Double Field Theory at SL(2) angles

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w/ F. Ciceri & G. Inverso (and G. Dibitetto & J. Melgarejo)

Duality covariant approaches to strings

- Different strings related by dualities: IIA/IIB T-duality, IIB S-duality, …

- *String dualities* realised as global symmetries in SUGRA

lower-dimensional phenomenon vs **higher-dimensional** phenomenon

embedding tensor, non-geometry… ß-supergravity, γ-supergravity…

Today's talk : **Extended Field Theories** [extend internal coords to transform under duality]

- Double Field Theory (DFT) ➡ Orthogonal groups O(d,d) [half-max SUGRA (T-duality)]
- Exceptional Field Theory (EFT) ➡ Exceptional groups $E_{d+1(d+1)}$ [max SUGRA (U-duality)]

[Siegel '93] [Hull & Zwiebach (Hohm) '09 '10] [Hohm & Samtleben '13]

Dualities in SUGRA and Extended Field Theory

D	Maximal sugra / EFT	Half-maximal sugra	DFT
9	$\mathbb{R}^+ \times \mathrm{SL}(2)$	$\mathbb{R}^+ \times \mathrm{O}(1,1+n)$	$\mathbb{R}^+ \times \mathrm{O}(1,1+n)$
8	$\mathrm{SL}(2) \times \mathrm{SL}(3)$	$\mathbb{R}^+ \times \mathrm{O}(2,2+n)$	$\mathbb{R}^+ \times \mathrm{O}(2,2+n)$
7	$\mathrm{SL}(5)$	$\mathbb{R}^+ \times \mathrm{O}(3,3+n)$	$\mathbb{R}^+ \times \mathrm{O}(3,3+n)$
6	$\mathrm{SO}(5,5)$	$\mathbb{R}^+ \times \mathrm{O}(4,4+n)$	$\mathbb{R}^+ \times \mathrm{O}(4,4+n)$
5	$\mathrm{E}_{6(6)}$	$\mathbb{R}^+ \times \mathrm{O}(5,5+n)$	$\mathbb{R}^+ \times \mathrm{O}(5,5+n)$
4	$\mathrm{E}_{7(7)}$	$\mathrm{SL}(2) \times \mathrm{O}(6,6+n)$	$\mathbb{R}^+ \times \mathrm{O}(6,6+n)$
3	$\mathrm{E}_{8(8)}$	$\mathrm{O}(8,8+n)$	$\mathbb{R}^+ \times \mathrm{O}(7,7+n)$

Duality groups of half-maximal SUGRA and DFT differ for $\mathbf{D} \leq \mathbf{4}$

* n = additional vector multiplets

… in this talk we will look at D=4 :

EFT with $E_{7(7)}$ duality group

[Hohm & Samtleben '13]

↓

SL(2)-DFT with SL(2) x O(6,6+n) duality group

[arXiv:1612.05230]

↓

DFT with R^+ x O(6,6+n) duality group

[Siegel '93]
[Hull & Zwiebach '09]
[Hohm, Hull & Zwiebach '10]
[Hohm & Kwak '11]

$E_{7(7)}$-EFT

[momentum, winding, …]

- Space-time : external (D=4) + **generalised internal** ($y^{\mathcal{M}}$ coordinates in **56** of $E_{7(7)}$)

Generalised diffs = *ordinary internal diffs* + internal *gauge transfos*

- Generalised Lie derivative built from an $E_{7(7)}$-invariant structure *Y*-tensor

$$\mathbb{L}_{\Lambda} U^{\mathcal{M}} = \Lambda^{\mathcal{N}} \partial_{\mathcal{N}} U^{\mathcal{M}} - U^{\mathcal{N}} \partial_{\mathcal{N}} \Lambda^{\mathcal{M}} + Y^{\mathcal{M}\mathcal{N}}{}_{\mathcal{P}\mathcal{Q}} \, \partial_{\mathcal{N}} \Lambda^{\mathcal{P}} \, U^{\mathcal{Q}}$$

