# Flux vacua from gauged supergravities

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with G. Dibitetto and D. Roest: arXiv:1102.0239

arXiv:1104.3587

arXiv:11xx.xxxx (...in progress)

#### The footprint of extra dimensions

- Four dimensional supergravity theories appear when compactifying string theory
- Fluctuations of the internal space around a fixed geometry translates into massless 4d scalar fields known as "moduli"

$$\mathcal{L} = \frac{1}{2} R - \frac{1}{2} \partial_{\mu} \phi_i \, \partial^{\mu} \phi^i$$

Deviations from GR !!

massless scalars = long range interactions (precision tests of GR)

Linking strings to observations — Mechanisms to stabilise moduli !!

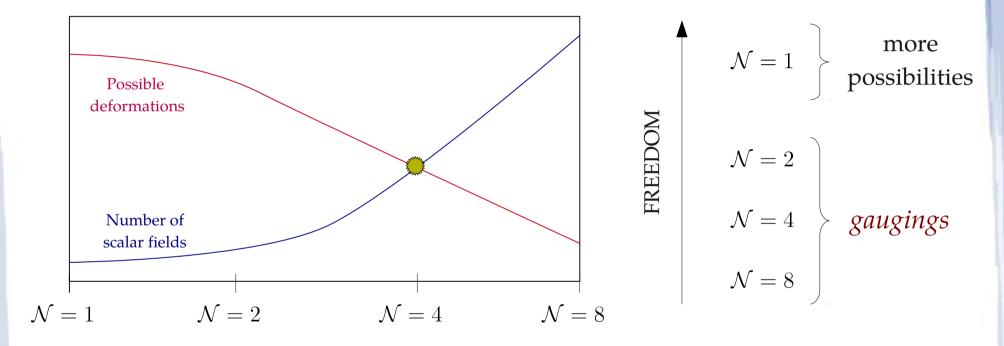
$$V(\phi) = m_{ij}^2 \, \phi^i \phi^j + \dots$$

\* Moduli VEVs  $\langle \phi \rangle = \phi_0$  determine 4d physics  $\langle g_s \text{ and } \mathrm{Vol}_{int} \rangle$ 

$$\Lambda_{c.c} \equiv V(\phi_0)$$
  $g_s \ ext{and} \ ext{Vol}_{in}$  fermi masses

### How to deform massless theories to have $V(\phi) \neq 0$ ?

Supersymmetry dictates what deformations are allowed



gaugings = part of the global symmetry is promoted to local ( gauge )

Gauged supergravities can be systematically studied

[Nicolai, Samtleben '00]

[de Wit, Samtleben, Trigiante '02 '05 '07]

[Schon, Weidner '06]

embedding tensor formalism

#### The embedding tensor formalism (I)

- \* Reducing 10d supergravities down to 4d yields *ungauged* supergravities with global symmetry  $G \implies$  duality group in 4d
- \* The scalar sector parameterises the coset space  $\mathcal{M} = G/H$  where H is the maximal compact subgroup of G

$$\mathcal{N} = 8$$

$$G = E_7 \quad H = SU(8)$$

$$\mathcal{N} = 4$$

$$G = SL(2) \times SO(6, 6)$$

$$H = SO(2) \times SO(6) \times SO(6)$$

- \* Abelian gauge fields  $A_{\mu}^{\mathcal{F}}$  in the fundamental rep. of G
- \* *Gauging* : a subgroup  $G_0 \subset G$  is promoted to local symmetry yielding a *gauged* supergravity

$$abla_{\mu} \longrightarrow \nabla_{\mu} - g \, A_{\mu}^{\mathcal{F}} \, \Theta_{\mathcal{F}}^{\mathcal{A}} \, t_{\mathcal{A}} \qquad \text{where} \quad \begin{cases} \mathcal{F} \equiv \text{fund} \\ \mathcal{A} \equiv \text{adj} \end{cases} \text{ reps of G}$$
embedding tensor

### The embedding tensor formalism (II)

 $^{\triangleright}$  The embedding tensor  $\Theta_{\mathcal{F}}{}^{\mathcal{A}}$  encodes any possible deformation of the theory (gauging)

$$\mathcal{F} \otimes \mathcal{A} = rep_1 \oplus rep_2 \oplus \dots$$
 of  $G$ 

- Consistency of the gauging implies
  - *i* ) Supersymmetry  $\implies$  Linear constraints on  $\Theta$  (some  $r \not\sim q_i$ )
  - ii) Gauge algebra  $\implies$  Quadratic constraints on  $\Theta$
- \* The *gauging* also induces a non-trivial scalar potential  $V(\Theta, \mathcal{M})$  for the scalar fields  $\mathcal{M} = G/H$
- > Stringy features (T-S-... dualities) implemented at the supergravity level!!

