Holographic & geometric aspects of electromagnetic duality in supergravity

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Based on 1907.04177 , 1907.11681 and 1910.06866

Geometry and Duality

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Outlook

- Electric-magnetic duality in N=8 supergravity
- M-theory
- Massive Type IIA :   Flowing to N = 3 CS-matter theory
- Type IIB : S-folds and interface SYM

- Electric-magnetic duality in N=8 supergravity

N=8 supergravity in 4D

- SUGRA : metric + 8 gravitini + 28 vectors + 56 dilatini + 70 scalars
  (s = 2) (s = 3/2) (s = 1) (s = 1/2) (s = 0)

Ungauged (abelian) supergravity: Reduction of M-theory on a *torus* T^7 down to 4D produces $N = 8$ supergravity with $G = U(1)^{28}$

[$E_{7(7)}$ symmetry]

[Cremmer, Julia '79]

Gauged (non-abelian) supergravity:

- Reduction of M-theory on a *sphere* S^7 down to 4D produces $N = 8$ supergravity with $G = SO(8)$ [de Wit, Nicolai '82]

- Reduction of M-theory on S^1 (Type IIA) and subsequently on S^6 down to 4D produces $N = 8$ supergravity with $G = ISO(7) = SO(7) \ltimes \mathbb{R}^7$ [Hull '84]

- Reduction of Type IIB on S^5 and subsequently on S^1 down to 4D produces $N = 8$ supergravity with $G = [\, SO(1,1) \times SO(6)\,] \ltimes \mathbb{R}^{12}$ [Inverso, Samtleben, Trigiante '16]

✱ These gauged supergravities believed to be unique for 30 years…

Electric-magnetic deformations

• Uniqueness historically inherited from the connection with NH geometries of branes and SCFT's

Type IIB : $AdS_5 \times S^5$ (D3-brane ~ N = 4 SYM in 4d) [Maldacena '97]

M-theory : $AdS_4 \times S^7$ (M2-brane ~ ABJM theory in 3d)

[Aharony, Bergman, Jafferis, Maldacena '08]

• N=8 supergravity in 4D admits a deformation parameter c yielding inequivalent theories. It is an electric/magnetic deformation

$$D = \partial - g\,(A^{\text{elec}} - c\,\tilde{A}_{\text{mag}})$$

g = 4D gauge coupling
c = deformation param.

[Dall'Agata, Inverso, Trigiante '12]

• There are two generic situations :

1) Family of $SO(8)_c$ theories : $c = [0, \sqrt{2} - 1]$ is a continuous parameter [similar for $SO(p,q)_c$]

2) Family of $CSO(p,q,r)_c$ theories : $c = 0$ or 1 is an (on/off) parameter

[Dall'Agata, Inverso, Marrani '14]

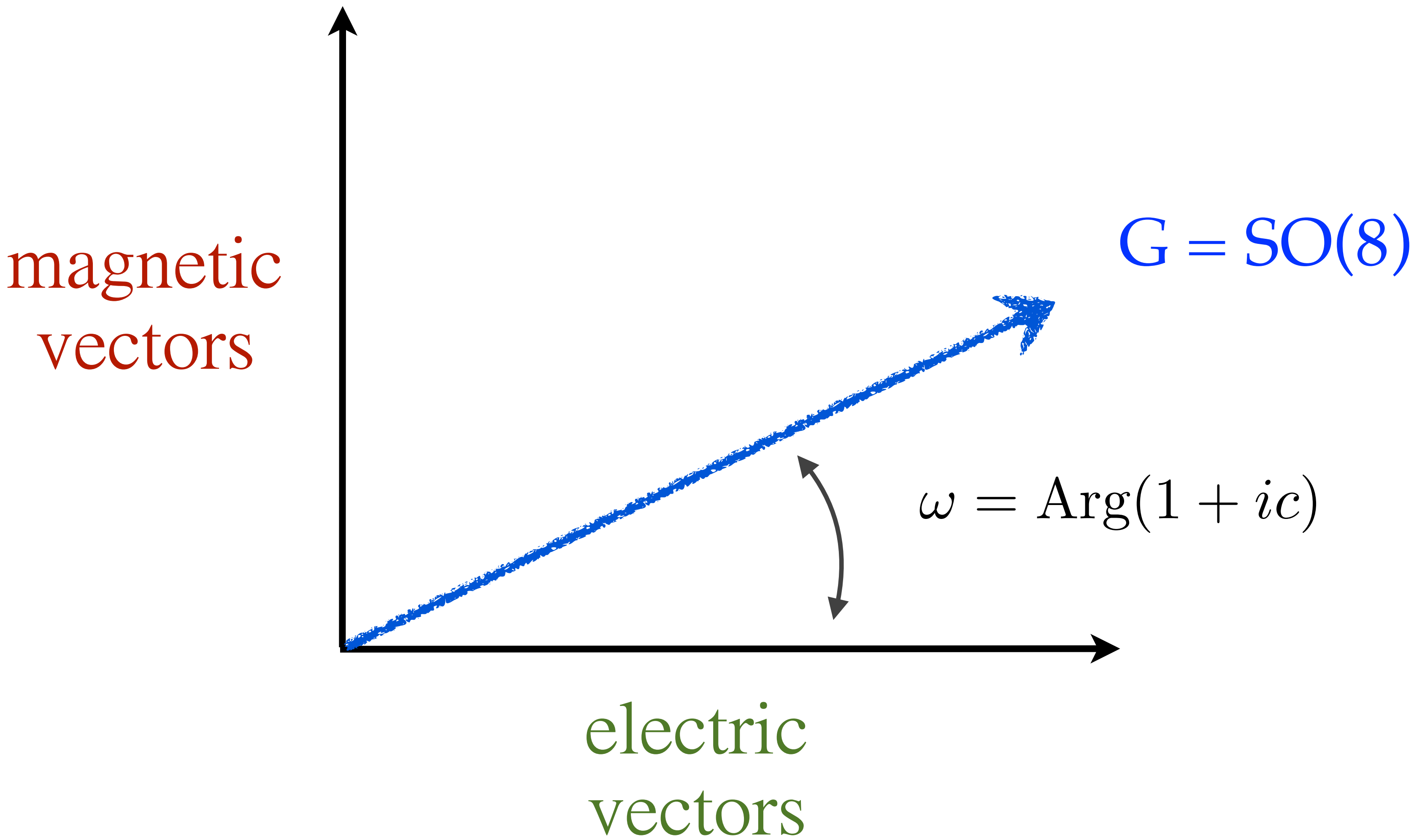
The questions arise:

- Does such an electric/magnetic deformation of 4D maximal supergravity enjoy a string/M-theory origin, or is it just a 4D feature ?

- For deformed 4D supergravities with supersymmetric AdS_4 vacua, are these AdS_4/CFT_3-dual to any identifiable 3d CFT ?

M-theory

$SO(8)_c$ theories : physical meaning in 4D



$$D = \partial - g\,(A^{\mathrm{elec}} - c\,\tilde{A}_{\mathrm{mag}})$$

SO(8)$_c$ theories : physical meaning in 11D …

?

Obstruction for SO(8)$_c$, *cf*. [de Wit, Nicolai ’13]

[Lee, Strickland-Constable, Waldram ’15]

$SO(8)_c$ theories : holographic AdS_4/CFT_3 meaning …

?

