Dyonic $\mathcal{N}=8$ supergravity from IIA strings and its Chern-Simons duals

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Electric/magnetic duality in maximal supergravity

While electromagnetic duality is a symmetry of many supergravity theories, this is not the case for the maximal ($\mathcal{N}=8$) gauged theory. It was recently shown that this rotation leads to a one-parameter family of $\mathrm{SO}(8)_c$ supergravities, with parameter c, and similarly for other gauge groups, like its contraction $\mathrm{ISO}(7)_c = \mathrm{SO}(7) \ltimes \mathbb{R}^7_c$. In the latter case, only the seven tranlations are gauged dyonically and the parameter c turns out to be a discrete (on/off) deformation [1].

The questions arise:

Does such an electric/magnetic deformation of maximal supergravity enjoy a string/M-theory origin, or is it just a four-dimensional feature?

For deformed supergravities with supersymmetric anti-de-Sitter vacua (AdS), are these $\mathrm{AdS}_4/\mathrm{CFT}_3$-dual to any identifiable three-dimensional superconformal field theory?

Dyonic ISO(7) supergravity [2]

• Using the embedding tensor formalism [3], the (bosonic) Lagrangian of the dyonic ISO(7)-gauged theory contains scalars $\mathcal{M}_{\mathbb{MN}}(\phi)$ parameterising $\mathrm{E}_{7(7)}/\mathrm{SU}(8)$, electric vectors $(\mathcal{A}^{IJ}=\mathcal{A}^{[IJ]}, \mathcal{A}^I)$, magnetic vectors $\tilde{\mathcal{A}}_I$, two-form fields $\mathcal{B}^I$ and the metric $g_{\mu\nu}$. It reads

$$\begin{aligned}\mathcal{L} \,=\,& R\,\mathrm{vol}_4 - \tfrac{1}{48}\, D\mathcal{M}_{\mathbb{MN}} \wedge * D\mathcal{M}^{\mathbb{MN}} + \tfrac{1}{2}\mathcal{I}_{\Lambda\Sigma}\,\mathcal{H}^{\Lambda}_{(2)} \wedge *\mathcal{H}^{\Sigma}_{(2)} - \tfrac{1}{2}\mathcal{R}_{\Lambda\Sigma}\,\mathcal{H}^{\Lambda}_{(2)}\wedge\mathcal{H}^{\Sigma}_{(2)} \\ &- V\,\mathrm{vol}_4 + g\,c\left[\mathcal{B}^I \wedge \left(\tilde{\mathcal{H}}_{(2)I} - \tfrac{g}{2}\,\delta_{IJ}\,\mathcal{B}^J\right) - \tfrac{1}{4}\tilde{\mathcal{A}}_I\wedge\tilde{\mathcal{A}}_J\wedge\left(d\mathcal{A}^{IJ} + \tfrac{g}{2}\,\delta_{KL}\,\mathcal{A}^{IK}\wedge\mathcal{A}^{JL}\right)\right],\end{aligned}$$

where $\mathbb{M}=1,\dots,56$ and $I=1,\dots,7$ are fundamental $\mathrm{E}_{7(7)}$ and SL(7) indices, respectively. The index $\Lambda=1,\dots,28$ collectively runs over the 21+7 electric field strenghts $(\mathcal{H}^{IJ}_{(2)}, \mathcal{H}^{I}_{(2)})$. The covariant derivative takes the form $D = d - g\,\mathcal{A}^{IJ}\,t_{[I}{}^{K}\,\delta_{J]K} + g\,(\delta_{IJ}\,\mathcal{A}^I - c\,\tilde{\mathcal{A}}_J)\,t_8{}^J$, gauging dyonically the $\mathbb{R}^7 \subset \mathrm{ISO}(7)$ generators $t_8{}^J$. Finally, the scalar potential reads

$$V = \frac{g^2}{168}\, X_{\mathbb{MP}}{}^{\mathbb{R}} X_{\mathbb{NQ}}{}^{\mathbb{S}} \mathcal{M}^{\mathbb{MN}}\left(\mathcal{M}^{\mathbb{PQ}}\mathcal{M}_{\mathbb{RS}} + 7\,\delta^{\mathbb{P}}_{\mathbb{S}}\,\delta^{\mathbb{Q}}_{\mathbb{R}}\right),$$

and depends on the scalars $\mathcal{M}_{\mathbb{MN}}$, the embedding tensor $X_{\mathbb{MN}}{}^{\mathbb{P}}(c)$ specifying the dyonic gauging of $\mathrm{ISO}(7)\subset\mathrm{E}_{7(7)}$, and the gauge coupling constant g.

