Generalised fluxes, moduli fixing and cosmological implications

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PhD thesis

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Outline

1) Introduction to flux moduli stabilisation scenarios

2) Flux vacua in T-duality invariant supergravity models

3) Some insights into the structure of T/S-duality invariant supergravity models

4) Conclusions

Introduction to flux moduli stabilisation scenarios

Type II ten dimensional supergravities

- Supergravity theories arise as low energy limits of String Theory,

$$\text{System size } L \gg l_s$$

- Low energy limit of type II superstrings $\Rightarrow \mathcal{N} = 2$ supersymmetric field theories in 10 dimensions.

- Type II SUGRA bosonic field content:
  - NS-NS universal sector: $G, \ B_2, \ \varphi$
  - R-R sector: $C_1, \ C_3$ (type IIA) or $C_0, \ C_2, \ C_4$ (type IIB)

- Superstring theories then necessarily predict the existence of extra dimensions as well as supersymmetry (not observed yet):
  - Compactification from 10 to 4 dimensions
  - Mechanisms of SUSY breaking

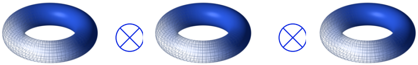
Reducing dimensions and supersymmetries

- Reducing type II supergravities on a six-torus $\mathbb{T}^6$ of characteristic size R,

System size $L \gg R \gg l_s$

$\Rightarrow \mathcal{N} = 8$ supersymmetric effective field theory in 4d.

- Reducing type II supergravities on $\frac{\mathbb{T}^6}{\mathbb{Z}_2 \times \mathbb{Z}_2}$ orbifold,
  - the action of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold group automatically implies the factorisation $\mathbb{T}^6 = \mathbb{T}^2_1 \times \mathbb{T}^2_2 \times \mathbb{T}^2_3$.



$\Rightarrow \mathcal{N} = 2$ supersymmetric effective field theory in 4d.

- Having an orientifold of the $\frac{\mathbb{T}^6}{(\mathbb{Z}_2 \times \mathbb{Z}_2)}$ orbifold entails modding out the theory by:

$i)$ Worldsheet symmetry $ii)$ Internal space involution σ

$\Rightarrow \mathcal{N} = 1$ supersymmetric effective field theory in 4d.

$\frac{\mathbb{T}^6}{(\mathbb{Z}_2 \times \mathbb{Z}_2)}$ IIB orientifold with O3/O7-planes

- Modding out the IIB theory by the worldsheet symmetry $(-1)^{F_L}\,\Omega_p$:

	EVEN	ODD
$(-1)^{F_L}\,\Omega_p$	$G\,,\,\varphi\,,\,C_0\,,\,C_4$	$B_2\,,\,C_2 \rightarrow$ Projected out

- Orientifold group action: Combined action of the internal space involution $\sigma = -\mathbb{I}_6$ and the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold group elements $\{\mathbb{1}, \theta_1, \theta_2, \theta_3\}$:
  - $\sigma \mathbb{1}$ creates O3-planes located at its fixed points.
  - $\sigma \theta_I$ creates $O7_I$-planes located at its fixed 4-cycles.

- Extensions of G.R. involving 4d complex scalar fields a.k.a moduli fields.
  - Geometric moduli associated to each $\mathbb{T}_I^2$ $(U_I \in \mathbb{C}\,,\, A_I \in \mathbb{R})$
    - Kähler: $T_I = \int_{M_6} C_4 \wedge \omega_I + i\, e^{-\varphi}\, A_J A_K$
    - Complex structure: U_I
  - Axio-dilaton: $S \equiv C_0 + i\, e^{-\varphi}$ determining $g_s = e^{\langle \varphi \rangle} = \dfrac{1}{\text{Im}S}$.

The moduli problem

- Lagrangian density for the moduli fields:

$$\mathcal{L}_{moduli} = K_{ij}\, \partial_\mu \Phi_i\, \partial^\mu \bar{\Phi}_j - V(\Phi) \qquad \text{with} \qquad V(\Phi) = 0$$

where the metric in the field space (Kähler metric),

$$K_{ij} = \frac{\partial^2 K}{\partial \Phi_i\, \partial \bar{\Phi}_j} \qquad \text{with} \qquad \Phi \equiv (\, S\,,\, T_1\,,\, T_2\,,\, T_3\,,\, U_1\,,\, U_2\,,\, U_3\,)\ ,$$

is computed from the Kähler potential

$$K = -\,\log\left(-i\,(S-\bar{S})\right) - \sum_{I=1}^{3} \log\left(-i\,(T_I-\bar{T}_I)\right) - \sum_{I=1}^{3} \log\left(-i\,(U_I-\bar{U}_I)\right)$$

- One of the main challenges of String Phenomenology is to find mechanisms for the moduli to be stabilised.
  - Problem: (i) massless moduli would mediate long range interactions. (ii) Physical quantities undetermined: g_s, internal space size and shape, vacuum energy,...
  - Challenges: moduli stabilisation in a de Sitter vacuum?, SUSY breaking?, inflation?...

$\Rightarrow$ How to induce $V(\Phi) \neq 0$? $\Longrightarrow$ Flux backgrounds !!

Background of gauge fluxes

- The orientifold involution action allows background fluxes $\bar{H}_3$ and $\bar{F}_3$ for the NS-NS and R-R 3-forms,

$$H_3 = dB_2 + \bar{H}_3 \qquad \text{and} \qquad F_3 = dC_2 - H_3 \wedge C_0 + \bar{F}_3$$

- Consistency conditions: O3/D3 sources needed to cancel a flux-induced tadpole for the R-R C_4 gauge potential.