Closure requires a **section constraint** : $Y^{\mathcal{P}\mathcal{Q}}{}_{\mathcal{M}\mathcal{N}} \, \partial_{\mathcal{P}} \otimes \partial_{\mathcal{Q}} = 0$

Two maximal solutions : M-theory (**7** dimensional) & Type IIB (**6** dimensional)

[**massless** theories]

Massive IIA arises as a *deformation of EFT*

[Ciceri, A.G. & Inverso '16]

[Hohm & Kwak '11 (sec const violated)]

[Romans '86]

[Cassani, de Felice, Petrini, Strickland-Constable & Waldram '16 (generalised geometry)]

$E_{7(7)}$-EFT

- $E_{7(7)}$-EFT action $\quad [\ \mathcal{D}_\mu = \partial_\mu - \mathbb{L}_{A_\mu}\ ]$

$$S_{\text{EFT}} \;=\; \int d^4x\, d^{56}y\, e\, \Big[\, \hat{R} + \tfrac{1}{48}\, g^{\mu\nu}\, \mathcal{D}_\mu \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \mathcal{D}_\nu \mathcal{M}_{\mathcal{M}\mathcal{N}} - \tfrac{1}{8}\, \mathcal{M}_{\mathcal{M}\mathcal{N}}\, \mathcal{F}^{\mu\nu\mathcal{M}} \mathcal{F}_{\mu\nu}{}^{\mathcal{N}} + e^{-1}\, \mathcal{L}_{\text{top}} - V_{\text{EFT}}(\mathcal{M}, g)\, \Big]$$

with *field strengths* & *potential term* given by

$$\mathcal{F}_{\mu\nu}{}^{\mathcal{M}} \;=\; 2\, \partial_{[\mu} A_{\nu]}{}^{\mathcal{M}} - \big[A_\mu, A_\nu\big]_{\text{E}}^{\mathcal{M}} + \text{two-form terms}$$

(tensor hierarchy)

$$V_{\text{EFT}}(\mathcal{M}, g) \;=\; -\tfrac{1}{48}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_\mathcal{M} \mathcal{M}^{\mathcal{K}\mathcal{L}}\, \partial_\mathcal{N} \mathcal{M}_{\mathcal{K}\mathcal{L}} + \tfrac{1}{2}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_\mathcal{M} \mathcal{M}^{\mathcal{K}\mathcal{L}}\, \partial_\mathcal{L} \mathcal{M}_{\mathcal{N}\mathcal{K}} - \tfrac{1}{2}\, g^{-1} \partial_\mathcal{M} g\, \partial_\mathcal{N} \mathcal{M}^{\mathcal{M}\mathcal{N}} - \tfrac{1}{4}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, g^{-1} \partial_\mathcal{M} g\, g^{-1} \partial_\mathcal{N} g - \tfrac{1}{4}\, \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \partial_\mathcal{M} g^{\mu\nu}\, \partial_\mathcal{N} g_{\mu\nu}$$

- Two-derivative potential : **ungauged** N=8 D=4 SUGRA when $\Phi(x,y) = \Phi(x)$

From $E_{7(7)}$-EFT to SL(2)-DFT

- Halving EFT with $E_{7(7)}$ symmetry to obtain SL(2)-DFT with SL(2) x O(6,6) symmetry

$$
\begin{array}{ccc}
E_{7(7)} & \rightarrow & SL(2) \times SO(6,6) \\
\mathbf{56} & \rightarrow & (\mathbf{2},\mathbf{12}) + (\mathbf{1},\mathbf{32}) \\
\underbrace{y^{\mathcal{M}}}_{\text{EFT}} & \rightarrow & \underbrace{y^{\alpha M}}_{\text{SL(2)-DFT}} + \cancel{y^{A}}
\end{array}
$$

α = (+ , -) vector index of SL(2)

M vector index of SO(6,6)

A M-W spinor index of SO(6,6)

[see Dibitetto, A.G. & Roest '11 for SUGRA]

via a **Z_2 truncation** (*vector* = +1 , *spinor* = -1) on coordinates, fields, etc.

