Supersymmetric S-folds

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Based on 1907.04177 (+ work in progress) with Colin Sterckx (+ Mario Trigiante)

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Outlook

- Electric-magnetic duality in N=8 supergravity
- Supersymmetric S-folds

- Electric-magnetic duality in N=8 supergravity

N=8 supergravity in 4D

- SUGRA : metric + 8 gravitini + 28 vectors + 56 dilatini + 70 scalars

  (s = 2) (s = 3/2) (s = 1) (s = 1/2) (s = 0)

Ungauged (abelian) supergravity: Reduction of M-theory on a *torus* T^7 down to 4D produces $N = 8$ supergravity with $G = U(1)^{28}$

[$E_{7(7)}$ symmetry]

[Cremmer, Julia '79]

Gauged (non-abelian) supergravity:

- M-theory on S^7 produces $N = 8$ supergravity with $G = SO(8)$ [de Wit, Nicolai '82]

- Type IIA on S^6 produces $N = 8$ supergravity with $G = ISO(7) = SO(7) \ltimes \mathbb{R}^7$ [Hull '84]

- Type IIB on $S^5 \times S^1$ produces $N = 8$ supergravity with $G = [\,SO(1,1) \times SO(6)\,] \ltimes \mathbb{R}^{12}$

[Inverso, Samtleben, Trigiante '16]

✱ These supergravities believed to be unique for 30 years…

Electric-magnetic deformations

• Uniqueness historically inherited from the connection with NH geometries of branes and SCFT's

Type IIB : $AdS_5 \times S^5$ (D3-brane ~ N = 4 SYM in 4d) [Maldacena '97]

M-theory : $AdS_4 \times S^7$ (M2-brane ~ ABJM theory in 3d)

[Aharony, Bergman, Jafferis, Maldacena '08]

• N=8 supergravity in 4D admits a deformation parameter c yielding inequivalent theories. It is an electric/magnetic deformation

$$\boxed{D = \partial - g\,(A^{\text{elec}} - c\,\tilde{A}_{\text{mag}})}$$

g = 4D gauge coupling
c = deformation param.

[Dall'Agata, Inverso, Trigiante '12]

• There are two generic situations :

1) Family of $SO(8)_c$ theories : $c = [0, \sqrt{2} - 1]$ is a continuous parameter [similar for $SO(p,q)_c$]

2) Family of $CSO(p,q,r)_c$ theories : $c = 0$ or 1 is an (on/off) parameter

[Dall'Agata, Inverso, Marrani '14]

The questions arise:

- Does such an electric/magnetic deformation of 4D maximal supergravity enjoy a string/M-theory origin, or is it just a 4D feature ?

- For deformed 4D supergravities with supersymmetric AdS_4 vacua, are these AdS_4/CFT_3-dual to any identifiable 3d CFT ?

M-theory

electric / magnetic deformation	higher-dimensional origin	Holographic AdS_4 / CFT_3 dual ?
✓	?	?

Obstruction for $SO(8)_c$, *cf.* [de Wit, Nicolai '13]

[Lee, Strickland-Constable, Waldram '15]

(massive) Type IIA

electric/magnetic deformation	higher-dimensional origin	Holographic AdS_4/CFT_3 dual ?
✓	✓	✓

$$\boxed{g\,c \;=\; \hat{F}_{(0)} \;=\; k/(2\pi\ell_s)}$$

[AG, Jafferis, Varela '15]

Type IIB

electric/magnetic deformation	higher-dimensional origin	Holographic AdS_4/CFT_3 dual ?
✓	✓	✓

Dyonically-gauged $[\,SO(1,1) \times SO(6)\,] \ltimes R^{12}$ supergravity

- Higher-dimensional origin as Type IIB on $S^1 \times S^5$ [Inverso, Samtleben, Trigiante '16]

- New AdS_4 vacuum with N=4 & SO(4) symmetry [Gallerati, Samtleben, Trigiante '14]

- Holographic expectation: N=4 interface SYM theory with SO(4) symmetry & Janus solutions

