Some aspects of new maximal supergravity

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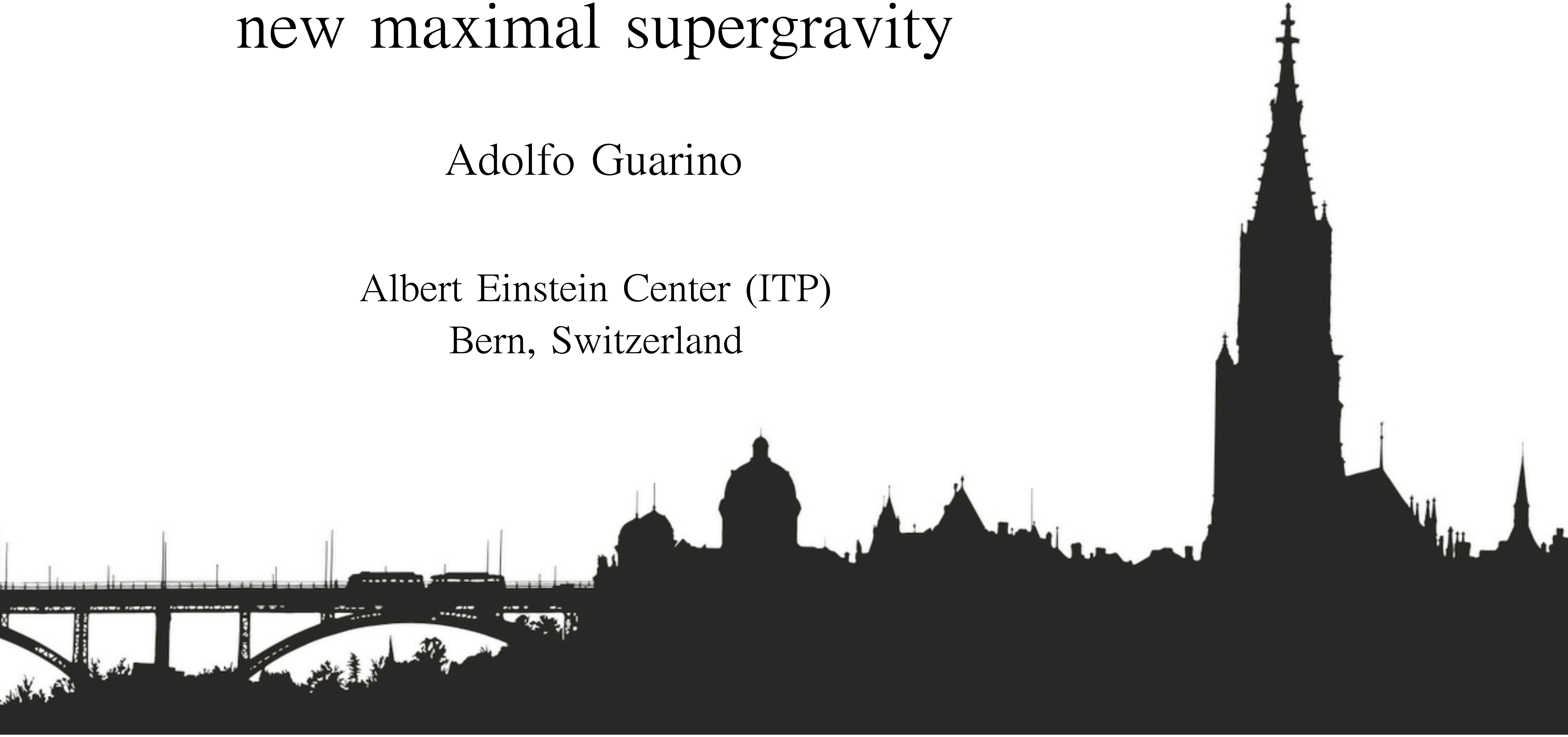
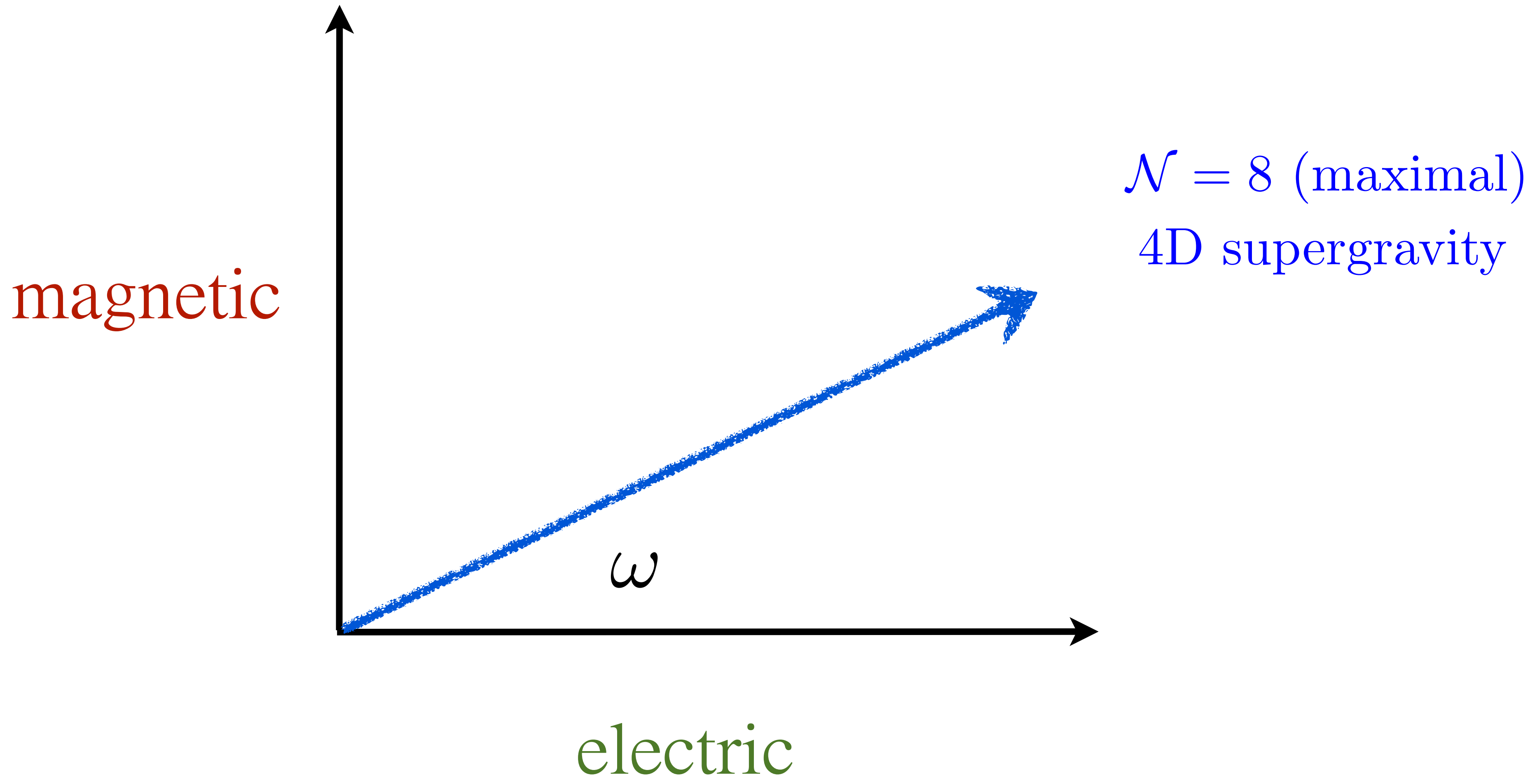