• We can describe the dynamics of the theory by using a (restricted) SL(7)-covariant tensor hierarchy of 4D fields. Apart from the metric and the scalars, there are

$$\begin{array}{rll} \mathbf{21}'+\mathbf{7}'+\mathbf{21}+\mathbf{7} & \text{vectors}: & \mathcal{A}^{IJ}\,,\quad \mathcal{A}^I\,,\quad \tilde{\mathcal{A}}_{IJ}\,,\quad \tilde{\mathcal{A}}_I\,, \\ \mathbf{48}+\mathbf{7}'+\mathbf{1} & \text{two-forms}: & \mathcal{B}_I{}^J\,,\quad \mathcal{B}^I\,,\quad \mathcal{B}\,, \\ \mathbf{28}'+\mathbf{1} & \text{three-forms}: & \mathcal{C}^{IJ}\,,\quad \mathcal{C}\,, \end{array}$$

endowed with duality relations that transfer degrees of freedom among different fields

$$\begin{aligned} \tilde{\mathcal{H}}_{(2)IJ} &= -\tfrac{1}{2}\mathcal{I}_{[IJ][KL]} * \mathcal{H}^{KL}_{(2)} - \mathcal{I}_{[IJ][K8]} * \mathcal{H}^{K}_{(2)} + \tfrac{1}{2}\mathcal{R}_{[IJ][KL]}\mathcal{H}^{KL}_{(2)} + \mathcal{R}_{[IJ][K8]}\mathcal{H}^{K}_{(2)}\,, \\ \tilde{\mathcal{H}}_{(2)I} &= -\tfrac{1}{2}\mathcal{I}_{[I8][KL]} * \mathcal{H}^{KL}_{(2)} - \mathcal{I}_{[I8][K8]} * \mathcal{H}^{K}_{(2)} + \tfrac{1}{2}\mathcal{R}_{[I8][KL]}\mathcal{H}^{KL}_{(2)} + \mathcal{R}_{[I8][K8]}\mathcal{H}^{K}_{(2)}\,, \\ \mathcal{H}_{(3)I}{}^{J} &= -\tfrac{1}{12}(t_I{}^J)_{\mathbb{M}}{}^{\mathbb{P}}\,\mathcal{M}_{\mathbb{NP}} * D\mathcal{M}^{\mathbb{MN}} - \tfrac{1}{7}\,\delta_I^J\,(\text{trace})\,, \\ \mathcal{H}_{(3)}{}^{I} &= -\tfrac{1}{12}(t_8{}^I)_{\mathbb{M}}{}^{\mathbb{P}}\,\mathcal{M}_{\mathbb{NP}} * D\mathcal{M}^{\mathbb{MN}}\,, \\ \mathcal{H}_{(3)} &= \tfrac{1}{12}(t_8{}^8)_{\mathbb{M}}{}^{\mathbb{P}}\,\mathcal{M}_{\mathbb{NP}} * D\mathcal{M}^{\mathbb{MN}}\,, \\ \mathcal{H}^{IJ}_{(4)} &= \tfrac{1}{84}X_{\mathbb{NQ}}{}^{\mathbb{S}}\left((t_K{}^{(I|})_{\mathbb{P}}{}^{\mathbb{R}}\mathcal{M}^{|J)K\,\mathbb{N}} + (t_8{}^{(I|})_{\mathbb{P}}{}^{\mathbb{R}}\mathcal{M}^{|J)8\,\mathbb{N}}\right)\left(\mathcal{M}^{\mathbb{PQ}}\mathcal{M}_{\mathbb{RS}} + 7\,\delta^{\mathbb{P}}_{\mathbb{S}}\,\delta^{\mathbb{Q}}_{\mathbb{R}}\right)\mathrm{vol}_4\,, \\ \mathcal{H}_{(4)} &= \tfrac{1}{84}X_{\mathbb{NQ}}{}^{\mathbb{S}}(t_8{}^K)_{\mathbb{P}}{}^{\mathbb{R}}\mathcal{M}_{8K}{}^{\mathbb{N}}\mathcal{M}^{\mathbb{PQ}}\mathcal{M}_{\mathbb{RS}}\,\mathrm{vol}_4\,. \end{aligned}$$