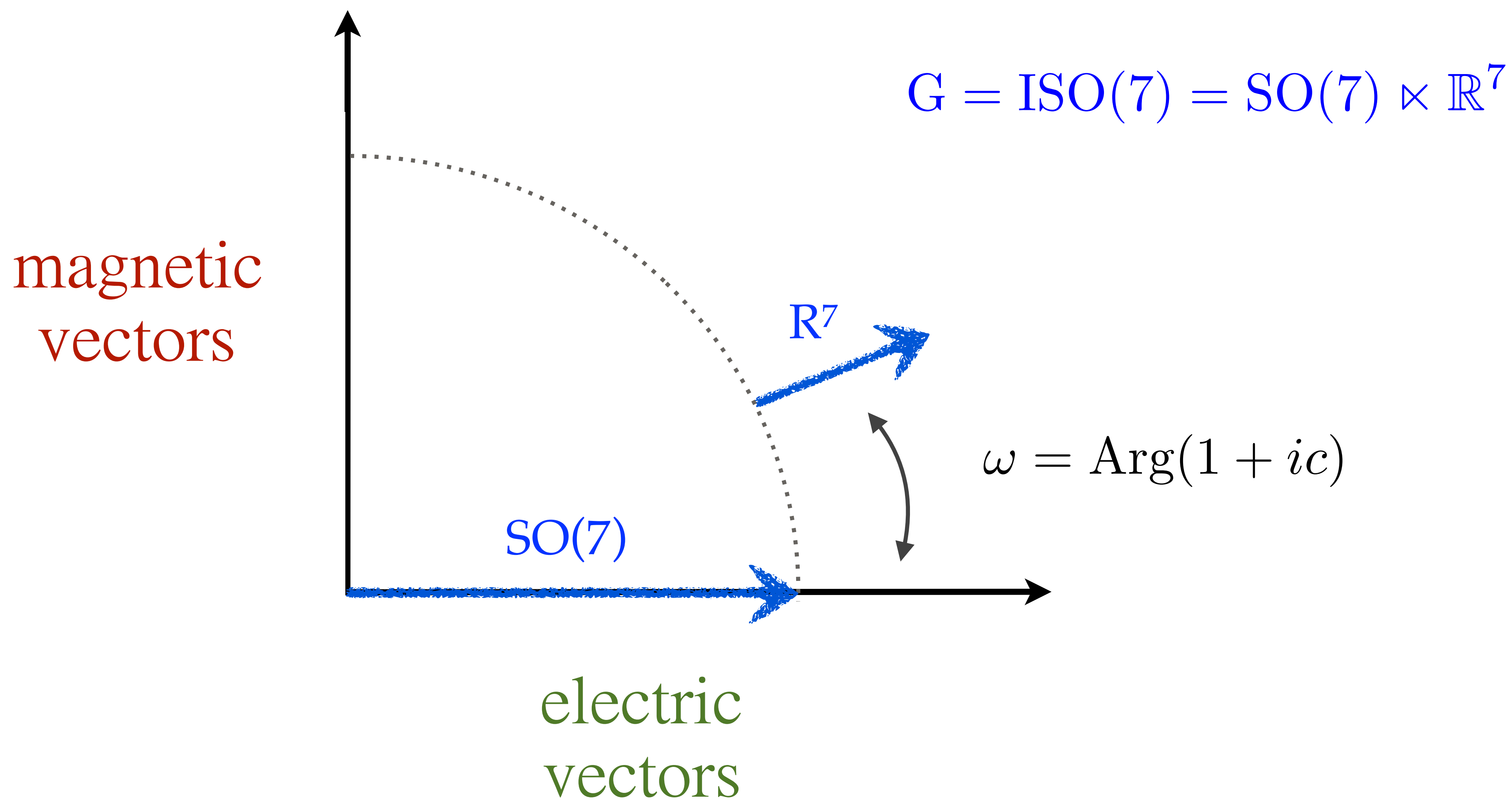
Massive Type IIA

electric/magnetic deformation	higher-dimensional origin	Holographic AdS_4/CFT_3 dual ?
✓	✓	✓

$$g\,c \;=\; \hat{F}_{(0)} \;=\; k/(2\pi\ell_s)$$

[AG, Jafferis, Varela '15]

Why ISO(7)$_c$ works ?



$$D = \partial - g\, A^{\mathrm{elec}}_{\mathrm{SO}(7)} - g\,(A^{\mathrm{elec}}_{\mathbb{R}^7} - c\, \tilde{A}_{\mathbb{R}^7\,\mathrm{mag}})$$

4D : Supersymmetric AdS_4 solutions

SUSY	bos. sym.	M^2L^2
$\mathcal{N}=3$	SO(4)	$3(1\pm\sqrt{3})^{(1)}\ ,\ (1\pm\sqrt{3})^{(6)}\ ,\ -\frac{9}{4}^{(4)}\ ,\ -2^{(18)}\ ,\ -\frac{5}{4}^{(12)}\ ,\ 0^{(22)}$ $(3\pm\sqrt{3})^{(3)}\ ,\ \frac{15}{4}^{(4)}\ ,\ \frac{3}{4}^{(12)}\ ,\ 0^{(6)}$
$\mathcal{N}=2$	U(3)	$(3\pm\sqrt{17})^{(1)}\ ,\ -\frac{20}{9}^{(12)}\ ,\ -2^{(16)}\ ,\ -\frac{14}{9}^{(18)}\ ,\ 2^{(3)}\ ,\ 0^{(19)}$ $4^{(1)}\ ,\ \frac{28}{9}^{(6)}\ ,\ \frac{4}{9}^{(12)}\ ,\ 0^{(9)}$
$\mathcal{N}=1$	G_2	$(4\pm\sqrt{6})^{(1)}\ ,\ -\frac{1}{6}(11\pm\sqrt{6})^{(27)}\ ,\ 0^{(14)}$ $\frac{1}{2}(3\pm\sqrt{6})^{(7)}\ ,\ 0^{(14)}$
$\mathcal{N}=1$	SU(3)	$(4\pm\sqrt{6})^{(2)}\ ,\ -\frac{20}{9}^{(12)}\ ,\ -2^{(8)}\ ,\ -\frac{8}{9}^{(12)}\ ,\ \frac{7}{9}^{(6)}\ ,\ 0^{(28)}$ $6^{(1)}\ ,\ \frac{28}{9}^{(6)}\ ,\ \frac{25}{9}^{(6)}\ ,\ 2^{(1)}\ ,\ \frac{4}{9}^{(6)}\ ,\ 0^{(8)}$

[Gallerati, Samtleben, Trigiante '14]

[AG, Jafferis, Varela '15]

[Borghese, AG, Roest '12]

[AG, Varela '15]

✦ $\mathcal{N}=2$ & $\mathcal{N}=3$ solutions will play a central role in holography !!

[Continuous R-symmetry]

