Towards disentangling the landscape of extended supergravities

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with G. Dibitetto and D. Roest : arXiv:1102.0239
arXiv:1104.3587
arXiv:11xx.xxxx (in progress)

The footprint of extra dimensions

- Four dimensional supergravity theories appear when compactifying string theory

- Fluctuations of the internal space around a fixed geometry translates into massless 4d scalar fields known as ``*moduli*"

$$\mathcal{L} = \frac{1}{2} R - \frac{1}{2} \partial_\mu \phi_i \, \partial^\mu \phi^i$$

Deviations from GR !!

massless scalars = long range interactions (precision tests of GR)

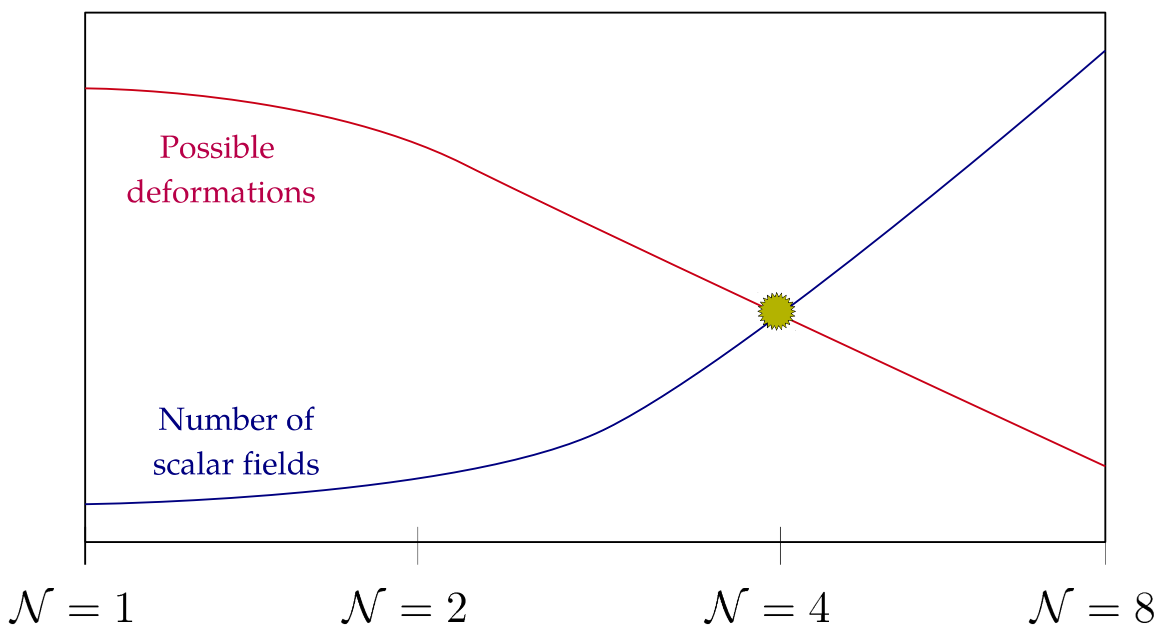
Linking strings to observations → Mechanisms to stabilise moduli !!

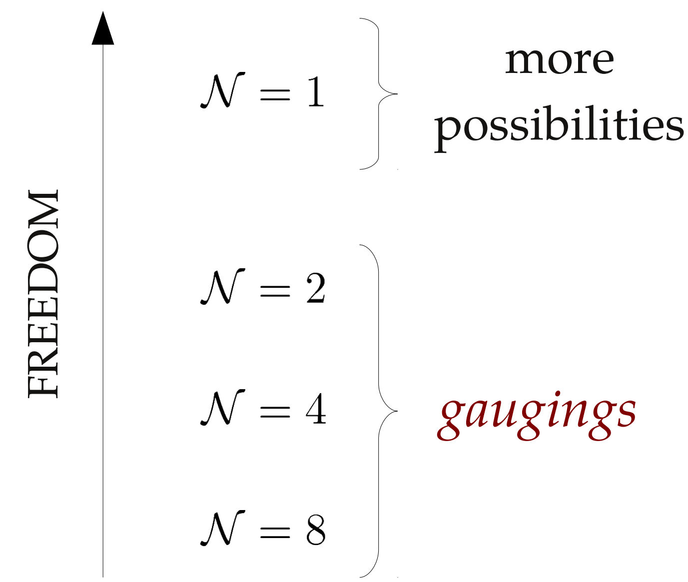
$$V(\phi) = m_{ij}^2 \, \phi^i \phi^j + \ldots$$