9 October 2013, Uppsala

Based on : arXiv:1209.3003 , arXiv:1302.6057 with A. Borghese & D. Roest

arXiv:13XX.XXXX A.G in progress...

This talk is about the consequences of U(1)-orientating a theory...



R-symmetry : U(1) yes or no?

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- Dimensional reduction of 10D SYM produces N=4 SYM

[Brink, Scherk & Schwarz ‘76]

$$L_{10D} = -\frac{1}{2}F^2 + \frac{i}{2}\,\bar{\lambda}\,\not{D}\lambda$$

$i = 1, .., 4$ ⇒

$$\begin{aligned} L_{4D} \;=\; & -\frac{1}{2}F^2 + i\,\bar{\lambda}_i\,\not{D}\lambda^i + \frac{1}{2}(D\phi_{ij})^2 \\ & -\frac{i}{2}g\,(f\,\phi^{ij}\bar{\lambda}_i\lambda_j + c.c) \\ & -\frac{1}{4}g^2\,(f\,\phi_{ij}\phi_{kl})^2 \end{aligned}$$

reduction = fermi masses + scalar potential

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[Cremmer & Julia '78, '79]

- Analogous results for N=8 gauged SUGRAs from M/Type II reductions with fluxes

$[f \leftrightarrow H_3, F_p, \omega, \ldots]$

> Reality condition on the 70 scalars :

$$\phi^*_{IJKL} = \phi^{IJKL} = \tfrac{1}{24}\, \epsilon^{IJKLMNPQ} \phi_{MNPQ}$$

R-symmetry group is SU(8) and not U(8) !!