[Bak, Gutperle, Hirano '03 (**N = 0**)]
[Clark, Freedman, Karch, Schnabl '04]
[D'Hoker, Ester, Gutperle '07, '07 (**N = 4**)]
[Gaiotto, Witten '08]
[Assel, Tomasiello '18 (**N = 3 , 4**)]

- Classification of (original) interface SYM theories

N=4 & SO(4) N=2 & SU(2) × U(1) N=1 & SU(3) N=0 & SO(6)

[D'Hoker, Ester, Gutperle '06 (**N = 1 , 2 , 4**)]

Question : *Simple analytic* holographic duals for the N = 0, 1, 2 interface SYM theories with SO(6) , SU(3) and SU(2) × U(1) internal symmetry using a bottom-up approach ?

A truncation : SU(3) invariant subsector

[Warner '83]

- Truncation : Retaining the fields and couplings which are invariant (singlets) under the action of a subgroup $G_0 \subset [\,\mathrm{SO}(1,1) \times \mathrm{SO}(6)\,] \ltimes \mathbb{R}^{12}$

  - SU(8) R-symmetry branching : gravitini $\mathbf{8} \rightarrow \mathbf{1} + \mathbf{1} + \mathbf{3} + \bar{\mathbf{3}}$ ➡ N = 2 SUSY

  - Scalars fields : $\mathbf{70} \rightarrow \mathbf{1}\ (\times 6) + \text{non-singlets}$ ➡ 6 real scalars $(\varphi\,,\,\chi\;,\;\phi\,,\,\sigma\,,\,\zeta\,,\,\tilde{\zeta}\,)$

  - Vector fields : $\mathbf{56} \rightarrow \mathbf{1}\,(\times 4) + \text{non-singlets}$ ➡ vectors $(\,A^0, A^1\,;\,\tilde{A}_0, \tilde{A}_1\,)$

- N = 2 gauged supergravity with $\mathrm{G} = \mathrm{SO}(1,1)_m \times \mathrm{U}(1)_e$ with 1 vector & 1 hypermultiplet

$$\mathcal{M}_{\text{scalar}} = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(2,1)}{\mathrm{U}(2)}$$

AdS$_4$ vacua $\quad (c \neq 0)$

[AG, Sterckx '19]

- **N=0 & SO(6) vacuum** [1 free parameter]

$$\chi = \text{free} \quad , \quad e^{-\varphi} = \frac{c}{\sqrt{2}} \, , \qquad e^{2\phi} = \frac{1}{\sqrt{1-\sigma^2}} \quad , \quad \sigma \in (-1,1) \quad , \quad |\vec{\zeta}|^2 = 0$$

… it turns out to be perturbatively unstable !!

- **N=1 & SU(3) vacuum** [2 free parameters]

$$\chi = 0 \quad , \quad e^{-\varphi} = \frac{\sqrt{5}c}{3} \, , \qquad e^{2\phi} = \frac{6}{5}\frac{1}{\sqrt{1-\sigma^2}} \quad , \quad \sigma \in (-1,1) \quad , \quad |\vec{\zeta}|^2 = \tfrac{2}{3}\sqrt{1-\sigma^2}$$

… the compact U(1)$_e$ symmetry broken by $|\vec{\zeta}|^2 \neq 0$ (charged)

Next step : **Uplift to Type IIB on R × S^5 using E$_{7(7)}$-EFT** [Hohm, Samtleben '13]

$\mathcal{N}=0$ & $\mathcal{N}=1$ supersymmetric S-folds

[AG, Sterckx '19]

$$ds_{10}^2 = \tfrac{1}{2}\sqrt{Y}\, e^{\varphi}\, ds^2_{\mathrm{AdS}_4} + \sqrt{Y}\, e^{-2\varphi}\, d\eta^2 + \frac{1}{\sqrt{Y}}\left[ds^2_{\mathbb{CP}^2} + Y\, \boldsymbol{\eta}^2 \right]$$

$$\widetilde{F}_5 = dC + \tfrac{1}{2}\,\epsilon_{\alpha\beta}\,\mathbb{B}^{\alpha}\wedge\mathbb{H}^{\beta} = \left(4 + \frac{6\,(1-Y)}{Y}\right) Y^{\frac{3}{4}}\,(1+\star)\,\mathrm{vol}_5$$