10D : ISO(7)$_c$ into type IIA supergravity

[AG, Varela '15]

$$d\hat{s}^2_{10} = \Delta^{-1}\, ds^2_4 \, + g_{mn}\, Dy^m\, Dy^n \, ,$$

$$\begin{aligned}\hat{A}_{(3)} &= \mu_I \mu_J \left(\mathcal{C}^{IJ} + \mathcal{A}^I \wedge \mathcal{B}^J + \tfrac{1}{6}\mathcal{A}^{IK} \wedge \mathcal{A}^{JL} \wedge \tilde{\mathcal{A}}_{KL} + \tfrac{1}{6}\mathcal{A}^I \wedge \mathcal{A}^{JK} \wedge \tilde{\mathcal{A}}_K\right) \\ &\quad + g^{-1}\left(\mathcal{B}_J{}^I + \tfrac{1}{2}\mathcal{A}^{IK} \wedge \tilde{\mathcal{A}}_{KJ} + \tfrac{1}{2}\mathcal{A}^I \wedge \tilde{\mathcal{A}}_J\right) \wedge \mu_I D\mu^J + \tfrac{1}{2} g^{-2}\, \tilde{\mathcal{A}}_{IJ} \wedge D\mu^I \wedge D\mu^J \\ &\quad - \tfrac{1}{2}\, \mu_I\, B_{mn}\, \mathcal{A}^I \wedge Dy^m \wedge Dy^n + \tfrac{1}{6} A_{mnp}\, Dy^m \wedge Dy^n \wedge Dy^p \, ,\end{aligned}$$

$$\hat{B}_{(2)} = -\mu_I \left(\mathcal{B}^I + \tfrac{1}{2}\mathcal{A}^{IJ} \wedge \tilde{\mathcal{A}}_J\right) - g^{-1}\, \tilde{\mathcal{A}}_I \wedge D\mu^I + \tfrac{1}{2} B_{mn}\, Dy^m \wedge Dy^n \, ,$$

$$\hat{A}_{(1)} = -\mu_I\, \mathcal{A}^I + A_m\, Dy^m \, .$$

where we have defined : $\quad Dy^m \equiv dy^m + \tfrac{1}{2}\, g\, K^m_{IJ}\, \mathcal{A}^{IJ} \quad , \quad D\mu^I \equiv d\mu^I - g\, \mathcal{A}^{IJ} \mu_J$

The scalars are embedded as

$$\begin{aligned} g^{mn} &= \tfrac{1}{4}\, g^2\, \Delta\, \mathcal{M}^{IJ\,KL}\, K^m_{IJ}\, K^n_{KL} \quad , \quad & B_{mn} &= -\tfrac{1}{2}\, \Delta\, g_{mp}\, K^p_{IJ}\, \partial_n \mu^K\, \mathcal{M}^{IJ}{}_{K8} \, , \\ A_m &= \tfrac{1}{2}\, g\, \Delta\, g_{mn}\, K^n_{IJ}\, \mu_K\, \mathcal{M}^{IJ\,K8} \quad , \quad & A_{mnp} &= \tfrac{1}{8}\, g\, \Delta\, g_{mq}\, K^q_{IJ}\, K^{KL}_{np}\, \mathcal{M}^{IJ}{}_{KL} + A_m B_{np} \, . \end{aligned}$$

$\mathcal{N}=2$ solution of massive type IIA

[AG, Jafferis, Varela '15]

- $\mathcal{N}=2$ & $\mathrm{SU}(3)_{\mathrm{F}} \times \mathrm{U}(1)_{\psi}$ AdS$_4$ point of the ISO(7)$_c$ theory

$$d\hat{s}^2_{10} = L^2 \frac{(3+\cos 2\alpha)^{\frac{1}{2}}}{(5+\cos 2\alpha)^{-\frac{1}{8}}} \left[ds^2(\mathrm{AdS}_4) + \frac{3}{2}\, d\alpha^2 + \frac{6\sin^2\alpha}{3+\cos 2\alpha}\, ds^2(\mathbb{CP}^2) + \frac{9\sin^2\alpha}{5+\cos 2\alpha}\, \boldsymbol{\eta}^2 \right] ,$$

$$e^{\hat{\phi}} = e^{\phi_0} \frac{(5+\cos 2\alpha)^{3/4}}{3+\cos 2\alpha} \quad , \quad \hat{H}_{(3)} = 24\sqrt{2}\, L^2\, e^{\frac{1}{2}\phi_0} \frac{\sin^3\alpha}{(3+\cos 2\alpha)^2}\, \boldsymbol{J}\wedge d\alpha \, ,$$

$$L^{-1} e^{\frac{3}{4}\phi_0} \hat{F}_{(2)} = -4\sqrt{6}\, \frac{\sin^2\alpha\cos\alpha}{(3+\cos 2\alpha)(5+\cos 2\alpha)}\, \boldsymbol{J} - 3\sqrt{6}\, \frac{(3-\cos 2\alpha)}{(5+\cos 2\alpha)^2} \sin\alpha \; d\alpha\wedge\boldsymbol{\eta} \, ,$$

$$L^{-3} e^{\frac{1}{4}\phi_0} \hat{F}_{(4)} = 6\, \mathrm{vol}_4 + 12\sqrt{3}\, \frac{7+3\cos 2\alpha}{(3+\cos 2\alpha)^2} \sin^4\alpha \; \mathrm{vol}_{\mathbb{CP}^2} + 18\sqrt{3}\, \frac{(9+\cos 2\alpha)\sin^3\alpha\cos\alpha}{(3+\cos 2\alpha)(5+\cos 2\alpha)}\, \boldsymbol{J}\wedge d\alpha\wedge\boldsymbol{\eta} \, ,$$

where we have introduced the quantities $L^2 \equiv 2^{-\frac{5}{8}}\, 3^{-1}\, g^{-2}\, c^{\frac{1}{12}}$ and $e^{\phi_0} \equiv 2^{\frac{1}{4}}\, c^{-\frac{5}{6}}$

✦ The angle $0 \le \alpha \le \pi$ locally foliates $\mathbf{S}^6$ with $\mathbf{S}^5$ regarded as Hopf fibrations over $\mathbb{CP}^2$

$\mathcal{N}=3$ solution of massive type IIA

[Pang, Rong '15]
[also De Luca et al '18]

- $\mathcal{N}=3$ & $\mathrm{SU}(2)_{\mathrm{F}} \times \mathrm{SO}(3)_d$ AdS$_4$ point of the ISO(7)$_c$ theory

$$d\hat{s}^2_{10} = L^2\,(3+\cos 2\alpha)^{1/8}\Big(3\cos^4\alpha+3\cos^2\alpha+2\Big)^{1/4}\Big[ds^2(\mathrm{AdS}_4)+\frac{2(3+\cos 2\alpha)\cos^2\alpha}{3\cos^4\alpha+3\cos^2\alpha+2}\,\delta_{ij}D\tilde{\mu}^i D\tilde{\mu}^j+2\,d\alpha^2+\frac{8\sin^2\alpha}{3+\cos 2\alpha}\,d\tilde{s}^2(S^3)\Big]$$

$$e^{\hat{\phi}} = e^{\phi_0}\,\frac{(3+\cos 2\alpha)^{3/4}}{\big(3\cos^4\alpha+3\cos^2\alpha+2\big)^{1/2}}$$

$$L^{-2}e^{-\frac{1}{2}\phi_0}\,\hat{B}_{(2)} = -\frac{2}{\sqrt{3}}\,\sin\alpha\,d\alpha\wedge\tilde{\mu}_i\,\rho^i+\frac{(5+3\cos 2\alpha)\cos^3\alpha}{\sqrt{3}\,(3\cos^4\alpha+3\cos^2\alpha+2)}\,\epsilon_{ijk}\,\tilde{\mu}^i D\tilde{\mu}^j\wedge D\tilde{\mu}^k$$
$$+\frac{4\sin^2\alpha\cos\alpha}{\sqrt{3}\,(3+\cos 2\alpha)}\,D\tilde{\mu}_i\wedge\rho^i+\frac{(7+\cos 2\alpha)\sin^2\alpha\cos\alpha}{\sqrt{3}\,(3+\cos 2\alpha)^2}\,\epsilon_{ijk}\,\tilde{\mu}^i\rho^j\wedge\rho^k\,,$$