- Moduli VEVs $\langle \phi \rangle = \phi_0$ determine 4d physics
  - $\Lambda_{c.c} \equiv V(\phi_0)$
  - g_s and Vol_{int}
  - fermi masses

How to deform massless theories to have $V(\phi) \neq 0$?

- Supersymmetry dictates what deformations are allowed





gaugings = part of the global symmetry is promoted to local (*gauge*)

- Gauged supergravities can be systematically studied

[Nicolai, Samtleben '00]

[de Wit, Samtleben, Trigiante '02 '05 '07]

[Schon, Weidner '06]

embedding tensor formalism

The embedding tensor formalism (I)

- Reducing 10d supergravities down to 4d yields *ungauged* supergravities with global symmetry G ⇨ duality group in 4d

- Abelian gauge fields $A_\mu^{\mathcal{F}}$ in the fundamental rep. of G

- The scalar sector parameterises the coset space $\mathcal{M} = G/H$ where H is the maximal compact subgroup of G

$\mathcal{N} = 8$

$G = E_7 \qquad H = SU(8)$

$\mathcal{N} = 4$

$G = SL(2) \times SO(6,6)$

$H = SO(2) \times SO(6) \times SO(6)$

- *Gauging* : a subgroup $G_0 \subset G$ is promoted to local symmetry yielding a *gauged* supergravity

$$\nabla_\mu \longrightarrow \nabla_\mu - g\, A_\mu^{\mathcal{F}}\, \Theta_{\mathcal{F}}{}^{\mathcal{A}}\, t_{\mathcal{A}}$$

embedding tensor

where $\begin{cases} \mathcal{F} \equiv \text{fund} \\ \mathcal{A} \equiv \text{adj} \end{cases}$ reps of G

The embedding tensor formalism (II)

- The embedding tensor $\Theta_{\mathcal{F}}{}^{\mathcal{A}}$ encodes any possible deformation of the theory (*gauging*)

$$\mathcal{F} \otimes \mathcal{A} = rep_1 \oplus rep_2 \oplus \ldots \qquad \text{of } G$$

- Consistency of the *gauging* implies

  i) Supersymmetry ⇒ Linear constraints on Θ (some ~~rep_i~~)

  ii) Gauge algebra ⇒ Quadratic constraints on Θ

- The *gauging* also induces a non-trivial scalar potential $V(\Theta, \mathcal{M})$ for the scalar fields $\mathcal{M} = G/H$

- *Stringy features (T-S-... dualities) implemented at the supergravity level !!*

A chain of theories

[Dibitetto, A.G, Roest in progress]

$G = E_7$

e.t comp = 912
vectors = 56
scalars = 133

$\mathcal{N} = 8$

$\xrightarrow{SO(3)}$

$G = SL(2) \times G_2$

e.t comp = (2,1) + (4,14) + (2,7)
vectors = (4,1)
scalars = (3,1) + (1,14)

$\mathcal{N} = 2$

$\downarrow \mathbb{Z}_2$ $\qquad$ $\mathbb{Z}_2 \downarrow$

$G = SL(2) \times SO(6,6)$

e.t comp = (2,12) + (2,220)
vectors = (2,12)
scalars = (3,1) + (1,66)

$\mathcal{N} = 4$

$\xrightarrow{SO(3)}$

$G = SL(2) \times SL(2) \times SL(2)$

e.t comp = (2,2,2) + (2,4,4)
vectors = 0
scalars = (3,1,1) + (1,3,1)+(1,1,3)

$\mathcal{N} = 1$

* reps of G

Half-maximal supergravities in 4d

[Schon, Weidner '06]

$$G = SL(2) \times SO(6,6)$$

e.t comp = (2,12) + (2,220)
vectors = (2,12)
scalars = (3,1) + (1,66)

$$\mathcal{N} = 4$$

Questions :

- Can the whole vacuum structure be charted in $\mathcal{N} = 4$ theories ?
- Are there connections in the landscape of vacua ?