- Gauge fluxes induce a $\mathcal{N}=1$ superpotential for the axiodilaton S and the complex structure moduli U_I,

$$W(S, U_I) = \int_{\mathrm{M}_6} (\bar{F}_3 - S\,\bar{H}_3) \wedge \Omega \overset{\mathbb{T}^6/\mathbb{Z}_2\times\mathbb{Z}_2}{=} P_1(U_I) + S\,P_2(U_I)\ ,$$

$\Rightarrow$ Non-vanishing $\mathcal{N}=1$ scalar potential !!

$$V(\Phi) = e^K \sum_{\Phi=S,T_I,U_I} K^{\Phi\bar{\Phi}} |D_\Phi W|^2 - 3|W|^2$$

where $D_\Phi W = \partial_\Phi W + W\,\partial_\Phi K$

- How to stabilise the Kähler moduli T_I? $\Rightarrow$ generalised fluxes, non-perturbative effects, ...

Font, Ibáñez, Lüst and Quevedo [Phys.Lett.B245:401-408,1990]
Achucarro, de Carlos, Casas and Doplicher [arXiv:hep-th/0601190]

T-duality and generalised background fluxes

- Type II reductions on $\mathbb{T}^6$ come out with an $\text{SO}(6,6)$ target space duality, the so-called T-duality, relating IIB and IIA compactifications.

- T-duality transformations upon $\bar{H}_3$ give rise to generalised fluxes

$$\bar{H}_{abc} \xrightarrow{T_a} \omega_{bc}^{a} \xrightarrow{T_b} Q_{c}^{ab} \xrightarrow{T_c} R^{abc}$$

  involving metric fluxes ω and *non-geometric* Q and R fluxes.

- Type IIB orientifold with O3/O7-planes $\Rightarrow$ Only $\bar{H}_3$ and Q fluxes.

- These fluxes define a 12d flux algebra $\mathfrak{g}$ spanned by:
  - X^a gauge generators coming from the reduction of B_2.
  - Z_a isometry generators coming from the reduction of G

$$\left[X^a, X^b\right] = Q_c^{ab}\, X^c \quad , \quad \left[Z_a\, , X^b\right] = Q_a^{bc}\, Z_c \quad , \quad [Z_a\, , Z_b] = \bar{H}_{abc}\, X^c$$

Shelton, Taylor and Wecht [arXiv:hep-th/0508133]

with $(a = 1, \ldots, 6)$, playing the role of structure constants.

- Quadratic Jacobi identities descend from the algebra,

$$Q^2 = 0 \qquad \text{and} \qquad \bar{H}_3\, Q = 0 \ ,$$

with the contractions of indices $Q_x^{[ab}\, Q_d^{c]x} = 0$ and $\bar{H}_{x[bc}\, Q_{d]}^{ax} = 0$.

- Flux-induced tadpoles: O3/D3 and O7/D7 sources needed to respectively cancel flux-induced tadpoles for the R-R C_4 and C_8 gauge potentials

$$\int C_4 \wedge \bar{H}_3 \wedge \bar{F}_3 \qquad \text{and} \qquad \int C_8 \wedge (Q \cdot \bar{F}_3)$$

- $\mathcal{N} = 1$ flux-induced superpotential depending on all the type IIB moduli fields,

$$\begin{aligned} W(S, T_I, U_I) &= \int_{\mathrm{M}_6} \Big((\bar{F}_3 - S\, \bar{H}_3) + Q \cdot \mathcal{J} \Big) \wedge \Omega \overset{\mathbb{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2}{=} \\ &= P_1(U_I) + S\, P_2(U_I) + \sum_{K=1}^{3} P_3^{(K)}(U_I)\, T_K \end{aligned}$$

- Effective theory invariant under $\mathrm{SL}(2)_U$ modular transformations on the complex structure moduli

$$U_I \to \frac{\alpha_I\, U_I + \beta_I}{\gamma_I\, U_I + \delta_I} \qquad \text{provided} \qquad \bar{F}_3 \to \bar{F}_3' \ , \quad \bar{H}_3 \to \bar{H}_3' \ , \quad Q \to Q'$$

S-duality and generalised background fluxes

- Type IIB non-perturbative $\text{SL}(2)_S$ self-duality, the so-called S-duality, acting on S and the gauge fluxes $(\bar{F}_3, \bar{H}_3)$ as

$$S \rightarrow \frac{a\,S+b}{c\,S+d} \quad , \quad \begin{pmatrix} \bar{F}_3 \\ \bar{H}_3 \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \bar{F}_3 \\ \bar{H}_3 \end{pmatrix}$$

$\Rightarrow$ A new P flux has to be introduced to build an $\text{SL}(2)_S$-doublet (Q, P) of non-geometric fluxes. Aldazabal, Cámara, Font and Ibáñez [arXiv:hep-th/0602089]

- $\mathcal{N}=1$ flux-induced superpotential,

$$\begin{aligned} W(S, T_I, U_I) &= \int_{M_6} \Big((\bar{F}_3 - S\,\bar{H}_3) + (Q - S\,P) \cdot \mathcal{J} \Big) \wedge \Omega \qquad \mathbb{T}^6/\mathbb{Z}_2\times\mathbb{Z}_2 \\ &= P_1(U_I) + S\,P_2(U_I) + \sum_{K=1}^{3} \Big(P_3^{(K)}(U_I) + S\,P_4^{(K)}(U_I) \Big)\, T_K \end{aligned}$$

- Effective theory now invariant under $\text{SL}(2)_{U,S}$ modular transformations

$$U_I \rightarrow \frac{\alpha\,U_I + \beta}{\gamma\,U_I + \delta} \quad \text{and} \quad S \rightarrow \frac{a\,S+b}{c\,S+d}$$

provided $(\bar{F}_3, \bar{H}_3) \rightarrow (\bar{F}'_3, \bar{H}'_3)$ and $(Q, P) \rightarrow (Q', P')$.