$$\mathbb{B}^{\alpha} = A^{\alpha}{}_{\beta}\,\mathfrak{b}^{\beta} = -\tfrac{1}{2}\,Y^{-1}\,A^{\alpha}{}_{\beta}\,\epsilon^{\beta\gamma}\,H_{\gamma\delta}\,\boldsymbol{\Omega}^{\delta}$$

$$m_{\alpha\beta} = (A^{-t})_{\alpha}{}^{\gamma}\,\mathfrak{m}_{\gamma\delta}\,(A^{-1})^{\delta}{}_{\beta}$$

with $\quad Y = 1 + \dfrac{1}{4}\,e^{2\phi}\,|\vec{\zeta}|^2 \quad$ and $\quad A^{\alpha}{}_{\beta} \equiv \begin{pmatrix} \sqrt{1+\tilde{y}^2} & \tilde{y} \\ \tilde{y} & \sqrt{1+\tilde{y}^2} \end{pmatrix} = \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix}$

[Bak, Gutperle, Hirano '03] **unstable !!**

[(hyperbolic) SO(1,1)-twist over S¹ $\leftrightarrow$ -ST^k monodromy ($k > 2$)]

N=0 & SO(6)

$$\mathfrak{m}_{\gamma\delta} = \frac{1}{\sqrt{1-\sigma^2}}\begin{pmatrix} 1 & -\sigma \\ -\sigma & 1 \end{pmatrix}$$

$$\mathfrak{b}^{\beta} = 0 \qquad Y = 1$$

No untwisted limit !!
(genuinely dyonic)

N=1 & SU(3)

$$\mathfrak{m}_{\gamma\delta} = \frac{1}{\sqrt{1-\sigma^2}}\begin{pmatrix} 1 & -\sigma \\ -\sigma & 1 \end{pmatrix}$$

$$\mathfrak{b}^{\beta} \neq 0 \qquad Y = \frac{6}{5}$$

$\mathcal{N} = 2$ supersymmetric S-folds

[AG, Sterckx, Trigiante to appear]

Symmetry : SU(2) x U(1)$_\sigma$

(genuinely dyonic)

$$ds^2 = \tfrac{1}{2}\,\Delta^{-1}\left[ds^2_{\text{AdS}_4} + d\eta^2 + d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\left(\sigma_2^2 + 8\,\Delta^4\,\left(\sigma_1^2 + \sigma_3^2\right)\right)\right]$$

$$\Delta^{-4} = 6 - 2\cos(2\theta)$$

$$\widetilde{F}_5 = 4\,\Delta^4 \sin\theta\,\cos^3\theta\,(1+\star)\Big[3\,d\theta\wedge d\phi\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3$$
$$-\,d\eta\wedge\Big(\cos(2\phi)\,d\theta - \tfrac{1}{2}\sin(2\theta)\,\sin(2\phi)\,d\phi\Big)\wedge\sigma_1\wedge\sigma_2\wedge\sigma_3\Big]$$

$\mathbb{B}^\alpha = A^\alpha{}_\beta\,\mathfrak{b}^\beta$ with

$$\mathfrak{b}_1 = \tfrac{1}{\sqrt{2}}\cos\theta\left[\Big(\sin\phi\,d\theta + \tfrac{1}{2}\sin(2\theta)\,d(\sin\phi)\Big)\wedge\sigma_2 + \sin\phi\,\frac{4\,\sin(2\theta)}{6-2\cos(2\theta)}\,\sigma_1\wedge\sigma_3\right]$$

$$\mathfrak{b}_2 = \tfrac{1}{\sqrt{2}}\cos\theta\left[\Big(\cos\phi\,d\theta + \tfrac{1}{2}\sin(2\theta)\,d(\cos\phi)\Big)\wedge\sigma_2 + \cos\phi\,\frac{4\,\sin(2\theta)}{6-2\cos(2\theta)}\,\sigma_1\wedge\sigma_3\right]$$