• Closed set of Bianchi identities $D\mathcal{H}_{(n)}$ and Hodge-duality relations in four dimensions.

tensor hierarchy + duality relations = **duality hierarchy**

[1] *Symplectic Deformations of Gauged Maximal Supergravity.* G. Dall'Agata, G. Inverso and Alessio Marrani. JHEP 1407(2014)133.

[2] *Dyonic ISO(7) supergravity and the duality hierarchy.* Adolfo Guarino and Oscar Varela. arXiv:1508.04432.

[3] *The maximal $D=4$ supergravities.* Bernard de Wit, Henning Samtleben and Mario Trigiante. JHEP 0706(2007)049 .

[4] *Consistent $\mathcal{N}=8$ truncation of massive IIA on S^6.* Adolfo Guarino and Oscar Varela. To appear.

[5] *String Theoretic Origin of Dyonic $\mathcal{N}=8$ Supergravity and its Simple Chern-Simons Duals.* Adolfo Guarino, Daniel L. Jafferis and Oscar Varela. Phys.Rev.Lett.115(2015)091601.

[6] *Towards the F-Theorem: $\mathcal{N}=2$ Field Theories on the Three-Sphere.* Daniel L. Jafferis, Igor R. Klebanov, Silviu S. Pufu, Benjamin R. Safdi. JHEP1106(2011)102

The Freund-Rubin term $\hat{F}_{(4)} = \mathcal{H}^{IJ}_{(4)}\,\mu_I\,\mu_J + \dots$, takes the compact form

$$\begin{aligned}\mathcal{H}^{IJ}_{(4)}\,\mu_I\,\mu_J \,=\,& -\tfrac{1}{3g}\,V\,\mathrm{vol}_4 \\ &+ \tfrac{1}{84g}\left(D\mathcal{H}_3 - 7\,\mathcal{H}^{IJ}_{(2)}\wedge\tilde{\mathcal{H}}_{(2)IJ} - 7\,\mathcal{H}^{I}_{(2)}\wedge\tilde{\mathcal{H}}_{(2)I}\right) \\ &- \tfrac{1}{2g}\left(D\mathcal{H}_{(3)I}{}^{J} - \mathcal{H}^{JK}_{(2)}\wedge\tilde{\mathcal{H}}_{(2)IK} - \mathcal{H}^{J}_{(2)}\wedge\tilde{\mathcal{H}}_{(2)I}\right)\mu^I\,\mu_J\,.\end{aligned}$$

Non-linear embedding into massive IIA on S^6 [4]

The non-linear embedding of the 4D (restricted) tensor hierarchy into the 10D type IIA fields reads

$$\begin{aligned} d\hat{s}^2_{10} &= \Delta^{-1}\,ds^2_4 + g_{mn}\,Dy^m\,Dy^n\,, \\ \hat{B}_{(2)} &= -\mu_I\left(\mathcal{B}^I + \tfrac{1}{2}\mathcal{A}^{IJ}\wedge\tilde{\mathcal{A}}_J\right) - g^{-1}\,\tilde{\mathcal{A}}_I\wedge D\mu^I + \tfrac{1}{2}\,B_{mn}\,Dy^m\wedge Dy^n\,, \\ e^{-\frac{3}{2}\hat{\phi}} &= \Delta\,\mathcal{M}^{I8\,J8}\,\mu_I\,\mu_J - g^{mn}\,A_m\,A_n\,, \\ \hat{A}_{(1)} &= -\mu_I\,\mathcal{A}^I + A_m\,Dy^m\,, \\ \hat{A}_{(3)} &= \mu_I\,\mu_J\left(\mathcal{C}^{IJ} + \mathcal{A}^I\wedge\mathcal{B}^J + \tfrac{1}{6}\,\mathcal{A}^{IK}\wedge\mathcal{A}^{JL}\wedge\tilde{\mathcal{A}}_{KL} + \tfrac{1}{6}\,\mathcal{A}^I\wedge\mathcal{A}^{JK}\wedge\tilde{\mathcal{A}}_K\right) \\ &\quad + g^{-1}\left(\mathcal{B}_J{}^I + \tfrac{1}{2}\mathcal{A}^{IK}\wedge\tilde{\mathcal{A}}_{KJ} + \tfrac{1}{2}\mathcal{A}^I\wedge\tilde{\mathcal{A}}_J\right)\wedge\mu_I\,D\mu^J + \tfrac{1}{2}\,g^{-2}\,\tilde{\mathcal{A}}_{IJ}\wedge D\mu^I\wedge D\mu^J \\ &\quad - \tfrac{1}{2}\,\mu_I\,B_{mn}\,\mathcal{A}^I\wedge Dy^m\wedge Dy^n + \tfrac{1}{6}\,A_{mnp}\,Dy^m\wedge Dy^n\wedge Dy^p\,, \end{aligned}$$