- Which are the flux constraints in the T/S-duality invariant theory?
- Ansatz: To apply S-duality $SL(2)_S$ transformations upon the flux constraints of the T-duality invariant effective theory.

  Aldazabal, Cámara, Font and Ibáñez [arXiv:hep-th/0602089]
- Jacobi identities:

T-dual theory	T/S-dual theory	transformation
$Q^2 = 0$	$Q^2 = 0$, $P^2 = 0$, $QP + PQ = 0$	$SL(2)_S$-triplet
$\bar{H}_3 Q = 0$	$\bar{H}_3 Q - \bar{F}_3 P = 0$	$SL(2)_S$-singlet

- Flux-induced tadpoles:
  - (D7 , I7 , NS7)-branes coupling to the $SL(2)_S$-triplet $(C_8, C_8', \tilde{C}_8)$ of R-R gauge potentials.

    Bergshoeff, de Roo, Kerstan, Ortín and Riccioni [arXiv:hep-th/0601128]

    Bergshoeff, Hartong, Ortín and Roest [arXiv:hep-th/0612072]
  - D3-branes coupling to the $SL(2)_S$-singlet R-R gauge potential C_4.

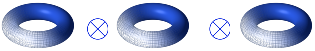
T-dual theory	T/S-dual theory	transformation
$\int C_8 \wedge (Q \cdot \bar{F}_3)$	$\int C_8 \wedge (Q \cdot \bar{F}_3)$, $\int C_8' \wedge (Q \cdot \bar{H}_3 + P \cdot \bar{F}_3)$, $\int \tilde{C}_8 \wedge (P \cdot \bar{H}_3)$	$SL(2)_S$-triplet
$\int C_4 \wedge \bar{H}_3 \wedge \bar{F}_3$	$\int C_4 \wedge \bar{H}_3 \wedge \bar{F}_3$	$SL(2)_S$-singlet

The isotropic supergravity models

- The $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry allows for a huge amount of flux parameters:

$$(\bar{F}_3\,,\,\bar{H}_3) + (Q\,,\,P) \;\longrightarrow\; (8+8) + (24+24) = 64 \text{ flux parameters !!}$$

- Additional $\mathbb{Z}_3$ (isotropy) symmetry on the fluxes under the exchange $1 \to 2 \to 3$ in the factorisation $\mathbb{T}^6 = \mathbb{T}_1^2 \times \mathbb{T}_2^2 \times \mathbb{T}_3^2$.



$$(e.g.\ \bar{H}_{|--} = \bar{H}_{-|-} = \bar{H}_{--|})$$

- Isotropic flux backgrounds reduce the number of flux parameters

$$(\bar{F}_3\,,\,\bar{H}_3) + (Q\,,\,P) \;\longrightarrow\; (4+4) + (8+8) = 24 \text{ flux parameters !!}$$

and are compatible with vacua in which

$$T_1 = T_2 = T_3 \equiv T \qquad \text{and} \qquad U_1 = U_2 = U_3 \equiv U\ .$$

- (S, T, U) SUGRA models

$$K = -\log\left(-i\,(S-\bar{S})\right) - 3\,\log\left(-i\,(T-\bar{T})\right) - 3\,\log\left(-i\,(U-\bar{U})\right)$$

$$W = \underbrace{P_1(U)}_{\bar{F}_3} + S\,\underbrace{P_2(U)}_{\bar{H}_3} + T\left(\underbrace{P_3(U)}_{Q} + S\,\underbrace{P_4(U)}_{P}\right)$$

Flux vacua in T-duality invariant SUGRA models
$(Q \neq 0$ and $P = 0)$

Based on the collaborations:

A. Font, A. G. and J. Moreno	[arXiv:0809.3748 [hep-th]]
B. de Carlos, A.G. and J. Moreno	[arXiv:0907.5580 [hep-th]]
B. de Carlos, A.G. and J. Moreno	[arXiv:0911.2876 [hep-th]]

- The models are specified by the Kähler potential and the superpotential given by

$$K = -\log\left(-i\,(S-\bar{S})\right) - 3\,\log\left(-i\,(T-\bar{T})\right) - 3\,\log\left(-i\,(U-\bar{U})\right)$$

$$W = \underbrace{P_1(U)}_{\bar{F}_3} + S\,\underbrace{P_2(U)}_{\bar{H}_3} + T\,\underbrace{P_3(U)}_{Q}$$

- Flux constraints:

Jacobi identities	(C_4, C_8) tadpole cancellations
$Q^2 = 0$	$\bar{H}_3 \wedge \bar{F}_3 = N_{O3} - N_{D3} \equiv N_3$
$\bar{H}_3 Q = 0$	$Q \cdot \bar{F}_3 = N_{D7} - N_{O7} \equiv N_7$

- Goals: Finding extrema $\left(\frac{\partial V}{\partial \Phi} = 0\right)$ of the (S, T, U) flux-induced scalar potential:
  - Supersymmetric Minkowski and AdS_4
  - Non-supersymmetric Minkowski/dS

- Computational approach: scanning of flux parameters + numerical minimisation procedure ⇒ Distribution of SUSY vacua.