$m_{\alpha\beta} = (A^{-t})_\alpha{}^\gamma\,\mathfrak{m}_{\gamma\delta}\,(A^{-1})^\delta{}_\beta$ with $\mathfrak{m}_{\gamma\delta} = 2\,\Delta^2\begin{pmatrix} 1+\sin^2\theta\,\cos^2\phi & -\frac{1}{2}\sin^2(\theta)\sin(2\phi) \\ -\frac{1}{2}\sin^2(\theta)\sin(2\phi) & 1+\sin^2\theta\,\sin^2\phi \end{pmatrix}$

Conclusions

- Dyonic N = 8 supergravity with ISO(7) and $[\mathrm{SO}(1,1)\times\mathrm{SO}(6)]\ltimes\mathrm{R}^{12}$ gaugings connected to massive IIA reductions on S^6 and type IIB reductions on $S^5\times S^1$

- Type IIB (S-folds) : 3d interface SYM theories with various (super) symmetries

  [$-ST^k$ monodromy ($k > 2$)]

$\mathcal{N} = 0$ & SO(6)
unstable !!

$\mathcal{N} = 1$ & SU(3)

$\mathcal{N} = 2$ & SU(2) × U(1)

- Brane set-up (7 branes)? , RG flows? , non-abelian T-duals? ….

Gracias !

Thank you !

Extra material

$E_{7(7)}$-EFT

[momentum, winding, …]

- Space-time : external (D=4) + **generalised internal** ($Y^{\mathcal{M}}$ coordinates in **56** of $E_{7(7)}$)

Generalised diffs = *ordinary internal diffs* + internal *gauge transfos*

[Coimbra, Strickland-Constable, Waldram '11]

- Generalised Lie derivative built from an $E_{7(7)}$-invariant structure Y-tensor

$$\mathbb{L}_{\Lambda} U^{\mathcal{M}} = \Lambda^{\mathcal{N}} \partial_{\mathcal{N}} U^{\mathcal{M}} - U^{\mathcal{N}} \partial_{\mathcal{N}} \Lambda^{\mathcal{M}} + Y^{\mathcal{M}\mathcal{N}}{}_{\mathcal{P}\mathcal{Q}} \, \partial_{\mathcal{N}} \Lambda^{\mathcal{P}} \, U^{\mathcal{Q}}$$

[no density term]

Closure requires a **section constraint** : $Y^{\mathcal{P}\mathcal{Q}}{}_{\mathcal{M}\mathcal{N}} \; \partial_{\mathcal{P}} \otimes \partial_{\mathcal{Q}} = 0$

Two maximal solutions : M-theory (**7** dimensional) & Type IIB (**6** dimensional)

[**massless** theories]

$y^{i=1\ldots5}$ (elec) , $\tilde{y}_1$ (mag)

$E_{7(7)}$-EFT

[Hohm, Samtleben '13]

- $E_{7(7)}$-EFT action $\quad [\ \mathcal{D}_\mu = \partial_\mu - \mathbb{L}_{A_\mu}\ ]$

$$S_{\text{EFT}} = \int d^4x\, d^{56}Y\, e\, \Big[\, \hat{R} + \tfrac{1}{48}\, g^{\mu\nu}\, \mathcal{D}_\mu \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \mathcal{D}_\nu \mathcal{M}_{\mathcal{M}\mathcal{N}} - \tfrac{1}{8}\, \mathcal{M}_{\mathcal{M}\mathcal{N}}\, \mathcal{F}^{\mu\nu\mathcal{M}} \mathcal{F}_{\mu\nu}{}^{\mathcal{N}} + e^{-1}\, \mathcal{L}_{\text{top}} - V_{\text{EFT}}(\mathcal{M}, g)\, \Big]$$

with *field strengths* & *potential term* given by

$$\mathcal{F}_{\mu\nu}{}^{\mathcal{M}} = 2\, \partial_{[\mu} A_{\nu]}{}^{\mathcal{M}} - \big[A_\mu, A_\nu\big]_{\text{E}}^{\mathcal{M}} + \text{two-form terms}$$