Half-maximal : symmetry and fields

- Global symmetry (duality) group $G = SL(2) \times SO(6,6)$

- Field content = supergravity multiplet + six vector multiplets

- Vectors $A_\mu^{\alpha M}$ in the fundamental of G

$\alpha = +, -$ is an electric-magnetic $SL(2)$ index
$M = 1, ..., 12$ is an $SO(6,6)$ index
} 24 vectors

- The scalar sector parameterises $\mathcal{M} = M_{\alpha\beta} \times M_{MN}$

  - 1 axion + 1 dilaton in SL(2)
  - 30 axions + 6 dilatons in SO(6,6)

  } 38 scalars

$$M_{\alpha\beta} \equiv e^{\phi} \begin{pmatrix} \chi^2 + e^{-2\phi} & \chi \\ \chi & 1 \end{pmatrix}$$

$$M_{MN} \equiv \begin{pmatrix} G^{-1} & -G^{-1} B \\ B\, G^{-1} & G - B\, G^{-1} B \end{pmatrix}$$

Half-maximal : *gaugings* and scalar potential

- A subgroup $G_0 \subset SL(2) \times SO(6,6)$ is promoted to local (*gauged*)
- *Gaugings* are classified by the embedding tensor parameters

$$\xi_{\alpha M} \in (\mathbf{2}, \mathbf{12}) \qquad \text{and} \qquad f_{\alpha MNP} \in (\mathbf{2}, \mathbf{220})$$

- Supersymmetry + gauge invariance determine the scalar potential

$$V = \frac{1}{64} f_{\alpha MNP} f_{\beta QRS} M^{\alpha\beta} \left[\frac{1}{3} M^{MQ} M^{NR} M^{PS} + \left(\frac{2}{3} \eta^{MQ} - M^{MQ} \right) \eta^{NR} \eta^{PS} \right] - \frac{1}{144} f_{\alpha MNP} f_{\beta QRS} \epsilon^{\alpha\beta} M^{MNPQRS} + \frac{3}{64} \xi_{\alpha}^{M} \xi_{\beta}^{N} M^{\alpha\beta} M_{MN}$$

To keep in mind : V is quadratic in the emb. tens. parameters

- 464 e.t. components + 38 scalars = TOO MUCH !!

The SO(3) truncation

- Keeping only fields and embedding tensor components invariant under the action of a subgroup $SO(3) \subset SO(6,6)$

$G = SL(2) \times SO(6,6)$

e.t comp = (2,12) + (2,220)
vectors = (2,12)
scalars = (3,1) + (1,66)

$\mathcal{N} = 4$

$\xrightarrow{SO(3)}$

$G = SL(2) \times SL(2) \times SL(2)$

e.t comp = (2,2,2) + (2,4,4)
vectors = 0
scalars = (3,1,1) + (1,3,1)+(1,1,3)

$\mathcal{N} = 1$

- R-symmetry group :

$$SU(4) \supset SO(3)$$

$$4 \longrightarrow 1 + 3$$

The SO(3) truncation : fields and *gaugings*

- Symmetry and fields :

  - global symmetry $G = SL(2) \times SO(2,2) = SL(2)^3$
  - $A_\mu^{\alpha M} = 0$ and $\xi_{\alpha M} = 0$ [Derendinger, Kounnas, Petropoulos, Zwirner '04]
  - scalar coset = 3 complex scalars = STU – models !!

$$\underbrace{S \equiv \chi + i\, e^{-\phi}}_{\text{SL(2)}} \quad , \quad \underbrace{T \equiv \chi_1 + i\, e^{-\varphi_1} \quad \text{and} \quad U \equiv \chi_2 + i\, e^{-\varphi_2}}_{\text{SO(2,2)}}$$

- SO(2,2) scalars : $G = e^{\varphi_2 - \varphi_1} \begin{pmatrix} \chi_2^2 + e^{-2\varphi_2} & -\chi_2 \\ -\chi_2 & 1 \end{pmatrix} \otimes \mathbb{I}_3 \quad , \quad B = \begin{pmatrix} 0 & \chi_1 \\ -\chi_1 & 0 \end{pmatrix} \otimes \mathbb{I}_3$