with the purely internal (scalar) components of the 10D fields given by

$$\begin{aligned} g^{mn} &= \tfrac{1}{4}\,g^2\,\Delta\,\mathcal{M}^{IJ\,KL}\,K^m_{IJ}\,K^n_{KL} \quad , \quad B_{mn} = -\tfrac{1}{2}\,\Delta\,g_{mp}\,K^p_{IJ}\,\partial_n\mu^K\,\mathcal{M}^{IJ}{}_{K8}\,, \\ A_m &= \tfrac{1}{2}\,g\,\Delta\,g_{mn}\,K^n_{IJ}\,\mu_K\,\mathcal{M}^{IJ\,K8} \quad , \quad A_{mnp} = \tfrac{1}{8}\,g\,\Delta\,g_{mq}\,K^q_{IJ}\,K^{KL}_{np}\,\mathcal{M}^{IJ}{}_{KL} + A_m B_{np}\,. \end{aligned}$$

We have used a unit radius S^6 parameterised as the locus $\delta_{IJ}\,\mu^I\,\mu^J = 1$ in $\mathbb{R}^7$, together with a set of Killing vectors $K_m{}^{IJ} = 2\,g^{-2}\,\mu^{[I}\partial_m\mu^{J]}$ and tensors $K_{mn}{}^{IJ} = 4\,g^{-2}\,\partial_{[m}\mu^I\partial_{n]}\mu^J$. Using the round S^6 metric $\mathring{g}_{mn} = g^{-2}\,\delta_{IJ}\,\partial_m\mu^I\,\partial_n\mu^J$, we have also defined

$$\Delta^2 \equiv \frac{\det\,g_{mn}}{\det\,\mathring{g}_{mn}} \quad , \quad Dy^m \equiv dy^m + \tfrac{1}{2}\,g\,K^m_{IJ}\,\mathcal{A}^{IJ} \quad , \quad D\mu^I \equiv d\mu^I - g\,\mathcal{A}^{IJ}\mu_J\,.$$

A new $\mathcal{N}=2$ solution of massive IIA [5]

There is an AdS_4 solution of the 4D theory preserving $\mathcal{N}=2$ supersymmetry and $\mathrm{U}(3)\subset\mathrm{ISO}(7)$ gauge symmetry, which uplifts to an analytic massive IIA solution of the form