- SL(2)-DFT generalised Lie derivative

[DFT corresponds to an α = + orientation]

$$
\mathbb{L}_{\Lambda} U^{\alpha M} = \Lambda^{\beta N} \partial_{\beta N} U^{\alpha M} - U^{\beta N} \partial_{\beta N} \Lambda^{\alpha M} + \eta^{MN} \eta_{PQ} \, \partial_{\beta N} \Lambda^{\beta P} \, U^{\alpha Q} + 2 \, \epsilon^{\alpha\beta} \, \epsilon_{\gamma\delta} \, \partial_{\beta N} \Lambda^{\gamma[M} \, U^{|\delta|N]}
$$

- SL(2)-DFT section constraints :

$$
\eta^{MN} \, \partial_{\alpha M} \otimes \partial_{\beta N} = 0 \quad , \quad \epsilon^{\alpha\beta} \, \partial_{\alpha[M|} \otimes \partial_{\beta|N]} = 0
$$

N=1 SUGRA in D=10 **as in DFT**

SL(2)-DFT with SL(2) x O(6,6) symmetry

- SL(2)-DFT action [$\mathcal{D}_\mu = \partial_\mu - \mathbb{L}_{A_\mu}$]

$$S_{\text{SL(2)-DFT}} = \int d^4x\, d^{24}y\, e\, \Big[\, \hat{R} + \tfrac{1}{4}\, g^{\mu\nu}\, \mathcal{D}_\mu M^{\alpha\beta}\, \mathcal{D}_\nu M_{\alpha\beta} + \tfrac{1}{8}\, g^{\mu\nu}\, \mathcal{D}_\mu M^{MN}\, \mathcal{D}_\nu M_{MN} - \tfrac{1}{8}\, M_{\alpha\beta}\, M_{MN}\, \mathcal{F}^{\mu\nu\,\alpha M} \mathcal{F}_{\mu\nu}{}^{\beta N} + e^{-1}\, \mathcal{L}_{\text{top}} - V_{\text{SL(2)-DFT}}(M, g)\,\Big]$$

with *field strengths* & *potential term* given by

$$\mathcal{F}_{\mu\nu}{}^{\alpha M} = 2\,\partial_{[\mu} A_{\nu]}{}^{\alpha M} - \big[A_\mu, A_\nu\big]_{\text{S}}^{\alpha M} + \text{two-form terms}$$

(tensor hierarchy)

$$\begin{aligned} V_{\text{SL(2)-DFT}}(M,g) \;=\;& M^{\alpha\beta} M^{MN} \Big[-\tfrac{1}{4}\,(\partial_{\alpha M} M^{\gamma\delta})(\partial_{\beta N} M_{\gamma\delta}) - \tfrac{1}{8}\,(\partial_{\alpha M} M^{PQ})(\partial_{\beta N} M_{PQ}) \\ & \quad + \tfrac{1}{2}\,(\partial_{\alpha M} M^{\gamma\delta})(\partial_{\delta N} M_{\beta\gamma}) + \tfrac{1}{2}(\partial_{\alpha M} M^{PQ})(\partial_{\beta Q} M_{NP})\Big] \\ +\;& \tfrac{1}{2}\, M^{MN} M^{PQ} (\partial_{\alpha M} M^{\alpha\delta})(\partial_{\delta Q} M_{NP}) + \tfrac{1}{2}\, M^{\alpha\beta} M^{\gamma\delta} (\partial_{\alpha M} M^{MQ})(\partial_{\delta Q} M_{\beta\gamma}) \\ -\;& \tfrac{1}{4}\, M^{\alpha\beta}\, M^{MN}\, \big[g^{-1}(\partial_{\alpha M} g)\, g^{-1}(\partial_{\beta N} g) + (\partial_{\alpha M} g^{\mu\nu})\,(\partial_{\beta N} g_{\mu\nu})\big] \\ -\;& \tfrac{1}{2}\, g^{-1}\,(\partial_{\alpha M} g)\,\partial_{\beta N}(M^{\alpha\beta} M^{MN}) \end{aligned}$$