- The *gaugings* $G_0 \subset G$ and the scalar potential $V(S,T,U)$ specified by the embedding tensor

$$f_{\alpha MNP} = 40 \text{ components}$$

The SO(3) truncation : *gaugings* from fluxes

- String embedding as flux compactification

$$f_{\alpha MNP} \;=\; \text{generalised fluxes}$$

Example: SO(3) truncation ⟺ type II orientifolds of $\mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$

- Type IIB fluxes and embedding tensor

$$f_{+mnp} = \tilde{F}'_{mnp} \quad , \quad f_{+mn}{}^{p} = Q'_{mn}{}^{p} \quad , \quad f_{+}{}^{mn}{}_{p} = Q^{mn}{}_{p} \quad , \quad f_{+}{}^{mnp} = \tilde{F}^{mnp} \quad ,$$

$$f_{-mnp} = \tilde{H}'_{mnp} \quad , \quad f_{-mn}{}^{p} = P'_{mn}{}^{p} \quad , \quad f_{-}{}^{mn}{}_{p} = P^{mn}{}_{p} \quad , \quad f_{-}{}^{mnp} = \tilde{H}^{mnp} \quad ,$$

[Dibitetto, Linares, Roest '10]

* index splitting $M = ({}_{m} \; , \; {}^{m})$

- Perfect matching with flux-induced superpotential (up to Q.C)

$$W_{flux} = (P_F - P_H\, S) + 3\,T\,(P_Q - P_P\, S) + 3\,T^2(P_{Q'} - P_{P'}\, S) + T^3\,(P_{F'} - P_{H'}\, S)$$

The SO(3) truncation : *gaugings* consistency

- Closure of the gauge algebra = quadratic constraints on $f_{\alpha MNP}$

gaugings

$$A_\mu = A_\mu^{\alpha M}\, T_{\alpha M}$$

$$[T_{\alpha M}, T_{\beta N}] = f_{\alpha\, MN}{}^{P}\; T_{\beta P}$$

➡

Quadratic Constraints

$$\epsilon^{\alpha\beta}\, f_{\alpha MNR}\, f_{\beta PQ}{}^{R} = 0$$

$$f_{\alpha R[MN}\, f_{\beta PQ]}{}^{R} = 0$$

- String theory :

quadratic constraints = B.I. for gauge fields + vanishing of the flux-induced tadpoles for sources breaking $\mathcal{N}=4$

Example : Type IIB orientifolds with O3/O7-planes

- (H , F) fluxes : Unconstrained D3-brane flux-induced tadpole
- (H , F , Q) fluxes : Vanishing of D7-brane flux-induced tadpole
- (H , F , Q , P) fluxes : Vanishing of D7 , NS7 and I7 flux-induced tadpoles
- ??

We would like to . . .

1) build all the consistent SO(3)-invariant gaugings specified by $f_{\alpha MNP}$ by solving the quadratic constraints

$$\epsilon\, f\, f = 0 \qquad \text{and} \qquad f\, f = 0$$

2) compute all the SO(3)-invariant extrema of the f-induced scalar potential $V(f\,,\,\Phi)$ by solving the extremisation conditions

$$\left.\frac{\partial V}{\partial \Phi}\right|_{\Phi_0} = 0 \qquad \text{with} \qquad \Phi \equiv (\,S\,,\,T\,,\,U\,)$$