$I = 1, \ldots, 8$

An extra U(1) in N=8 gauged supergravity

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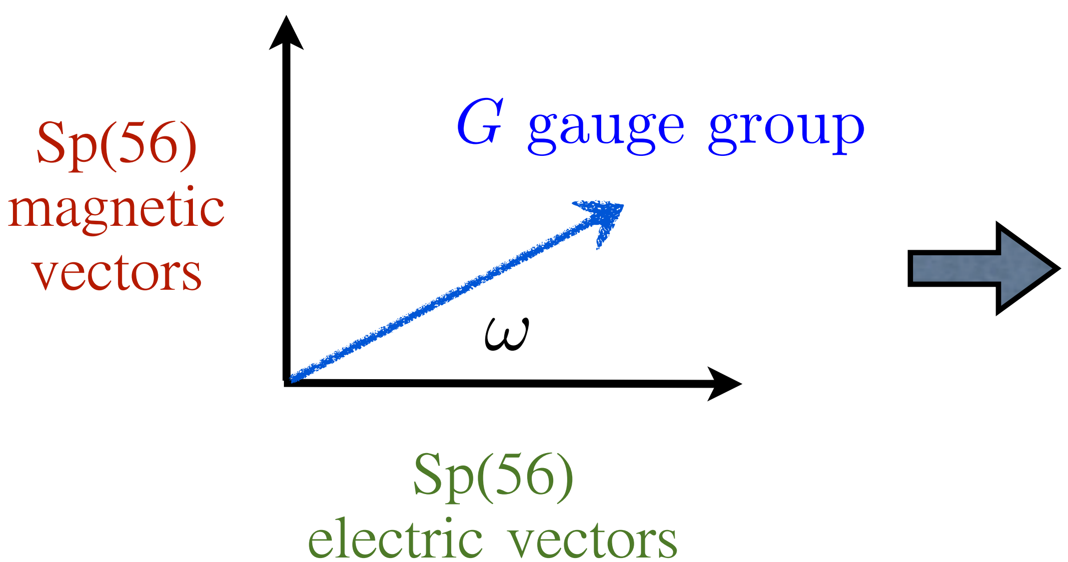
[Dall' Agata, Inverso & Trigiante '12]

- Recently, an extra U(1) rotation outside the R-symmetry group SU(8) has been identified and used to orientate G inside the Sp(56) group of electromagnetic transf.



Covariant derivative :

$$D_\mu \phi = \partial_\mu \phi + \left(\cos\omega \, A_\mu^{(\text{electric})} + \sin\omega \, A_\mu^{(\text{magnetic})} \right) \phi$$

> Therefore : $\omega = 0$ (electric) , $\omega = \frac{\pi}{2}$ (magnetic) and $0 < \omega < \frac{\pi}{2}$ (dyonic)

There is a one-parameter family of new maximal supergravities !!

...so what are the consequences of this U(1) ??

In this talk we will :

1) use the embedding-tensor formalism to compute the ω-dependent scalar potential and analyse its critical points

[de Wit, Samtleben & Trigiante '07]
[Dall' Agata, Inverso & Trigiante '12]
[Borghese, A.G , & Roest '13]

2) compute fermion mass terms to track singular solutions

[Borghese, A.G , & Roest '12, '13]

3) build domain-wall solutions

[A.G , in progress...]

An ω-family of new maximal supergravities :

scalar potential & critical points

Gaugings, embedding tensor & scalar potential

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$$A_\mu^M = \Theta^M{}_\alpha \, t^\alpha \quad \Rightarrow \quad [A^M, A^N] = X^{MN}{}_P \, A^P \qquad \text{with} \qquad X^{MN}{}_P = \Theta^M{}_\alpha \, [t^\alpha]^N{}_P$$

$[\, M = 1, \ldots, 56 \,]$

$[\, \alpha = 1, \ldots, 133 \,]$

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Scalar potential : Straightforward once the embbeding tensor $\Theta^M{}_\alpha(\omega)$ is known

$$V = \frac{1}{672} X_{MNP} \, X_{QRS} \left(M^{MQ} \, M^{NR} \, M^{PS} + 7 \, M^{MQ} \, \Omega^{NR} \, \Omega^{PS} \right)$$

where $M(\phi) \in \frac{E_7}{SU(8)}$ contains the 70 scalar fields of the theory