#### A chain of theories in 4d

[Dibitetto, A.G, Roest in progress]

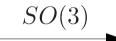
$$G = E_7$$

e.t 
$$comp = 912$$

$$vectors = 56$$

$$scalars = 133$$

$$\mathcal{N}=8$$



$$G = SL(2) \times G_2$$

e.t comp = 
$$(2,1) + (4,14) + (2,7)$$

vectors = 
$$(4,1)$$

$$scalars = (3,1) + (1,14)$$

$$\mathcal{N}=2$$

 $\mathbb{Z}_2$ 

$$G = SL(2) \times SO(6,6)$$

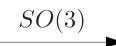
e.t comp = 
$$(2,12) + (2,220)$$

vectors = 
$$(2,12)$$

$$scalars = (3,1) + (1,66)$$



$$\mathcal{N}=4$$



$$G = SL(2) \times SL(2) \times SL(2)$$

e.t comp = 
$$(2,2,2) + (2,4,4)$$

 $\mathbb{Z}_2$ 

$$vectors = 0$$

scalars = 
$$(3,1,1) + (1,3,1) + (1,1,3)$$

$$\mathcal{N}=1$$

## Half-maximal supergravities in 4d

[Schon, Weidner '06]

$$G = SL(2) \times SO(6, 6)$$
  
e.t comp =  $(2,12) + (2,220)$   
vectors =  $(2,12)$   
scalars =  $(3,1) + (1,66)$   
 $\mathcal{N} = 4$ 

#### Questions:

- <sup>></sup> Can the whole vacuum structure be charted in  $\mathcal{N}=4$  theories?
- > Are there connections in the landscape of vacua?

# Half-maximal: symmetry and fields

Figure 6. Global symmetry (duality) group  $G = SL(2) \times SO(6,6)$ 

$$G = SL(2) \times SO(6,6)$$

- Field content = supergravity multiplet + six vector multiplets
- Vectors  $A_{\mu}^{\alpha M}$  in the fundamental of G

$$\alpha = +, -$$
 is an electric-magnetic  $SL(2)$  index  $M = 1, ..., 12$  is an  $SO(6,6)$  index

\* The scalar sector parameterises  $\mathcal{M} = M_{\alpha\beta} \times M_{MN}$ 

$$M_{\alpha\beta} \equiv e^{\phi} \begin{pmatrix} \chi^2 + e^{-2\phi} & \chi \\ \chi & 1 \end{pmatrix}$$

$$M_{MN} \equiv \left( \begin{array}{ccc} G^{-1} & -G^{-1} B \\ B G^{-1} & G - B G^{-1} B \end{array} \right)$$

# Half-maximal: gaugings and scalar potential

- \* A subgroup  $G_0 \subset SL(2) \times SO(6,6)$  is promoted to local ( *gauged* )
- Gaugings are classified by the embedding tensor parameters

$$\xi_{\alpha M} \in ({f 2},{f 12})$$
 and  $f_{\alpha MNP} \in ({f 2},{f 220})$ 

> Supersymmetry + gauge invariance determine the scalar potential

$$V = \frac{1}{64} f_{\alpha MNP} f_{\beta QRS} M^{\alpha \beta} \left[ \frac{1}{3} M^{MQ} M^{NR} M^{PS} + \left( \frac{2}{3} \eta^{MQ} - M^{MQ} \right) \eta^{NR} \eta^{PS} \right] - \frac{1}{144} f_{\alpha MNP} f_{\beta QRS} \epsilon^{\alpha \beta} M^{MNPQRS} + \frac{3}{64} \xi_{\alpha}^{M} \xi_{\beta}^{N} M^{\alpha \beta} M_{MN}$$

To keep in mind: V is quadratic in the emb. tens. parameters

> 464 e.t. components + 38 scalars = TOO MUCH!!