  Shelton, Taylor and Wecht [arXiv:hep-th/0607015]

- Our approach: To make extensive use of the isotropic orbifold symmetries as well as of the flux-algebra structure.

Sketch of our approach

Non-geometric Q flux
(8 flux parameters)

"Q-algebra"
$[X^a, X^b] = Q^{ab}_c X^c$
$Q^2 = 0$

Five possible Q-algebras !!
compatible with the
$\mathbb{Z}_2 \times \mathbb{Z}_2$ isotropic orbifold

Q-algebra embedding

$$\Gamma \equiv \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

Modular transformation on U

$$\mathcal{Z} \equiv \Gamma\, U = \frac{\alpha\, U + \beta}{\gamma\, U + \delta}$$

Γ-dependence drops out !

$K(S,T,U) \longrightarrow \mathcal{K}(\mathcal{S},\mathcal{T},\mathcal{Z})$

$W(S,T,U) \longrightarrow \mathcal{W}(\mathcal{S},\mathcal{T},\mathcal{Z}\ ;\ \theta_\epsilon, \theta_\xi)$

NS-NS $\bar{H}_3$ flux
(4 flux parameters)

$\bar{H}_3 Q = 0$

$(\epsilon_1, \epsilon_2) \equiv |\epsilon|\, e^{i\,\theta_\epsilon}$

Rescaling $S \to \mathcal{S}$
$|\epsilon|$ drops out!

"Flux algebra"
$\theta_\epsilon \in [0, 2\pi]$

R-R $\bar{F}_3$ flux
(4 flux parameters)

real shifts on
S and T

$(\xi_3, \xi_7) \equiv |\xi|\, e^{i\,\theta_\xi}$

Rescaling $T \to \mathcal{T}$
$|\xi|$ drops out!

(N_3, N_7) sources
$\theta_\xi \in [0, 2\pi]$

Flux algebras and SUGRA models

- The set of allowed Q-algebras accordingly to the orbifold symmetries:

$$\mathfrak{so}(3,1) \quad , \quad \mathfrak{so}(4) \quad , \quad \mathfrak{su}(2)+\mathfrak{u}(1)^3 \quad , \quad \mathfrak{iso}(3) \quad \text{and} \quad \mathfrak{nil}$$

$\Rightarrow$ Five non-equivalent Q-algebras $\Rightarrow$ Splitting of SUGRA models.

- Provided a particular Q-algebra, its associated effective SUGRA model is encoded into two phases $(\theta_\epsilon, \theta_\xi)$.

$$\mathcal{K} = -3\,\log\left(-i\,(\mathcal{Z}-\bar{\mathcal{Z}})\right) - \log\left(-i\,(\mathcal{S}-\bar{\mathcal{S}})\right) - 3\,\log\left(-i\,(\mathcal{T}-\bar{\mathcal{T}})\right)$$

$$\mathcal{W} = \mathcal{P}_1(\mathcal{Z},\theta_\epsilon,\theta_\xi) + \mathcal{S}\,\mathcal{P}_2(\mathcal{Z},\theta_\epsilon) + \mathcal{T}\,\mathcal{P}_3(\mathcal{Z})$$

Q-algebra	$\mathcal{P}_3(\mathcal{Z})/3$	$\mathcal{P}_2(\mathcal{Z},\theta_\epsilon)$	$\mathcal{P}_1(\mathcal{Z},\theta_\epsilon,\theta_\xi)$
$\mathfrak{so}(3,1)$	$-\mathcal{Z}(\mathcal{Z}^2+1)$	$-c_\epsilon\,\mathcal{Z}^3 - 3\,s_\epsilon\,\mathcal{Z}^2 + 3\,c_\epsilon\,\mathcal{Z} + s_\epsilon$	
$\mathfrak{so}(4)$	$\mathcal{Z}(\mathcal{Z}^2-1)$	$c_\epsilon\,\mathcal{Z}^3 + 3\,s_\epsilon\,\mathcal{Z}^2 + 3\,c_\epsilon\,\mathcal{Z} + s_\epsilon$	
$\mathfrak{su}(2)+\mathfrak{u}(1)^3$	$\mathcal{Z}$	$c_\epsilon\,\mathcal{Z}^3 + s_\epsilon$	$s_\xi\,\widetilde{\mathcal{P}}_3(\mathcal{Z}) - c_\xi\,\widetilde{\mathcal{P}}_2(\mathcal{Z})$
$\mathfrak{iso}(3)$	$-\mathcal{Z}$	$3\,c_\epsilon\,\mathcal{Z} + s_\epsilon$	
$\mathfrak{nil}$	1	$c_\epsilon - 3\,s_\epsilon\,\mathcal{Z}$	

- Any consistent flux model can be brought to one of the above five models by applying a modular transformation $\mathcal{Z} = \Gamma\,U$.

- Provided a particular Q-algebra, its extension to a complete $(\bar{H}_3, Q)$ flux algebra $\mathfrak{g}$ is encoded into the value of the phase $\theta_\epsilon \in [0, 2\pi]$.