Truncating the scalar sector: 70 scalars are intractable

- Truncate most of the 70 scalars and look for critical points of $V(\phi)$ with large residual symmetry groups $G_0 \subset G$

Some relevant truncations :

- the $G_0 = G_2$ invariant sector ⇒ N = 1 , 2 real scalars

- the $G_0 = D_4 \times SO(4)$ invariant sector ⇒ N = 0 , 2 real scalar

Simple and interesting!!

- the $G_0 = SU(3)$ invariant sector ⇒ N = 2 , 6 real scalar

(last part of the talk)

Example 1 : G_2 invariant sector of G = SO(8)

[Dall' Agata, Inverso & Trigiante '12]
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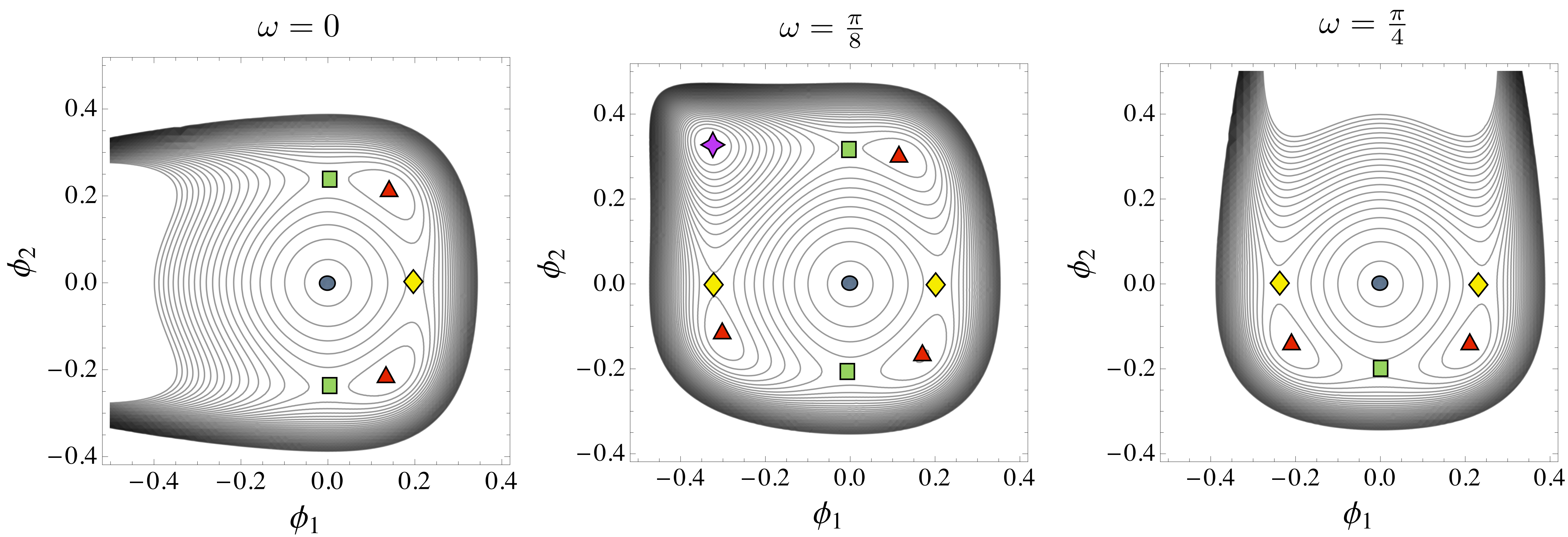

critical point	residual sym G_0	SUSY	Stability
●	$SO(8)$	$\mathcal{N} = 8$	✓
■	$SO(7)_-$	$\mathcal{N} = 0$	×
◆	$SO(7)_+$	$\mathcal{N} = 0$	×
▲	G_2	$\mathcal{N} = 1$	✓
✦	G_2	$\mathcal{N} = 0$	✓