### The SO(3) truncation

\* Keeping only fields and embedding tensor components invariant under the action of a subgroup  $SO(3) \subset SO(6,6)$ 

$$G = SL(2) \times SO(6,6)$$
e.t comp = (2,12) + (2,220) vectors = (2,12) scalars = (3,1) + (1,66) 
$$\mathcal{N} = 4$$

$$SO(3)$$

$$SO(3)$$
e.t comp = (2,2,2) + (2,4,4) vectors = 0 scalars = (3,1,1) + (1,3,1) + (1,1,3) 
$$\mathcal{N} = 1$$

R-symmetry group:

$$SU(4) \supset SO(3)$$

$$4 \longrightarrow 1 + 3$$

# The SO(3) truncation: fields and gaugings

Symmetry and fields :

[Derendinger, Kounnas, Petropoulos, Zwirner '04]

- global symmetry  $G = SL(2)_S \times SL(2)_T \times SL(2)_U$
- $A_{\mu}^{\alpha M} = 0$  and  $\xi_{\alpha M} = 0$
- scalar coset = 3 complex scalars = STU models!!

$$S \equiv \chi + i e^{-\phi}$$
 ,  $T \equiv \chi_1 + i e^{-\varphi_1}$  and  $U \equiv \chi_2 + i e^{-\varphi_2}$ 

$$SL(2)$$

$$SO(6,6)$$

\* The *gaugings*  $G_0 \subset G$  and the scalar potential V(S,T,U) are specified by the embedding tensor

$$f_{\alpha MNP} = 40$$
 components

# The SO(3) truncation: gaugings from fluxes

String embedding as flux compactification

$$f_{\alpha MNP}$$
 = generalised fluxes

**Example:** SO(3) truncation  $\Leftrightarrow$  type II orientifolds of  $\mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ 

> Type IIB fluxes and embedding tensor

$$f_{+mnp} = \tilde{F'}_{mnp}$$
 ,  $f_{+mn}^{\ p} = {Q'}_{mn}^{\ p}$  ,  $f_{+mn}^{\ mn} = {Q}_{p}^{\ mn}$  ,  $f_{+mn}^{\ mn} = \tilde{F}^{mnp}$  ,  $f_{-mnp} = \tilde{H'}_{mnp}$  ,  $f_{-mn}^{\ p} = P'_{mn}^{\ p}$  ,  $f_{-mn}^{\ mn} = P^{mn}_{\ p}$  ,  $f_{-mnp}^{\ mn} = \tilde{H}^{mnp}$  ,

[Dibitetto, Linares, Roest '10]

\* index splitting M = (m, m)

Perfect matching with flux-induced superpotential (up to Q.C)

$$W_{flux} = (P_F - P_H S) + 3T(P_Q - P_P S) + 3T^2(P_{Q'} - P_{P'} S) + T^3(P_{F'} - P_{H'} S)$$

# The SO(3) truncation: gaugings consistency

• Closure of the gauge algebra = quadratic constraints on  $f_{\alpha MNP}$ 

$$gaugings$$

$$A_{\mu} = A_{\mu}^{\alpha M} T_{\alpha M}$$

$$[T_{\alpha M}, T_{\beta N}] = f_{\alpha MN}^{P} T_{\beta P}$$

$$Quadratic Constraints$$

$$\epsilon^{\alpha \beta} f_{\alpha MNR} f_{\beta PQ}^{R} = 0$$

$$f_{\alpha R[MN} f_{\beta PQ]}^{R} = 0$$

> String theory :

quadratic constraints = 
$$\frac{\text{B.I. for gauge fields} + \text{vanishing of the flux-induced tadpoles for sources breaking } \mathcal{N} = 4$$

Example: Type IIB orientifolds with O3/O7-planes

- $\bullet$  (H, F) fluxes: Unconstrained D3-brane flux-induced tadpole
- (H, F, Q) fluxes : Vanishing of D7-brane flux-induced tadpole
- $\bullet$  (H  $\,,\,F$   $,\,Q$   $,\,P)$  fluxes : Vanishing of D7  $,\,NS7$  and I7 flux-induced tadpoles
- ??