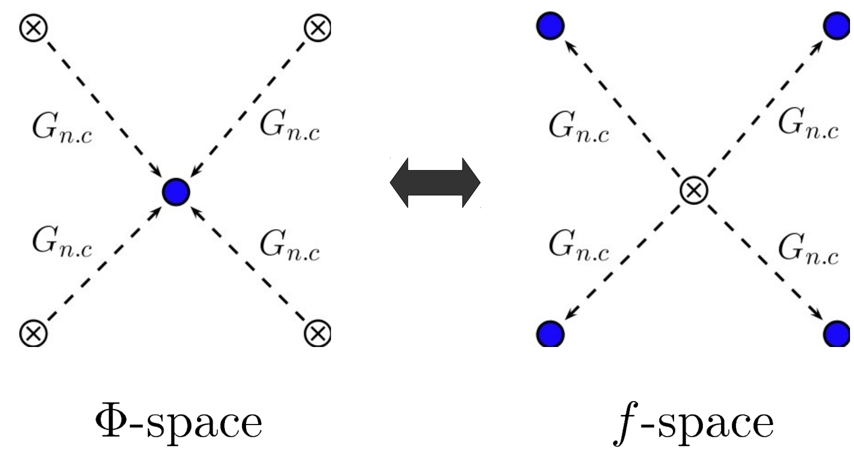
3) check stability of these extrema with respect to fluctuations of all the 38 scalars of half-maximal supergravity

4) identify the gauge group G_0 underlying all the different solutions

. . . but is this doable ?

Strategy and tools

- Idea : use the global symmetry group (non-compact part) to bring any field solution back to the origin !!



- At the origin everything is simply quadratic in the $f_{\alpha MNP}$ parameters

$$V(\Phi) = \sum_{\text{terms}} f\, f\;\; \Phi^{\text{high degree}}$$

so then,

computing the vacua structure = multivariate polynomial system $I = \langle\, \partial_\Phi V|_{\Phi_0}\;\; ,\;\; \epsilon\, f\, f\;\; ,\;\; f\, f\,\rangle$

⇒ Algebraic Geometry techniques !!

[Gray, He, Lukas '06]

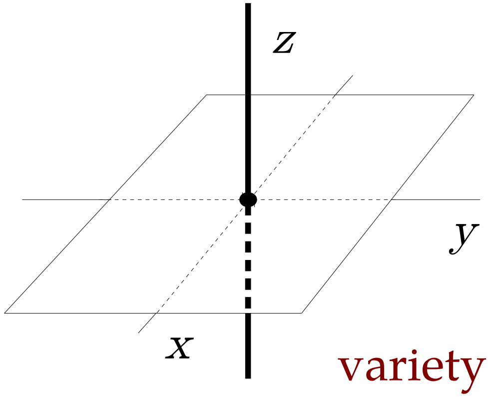
Basics of Algebraic Geometry

- Algebraic Geometry studies multivariate polynomial system and their link to geometry

$$I = \langle P_1 \, , \, P_2 \rangle$$

$$\begin{aligned} P_1(x,y,z) &= x\,z \\ P_2(x,y,z) &= y\,z \end{aligned}$$

algebraic system ⟺



variety

- GTZ prime decomposition (analogous to integers dec. 30 = 2 x 3 x 5)

$$I = J_1 \cap J_2 \quad \text{where} \quad \begin{cases} J_1 = \langle z \rangle \longrightarrow xy\text{-plane} \\ J_2 = \langle x \, , \, y \rangle \longrightarrow z\text{-axis} \end{cases}$$

$$J_1 \cap J_2 \quad \longleftrightarrow \quad V(J_1) \cup V(J_2)$$

algebra-geometry dictionary

- Applying the above procedure to our problem involving fluxes

$$I = \langle \, \partial_\Phi V|_{\Phi_0} \quad , \quad \epsilon\, f\, f \quad , \quad f\, f \, \rangle$$

$$I = J_1 \cap J_2 \cap \ldots \cap J_n$$

Splitting of the landscape into n disconnected pieces !!

[*Singular* project, 97]

An example : type IIA with metric fluxes

- Testing the method with type IIA orientifold models including gauge fluxes and a metric flux

[Dall'Agatta, Villadoro, Zwirner '09]

$$\left(F_{p=0,2,4,6} \quad , \quad H_3 \right) \quad + \quad \omega \quad \subset \quad f_{\alpha MNP}$$

Q.C. of *gaugings* = B.I. + tadpoles cancellation

- Subset of embedding tensor components closed under $G_{n.c}$
  - Fields can still be set at the origin without lost of generality
  - Stability with respect to fluctuations around the origin can be computed

[Borghese, Roest '10]

- Vacua structure of these type IIA orientifolds

16 AdS_4 critical points **+** Mink of dim=1 → GKP after three T-dualities

[Giddings, Kachru, Polchinski '02]