$$L^{-3}e^{\frac{1}{4}\phi_0}\,\hat{A}_{(3)} = \frac{2\sqrt{2}}{\sqrt{3}}\,\sin\alpha\cos\alpha\,d\alpha\wedge\epsilon_{ijk}\,\tilde{\mu}^i D\tilde{\mu}^j\wedge\rho^k$$
$$-\frac{2\sqrt{2}\,\sin^2\alpha\cos^2\alpha}{\sqrt{3}\,(3\cos^4\alpha+3\cos^2\alpha+2)}\,\epsilon_{ijk}\,D\tilde{\mu}^i\wedge D\tilde{\mu}^j\wedge\rho^k$$
$$+\frac{4\sqrt{2}\,\sin^2\alpha\cos^2\alpha}{\sqrt{3}\,(3+\cos 2\alpha)}\,\tilde{\mu}_i D\tilde{\mu}_j\wedge\rho^i\wedge\rho^j$$
$$-\frac{2\sqrt{2}\,(2+\cos 2\alpha)\sin^4\alpha}{3\sqrt{3}\,(3+\cos 2\alpha)^2}\,\epsilon_{ijk}\,\rho^i\wedge\rho^j\wedge\rho^k\,,$$

$$L^{-1}e^{\frac{3}{4}\phi_0}\,\hat{A}_{(1)} = \sqrt{2}\,\frac{\sin^2\alpha\cos\alpha}{3+\cos 2\alpha}\,\tilde{\mu}_i\,\rho^i\,.$$

where we have introduced the quantities $L^2 \equiv 2^{-\frac{31}{12}}3^{\frac{3}{8}}g^{-2}c^{\frac{1}{12}}$ and $e^{\phi_0} \equiv 2^{-\frac{1}{6}}3^{\frac{1}{4}}c^{-\frac{5}{6}}$

✦ The angle $0 \le \alpha \le \pi/2$ so that S^6 is topologically described as the join of S^2 and S^3 with S^3 regarded as a Hopf fibration over $\mathbb{CP}^1$

3D : CFT_3 duals

[Schwarz '04]
[Gaiotto, Tomasiello '09]
[AG, Jafferis, Varela '15]

- 3d SYM with simple group SU(N) + CS term (level k) ➡ super CS-matter theory !!

[$N_f = 3$ chiral fields]

$$\mathcal{N} = 2 \;\; \& \;\; \mathrm{SU}(3)_{\mathrm{F}}$$

$$\mathcal{W}_{\mathcal{N}=2} = \mathrm{tr}\left([\Phi^1, \Phi^2]\, \Phi^3 \right)$$

[$N_f = 2$ chiral fields]

$$\mathcal{N} = 3 \;\; \& \;\; \mathrm{SU}(2)_{\mathrm{F}}$$

$$\mathcal{W}_{\mathcal{N}=3} = \frac{2\pi}{k}\, \mathrm{tr}\left([\Phi^1, \Phi^2] \right)^2$$

- **Perfect matching :** 3d field theory *vs* gravitations free energy

3d free energy F = - Log(Z) computed via localisation ($N \gg k$)

[Pestun '07] [Kapustin, Willett, Yaakov '09]
[Jafferis '10] [Jafferis, Klebanov, Pufu, Safdi '11]
[Closset, Dumitrescu, Festuccia, Komargodski '12 '13]

=

gravitational free energy computed from the warp factor in the massive IIA solutions

[Emparan, Johnson, Myers '99]

N=2 : [AG, Jafferis, Varela '15]

N=3 : [Pang, Rong '15]

3D : CFT$_3$ duals

[Schwarz '04]
[Gaiotto, Tomasiello '09]
[AG, Jafferis, Varela '15]

- 3d SYM with simple group SU(N) + CS term (level k) ⇒ super CS-matter theory !!

[$N_f = 3$ chiral fields]

$$\mathcal{N} = 2 \;\; \& \;\; \mathrm{SU}(3)_{\mathrm{F}}$$

$$\mathcal{W}_{\mathcal{N}=2} = \mathrm{tr}\left([\Phi^1, \Phi^2]\, \Phi^3 \right)$$

UV to IR ??

Gaiotto—Yin type

[Gaiotto, Yin '07]

[$N_f = 2$ chiral fields]

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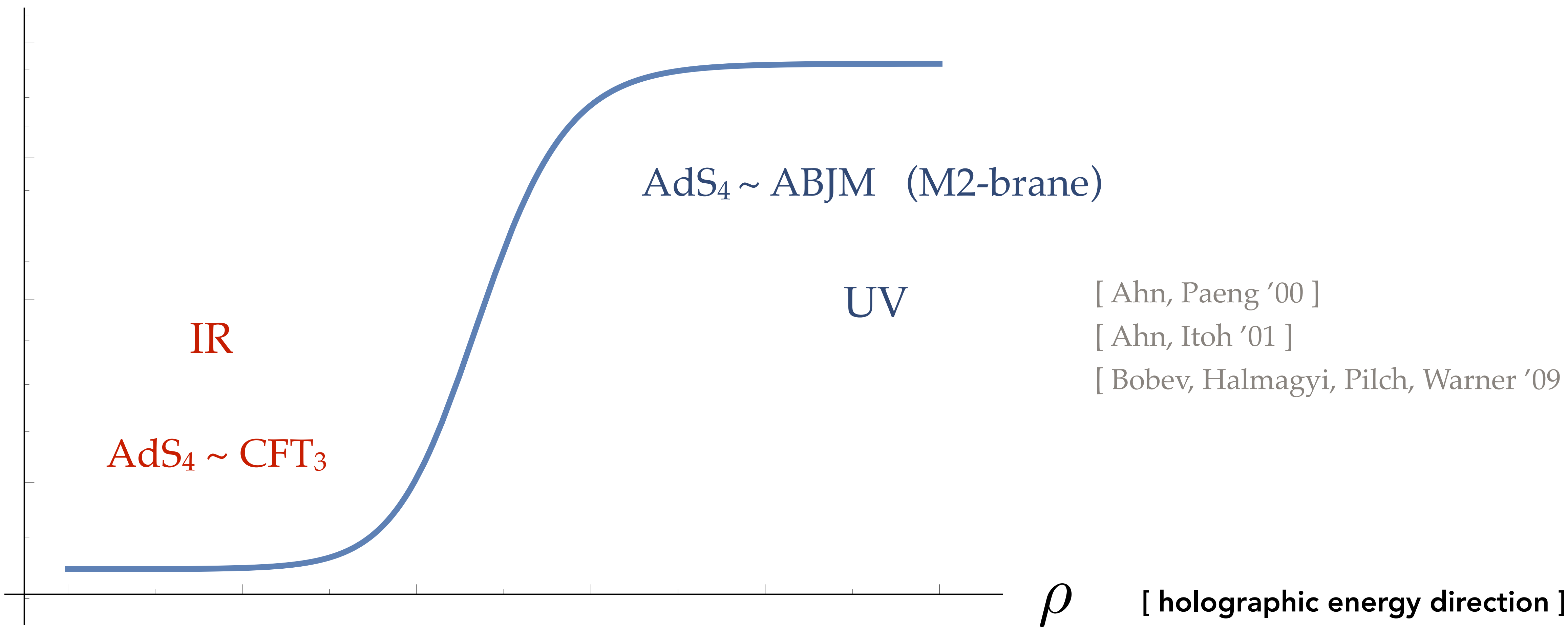
N=3 : [Pang, Rong '15]

Holographic description of RG flows

[Boonstra, Skenderis, Townsend '98]

- RG flows are described holographically as non-AdS_4 solutions in gravity

- RG flows on M2-brane : SO(8)-gauged sugra from M-theory on S^7



[Ahn, Paeng '00]
[Ahn, Itoh '01]
[Bobev, Halmagyi, Pilch, Warner '09]

- RG flows on D3-brane : SO(6)-gauged sugra from type IIB on S^5 and N=4 SYM in 4D

[Freedman, Gubser, Pilch, Warner '99]
[Pilch, Warner '00] [Benini, Bobev '12,'13]

Holographic RG flows on the D2-brane

- D2-brane :