#### We would like to . . .

1) build all the consistent SO(3)-invariant gaugings specified by  $f_{\alpha MNP}$  by solving the quadratic constraints

$$\epsilon f f = 0$$
 and  $f f = 0$ 

2) compute all the SO(3)-invariant extrema of the f-induced scalar potential  $V(f,\Phi)$  by solving the extremisation conditions

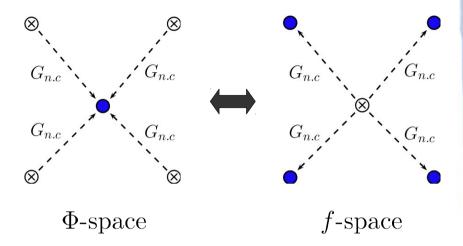
$$\left. \frac{\partial V}{\partial \Phi} \right|_{\Phi_0} = 0 \qquad \text{with} \qquad \Phi \equiv (S, T, U)$$

- 3) check stability of these extrema with respect to fluctuations of all the 38 scalars of half-maximal supergravity
- 4) identify the gauge group  $G_0$  underlying all the different solutions

#### ... but is this doable?

# Strategy and tools

Idea: use the global symmetry group (non-compact part) to bring any field solution back to the origin!!



\* At the origin everything is simply quadratic in the  $f_{\alpha MNP}$  parameters

$$V(\Phi) = \sum_{\text{terms}} f f \Phi^{\text{high degree}}$$

so then,

multivariate polynomial system 
$$I = \langle \; \partial_\Phi V |_{\Phi_0} \;\;,\; \epsilon \, f \, f \;,\; f \, f \; \rangle$$



Algebraic Geometry techniques!!

# Basics of Algebraic Geometry

Algebraic Geometry studies multivariate polynomial system and their link to geometry

$$I = \langle P_1 \, , \, P_2 \rangle$$
 $P_1(x,y,z) = x z$ 
 $P_2(x,y,z) = y z$ 

algebraic system variety

Fig. GTZ prime decomposition (analogous to integers dec.  $30 = 2 \times 3 \times 5$ )

$$I = J_1 \cap J_2$$
 where 
$$\begin{cases} J_1 = \langle z \rangle \longrightarrow xy\text{-plane} \\ J_2 = \langle x, y \rangle \longrightarrow z\text{-axis} \end{cases}$$



$$J_1 \cap J_2 \longleftrightarrow V(J_1) \cup V(J_2)$$

algebra-geometry dictionary

Applying the above procedure to our problem involving fluxes

$$I = \langle \partial_{\Phi} V |_{\Phi_0}$$
,  $\epsilon f f$ ,  $f f \rangle$  Splitting of the landscape into  $n$  disconnected pieces



into n disconnected pieces!!

## An example: type IIA with metric fluxes

> Testing the method with type IIA orientifold models including gauge fluxes and a metric flux [Dall'Agatta, Villadoro, Zwirner '09]

$$\left( \left( F_{p=0,2,4,6} , H_3 \right) + \omega \subset f_{\alpha MNP} \right)$$

Q.C. of gaugings = B.I. + tadpoles cancellation

- $\triangleright$  Subset of embedding tensor components closed under  $G_{n.c}$ 
  - Fields can still be set at the origin without lost of generality
  - Stability with respect to fluctuations around the origin can be computed [Borghese, Roest '10]

Vacua structure of these type IIA orientifolds

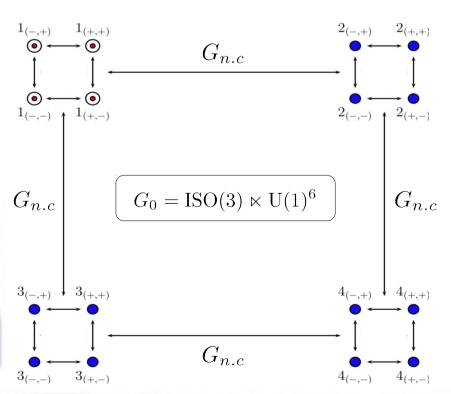


### The 16 critical points

An AdS<sub>4</sub> landscape

$$16 = 4 + 4 + 4 + 4$$

Extra vanishing of ALL the flux-induced tadpoles!!