- Two-derivative potential : **ungauged** N=4 D=4 SUGRA when $\Phi(x,y) = \Phi(x)$

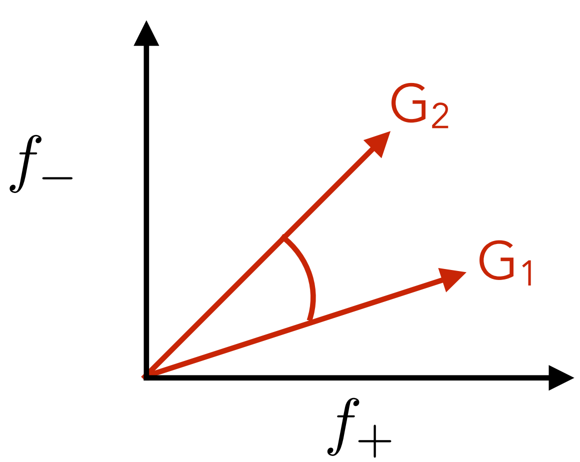
SL(2) angles & moduli stabilisation

- Scherk-Schwarz (SS) reductions with SL(2) x O(6,6) twist matrices $U_{\alpha M}{}^{\beta N} = e^{\lambda}\, e_{\alpha}{}^{\beta}\, U_M{}^N$ yield N=4 , D=4 gaugings [Schön & Weidner '06]

$$\begin{aligned} f_{\alpha MNP} &= -3\, e^{-\lambda}\, e_{\alpha}{}^{\beta}\, \eta_{Q[M}\, U_N{}^R\, U_{P]}{}^S\ \partial_{\beta R} U_S{}^Q \\ \xi_{\alpha M} &= 2\, U_M{}^N \partial_{\beta N}(e^{-\lambda} e_{\alpha}{}^{\beta}) \end{aligned}$$

[de Roo & Wagemans '85]

- *Moduli stabilisation* **requires** gaugings G = G_1 x G_2 at **relative** SL(2) angles



(sec. constraint violated)

$$\epsilon^{\alpha\beta}\, \partial_{\alpha[M|} \otimes \partial_{\beta|N]} \neq 0$$

- Dependence on **both** type + & type - coordinates [**not** possible in DFT]

Example : SO(4) x SO(4) gaugings and non-geometry

- SS with $U(y^{\alpha M}) \in \mathrm{O}(6,6)$: Half of the coords of type + & half of type -

- SL(2)-superposition of *two chains* of **non-geometric** fluxes $(\, H \, , \, \omega \, , \, Q \, , \, R \,)_{\pm}$

f_{+}	$f_{+mnp} = H^{+}{}_{mnp}$, $f_{+mn\bar{p}} = \omega^{+}{}_{mn}{}^{p}$, $f_{+\bar{m}\bar{n}p} = Q^{+mn}{}_{p}$, $f_{+\bar{m}\bar{n}\bar{p}} = R^{+mnp}$
f_{-}	$f_{-mnp} = H^{-}{}_{mnp}$, $f_{-mn\bar{p}} = \omega^{-}{}_{mn}{}^{p}$, $f_{-\bar{m}\bar{n}p} = Q^{-mn}{}_{p}$, $f_{-\bar{m}\bar{n}\bar{p}} = R^{-mnp}$

Most general family (8 params) of SO(4) x SO(4) gaugings of N=4 SUGRA

- SO(4) x SO(4) SUGRA : **AdS$_4$** & **dS$_4$** vacua (sphere/hyperboloid reductions)

[de Roo, Westra, Panda & Trigiante '03] [Dibitetto, A.G. & Roest '12]

- ``Hybrid ±" sources to cancel flux-induced tadpoles : SL(2)-dual NS-NS branes

Summary & Future directions

- SL(2)-DFT captures the duality group of N=4 SUGRA in D=4
- SL(2)-DFT sec. constraints : N=1 SUGRA in D=10 & N=(2,0) SUGRA in D=6
- SL(2)-DFT action extendable to SL(2) x SO(6,6+n) and *deformable as EFT*

[Ciceri, A.G. & Inverso '16]