Q-algebra	Flux algebra $\mathfrak{g}$	θ_ϵ
$\mathfrak{so}(3,1)$	$\mathfrak{so}(3,1) + \mathfrak{so}(3,1)$	$[0, 2\pi]$
$\mathfrak{so}(4)$	$\mathfrak{so}(3,1) + \mathfrak{so}(3,1)$	$(-\frac{\pi}{4}, \frac{\pi}{4})$
	$\mathfrak{so}(4) + \mathfrak{so}(4)$	$(\frac{3\pi}{4}, -\frac{3\pi}{4})$
	$\mathfrak{so}(3,1) + \mathfrak{iso}(3)$	$-\frac{\pi}{4}, \frac{\pi}{4}$
	$\mathfrak{so}(3,1) + \mathfrak{so}(4)$	$(\frac{\pi}{4}, \frac{3\pi}{4}) \cup (-\frac{3\pi}{4}, -\frac{\pi}{4})$
	$\mathfrak{iso}(3) + \mathfrak{so}(4)$	$\frac{3\pi}{4}, -\frac{3\pi}{4}$
$\mathfrak{su}(2) + \mathfrak{u}(1)^3$	$\mathfrak{so}(3,1) + \mathfrak{nil}$	$(\frac{3\pi}{2}, 0) \cup (0, \frac{\pi}{2})$
	$\mathfrak{so}(3,1) + \mathfrak{u}(1)^6$	0
	$\mathfrak{iso}(3) + \mathfrak{nil}$	$\frac{\pi}{2}, \frac{3\pi}{2}$
	$\mathfrak{so}(4) + \mathfrak{nil}$	$(\frac{\pi}{2}, \pi) \cup (\pi, \frac{3\pi}{2})$
	$\mathfrak{so}(4) + \mathfrak{u}(1)^6$	π
$\mathfrak{iso}(3)$	$\mathfrak{so}(3,1) \oplus_{\mathbb{Z}_3} \mathfrak{u}(1)^6$	$(\frac{3\pi}{2}, \frac{\pi}{2})$
	$\mathfrak{iso}(3) \oplus_{\mathbb{Z}_3} \mathfrak{u}(1)^6$	$\frac{\pi}{2}, \frac{3\pi}{2}$
	$\mathfrak{so}(4) \oplus_{\mathbb{Z}_3} \mathfrak{u}(1)^6$	$(\frac{\pi}{2}, \frac{3\pi}{2})$
$\mathfrak{nil}$	$\mathfrak{nil}_{12}(4)$	$(0, \pi) \cup (\pi, 2\pi)$
	$\mathfrak{nil}_{12}(2)$	$0, \pi$

Supersymmetric flux vacua

- Supersymmetric vacua are characterised by the vanishing of the F-terms:

$$D_{\mathcal{Z}}\mathcal{W} = D_{\mathcal{S}}\mathcal{W} = D_{\mathcal{T}}\mathcal{W} = 0$$

$\Rightarrow \quad V_0 = -3\, e^{\mathcal{K}_0} |\mathcal{W}_0|^2 \leq 0$ at any SUSY vacuum $\Rightarrow$ Mkw or AdS$_4$.

- Physical constraints have to be imposed over the moduli VEVs

$$g_s = \frac{1}{\text{Im}\mathcal{S}_0} > 0 \qquad \text{and} \qquad \text{Vol}_6 = \left(\frac{\text{Im}\mathcal{T}_0}{\text{Im}\mathcal{S}_0}\right)^{\frac{3}{2}} > 0$$

Supersymmetric Mkw vacua: They require $\mathcal{W}_0 = 0$ and then the roots alignment

$$\mathcal{P}_1(\mathcal{Z}_0) = \mathcal{P}_2(\mathcal{Z}_0) = \mathcal{P}_3(\mathcal{Z}_0) = 0 \quad \Longrightarrow \quad \text{Im}\mathcal{Z}_0 = \text{Im}U_0 = 0$$

resulting in an ill-defined internal space.

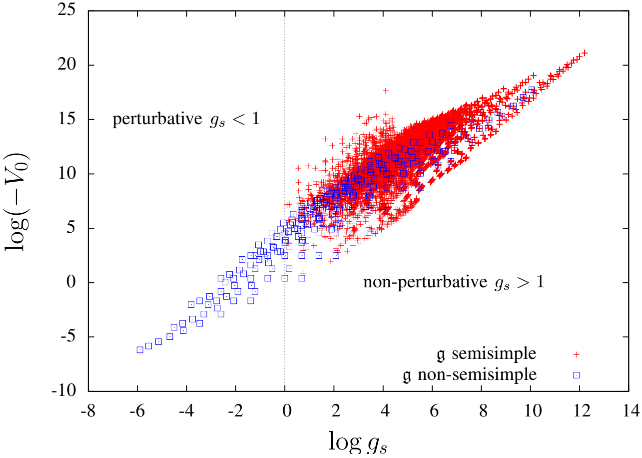
Supersymmetric AdS$_4$ vacua: They exist in all the five SUGRA models !!

Remarks on the supersymmetric AdS_4 landscape

- Set of layers of vacua associated to different flux algebras.
- There are regions in flux space with small characteristic data (g_s, V_0).
- Relation between the moduli VEVs and the R-R net charges (N_3, N_7).
- Non-semisimple Q-algebras:
  - In the $\mathfrak{nil}$ and $\mathfrak{iso}(3)$ models, SUSY forces one combination of the ReS and ReT axions to be unstabilised. These models come out with O3/O7 and D3/D7 sources respectively.
  - In the $\mathfrak{iso}(3)$ model it is possible to avoid sources altogether $(N_3 = N_7 = 0)$.
  - In the $\mathfrak{su}(2) + \mathfrak{u}(1)^3$ model, all the moduli are stabilised and D3/D7-branes are generically required.
- Semisimple Q-algebras:
  - All the moduli are stabilised.
  - Appearance of multiple vacua for certain flux configurations.
  - Richer structure of R-R charges, i.e. O3/D3 or O7/D7 sources can be avoided separately.