$1_{(\pm,\pm)}$	$2_{(\pm,\pm)}$	$3_{(\pm,\pm)}$	$4_{(\pm,\pm)}$
$\mathcal{N} = 1  \mathrm{SUSY}$ & FAKE SUSY	SUSY	SUSY	SUSY
stable	unstable	stable	stable
$m^2 = -2/3$	$m^2 = -4/5$	$m^2 > 0$	$m^2 > 0$
V= -1		V= -8/15	
v — -1	V = -32/27		V = -32/27

(\*)  $m^2 \equiv \text{lightest mode (B.F. bound} = -3/4)$ 

\* All the solutions are connected and correspond to  $\omega \equiv SU(2) \times SU(2)$ 

[Caviezel, Koerber, Körs, Lüst, Tsimpis/Wrase, Zagermann '08, '08]

Unique gauging

Unique theory

with 4 different vacua!!

# Lifting to maximal supergravity?

$$G = E_7$$
e.t comp = 912
vectors = 56
scalars = 133
$$\mathcal{N} = 8$$

$$G = SL(2) \times SO(6,6)$$
 e.t comp =  $(2,12) + (2,220)$  vectors =  $(2,12)$  scalars =  $(3,1) + (1,66)$   $\mathcal{N} = 4$ 

#### Question:

Is the vanishing of ALL the flux-induced tadpoles enough for the geometric IIA solutions to lift to a maximal supergravity theory?

# When half-becomes maximal supergravity (I)

[Dibitetto, A.G., Roest '11]

See the maximal theory with the half-maximal ''glasses' and then modding out by a  $\mathbb{Z}_2$  orientifold

 $^{\flat}$  Gauge algebra of the maximal theory with embedding tensor  $X(f,\xi)$ 

$$[A_B, A_B] = X_{BB}{}^B A_B + X_{BB}{}^F A_F$$

$$[A_B, A_F] = X_{BF}{}^B A_B + X_{BF}{}^F A_F$$

$$[A_F, A_F] = X_{FF}{}^B A_B + X_{FF}{}^F A_F$$
Jacobi identities =  $QC_{\mathcal{N}=8}$ 

Components with an odd number of fermionic indices projected out !!

# When half-becomes maximal supergravity (II)

For a half-maximal to be embeddable in a maximal theory

$$QC_{\mathcal{N}=8} = QC_{\mathcal{N}=4} + \text{extra conditions for the lifting}$$

The extra conditions are

$$f_{\alpha MNP} f_{\beta}^{MNP} = 0 \quad \text{and} \quad \epsilon^{\alpha \beta} f_{\alpha [MNP} f_{\beta QRS]} \Big|_{SD} = 0$$

$$(3,1) \quad (1,462')$$

> Type IIB : flux-induced tadpoles for dual sources ?

$$H_3 \wedge F_3 \subset (1,462')$$
 which objects fill the rep?

D3-brane tadpole

# An example: lifting of geometric type IIA

All the 16 geom. IIA solutions lift to maximal supergravity

Fake SUSY becomes SUSY

$$SU(8) \rightarrow SU(4) \times SU(4)$$
 $\mathcal{N}=8$ 
 $\mathcal{N}=4$ 
 $SUSY$ 
 $\mathcal{N}=4$ 
 $\mathcal{N}=4$ 
 $\mathcal{N}=4$ 
 $\mathcal{N}=4$ 
 $\mathcal{N}=4$ 

$\boxed{1_{(\pm,\pm)}}$	$2_{(\pm,\pm)}$	$3_{(\pm,\pm)}$	$4_{(\pm,\pm)}$
$\mathcal{N} = 1$ SUSY	SUSY	SUSY	SUSY
stable	unstable	stable	unstable
$m^2 = -2/3$	$m^2 = -4/5$	$m^2 > 0$	$m^2 = -4/3$
T7 1		V= -8/15	
V= -1	V = -32/27		V = -32/27

(\*)  $m^2 \equiv \text{lightest mode (B.F. bound} = -3/4)$ 

Computing the mass of the 70 scalars SUSY and stable minimum !!

Underlying gaugings in maximal supergravity (28 vectors) ??