- Non-geometric gaugings at non-trivial SL(2) angles : *full moduli AdS_4 stabilisation*

[**not** possible in DFT]

- Flux formulation of SL(2)-DFT : sec. cons violating terms & SL(2)-NS-NS branes

[Aldazabal, Graña, Marqués & Rosabal '13]

- Cosmological applications of SL(2)-DFT (stable dS_4 , inflation, …)

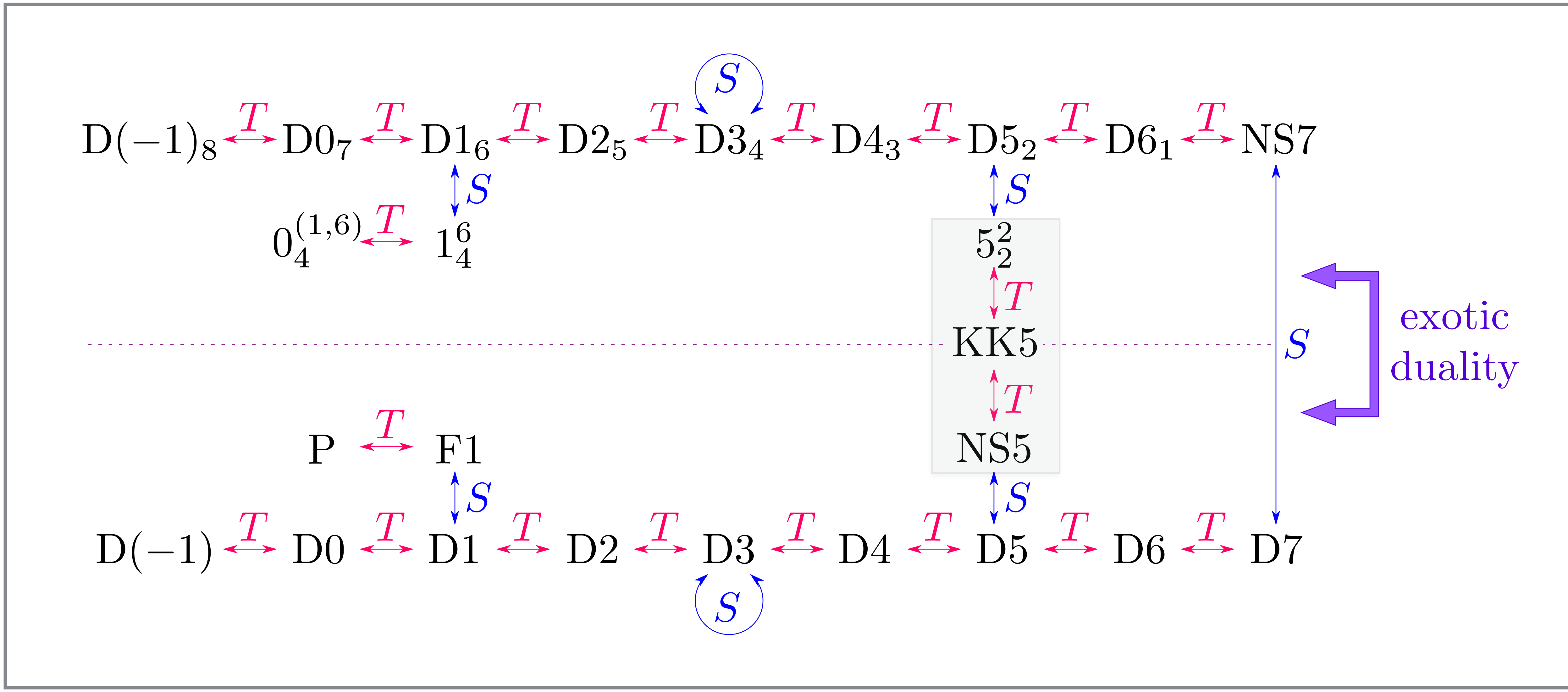
[Hassler, Lüst & Massai '14]

Grazie mille!!

Thanks a lot!!