The 16 critical points

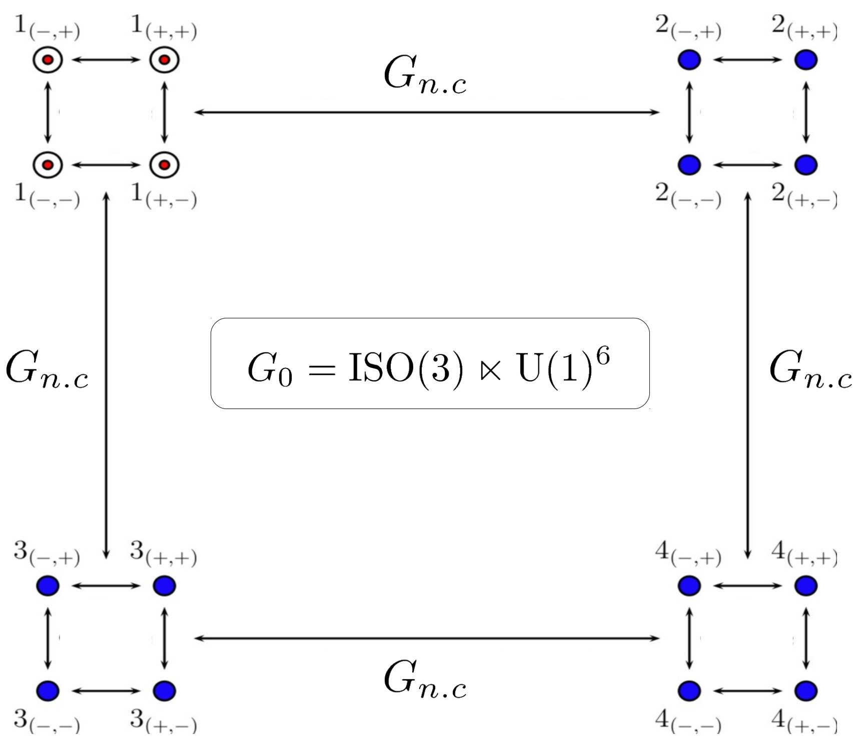
- An AdS$_4$ landscape

$$16 = 4 + 4 + 4 + 4$$

- Extra vanishing of ALL the flux-induced tadpoles !!

$1_{(\pm,\pm)}$	$2_{(\pm,\pm)}$	$3_{(\pm,\pm)}$	$4_{(\pm,\pm)}$
$\mathcal{N}=1$ SUSY & FAKE SUSY	~~SUSY~~	~~SUSY~~	~~SUSY~~
stable	unstable	stable	stable
m^2 = -2/3	m^2 = -4/5	$m^2 > 0$	$m^2 > 0$
V= -1	V= -32/27	V= -8/15	V= -32/27

$(*)\ m^2 \equiv$ lightest mode (B.F. bound $= -3/4$)



- All the solutions are connected and correspond to $\omega \equiv SU(2) \times SU(2)$

[Caviezel, Koerber, Körs, Lüst, Tsimpis/Wrase, Zagermann '08, '08]

- Unique *gauging*

Unique theory

with 4 different vacua !!

Lifting to maximal supergravity ?

$G = E_7$

e.t comp = 912
vectors = 56
scalars = 133

$\mathcal{N} = 8$

$\uparrow \; \mathbb{Z}_2$

$G = SL(2) \times SO(6,6)$

e.t comp = (2,12) + (2,220)
vectors = (2,12)
scalars = (3,1) + (1,66)

$\mathcal{N} = 4$

Question :

- Is the vanishing of ALL the flux-induced tadpoles enough for the geometric IIA solutions to lift to a maximal supergravity theory?