(...in progress)

#### Conclusions

- > Some progress towards disentangling the landscape of extended supergravities can still be done without performing statistics of vacua
- The approach relies on the combined use of global symmetries and of algebraic geometry techniques
- As a warming-up, the complete vacua structure of simple type IIA orientifold theories can be worked out revealing some odd features:
  - *i)* connections between vacua
  - *ii)*  $\mathcal{N} = 8$  lifting of the entire vacua structure
  - *iii*) stability without supersymmetry

#### For the future :

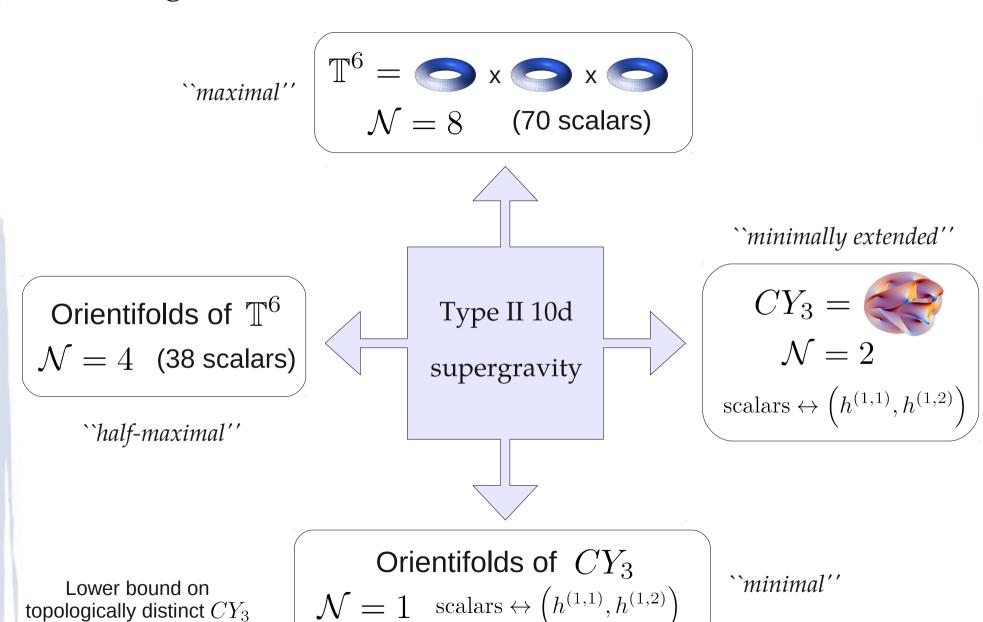
- Beyond the geometric limit: non-geometric backgrounds, dual branes . . .
- de Sitter in extended supergravity (maybe  $\mathcal{N}=2$  ) and links to Cosmology
- Higher dimensional origin of gaugings : DFT, Generalised Geometry . . .

Thanks!!

Extra material...

#### Internal geometries and massless theories . . .

30108



### Gaugings and their higher-dimensional origin

 $^{\triangleright} \mathcal{N} = 8$ : Gauging a subgroup of the global symmetry  $G = E_7$ 

Internal space extension



Exceptional Generalised Geometry?

[Pacheco, Waldram '08, Grana, Louis, Sim, Waldram '09]

[Aldazabal, Andrés, Cámara, Grana '10]

 $^{*}$   $\mathcal{N}=4$  : Gauging a subgroup of the global symmetry  $\mathit{G}=\mathit{SL}(2)\times\mathit{SO}(6,6)$ 

Internal space extension



Doubled/Generalised Geometry?

[Hitchin '02, Gualtieri '04] [Hull '04, '06]



String compactifications including generalised flux backgrounds !!