Extra material

A family of exotic branes



[Sakatani '15]

SO(4) x SO(4) twist matrices

- O(6,6) twist : $U_M{}^{\underline{N}}(y^{\alpha M}) = \begin{pmatrix} \mathbb{I}_6 & 0_6 \\ \beta & \mathbb{I}_6 \end{pmatrix} \begin{pmatrix} \mathbb{I}_6 & b \\ 0_6 & \mathbb{I}_6 \end{pmatrix} \begin{pmatrix} u & 0_6 \\ 0_6 & u^{-t} \end{pmatrix} = \begin{pmatrix} u_m{}^{\underline{n}} & b_{mp}\,(u^{-t})^p{}_{\underline{n}} \\ \beta^{mp}\,u_p{}^{\underline{n}} & (u^{-t})^m{}_{\underline{n}} + \beta^{mp}\,b_{pq}\,(u^{-t})^q{}_{\underline{n}} \end{pmatrix}$

where $\beta^{mn} = \begin{pmatrix} (\beta_{(1)})^{ab} & 0_3 \\ 0_3 & (\beta_{(2)})^{ij} \end{pmatrix}$, $b_{mn} = \begin{pmatrix} (b_{(1)})_{ab} & 0_3 \\ 0_3 & (b_{(2)})_{ij} \end{pmatrix}$, $u_m{}^{\underline{n}} = \begin{pmatrix} (u_{(1)})_a{}^{\underline{b}} & 0_3 \\ 0_3 & (u_{(2)})_i{}^{\underline{j}} \end{pmatrix}$

$$u_{(1),(2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2}\,(\cos Y_{(1),(2)} + \cos \widetilde{Y}_{(1),(2)}) & -\frac{1}{2}\,(\sin Y_{(1),(2)} + \sin \widetilde{Y}_{(1),(2)}) \\ 0 & \frac{1}{2}\,(\sin Y_{(1),(2)} + \sin \widetilde{Y}_{(1),(2)}) & \frac{1}{2}\,(\cos Y_{(1),(2)} + \cos \widetilde{Y}_{(1),(2)}) \end{pmatrix},$$

$$b_{(1),(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}\,\sin(Y_{(1),(2)} - \widetilde{Y}_{(1),(2)}) \\ 0 & -\frac{1}{2}\,\sin(Y_{(1),(2)} - \widetilde{Y}_{(1),(2)}) & 0 \end{pmatrix},$$

$$\beta_{(1),(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \tan\left(\frac{1}{2}\,(Y_{(1),(2)} - \widetilde{Y}_{(1),(2)})\right) \\ 0 & -\tan\left(\frac{1}{2}\,(Y_{(1),(2)} - \widetilde{Y}_{(1),(2)})\right) & 0 \end{pmatrix},$$

$$\begin{aligned} Y_{(1)} &= (\tilde{c}'_1 - a'_0)\,(y^{+1} - y^{+\bar{1}}) + (\tilde{d}'_1 - b'_0)\,(y^{-1} - y^{-\bar{1}}) \\ \widetilde{Y}_{(1)} &= (\tilde{c}'_1 + a'_0)\,(y^{+1} + y^{+\bar{1}}) + (\tilde{d}'_1 + b'_0)\,(y^{-1} + y^{-\bar{1}}) \\ Y_{(2)} &= (\tilde{c}'_2 - a'_3)\,(y^{+4} - y^{+\bar{4}}) + (\tilde{d}'_2 - b'_3)\,(y^{-4} - y^{-\bar{4}}) \\ \widetilde{Y}_{(2)} &= (\tilde{c}'_2 + a'_3)\,(y^{+4} + y^{+\bar{4}}) + (\tilde{d}'_2 + b'_3)\,(y^{-4} + y^{-\bar{4}}) \end{aligned}$$

Deformed EFT (XFT)

