}(6,6)$: Half of the coords of type + & half of type -
- SL(2)-superposition of *two chains* of **non-geometric** fluxes $(\; H \; , \; \omega \; , \; Q \; , \; R \;)_{\pm}$

f_{+}	$f_{+abc} = H^{(+)}{}_{abc}$, $f_{+\bar{a}\bar{b}c} = Q^{(+)ab}{}_{c}$	$f_{+ijk} = H^{(+)}{}_{ijk}$, $f_{+\bar{i}\bar{j}k} = Q^{(+)ij}{}_{k}$	$f_{+ab\bar{c}} = \omega^{(+)}{}_{ab}{}^{c}$, $f_{+\bar{a}\bar{b}\bar{c}} = R^{(+)abc}$	$f_{+ij\bar{k}} = \omega^{(+)}{}_{ij}{}^{k}$, $f_{+\bar{i}\bar{j}\bar{k}} = R^{(+)ijk}$
f_{-}	$f_{-ijk} = H^{(-)}{}_{ijk}$, $f_{-\bar{i}\bar{j}k} = Q^{(-)ij}{}_{k}$	$f_{-abc} = H^{(-)}{}_{abc}$, $f_{-\bar{a}\bar{b}c} = Q^{(-)ab}{}_{c}$	$f_{-ij\bar{k}} = \omega^{(-)}{}_{ij}{}^{k}$, $f_{-\bar{i}\bar{j}\bar{k}} = R^{(-)ijk}$	$f_{-ab\bar{c}} = \omega^{(-)}{}_{ab}{}^{c}$, $f_{-\bar{a}\bar{b}\bar{c}} = R^{(-)abc}$

Most general family (8 params) of SO(4) x SO(4) gaugings of N=4 sugra

- SO(4) x SO(4) N=4 sugra : **AdS** & **dS** vacua (sphere/hyperboloid reductions)

[de Roo, Westra, Panda & Trigiante '03]
[Dibitetto, A.G. & Roest '12]

- \`\`Hybrid ±" sources to cancel flux-induced tadpoles (QC) : *dual branes ?*

[Bergshoeff, de Roo, Kerstan, Ortín & Riccioni '06]

What next ?

- X constraint $X_{\mathcal{MN}}{}^{\mathcal{P}}\,\partial_{\mathcal{P}} = 0$ and mutually BPS states

[Bossard & Kleinschmidt '15]
[Bandos '15]

- SL(2)-DFT sec. constraints : N=1 SUGRA in D=10 & N=(2,0) SUGRA in D=6

- Flux formulation of SL(2)-DFT : sec. cons violating terms & dual NS-NS branes

[Bergshoeff, de Roo, Kerstan, Ortín & Riccioni '06]
[Aldazabal, Graña, Marqués & Rosabal '13]

- SL(2)-DFT and consistent truncations to *half-maximal* supergravity

- Cosmological applications of SL(2)-DFT (moduli stab, de Sitter, inflation, …)

[Hassler, Lüst & Massai '14]

Gracias !!

Application : massive IIA on S^{n-1} ($2 < n < 8$)

Massless IIA reductions on S^{n-1} to gauged maximal sugra : EFT framework

Massive IIA reductions on S^{n-1} to gauged maximal sugra : XFT framework

Question : when is a consistent reduction Ansatz for *massless* IIA also consistent for *massive* IIA ?

[Cvetic , Lü & Pope '00]
[Lee, Strickland-Constable & Waldram '14]
[Hohm & Samtleben '14]

Procedure :

a) *Massless* IIA : generalised twist matrices valued in SL(n)

b) Romans mass introduced as an X^R deformation

c) Consistency requires the *stabiliser* of X^R in $E_{n(n)}$ to contain SL(n)

Answer : Only **massive IIA on S^6** works (n=7) !!

[A.G. & Varela '15]

Extended (super) Poincaré superalgebra

- Central charges (internal symmetries) $\mathcal{Z}_{IJ} = (a^a_{IJ})\ T^a$

- The algebra :

$$[P_\mu, P_\nu] = 0 \qquad [M_{\mu\nu}, M_{\rho\sigma}] = i\,(\eta_{\nu\rho}\, M_{\mu\sigma} - \eta_{\nu\sigma}\, M_{\mu\rho} - \eta_{\mu\rho}\, M_{\nu\sigma} + \eta_{\mu\sigma}\, M_{\nu\rho})$$

$$[P_\mu, M_{\rho\sigma}] = i\,(\eta_{\mu\rho}\, P_\sigma - \eta_{\mu\sigma}\, P_\rho)$$

$$\left[T^a, T^b\right] = i f^{ab}_{\ \ c}\ T^c \qquad [T^a, P_\mu] = [T^a, M_{\mu\nu}] = 0$$

$$\left[\mathcal{Q}^I_\alpha, P_\mu\right] = \left[\bar{\mathcal{Q}}^I_{\dot\alpha}, P_\mu\right] = 0 \qquad \left[\mathcal{Q}^I_\alpha, T^a\right] = (b_a)^I{}_J\ \mathcal{Q}^J_\alpha \qquad \left[\bar{\mathcal{Q}}^I_{\dot\alpha}, T^a\right] = -\bar{\mathcal{Q}}^J_{\dot\alpha}\ (b_a)_J{}^I$$