$$\begin{aligned} d\hat{s}^2_{10} &= L^2\frac{(3+\cos 2\alpha)^{\frac{1}{2}}}{(5+\cos 2\alpha)^{-\frac{1}{8}}}\left[ds^2(\mathrm{AdS}_4) + \frac{3}{2}\,d\alpha^2 + \frac{6\sin^2\alpha}{3+\cos 2\alpha}\,ds^2(\mathbb{CP}^2) + \frac{9\sin^2\alpha}{5+\cos 2\alpha}\,\boldsymbol{\eta}^2\right], \\ e^{\hat{\phi}} &= e^{\phi_0}\,\frac{(5+\cos 2\alpha)^{3/4}}{3+\cos 2\alpha} \quad , \quad \hat{H}_{(3)} = 24\sqrt{2}\,L^2\,e^{\frac{1}{2}\phi_0}\,\frac{\sin^3\alpha}{(3+\cos 2\alpha)^2}\,\boldsymbol{J}\wedge d\alpha\,, \\ L^{-1}\,e^{\frac{3}{4}\phi_0}\,\hat{F}_{(2)} &= -4\sqrt{6}\,\frac{\sin^2\alpha\cos\alpha}{(3+\cos 2\alpha)(5+\cos 2\alpha)}\,\boldsymbol{J} - 3\sqrt{6}\,\frac{(3-\cos 2\alpha)}{(5+\cos 2\alpha)^2}\,\sin\alpha\;d\alpha\wedge\boldsymbol{\eta}\,, \\ L^{-3}\,e^{\frac{1}{4}\phi_0}\,\hat{F}_{(4)} &= 6\,\mathrm{vol}_4 \\ &\quad + 12\sqrt{3}\,\frac{7+3\cos 2\alpha}{(3+\cos 2\alpha)^2}\,\sin^4\alpha\;\mathrm{vol}_{\mathbb{CP}^2} + 18\sqrt{3}\,\frac{(9+\cos 2\alpha)\sin^3\alpha\cos\alpha}{(3+\cos 2\alpha)(5+\cos 2\alpha)}\,\boldsymbol{J}\wedge d\alpha\wedge\boldsymbol{\eta}\,, \end{aligned}$$

with $L^2 \equiv 2^{-\frac{5}{8}}\,3^{-1}\,g^{-2}\,c^{\frac{1}{12}}$ and $e^{\phi_0} \equiv 2^{\frac{1}{4}}\,c^{-\frac{5}{6}}$. The angle $0\le\alpha\le\pi$ locally foliates S^6 with S^5 leaves regarded as Hopf fibrations over $\mathbb{CP}^2$, with fibers squashed as a function of α. Also, $\boldsymbol{J}$ is the Kähler form of $\mathbb{CP}^2$ and $\eta = d\psi + \sigma$ with $0\le\psi\le 2\pi$ a coordinate along the fiber and $d\sigma = 2\boldsymbol{J}$.

CFT candidate and matching of free energies [5]

• We propose an $\mathcal{N}=2$ Chern-Simons-matter theory with simple gauge group SU(N), level k and only adjoint matter, as the CFT dual of the $\mathcal{N}=2$ massive IIA solution. The 3d free energy $F=-\log Z$, where Z is the partition function of the CFT on a Euclidean S^3, can be computed via localisation over supersymmetric configurations [6]

$$Z = \int\prod_{i=1}^{N}\frac{d\lambda_i}{2\pi}\,\prod_{i<j=1}^{N}\left(2\sinh^2\!\left(\frac{\lambda_i-\lambda_j}{2}\right)\right)\times\prod_{i,j=1}^{N}\left(\exp\!\left(\ell\!\left(\frac{1}{3}+\frac{i}{2\pi}(\lambda_i-\lambda_j)\right)\right)\right)^3 e^{\frac{ik}{4\pi}\sum\lambda_i^2}\,,$$

where λ_i are the Coulom branch parameters. In the $N\gg k$ limit, the result for the free energy is given by

$$F = \frac{3^{13/6}\pi}{40}\left(\frac{32}{27}\right)^{2/3} k^{1/3}N^{5/3}\,.$$

• The gravitational free energy of the massive IIA solution can be computed in terms of N using the charge quantisation condition $N = -(2\pi\ell_s)^{-5}\int_{S^6} e^{\frac{1}{2}\hat{\phi}}\hat{*}\hat{F}_{(4)} + \hat{B}_{(2)}\wedge d\hat{A}_{(3)} + \frac{1}{6}\,\hat{F}_{(0)}\,\hat{B}^3_{(2)}$ for D2-branes. Denoting by e^{2A} the warp factor in the metric of the IIA solution, one finds

$$F = \frac{16\pi^3}{(2\pi\ell_s)^8}\int_{S^6} e^{8A}\,\mathrm{vol}_6 = \frac{\pi}{5}\,2^{1/3}\,3^{1/6}\,k^{1/3}\,N^{5/3}\,,$$

in exact agreement with the gravitational result provided the 4D/10D/3d identification

$$g\,c \;=\; \hat{F}_{(0)} \;=\; k/(2\pi\ell_s)$$