- Generalised Lie derivative

[no density term]

$$\mathbb{L}_\Lambda U^{\mathcal{M}} \;=\; \Lambda^{\mathcal{N}}\partial_{\mathcal{N}}U^{\mathcal{M}} - U^{\mathcal{N}}\partial_{\mathcal{N}}\Lambda^{\mathcal{M}} + Y^{\mathcal{M}\mathcal{N}}{}_{\mathcal{P}\mathcal{Q}}\,\partial_{\mathcal{N}}\Lambda^{\mathcal{P}}\,U^{\mathcal{Q}}$$

in terms of an $E_{n(n)}$-invariant structure Y-tensor. Closure requires **sec. constraint**

- **Deformed** generalised Lie derivative

$$\widetilde{\mathbb{L}}_\Lambda U^{\mathcal{M}} \;=\; \Lambda^{\mathcal{N}}\partial_{\mathcal{N}}U^{\mathcal{M}} - U^{\mathcal{N}}\partial_{\mathcal{N}}\Lambda^{\mathcal{M}} + Y^{\mathcal{M}\mathcal{N}}{}_{\mathcal{P}\mathcal{Q}}\,\partial_{\mathcal{N}}\Lambda^{\mathcal{P}}\,U^{\mathcal{Q}} - \underbrace{X_{\mathcal{N}\mathcal{P}}{}^{\mathcal{M}}\,\Lambda^{N}\,U^{\mathcal{P}}}_{\text{non-derivative}}$$

in terms of an X deformation which is $E_{n(n)}$-algebra valued

- Closure & triviality of the Jacobiator require (*together with* ***sec. constraint***)

$$X_{\mathcal{M}\mathcal{N}}{}^{\mathcal{P}}\,\partial_{\mathcal{P}} = 0$$

X constraint

$$X_{\mathcal{M}\mathcal{P}}{}^{\mathcal{Q}}\,X_{\mathcal{N}\mathcal{Q}}{}^{\mathcal{R}} - X_{\mathcal{N}\mathcal{P}}{}^{\mathcal{Q}}\,X_{\mathcal{M}\mathcal{Q}}{}^{\mathcal{R}} + X_{\mathcal{M}\mathcal{N}}{}^{\mathcal{Q}}\,X_{\mathcal{Q}\mathcal{P}}{}^{\mathcal{R}} = 0$$

Quadratic constraint (gauged max. supergravity)

[*X* deformation *vs* embedding tensor]

X deformation : background fluxes & Romans mass

$$Y^{\mathcal{PQ}}{}_{\mathcal{MN}}\, \partial_{\mathcal{P}} \otimes \partial_{\mathcal{Q}} = 0$$

section constraint

$$X_{\mathcal{MN}}{}^{\mathcal{P}}\, \partial_{\mathcal{P}} = 0$$

X constraint

[algebraic system]

M-theory (**n** coords)

- SL(n) orbit
- Freund-Rubin param. (n = 4 and n = 7)
- *massless* IIA (subcase)

Type IIB (**n-1** coords)

- SL(n-1) orbit
- *p*-form fluxes compatible with SL(n-1)
- **SL(2)-triplet of 1-form flux** (includes compact SO(2))

+

New massive Type IIA (**n-1** coords)

- SL(n-1) orbit
- *p*-form fluxes compatible with SL(n-1)
- dilaton flux
- **Romans mass parameter** (*kills the M-theory coord*)

Massive Type IIA described in a purely geometric manner !!

[QC = flux-induced tadpoles]

$E_{7(7)}$-XFT action

- $E_{7(7)}$-XFT action $[\ \mathcal{D}_\mu = \partial_\mu - \widetilde{\mathbb{L}}_{A_\mu}\ ]$ [$y^{\mathcal{M}}$ coords in the **56** of $E_{7(7)}$]

$$S_{\text{XFT}} = \int d^4x\, d^{56}y\, e\, \Big[\, \hat{R} + \tfrac{1}{48}\, g^{\mu\nu}\, \mathcal{D}_\mu \mathcal{M}^{\mathcal{M}\mathcal{N}}\, \mathcal{D}_\nu \mathcal{M}_{\mathcal{M}\mathcal{N}} - \tfrac{1}{8}\, \mathcal{M}_{\mathcal{M}\mathcal{N}}\, \mathcal{F}^{\mu\nu\mathcal{M}} \mathcal{F}_{\mu\nu}{}^{\mathcal{N}} + e^{-1}\, \mathcal{L}_{\text{top}} - V_{\text{XFT}}(\mathcal{M}, g)\, \Big]$$