A flash of the AdS_4 landscape

- Q-algebra$\equiv \mathfrak{so}(4)$ model with $\Gamma = \begin{pmatrix} \alpha & 0 \\ 0 & \gamma \end{pmatrix}$, $N_3 = 16$, $N_7 = 0$ and integer fluxes (even) up to 20.



Non-supersymmetric Minkowski/de Sitter vacua

- The strategy: Our strategy entails finding the Mkw extrema of the scalar potential by solving

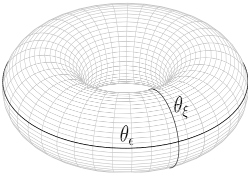
$$\left.\frac{\partial V(\Phi,\theta_\epsilon,\theta_\xi)}{\partial \Phi}\right|_{\Phi=\Phi_0} = 0 \qquad \text{plus} \qquad V(\Phi_0,\theta_\epsilon,\theta_\xi) = 0$$

and then looking for dS vacua continuously connected to them.

- The stabilisation of the $\mathcal{S}$ and $\mathcal{T}$ moduli in a Mkw vacuum can be worked out analytically.

- The stabilisation of the $\mathcal{Z}$ modulus has to be tackled numerically.

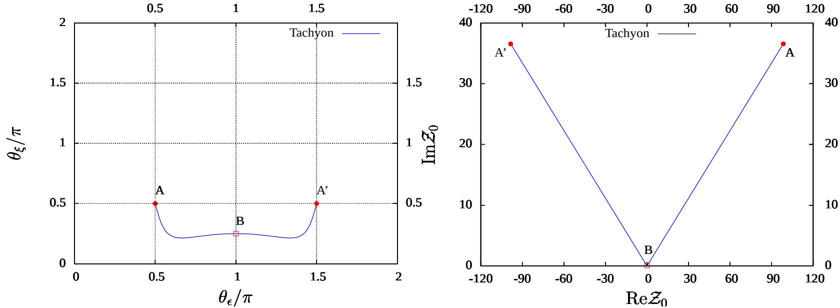
- The $V(\Phi_0,\theta_\epsilon,\theta_\xi) = 0$ condition relates the values of the $(\theta_\epsilon,\theta_\xi)$ phases at the Mkw vacua. Therefore, these vacua draw lines within the parameter space $(\theta_\epsilon,\theta_\xi)$.



Models based on the $\mathfrak{nil}$ Q-algebra.

- There are no solution of the extremisation conditions when demanding the vanishing of the cosmological constant $\Rightarrow$ These models are excluded to accommodate for Minkowski extrema.

Models based on the $\mathfrak{iso}(3)$ Q-algebra.

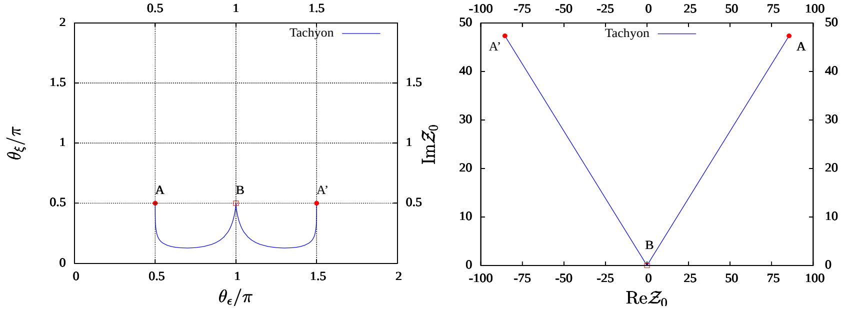


- Unstable Mkw extrema with an underlying $\mathfrak{g} = \mathfrak{so}(4) \oplus_{\mathbb{Z}_3} \mathfrak{u}(1)^6$ flux algebra and with O3/D7 sources.

– Points A and A': Underlying $\mathfrak{g} = \mathfrak{iso}(3) \oplus_{\mathbb{Z}_3} \mathfrak{u}(1)^6$ and $|\Phi_0| \to \infty$.

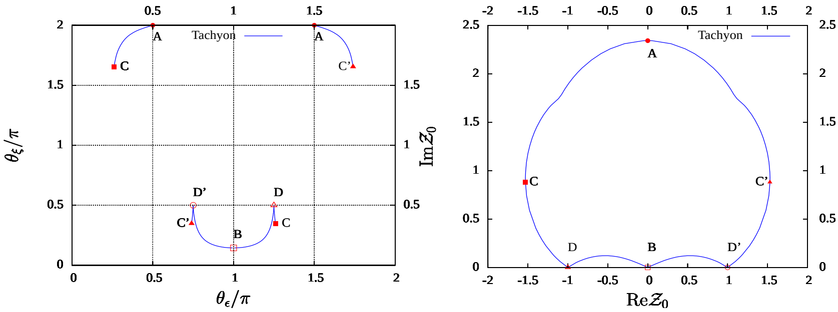
– Point B : Special point where $|\Phi_0| \to 0$ dynamically.

Models based on the $\mathfrak{su}(2)+\mathfrak{u}(1)^3$ Q-algebra.