# The SO(3) truncation: fields and gaugings

#### Symmetry and fields :

- global symmetry  $G = SL(2)_S \times SL(2)_T \times SL(2)_U$
- $A_{\mu}^{\alpha M} = 0$  and  $\xi_{\alpha M} = 0$

[Derendinger, Kounnas, Petropoulos, Zwirner '04]

• scalar coset = 3 complex scalars = STU - models !!

$$S \equiv \chi + i e^{-\phi}$$
 ,  $T \equiv \chi_1 + i e^{-\varphi_1}$  and  $U \equiv \chi_2 + i e^{-\varphi_2}$ 

$$SL(2)$$

$$SO(6,6)$$

-SO(6,6): 
$$G = e^{\varphi_2 - \varphi_1} \begin{pmatrix} \chi_2^2 + e^{-2\varphi_2} & -\chi_2 \\ -\chi_2 & 1 \end{pmatrix} \otimes \mathbb{I}_3$$
,  $B = \begin{pmatrix} 0 & \chi_1 \\ -\chi_1 & 0 \end{pmatrix} \otimes \mathbb{I}_3$ 

 $^{\triangleright}$  The *gaugings*  $G_0 \subset G$  and the scalar potential V(S,T,U) are specified by the embedding tensor

$$f_{\alpha MNP} = 40$$
 components

### De Sitter in extended supergravity

>  $\mathcal{N}=8$  : unstable dS solutions with SO(4,4) and SO(5,3) gaugings [Hull, Warner '85]

 $^{\triangleright} \mathcal{N} = 4$ : unstable dS solutions with gaugings at angles

[De Roo, Wagemans '85]

$$i$$
 )  $G_1 imes G_2$  gaugings with  $\begin{cases} G_i = SO(p_i,q_i) &, p_i+q_i=4 \ \\ G_i = CSO(p_i,q_i,r_i) &, p_i+q_i+r_i=4 \end{cases}$ 

ii)  $SO(3,1) \ltimes U(1)^6$  gauging

[De Roo, Westra, Panda, (Trigiante) '02, '03, '06]

[Dibitetto, A.G, Roest '11]

non-geometric fluxes in string theory!!

[Dibitetto, Linares, Roest '10]

>  $\mathcal{N}=2$  : stable dS solutions with  $SO(2,1)\times SO(3)$  gauging plus Fayet-Iliopoulos terms [Fré, Trigiante, Van Proeyen '03]

unclear origin in string theory!!

### De Sitter in minimal supergravity

> No-go theorems forbidding dS solutions in  $\mathcal{N}=1$  compactifications with gauge fluxes

$$V_o = -\frac{1}{9} \sum \bar{F}^2 \le 0$$
 AdS!!

[Hertzberg, Kachru, Taylor, Tegmark '07]

> Including more general fluxes : ( metric + non-geometric )

$$V_o = -\frac{1}{9} \sum \bar{F}^2 + \Delta V_{\text{metric}} + \Delta V_{\text{non-geom}}$$

*a)* metric fluxes **\ unstable** dS in type IIA models

[Caviezel, Koerber, Kors, Lust, Wrase, Zagerman '08]

*b*) non-geometric fluxes  $\longleftrightarrow$  stable dS in type IIA models

[de Carlos, A.G, Moreno '09, '10]

> Including D-branes to uplift an AdS solution

[Kachru, Kallosh, Linde, Trivedi '03]

*a)* D-terms from D-branes  $\iff$  stable dS in type IIB models

[Burgess, Kallosh, Quevedo '03]

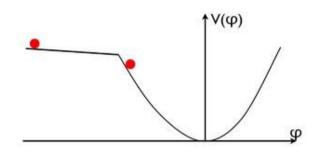
*b*) non-perturbative effects from D-branes  $\iff$  stable dS in type IIB

[Achúcarro, de Carlos, Casas, Doplicher '06]

### Cosmology from moduli?

' slow-roll inflation requires an almost flat dS saddle point of  $\,V(\phi)\,$  from which to start rolling down

$$\eta \equiv M_p^2 \left(\frac{V''}{V}\right) \ll 1$$



- $^{\flat}$  dS saddle points suffering from eta-problem, *i.e.*  $\eta \sim \mathcal{O}(1)$ 
  - i) gaugings in extended supergravity

[Kallosh, Linde, Prokushkin, Shmakova '01]

ii ) general fluxes in minimal supergravity

[Flauger, Paban, Robbings, Wrase '08] [de Carlos, A.G, Moreno '10]

 $^{>}$  dS saddle points with  $\eta \ll 1$  in minimal supergravity including non-perturbative effects  $\implies$  axion inflation !!

[Dimopoulos, Kachru, McGreevy, Wacker '05]