with *field strengths* & *potential* given by

(deformed tensor hierarchy)

$$\mathcal{F}_{\mu\nu}{}^{\mathcal{M}} = 2\, \partial_{[\mu} A_{\nu]}{}^{\mathcal{M}} + X_{[\mathcal{P}\mathcal{Q}]}{}^{\mathcal{M}}\, A_\mu{}^{\mathcal{P}} A_\nu{}^{\mathcal{Q}} - \big[A_\mu, A_\nu\big]_{\text{E}}^{\mathcal{M}} + \text{two-form terms}$$

$$V_{\text{XFT}}(\mathcal{M}, g, X) = V_{\text{EFT}}(\mathcal{M}, g) + \underbrace{\tfrac{1}{12}\, \mathcal{M}^{MN} \mathcal{M}^{KL} X_{MK}{}^{P}\, \partial_N \mathcal{M}_{PL}}_{\text{cross term}} + \underbrace{V_{\text{SUGRA}}(\mathcal{M}, X)}_{\text{gauged max. sugra}}$$

- Two-One-**Zero**-derivative potential : **gauged** 4D max. sugra when $\Phi(x, y) = \Phi(x)$

Extended (super) Poincaré superalgebra

- Central charges (internal symmetries) $\mathcal{Z}_{IJ} = (a^a_{IJ})\ T^a$

- The algebra :

$$[P_\mu, P_\nu] = 0 \qquad [M_{\mu\nu}, M_{\rho\sigma}] = i\,(\eta_{\nu\rho}\,M_{\mu\sigma} - \eta_{\nu\sigma}\,M_{\mu\rho} - \eta_{\mu\rho}\,M_{\nu\sigma} + \eta_{\mu\sigma}\,M_{\nu\rho})$$

$$[P_\mu, M_{\rho\sigma}] = i\,(\eta_{\mu\rho}\,P_\sigma - \eta_{\mu\sigma}\,P_\rho)$$

$$[T^a, T^b] = if^{ab}_{\ \ c}\ T^c \qquad [T^a, P_\mu] = [T^a, M_{\mu\nu}] = 0$$

$$[\mathcal{Q}^I_\alpha, P_\mu] = [\bar{\mathcal{Q}}^I_{\dot\alpha}, P_\mu] = 0 \qquad [\mathcal{Q}^I_\alpha, T^a] = (b_a)^I{}_J\ \mathcal{Q}^J_\alpha \qquad [\bar{\mathcal{Q}}^I_{\dot\alpha}, T^a] = -\bar{\mathcal{Q}}^J_{\dot\alpha}\ (b_a)_J{}^I$$

$$[\mathcal{Q}^I_\alpha, M_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu})_\alpha{}^\beta\ \mathcal{Q}^I_\beta \qquad [\bar{\mathcal{Q}}^I_{\dot\alpha}, M_{\mu\nu}] = -\frac{1}{2}\bar{\mathcal{Q}}^I_{\dot\beta}\ (\bar\sigma_{\mu\nu})^{\dot\beta}{}_{\dot\alpha}$$

$$\left\{\bar{\mathcal{Q}}^I_{\dot\alpha}, \bar{\mathcal{Q}}^J_{\dot\beta}\right\} = -2\,\epsilon_{\dot\alpha\dot\beta}\ \mathcal{Z}^{IJ\,\dagger} \qquad \left\{\mathcal{Q}^I_\alpha, \mathcal{Q}^J_\beta\right\} = 2\,\epsilon_{\alpha\beta}\ \mathcal{Z}^{IJ} \qquad \left\{\mathcal{Q}^I_\alpha, \bar{\mathcal{Q}}^J_{\dot\beta}\right\} = 2\ \delta^{IJ}\ (\sigma^\mu)_{\alpha\dot\beta}\ P_\mu$$