- Unstable Mkw extrema with an underlying $\mathfrak{g}=\mathfrak{so}(4)+\mathfrak{nil}$ flux algebra and with O3/D7 sources.
  - Points A and A': Underlying $\mathfrak{g}=\mathfrak{iso}(3)+\mathfrak{nil}$ and $|\Phi_0|\to\infty$.
  - Point B : Underlying $\mathfrak{g}=\mathfrak{so}(4)+\mathfrak{u}(1)^6$ and $|\Phi_0|\to 0$.

Models based on the $\mathfrak{so}(4)$ Q-algebra.



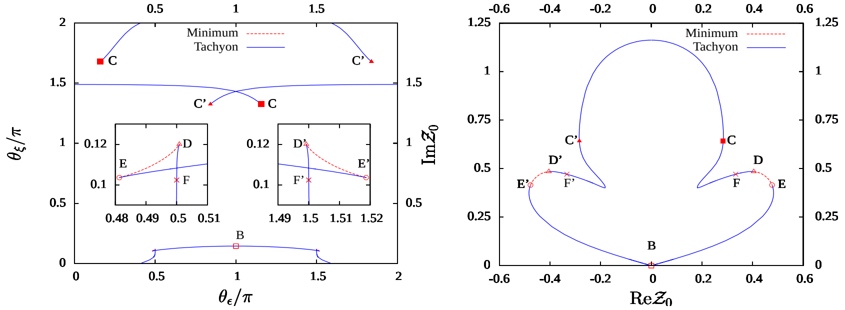
- Two branches of unstable Mkw extrema with different flux algebras:

  (1) Line $\overline{\text{DD'}}$ through point B: Underlying $\mathfrak{g} = \mathfrak{so}(4) + \mathfrak{so}(4)$ flux algebra. Special point B in which $\mathrm{Im}\Phi_0 \to 0$ dynamically.

  (2) Line $\overline{\text{DD'}}$ through point A: Underlying $\mathfrak{g} = \mathfrak{so}(3,1) + \mathfrak{so}(4)$ flux algebra. Special A, C and C' points where $\mathrm{Im}\mathcal{T}_0 \to 0$ dynamically.

– Points D and D': Underlying $\mathfrak{g} = \mathfrak{iso}(3) + \mathfrak{so}(4)$ and $\mathrm{Im}\mathcal{S}_0 \to \infty$, $\mathrm{Im}\mathcal{T}_0 \to 0$ and $\mathrm{Im}\mathcal{Z}_0 \to 0$.

Models based on the $\mathfrak{so}(3,1)$ Q-algebra.



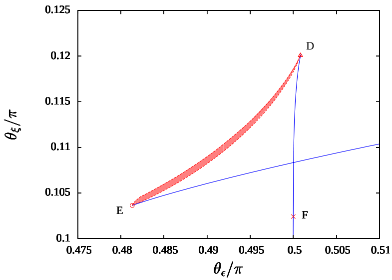
- There is an unique $\mathfrak{g} = \mathfrak{so}(3,1) + \mathfrak{so}(3,1)$ flux algebra.
  - Point B: Special point where $\mathrm{Im}\Phi_0 \to 0$ dynamically.
  - Points C and C' : Special points where $\mathrm{Im}\mathcal{S}_0 \to 0$ dynamically.

★ Lines $\overline{\mathrm{DE}}$ & $\overline{\mathrm{D'E'}}$: Stable Mkw vacua with O3-planes and D7-branes !!

- **Minkowski stable vacua** are continuously **connected to dS stable vacua via**

$$\theta_\xi^{(\mathrm{dS})} = \theta_\xi^{(\mathrm{Mkw})} + \delta\theta_\xi$$

up to some critical value $0 < \delta\theta_\xi < \delta\theta_\xi^{(\mathrm{crit})}$.



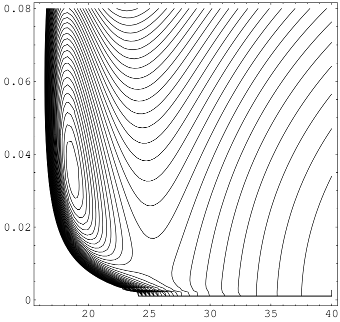
- **Supersymmetry broken** by all moduli,

$$F_{\mathcal{Z}} \neq 0\,, \quad F_{\mathcal{S}} \neq 0\,, \quad F_{\mathcal{T}} \neq 0$$

Gómez-Reino and Scrucca [arXiv:hep-th/0602246]

- **dS saddle point with η-problem**, $\eta \sim \mathcal{O}(10)$, close to the Mkw vacuum $\Rightarrow$ No *slow-roll* inflation.

Flauger, Paban, Robbins and Wrase [arXiv:hep-th/08123886]
Caviezel, Koerber, Körs, Lüst, Wrase and Zagermann [arXiv:hep-th/08123551]



- **Non-geometric type IIA duals** with O6/D6 $\Rightarrow$ avoiding no-go theorems.