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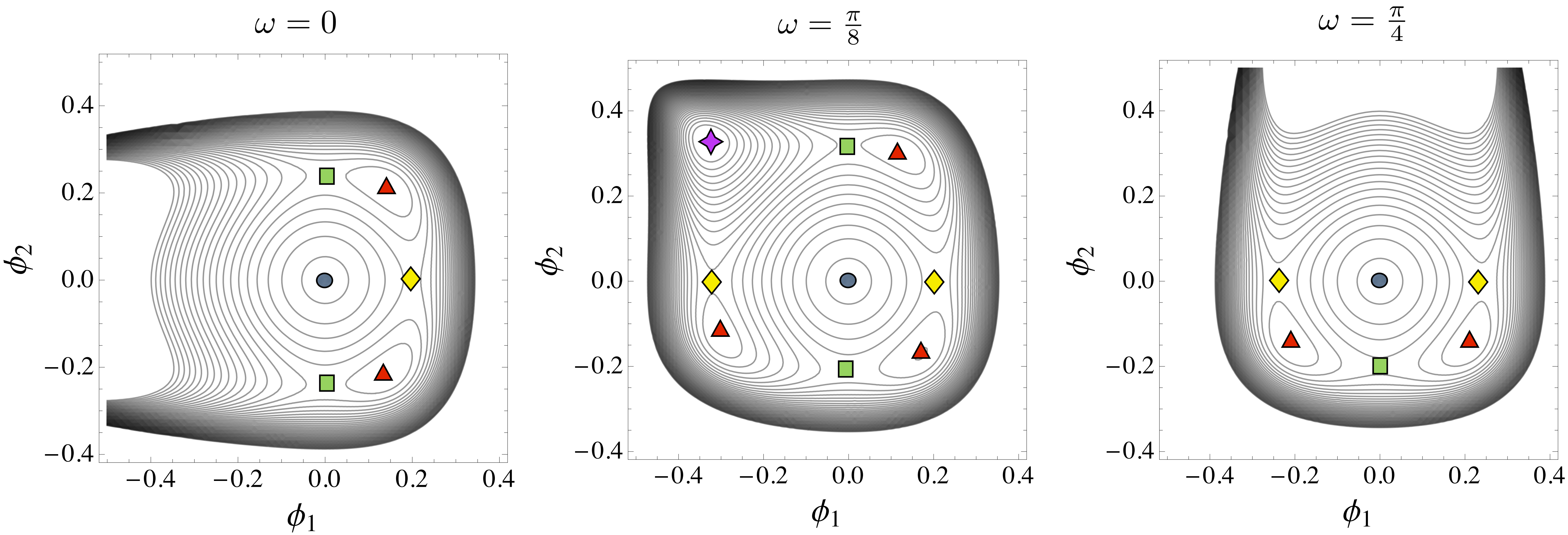

> Mass spectra insensitive to ω

> $\frac{\pi}{4}$-periodicity with transmutation of $SO(7)_\pm$

> Runaway of points at $\omega = n\,\frac{\pi}{4}$

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[Borghese, A.G & Roest '13]

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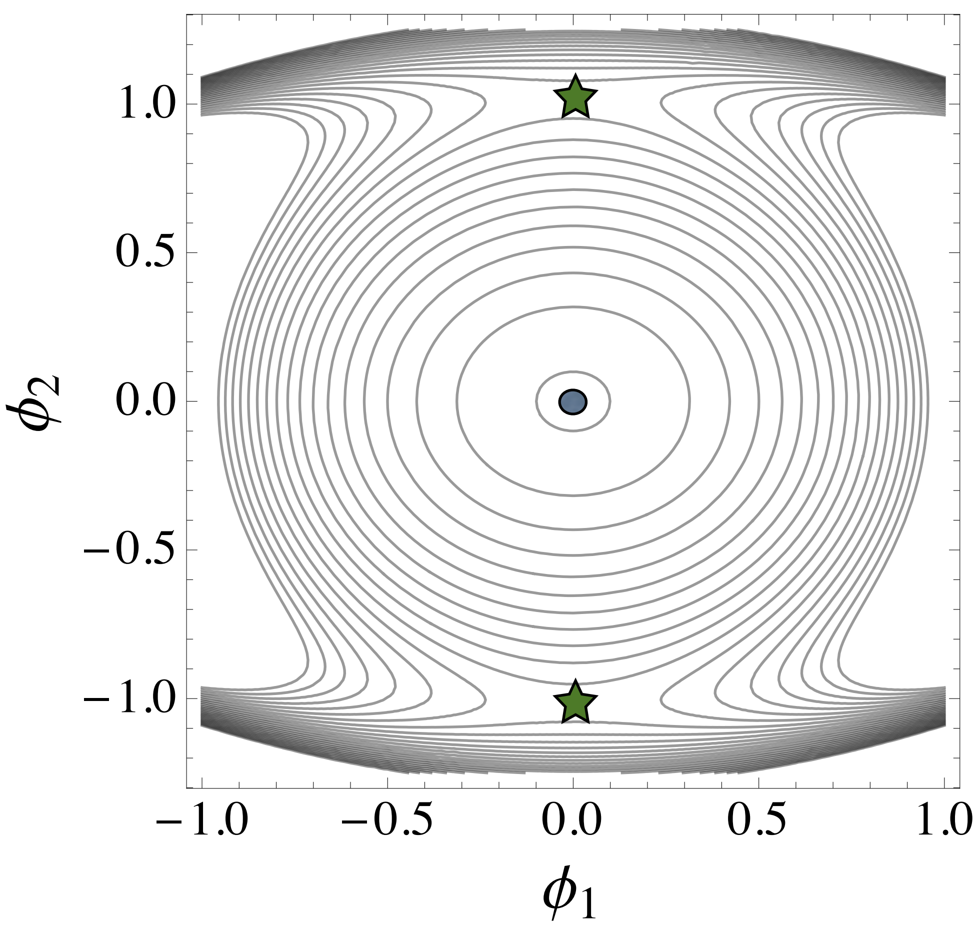
    i) the vectorial embedding : $8_v = (1,1) + (3,1) + (1,1) + (1,3)$