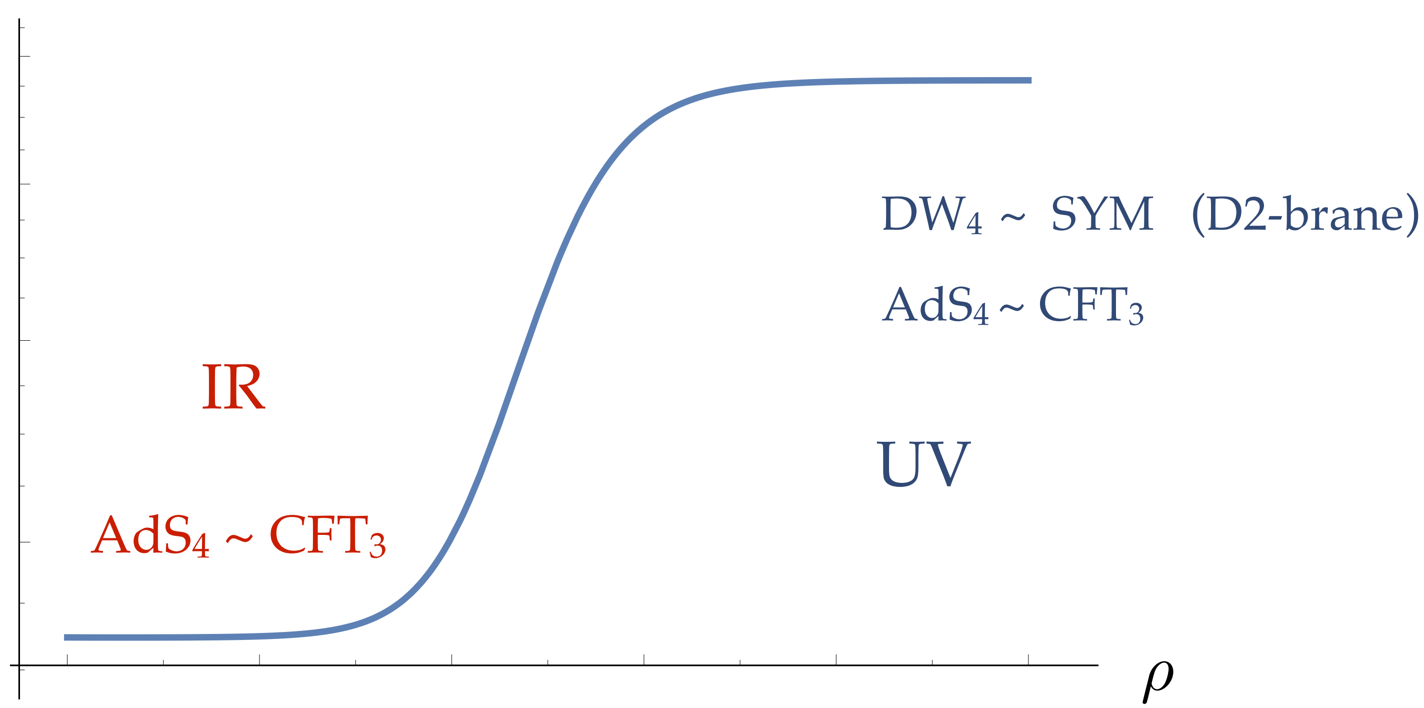
$$d\hat{s}_{10}^2 = e^{\frac{3}{4}\phi}\left(-e^{2U}dt^2 + e^{-2U}dr^2 + e^{2(\psi-U)}ds_{\Sigma_2}^2\right) + g^{-2}e^{-\frac{1}{4}\phi}ds_{S^6}^2$$

$$e^{\hat{\Phi}} = e^{\frac{5}{2}\phi}$$

$$\hat{F}_{(4)} = 5\,g\,e^{\phi}\,e^{2(\psi-U)}\,dt\wedge dr\wedge d\Sigma_2$$

with $e^{2U}\sim r^{\frac{7}{4}}$, $e^{2(\psi-U)}\sim r^{\frac{7}{4}}$ and $e^{\phi}\sim r^{-\frac{1}{4}}$ ⇒ DW_4 domain-wall (SYM)

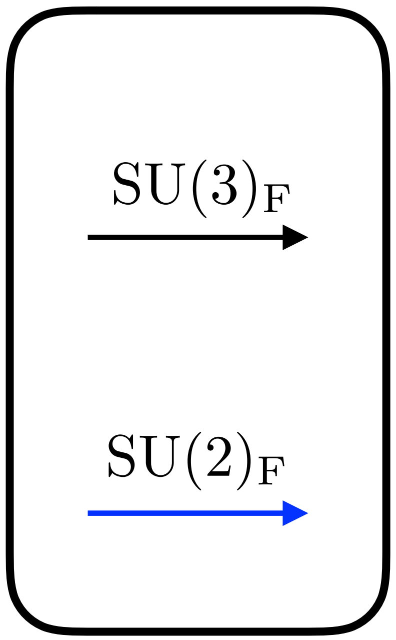
- RG flows on D2-brane : ISO(7)$_c$-gauged sugra from mIIA on S^6