$$\left[\mathcal{Q}^I_\alpha, M_{\mu\nu}\right] = \frac{1}{2}(\sigma_{\mu\nu})_\alpha{}^\beta\ \mathcal{Q}^I_\beta \qquad \left[\bar{\mathcal{Q}}^I_{\dot\alpha}, M_{\mu\nu}\right] = -\frac{1}{2}\bar{\mathcal{Q}}^I_{\dot\beta}\ (\bar\sigma_{\mu\nu})^{\dot\beta}{}_{\dot\alpha}$$

$$\left\{\bar{\mathcal{Q}}^I_{\dot\alpha}, \bar{\mathcal{Q}}^J_{\dot\beta}\right\} = -2\,\epsilon_{\dot\alpha\dot\beta}\ \mathcal{Z}^{IJ\,\dagger} \qquad \left\{\mathcal{Q}^I_\alpha, \mathcal{Q}^J_\beta\right\} = 2\,\epsilon_{\alpha\beta}\ \mathcal{Z}^{IJ} \qquad \left\{\mathcal{Q}^I_\alpha, \bar{\mathcal{Q}}^J_{\dot\beta}\right\} = 2\ \delta^{IJ}\ (\sigma^\mu)_{\alpha\dot\beta}\ P_\mu$$

SO(4) x SO(4) twist matrices

- O(6,6) twist : $U_M{}^{\underline{N}}(y^{\alpha M}) = \begin{pmatrix} \mathbb{I}_6 & 0_6 \\ \beta & \mathbb{I}_6 \end{pmatrix} \begin{pmatrix} \mathbb{I}_6 & b \\ 0_6 & \mathbb{I}_6 \end{pmatrix} \begin{pmatrix} u & 0_6 \\ 0_6 & u^{-t} \end{pmatrix} = \begin{pmatrix} u_m{}^{\underline{n}} & b_{mp}\,(u^{-t})^p{}_{\underline{n}} \\ \beta^{mp}\,u_p{}^{\underline{n}} & (u^{-t})^m{}_{\underline{n}} + \beta^{mp}\,b_{pq}\,(u^{-t})^q{}_{\underline{n}} \end{pmatrix}$

where $\beta^{mn} = \begin{pmatrix} (\beta_{(1)})^{ab} & 0_3 \\ 0_3 & (\beta_{(2)})^{ij} \end{pmatrix}$, $b_{mn} = \begin{pmatrix} (b_{(1)})_{ab} & 0_3 \\ 0_3 & (b_{(2)})_{ij} \end{pmatrix}$, $u_m{}^{\underline{n}} = \begin{pmatrix} (u_{(1)})_a{}^{\underline{b}} & 0_3 \\ 0_3 & (u_{(2)})_i{}^{\underline{j}} \end{pmatrix}$

$$u_{(1),(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2}\left(\cos Y_{(1),(2)} + \cos \widetilde{Y}_{(1),(2)}\right) & -\frac{1}{2}\left(\sin Y_{(1),(2)} + \sin \widetilde{Y}_{(1),(2)}\right) \\ 0 & \frac{1}{2}\left(\sin Y_{(1),(2)} + \sin \widetilde{Y}_{(1),(2)}\right) & \frac{1}{2}\left(\cos Y_{(1),(2)} + \cos \widetilde{Y}_{(1),(2)}\right) \end{pmatrix},$$

$$b_{(1),(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}\sin(Y_{(1),(2)} - \widetilde{Y}_{(1),(2)}) \\ 0 & -\frac{1}{2}\sin(Y_{(1),(2)} - \widetilde{Y}_{(1),(2)}) & 0 \end{pmatrix},$$

$$\beta_{(1),(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \tan\left(\frac{1}{2}\left(Y_{(1),(2)} - \widetilde{Y}_{(1),(2)}\right)\right) \\ 0 & -\tan\left(\frac{1}{2}\left(Y_{(1),(2)} - \widetilde{Y}_{(1),(2)}\right)\right) & 0 \end{pmatrix},$$

$$\begin{aligned} Y_{(1)} &= (\tilde{c}'_1 - a'_0)\,(y^{+1} - y^{+\bar{1}}) + (\tilde{d}'_1 - b'_0)\,(y^{-1} - y^{-\bar{1}}) \\ \widetilde{Y}_{(1)} &= (\tilde{c}'_1 + a'_0)\,(y^{+1} + y^{+\bar{1}}) + (\tilde{d}'_1 + b'_0)\,(y^{-1} + y^{-\bar{1}}) \\ Y_{(2)} &= (\tilde{c}'_2 - a'_3)\,(y^{+4} - y^{+\bar{4}}) + (\tilde{d}'_2 - b'_3)\,(y^{-4} - y^{-\bar{4}}) \\ \widetilde{Y}_{(2)} &= (\tilde{c}'_2 + a'_3)\,(y^{+4} + y^{+\bar{4}}) + (\tilde{d}'_2 + b'_3)\,(y^{-4} + y^{-\bar{4}}) \end{aligned}$$