(tensor hierarchy)

$$V_{\text{EFT}}(\mathcal{M}, g) = -\tfrac{1}{48}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_\mathcal{M} \mathcal{M}^{\mathcal{K}\mathcal{L}}\, \partial_\mathcal{N} \mathcal{M}_{\mathcal{K}\mathcal{L}} + \tfrac{1}{2}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_\mathcal{M} \mathcal{M}^{\mathcal{K}\mathcal{L}}\, \partial_\mathcal{L} \mathcal{M}_{\mathcal{N}\mathcal{K}} - \tfrac{1}{2}\, g^{-1} \partial_\mathcal{M} g\, \partial_\mathcal{N} \mathcal{M}^{\mathcal{M}\mathcal{N}} - \tfrac{1}{4}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, g^{-1} \partial_\mathcal{M} g\, g^{-1} \partial_\mathcal{N} g - \tfrac{1}{4}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_\mathcal{M} g^{\mu\nu}\, \partial_\mathcal{N} g_{\mu\nu}$$

- Two-derivative potential : **ungauged** N=8 D=4 SUGRA when $\Phi(x, Y) = \Phi(x)$

Generalised Scherk-Schwarz reductions

[Hohm, Samtleben ’14]

[Baguet, Hohm, Samtleben ’15]

+ 50% people in this workshop

- SL(8) twist (geometry) :

$$(U^{-1})_A{}^B = \left(\frac{\mathring{\rho}}{\hat{\rho}}\right)^{\frac{1}{2}} \begin{pmatrix} 1 & 0 & 0 & -\mathring{\rho}^{-2}\, c\, \tilde{y}_1 \\ 0 & \delta^{ij} + \hat{K}\, y^i\, y^j & -\lambda\, \hat{\rho}^2 y^i & 0 \\ 0 & -\lambda\, \hat{\rho}^2 y^j\, \hat{K} & \hat{\rho}^4 & 0 \\ -\mathring{\rho}^{-2}\, c\, \tilde{y}_1 & 0 & 0 & \mathring{\rho}^{-4}(1+\tilde{y}_1^2) \end{pmatrix}$$

- EFT fields = Twist × 4D fields :

$$\begin{aligned} g_{\mu\nu}(x,Y) &= \rho^{-2}(Y) g_{\mu\nu}(x) \\ \mathcal{M}_{MN}(x,Y) &= U_M{}^K(Y)\, U_N{}^L(Y)\, M_{KL}(x) \\ \mathcal{A}_\mu{}^M(x,Y) &= \rho^{-1}\, A_\mu{}^N(x)\, (U^{-1})_N{}^M(Y) \\ \mathcal{B}_{\mu\nu\,\alpha}(x,Y) &= \rho^{-2}(Y)\, U_\alpha{}^\beta(Y)\, B_{\mu\nu\,\beta}(x) \\ \mathcal{B}_{\mu\nu\,M}(x,Y) &= -2\,\rho^{-2}(Y)\, (U^{-1})_S{}^P(Y)\, \partial_M U_P{}^R(Y)\, (t^\alpha)_R{}^S\, B_{\mu\nu\,\alpha}(x) \end{aligned}$$

- Type IIB fields = EFT fields :

$$\begin{aligned} G^{mn} &= G^{1/2}\, \mathcal{M}^{mn} \;, \\ \mathbb{B}_{mn}{}^\alpha &= G^{1/2}\, G_{mp}\, \epsilon^{\alpha\beta}\, \mathcal{M}^p{}_{n\beta} \;, \\ m_{\alpha\beta} &= \tfrac{1}{6}\, G\, \left(\mathcal{M}^{mn}\, \mathcal{M}_{m\alpha\, n\beta} + \mathcal{M}^m{}_{k\alpha}\, \mathcal{M}^k{}_{m\beta} \right) \;, \\ C_{klmn} &= -\tfrac{1}{4}\, G^{1/2}\, G_{k\rho}\, \mathcal{M}^\rho{}_{lmn} + \tfrac{3}{8}\, \epsilon_{\alpha\beta}\, \mathbb{B}_{k[l}{}^\alpha\, \mathbb{B}_{mn]}{}^\beta \end{aligned}$$