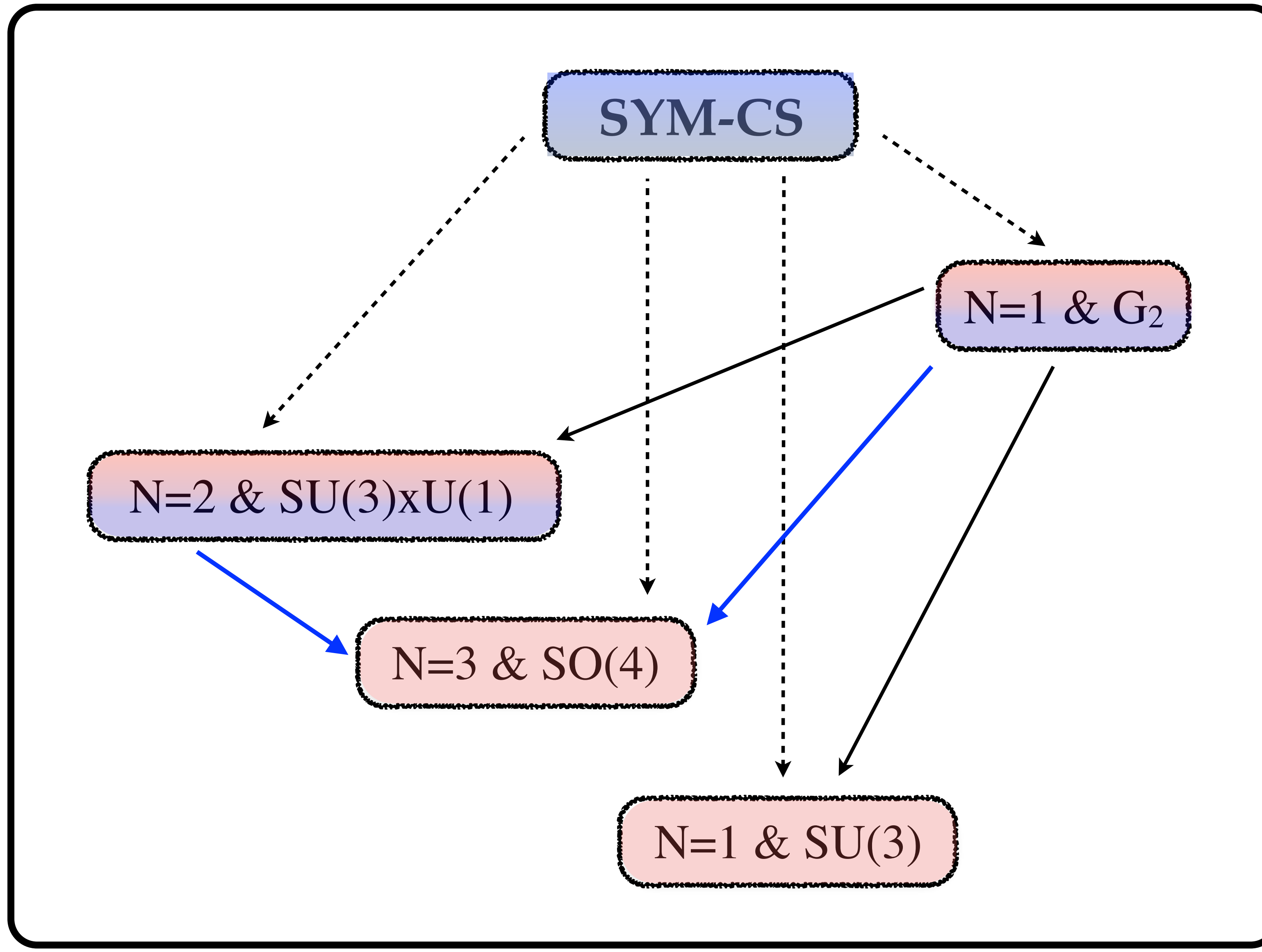


Supersymmetric domain-walls

[AG, Tarrío, Varela ’16, ’19]

subsectors

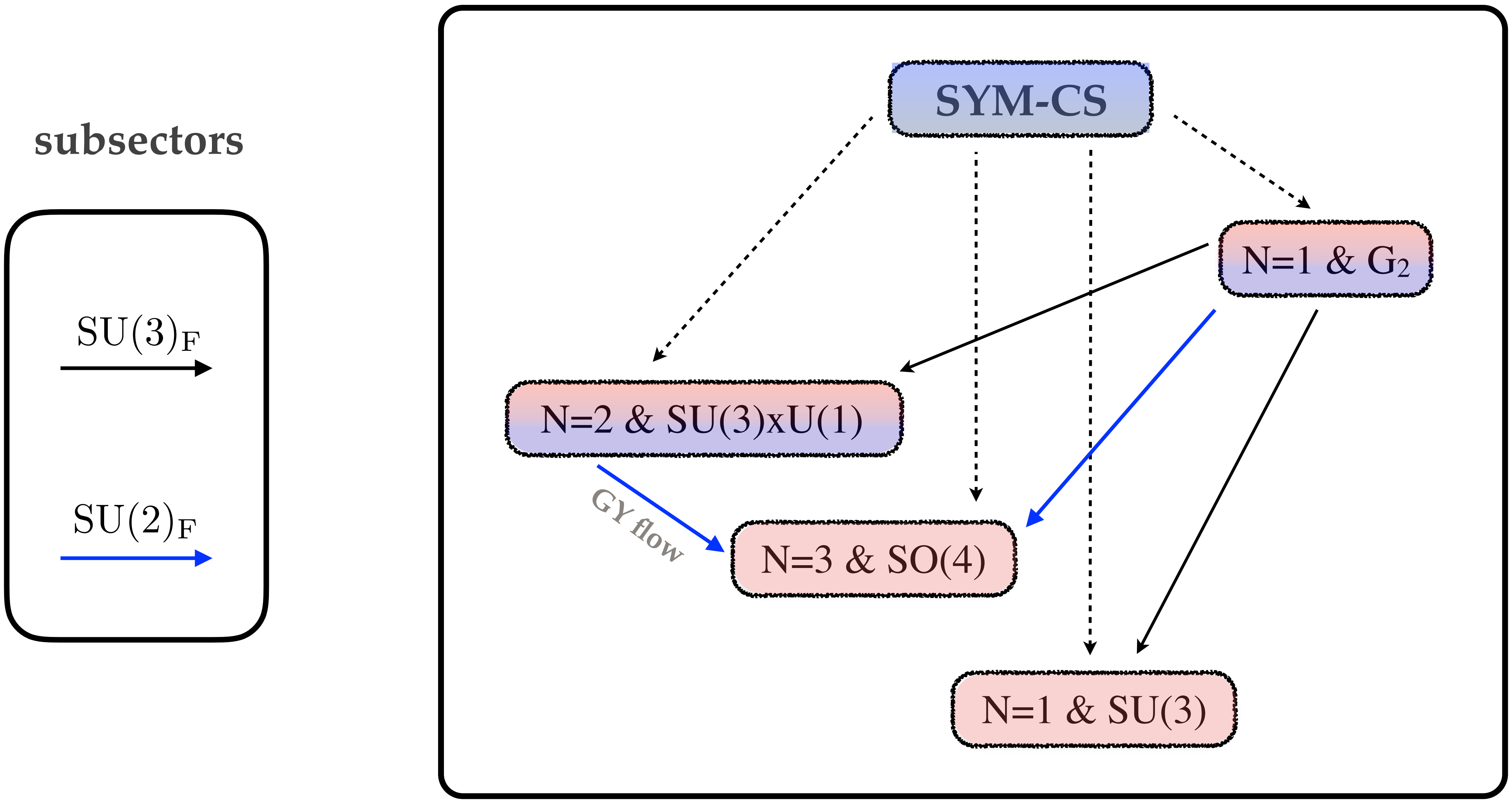




- BPS domain-wall solutions of the dyonic ISO(7)-gauged supergravity dual to RG flows from SYM-CS (dashed lines) and between CFT’s (solid lines)

Supersymmetric domain-walls

[AG, Tarrío, Varela '16, '19]



- BPS domain-wall solutions of the dyonic ISO(7)-gauged supergravity dual to RG flows from SYM-CS (dashed lines) and between CFT's (solid lines)

GY flow : field theory side

[Gaiotto, Yin '07]

- Free energy as a function of Δ_a for the chiral fields Φ^a

$$F = \frac{3\sqrt{3}\,\pi}{20\cdot 2^{1/3}}\left[1+\sum_{a=1}^{N_f}(1-\Delta_a)\left[1-2\,(1-\Delta_a)^2\right]\right]^{2/3} k^{1/3}\,N^{5/3}$$

[Jafferis '10]
[Jafferis, Klebanov, Pufu, Safdi '11]
[Fluder, Sparks '15]

- Marginality of $\mathcal{W}$ + F-extremisation

$N_f = 3$ chiral fields : $\Delta_1+\Delta_2+\Delta_3 = 2$

$$\mathcal{N}=2\ \ \&\ \ \mathrm{SU}(3)_\mathrm{F}$$

$$\mathcal{W}_{\mathcal{N}=2} = \mathrm{tr}\left([\Phi^1,\Phi^2]\,\Phi^3\right)$$

F-extremisation : $\Delta_1=\Delta_2=\Delta_3=\dfrac{2}{3}$

$N_f = 2$ chiral fields : $\Delta_1+\Delta_2 = 1$

$$\mathcal{N}=3\ \ \&\ \ \mathrm{SU}(2)_\mathrm{F}$$

$$\mathcal{W}_{\mathcal{N}=3} = \tfrac{2\pi}{k}\,\mathrm{tr}\left([\Phi^1,\Phi^2]\right)^2$$

F-extremisation : $\Delta_1=\Delta_2=\dfrac{1}{2}$

- Mass deforming $\mathcal{N}=2\ \&\ \mathrm{SU}(3)_\mathrm{F}$

$$\mathcal{W}_{\mathcal{N}=2\,,\,\mathrm{def}} = \mathrm{tr}\left([\Phi^1,\Phi^2]\,\Phi^3 + \underline{\tfrac{1}{2}\,\mu\,(\Phi^3)^2}\right)$$

$$2\,\Delta_3 = \frac{4}{3} < 2$$

UV to IR ($\mu \ll 1$) $\Longrightarrow$

$$\mathcal{W} = \tfrac{1}{2\mu}\,\mathrm{tr}\left([\Phi^1,\Phi^2]\right)^2$$

GY flow : gravity side (I)