When half- becomes maximal supergravity (I)

[Dibitetto, A.G., Roest '11]

- See the maximal theory with the half-maximal *''glasses''* and then modding out by a $\mathbb{Z}_2$ orientifold

$$E_7 \supset SL(2) \times SO(6,6)$$

$$\begin{array}{llll} \text{vectors}: & 56 & \longrightarrow & (2,12) + (1,32) \\ \text{scalars}: & 133 & \longrightarrow & (3,1) + (1,66) + (2,32') \\ \text{e.t:} & \underbrace{912}_{X} & \longrightarrow & \underbrace{(2,12)}_{\xi_{\alpha M}} + \underbrace{(2,220)}_{f_{\alpha MNP}} + (1,352') + (3,32) \end{array}$$

$$\left.\right\} \quad \begin{array}{llll} & & & \mathbb{Z}_2 \\ rep & \equiv & bos\ (B) & \to\ even \\ rep & \equiv & \underbrace{fermi\ (F)}_{SO(6,6)} & \to\ odd \end{array}$$

- Gauge algebra of the maximal theory with embedding tensor $X(f,\xi)$

$$\begin{array}{lll} [A_B, A_B] & = & X_{BB}{}^{B}\, A_B + X_{BB}{}^{F}\, A_F \\ [A_B, A_F] & = & X_{BF}{}^{B}\, A_B + X_{BF}{}^{F}\, A_F \\ [A_F, A_F] & = & X_{FF}{}^{B}\, A_B + X_{FF}{}^{F}\, A_F \end{array} \left.\right\} \quad \text{Jacobi identities} = QC_{\mathcal{N}=8}$$

- Components with an odd number of fermionic indices projected out !!

When half- becomes maximal supergravity (II)

- For a half-maximal to be embeddable in a maximal theory

$$QC_{\mathcal{N}=8} \quad = \quad QC_{\mathcal{N}=4} \quad + \quad \text{extra conditions for the lifting}$$

- The extra conditions are

$$\underbrace{f_{\alpha MNP}\, f_{\beta}{}^{MNP} = 0}_{(3,1)} \qquad \text{and} \qquad \underbrace{\epsilon^{\alpha\beta}\; f_{\alpha[MNP}\, f_{\beta QRS]}\,\big|_{\text{SD}} = 0}_{(1,462')}$$

- All the geometric IIA solutions do lift to maximal supergravity !!

- Type IIB : flux-induced tadpoles for dual sources ?

$$\underbrace{H_3 \wedge F_3}_{\text{D3-brane tadpole}} \subset (1, 462') \quad \Longrightarrow \quad \text{which objects fill the rep ?}$$

Conclusions

- Some progress towards disentangling the landscape of extended supergravities can still be done without performing statistics of vacua

- The approach relies on the combined use of global symmetries and of algebraic geometry techniques

- As a warming-up, the complete vacua structure of simple type IIA orientifold theories can be worked out revealing some odd features :

  i) stability without supersymmetry
  ii) connections between vacua
  iii) $\mathcal{N} = 8$ embedding of the entire vacua structure

- For the future :

- Stability of type IIA geometric solutions in maximal supergravity ?

- Beyond the geometric limit : non-geometric backgrounds, dual branes . . .

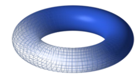
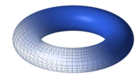
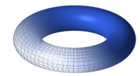
- Systematic search of de Sitter stable solutions in extended supergravity (maybe in $\mathcal{N} = 2$) and also links to Cosmology

Thanks !!

Extra material . . .

Internal geometries and massless theories . . .

``maximal''

$\mathbb{T}^6 =$  x  x 

$\mathcal{N} = 8$ (70 scalars)

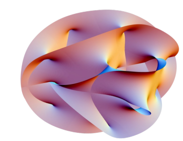
Type II 10d supergravity

``half-maximal''

Orientifolds of $\mathbb{T}^6$

$\mathcal{N} = 4$ (38 scalars)

``minimally extended''

$CY_3 =$ 

$\mathcal{N} = 2$

scalars $\leftrightarrow \left(h^{(1,1)}, h^{(1,2)}\right)$

``minimal''

Orientifolds of CY_3

$\mathcal{N} = 1$ scalars $\leftrightarrow \left(h^{(1,1)}, h^{(1,2)}\right)$

Lower bound on topologically distinct CY_3
30108

Gaugings and their higher-dimensional origin

- $\mathcal{N} = 8$: Gauging a subgroup of the global symmetry $G = E_7$

Internal space extension $\longleftrightarrow$ Exceptional Generalised Geometry ?