Hertzberg, Kachru, Taylor and Tegmark [arXiv:0711.2512]
Haque, Shiu, Underwood and Van Riet [arXiv:0810.5328]
Silverstein [arXiv:0712.1196]

Insights into T/S-duality invariant SUGRA models $(Q \neq 0 \quad \text{and} \quad P \neq 0)$

Based on the collaboration:

A. G. and G. J. Weatherill [arXiv:0811.2190 [hep-th]]

- The models are specified by the Kähler potential and the superpotential given by

$$K = -3\,\log\left(-i\,(U-\bar{U})\right) - \log\left(-i\,(S-\bar{S})\right) - 3\,\log\left(-i\,(T-\bar{T})\right)$$

$$W = \underbrace{P_1(U)}_{\bar{F}_3} + S\,\underbrace{P_2(U)}_{\bar{H}_3} + T\left(\underbrace{P_3(U)}_{Q} + S\,\underbrace{P_4(U)}_{P}\right)$$

- Flux constraints:

Jacobi identities	$(C_8, C_8', \tilde{C}_8)$ and C_4 tadpole cancellations
$Q^2 = 0$, $QP + PQ = 0$, $P^2 = 0$ $\bar{H}_3 Q - \bar{F}_3 P = 0$	$Q \cdot \bar{F}_3 = N_7$, $Q \cdot \bar{H}_3 + P \cdot \bar{F}_3 = N_7'$, $P \cdot \bar{H}_3 = \tilde{N}_7$ $\bar{H}_3 \wedge \bar{F}_3 = N_3$

- Objectives:
  - To extend the techniques developed for the T-duality invariant SUGRA models to the case of T/S-duality invariant ones.
  - To look for simple supersymmetric solutions.
- Tools: Algebraic Geometry techniques to solve the Jacobi identities.

Flux backgrounds and SUSY solutions

- Non-geometric (Q, P) backgrounds: The P flux can be implemented as deformations of the Q-algebra by an element of its second cohomology class.
  - $Q^2 = 0 \quad \Rightarrow \quad Q$-algebra whose embedding is encoded within Γ_Q.
  - $P^2 = 0 \quad \Rightarrow \quad P$-algebra whose embedding is encoded within Γ_P.
  - $QP + PQ = 0 \quad \Rightarrow \quad$ Restriction on the embedding matrices (Γ_Q, Γ_P).

  Using the Gianni-Trager-Zacharias GTZ primary decomposition algorithm implemented in the computer algebra system *Singular*, the solution to $QP + PQ = 0$ breaks down into various branches.

- Gauge $(\bar{F}_3, \bar{H}_3)$ backgrounds: The $\bar{H}_3 Q - \bar{F}_3 P = 0$ condition remains a linear system $\Rightarrow$ Analytical solution.

- The entire set of consistent flux backgrounds can be systematically built $\Rightarrow$ Supersymmetric AdS_4 and Minkowski vacua can be found !!

- Question: Can these solutions be lifted to $\mathcal{N} = 4$ gauged supergravities with electric-magnetic gaugings?

Schon and Weidner [arXiv:hep-th/0602024]

Lifting to $\mathcal{N}=4$

- The $\mathcal{N}=4$ flux algebra involving gauge $(\bar{H}_3, \bar{F}_3)$ and non-geometric (Q,P) fluxes is now known.

$$\text{Electric } (+): \quad f_{+}{}^{abc} = \tilde{F}^{abc} \quad , \quad f_{+a}{}^{bc} = Q_a^{bc}$$
$$\text{Magnetic } (-): \quad f_{-}{}^{abc} = \tilde{H}^{abc} \quad , \quad f_{-a}{}^{bc} = P_a^{bc}$$

$\Rightarrow$ Jacobi identities imply $QP = PQ = 0$ as well as absence of 7-branes.

Aldazabal, Cámara and Rosabal [arXiv:0811.2900 [hep-th]]

Dibitetto, Linares and Roest [arXiv:1001.3982 [hep-th]]

- All the supersymmetric Minkowski solutions we found can not be lifted to $\mathcal{N}=4$ solutions since:
  - They satisfy $QP + PQ = 0$ with $QP \neq 0$ and $PQ \neq 0$.
  - They need of 7-branes to cancel the flux-induced tadpoles.

- We also found solutions with $QP = PQ = 0$ and without requiring 7-branes to cancel flux-induced tadpoles:
  - Non-supersymmetric Mkw/dS solutions in the T-duality invariant setup.
  - Supersymmetric AdS$_4$ solutions in both T- and T/S-duality invariant setup.

- What electric-magnetic gaugings these solutions correspond to ?

de Roo, Westra and Panda [arXiv:hep-th/0606282]

Roest [arXiv:0902.0479 [hep-th]]

Conclusions

- A plethora of classical moduli extrema of the effective scalar potential take place in $\mathcal{N} = 1$ type II orientifold models including generalised fluxes.
- Understanding the generalised flux algebras underlying these flux-induced models becomes crucial for removing redundant degrees of freedom. This allows us to be exhaustive when performing a scanning of vacua.
- Algebraic Geometry Techniques can be applied either to find solutions to the generalised flux constraints or to find moduli extrema.
- Non-supersymmetric Mkw/dS extrema describe lines in the parameter space connecting points associated to either a special algebra or a moduli space singularity.

- Mkw/dS stable vacua appear in models based on the $\mathfrak{so}(3,1)+\mathfrak{so}(3,1)$ generalised flux algebra built upon an $\mathfrak{so}(3,1)$ Q-algebra. They require O3-planes and D7-branes to cancel flux-induced tadpoles.

- From a cosmological viewpoint, *slow-roll* inflation in the moduli sector seems not likely to take place in these generalised flux scenarios due to the persistence of an η-problem.

- It would be interesting to explore the origin of these supergravity solutions as electric-magnetic gaugings of half-maximal $\mathcal{N}=4$ supergravity arising from type II orientifold reductions. This would shed light on the lifting of these non-geometric backgrounds to ten dimensions.

...thank you all !!