- Domain-wall solution

UV: AdS4	holographic energy direction	IR: AdS4
$\mathcal{N}=2$ & $\mathrm{SU}(3)_{\mathrm{F}} \times \mathrm{U}(1)_{\psi}$	$\longrightarrow$	$\mathcal{N}=3$ & $\mathrm{SU}(2)_{\mathrm{F}} \times \mathrm{SO}(3)_{d}$

- Subsector of the $\mathrm{ISO}(7)$ theory capturing relevant/irrelevant deformations

UV:

Flavour	R-symmetry
$\mathrm{SU}(3)_{\mathrm{F}}$ $\times$	$\mathrm{U}(1)_{\psi}$
$\mathrm{SU}(2)_{\mathrm{F}} \times \mathrm{U}(1)_{\tau}$	$\times\ \mathrm{U}(1)_{\psi}$
$\mathrm{SU}(2)_{\mathrm{F}}$	$\times\ \mathrm{U}(1)_{\mathrm{d}}$

RG flow: $\xrightarrow{\mathrm{SU}(2)_{\mathrm{F}} \times \mathrm{U}(1)_{\mathrm{d}}}$

IR:

Flavour	R-symmetry
$\mathrm{SU}(2)_{\mathrm{F}}$	$\times\ \mathrm{SO}(3)_{\mathrm{d}}$
$\mathrm{SU}(2)_{\mathrm{F}}$	$\times\ \mathrm{U}(1)_{\mathrm{d}}$

- Minimal model with 4 chirals [+ identifications]

$$\mathrm{SU}(2)_{\mathrm{F}} \times \mathrm{U}(1)_{\mathrm{d}} \subset \mathrm{ISO}(7) \quad \Rightarrow$$

$$K = -2 \sum_{i=1}^{3} \log[-i(z_i - \bar{z}_i)] - \log[-i(z_4 - \bar{z}_4)]$$

$$W = g \left[c + 4\, z_1\, z_2\, z_3 \,+ \left(z_1^2 + z_2^2 + z_3^2 \right)\, z_4 \right]$$

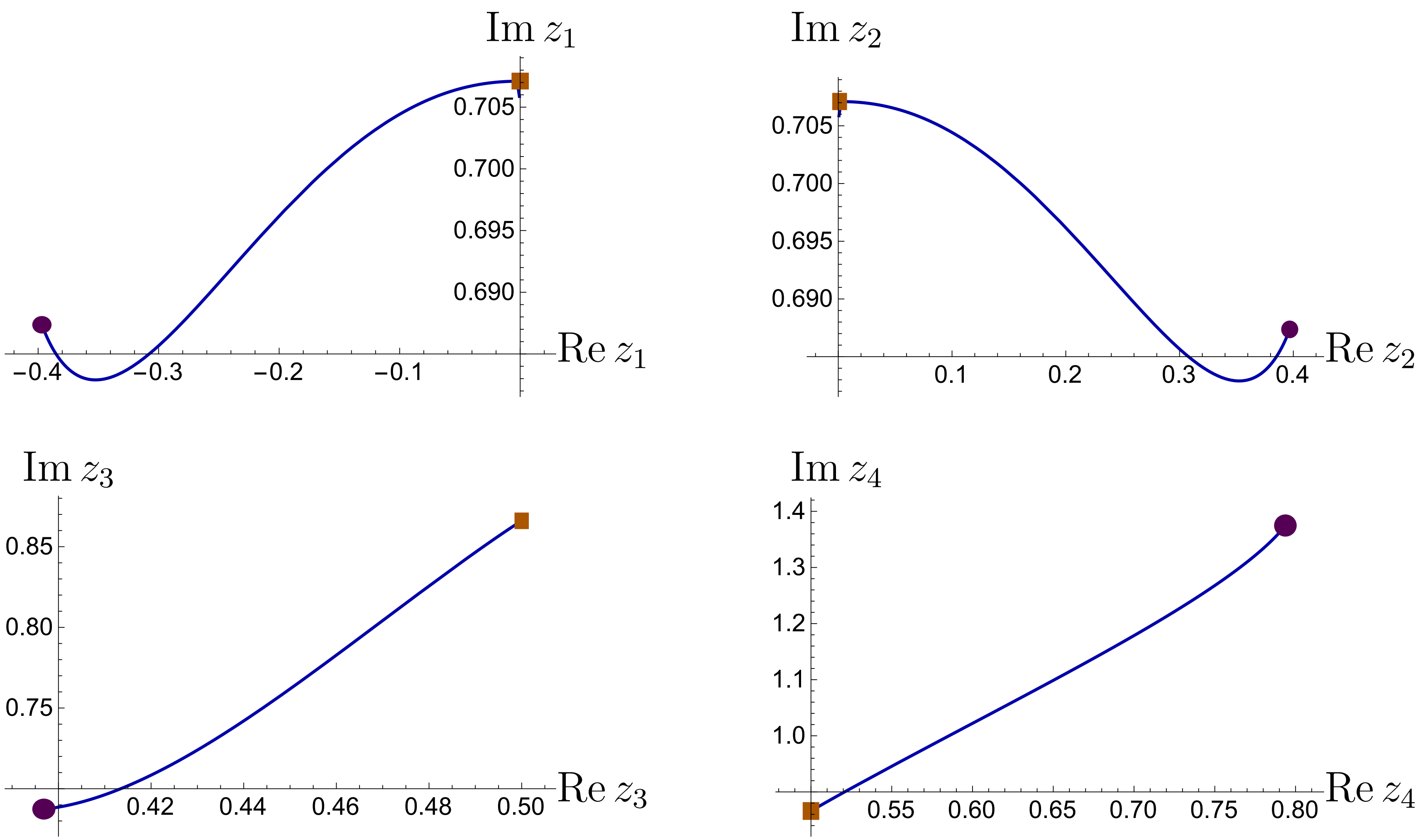
GY flow : gravity side (II)

- Domain-wall Ansatz : $ds^2 = e^{2A(\rho)} \eta_{\mu\nu} dx^\mu dx^\nu + d\rho^2$

[gravitino mass]

- BPS equations : $\partial_\rho A = 2\,\mathcal{W}$, $\partial_\rho z^I = -4\,K^{I\bar{J}}\,\partial_{\bar{z}_{\bar{J}}} \mathcal{W}$ with $\mathcal{W} = \frac{1}{2} e^{K/2} \left(W \overline{W}\right)^{1/2}$

Holographic dual of GY flow (numerical) ⟺ Interpolating massive IIA background



■ $\mathcal{N} = 2$ & $\mathrm{SU}(3)_{\mathrm{F}} \times \mathrm{U}(1)_\psi$

● $\mathcal{N} = 3$ & $\mathrm{SU}(2)_{\mathrm{F}}$

Type IIB

electric/magnetic deformation	higher-dimensional origin	Holographic AdS_4/CFT_3 dual ?
✓	✓	✓

Dyonically-gauged $[\,SO(1,1) \times SO(6)\,] \ltimes R^{12}$ supergravity

- Higher-dimensional origin as Type IIB on $S^1 \times S^5$

[Inverso, Samtleben, Trigiante '16]

[Gallerati, Samtleben, Trigiante '14]

- New AdS_4 vacuum with N=4 & SO(4) symmetry

- Holographic expectation: N=4 interface SYM theory with SO(4) symmetry & Janus solutions

[Bak, Gutperle, Hirano '03 (**N = 0**)]
[Clark, Freedman, Karch, Schnabl '04]
[D'Hoker, Ester, Gutperle '07, '07 (**N = 4**)]
[Gaiotto, Witten '08]
[Assel, Tomasiello '18 (**N = 3 , 4**)]

- Classification of (original) interface SYM theories

N=4 & SO(4) N=2 & SU(2) × U(1) N=1 & SU(3) N=0 & SO(6)

[D'Hoker, Ester, Gutperle '06 (**N = 1 , 2 , 4**)]

Question : *Simple analytic* holographic duals for the N = 0, 1, 2 interface SYM theories with SO(6) , SU(3) and SU(2) × U(1) internal symmetry using a bottom-up approach ?