[Pacheco, Waldram '08 , Grana, Louis, Sim, Waldram '09]

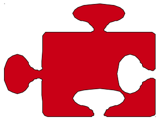
[Aldazabal, Andrés, Cámara, Grana '10]

- $\mathcal{N} = 4$: Gauging a subgroup of the global symmetry $G = SL(2) \times SO(6,6)$

Internal space extension $\longleftrightarrow$ Doubled/Generalised Geometry ?

[Hitchin '02, Gualtieri '04]

[Hull '04, '06]



pathological
internal spaces

String compactifications including generalised flux backgrounds !!

De Sitter in extended supergravity

- $\mathcal{N} = 8$: unstable dS solutions with $SO(4,4)$ and $SO(5,3)$ gaugings

[Hull, Warner '85]

- $\mathcal{N} = 4$: unstable dS solutions with gaugings at *angles*

[De Roo, Wagemans '85]

i) $G_1 \times G_2$ gaugings with
$$\begin{cases} G_i = SO(p_i, q_i) \quad , \quad p_i + q_i = 4 \\ G_i = CSO(p_i, q_i, r_i) \quad , \quad p_i + q_i + r_i = 4 \end{cases}$$

[De Roo, Westra, Panda, (Trigiante) '02, '03, '06]

ii) $SO(3,1) \ltimes U(1)^6$ gauging

[Dibitetto, A.G, Roest '11]

non-geometric fluxes in string theory !!

[Dibitetto, Linares, Roest '10]

- $\mathcal{N} = 2$: stable dS solutions with $SO(2,1) \times SO(3)$ gauging plus Fayet-Iliopoulos terms

[Fré, Trigiante, Van Proeyen '03]

unclear origin in string theory !!

De Sitter in minimal supergravity

- No-go theorems forbidding dS solutions in $\mathcal{N}=1$ compactifications with gauge fluxes

$$V_o = -\frac{1}{9}\sum \bar{F}^2 \leq 0 \quad \Rightarrow \quad \text{AdS !!}$$

[Hertzberg, Kachru, Taylor, Tegmark '07]

- Including **more general fluxes** : (metric + non-geometric)

$$V_o = -\frac{1}{9}\sum \bar{F}^2 + \Delta V_{\text{metric}} + \Delta V_{\text{non-geom}}$$

a) metric fluxes ⟷ unstable dS in type IIA models

[Caviezel, Koerber, Kors, Lust, Wrase, Zagerman '08]

b) non-geometric fluxes ⟷ stable dS in type IIA models

[de Carlos, A.G, Moreno '09, '10]

- Including D-branes to **uplift an AdS** solution

[Kachru, Kallosh, Linde, Trivedi '03]

a) D-terms from D-branes ⟷ stable dS in type IIB models

[Burgess, Kallosh, Quevedo '03]

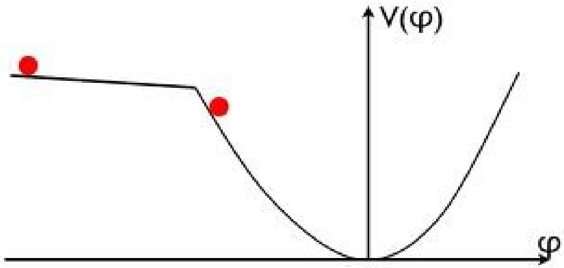
b) non-perturbative effects from D-branes ⟷ stable dS in type IIB

[Achúcarro, de Carlos, Casas, Doplicher '06]

Cosmology from moduli ?

- slow-roll inflation requires an almost flat dS saddle point of $V(\phi)$ from which to start rolling down

$$\eta \equiv M_p^2 \left(\frac{V''}{V} \right) \ll 1$$



- dS saddle points suffering from eta-problem, *i.e.* $\eta \sim \mathcal{O}(1)$

i) gaugings in extended supergravity [Kallosh, Linde, Prokushkin, Shmakova '01]

ii) general fluxes in minimal supergravity [Flauger, Paban, Robbings, Wrase '08] [de Carlos, A.G, Moreno '10]

- dS saddle points with $\eta \ll 1$ in minimal supergravity including non-perturbative effects $\Longrightarrow$ axion inflation !!

[Dimopoulos, Kachru, McGreevy, Wacker '05]