A truncation : SU(3) invariant subsector

[Warner '83]

- Truncation : Retaining the fields and couplings which are invariant (singlets) under the action of a subgroup $G_0 \subset [\,\mathrm{SO}(1,1) \times \mathrm{SO}(6)\,] \ltimes \mathbb{R}^{12}$

- SU(8) R-symmetry branching : gravitini $\mathbf{8} \to \mathbf{1} + \mathbf{1} + \mathbf{3} + \bar{\mathbf{3}}$ ➡ N = 2 SUSY

- Scalars fields : $\mathbf{70} \to \mathbf{1}\ (\times 6) + \text{non-singlets}$ ➡ 6 real scalars $(\varphi\,,\, \chi\,,\, \phi\,,\, \sigma\,,\, \zeta\,,\, \tilde{\zeta}\,)$

- Vector fields : $\mathbf{56} \to \mathbf{1}\,(\times 4) + \text{non-singlets}$ ➡ vectors $(\, A^0, A^1\,;\, \tilde{A}_0, \tilde{A}_1\,)$

- N = 2 gauged supergravity with $\mathrm{G} = \mathrm{SO}(1,1)_m \times \mathrm{U}(1)_e$ with 1 vector & 1 hypermultiplet

$$\mathcal{M}_{\text{scalar}} = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(2,1)}{\mathrm{U}(2)}$$

AdS$_4$ vacua $\quad (c \neq 0)$

[AG, Sterckx '19]

- **N=0 & SO(6) vacuum** [1 free parameter]

$$\chi = \text{free} \quad , \quad e^{-\varphi} = \frac{c}{\sqrt{2}} \, , \qquad e^{2\phi} = \frac{1}{\sqrt{1-\sigma^2}} \quad , \quad \sigma \in (-1,1) \quad , \quad |\vec{\zeta}|^2 = 0$$

… it turns out to be perturbatively unstable !!

- **N=1 & SU(3) vacuum** [2 free parameters]

$$\chi = 0 \quad , \quad e^{-\varphi} = \frac{\sqrt{5}c}{3} \, , \qquad e^{2\phi} = \frac{6}{5}\,\frac{1}{\sqrt{1-\sigma^2}} \quad , \quad \sigma \in (-1,1) \quad , \quad |\vec{\zeta}|^2 = \tfrac{2}{3}\sqrt{1-\sigma^2}$$

… the compact U(1)$_e$ symmetry broken by $|\vec{\zeta}|^2 \neq 0$ (charged)

Next step : **Uplift to Type IIB on R × S^5 using E$_{7(7)}$-EFT** [Hohm, Samtleben '13]

S-folds and (non-) supersymmetric Janus

[AG, Sterckx '19]

$$ds_{10}^2 = \tfrac{1}{2}\sqrt{Y}\,e^{\varphi}\,ds^2_{\text{AdS}_4} + \sqrt{Y}\,e^{-2\varphi}\,d\eta^2 + \frac{1}{\sqrt{Y}}\left[ds^2_{\mathbb{CP}^2} + Y\,\boldsymbol{\eta}^2\right]$$

$$\widetilde{F}_5 = dC + \tfrac{1}{2}\,\epsilon_{\alpha\beta}\,\mathbb{B}^{\alpha}\wedge\mathbb{H}^{\beta} = \left(4 + \frac{6\,(1-Y)}{Y}\right) Y^{\frac{3}{4}}\,(1+\star)\,\text{vol}_5$$

$$\mathbb{B}^{\alpha} = A^{\alpha}{}_{\beta}\,\mathfrak{b}^{\beta} = -\tfrac{1}{2}\,Y^{-1}\,A^{\alpha}{}_{\beta}\,\epsilon^{\beta\gamma}\,H_{\gamma\delta}\,\boldsymbol{\Omega}^{\delta}$$

$$m_{\alpha\beta} = (A^{-t})_{\alpha}{}^{\gamma}\,\mathfrak{m}_{\gamma\delta}\,(A^{-1})^{\delta}{}_{\beta}$$

with $Y = 1 + \dfrac{1}{4}\,e^{2\phi}\,|\vec{\zeta}|^2$ and $A^{\alpha}{}_{\beta} \equiv \begin{pmatrix} \sqrt{1+\tilde{y}^2} & \tilde{y} \\ \tilde{y} & \sqrt{1+\tilde{y}^2} \end{pmatrix} = \begin{pmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{pmatrix}$

[Bak, Gutperle, Hirano '03] **unstable !!**

[(hyperbolic) SO(1,1)-twist over S¹ ↔ -*STᵏ* monodromy (*k* > 2)]

N=0 & SO(6)

$$\mathfrak{m}_{\gamma\delta} = \frac{1}{\sqrt{1-\sigma^2}}\begin{pmatrix} 1 & -\sigma \\ -\sigma & 1 \end{pmatrix}$$

$$\mathfrak{b}^{\beta} = 0 \qquad Y = 1$$

No untwisted limit !!
(genuinely dyonic)

N=1 & SU(3)

$$\mathfrak{m}_{\gamma\delta} = \frac{1}{\sqrt{1-\sigma^2}}\begin{pmatrix} 1 & -\sigma \\ -\sigma & 1 \end{pmatrix}$$

$$\mathfrak{b}^{\beta} \neq 0 \qquad Y = \frac{6}{5}$$

Summary

- Dyonic N = 8 supergravity with ISO(7) and $[SO(1,1) \times SO(6)] \ltimes R^{12}$ gaugings connected to massive IIA reductions on S^6 and type IIB reductions on $R \times S^5$

- massive IIA : 3d CS-matter theories with simple gauge group SU(N) and adjoint matter

[$N_f = 3$ chiral fields]

$$\mathcal{N} = 2 \;\; \& \;\; \mathrm{SU}(3)_{\mathrm{F}}$$

$$\mathcal{W}_{\mathcal{N}=2} = \mathrm{tr}\left([\Phi^1, \Phi^2]\, \Phi^3\right)$$

UV to IR

Holographic dual of GY flow

[$N_f = 2$ chiral fields]

$$\mathcal{N} = 3 \;\; \& \;\; \mathrm{SU}(2)_{\mathrm{F}}$$

$$\mathcal{W}_{\mathcal{N}=3} = \frac{2\pi}{k}\, \mathrm{tr}\left([\Phi^1, \Phi^2]\right)^2$$

- Type IIB (S-folds) : 3d interface SYM theories with various (super) symmetries

[$-ST^k$ monodromy ($k > 2$)]

$$\mathcal{N} = 0 \;\&\; \mathrm{SO}(6)$$

unstable !!

$$\mathcal{N} = 1 \;\&\; \mathrm{SU}(3)$$

[see also Bobev, Gautason, Pilch, Suh, van Muiden '19]

- Brane set-up? , N=2 interface SYM? , RG flows? ….

Danke schön !

Thank you !