$\omega = 0$



[Warner '84]

[Fischbacher, Pilch & Warner '10]

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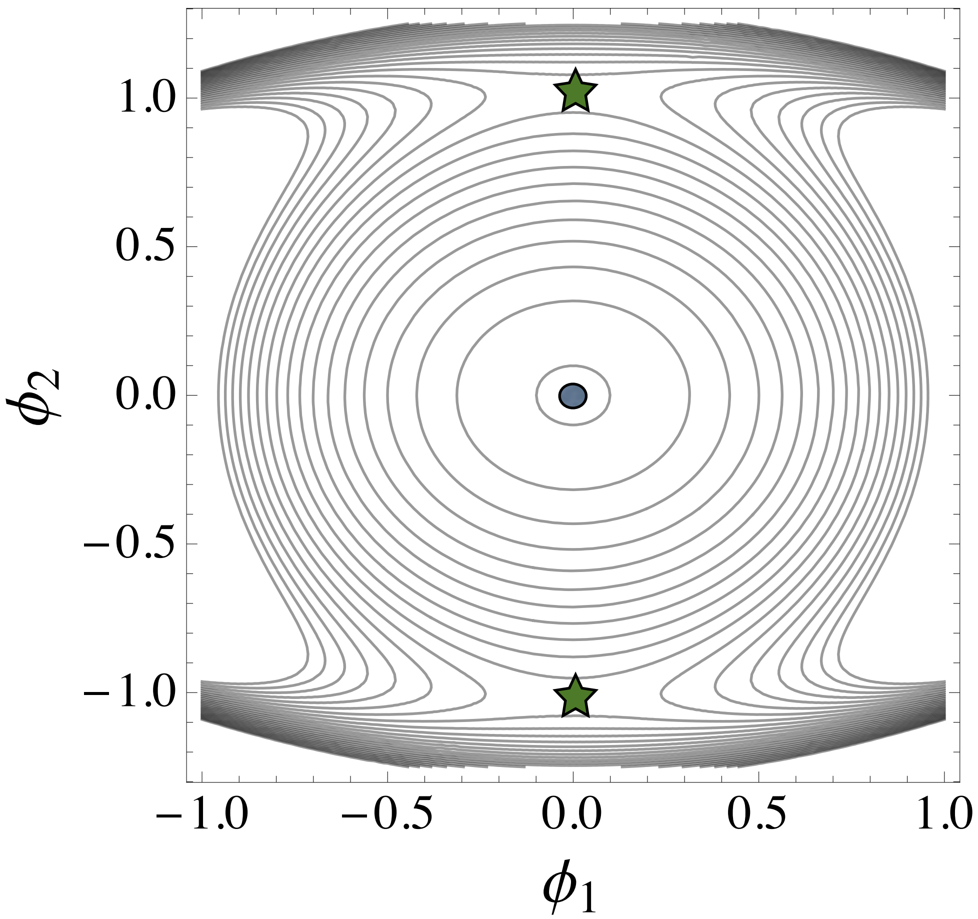
$\omega \in [0, \frac{\pi}{4}]$



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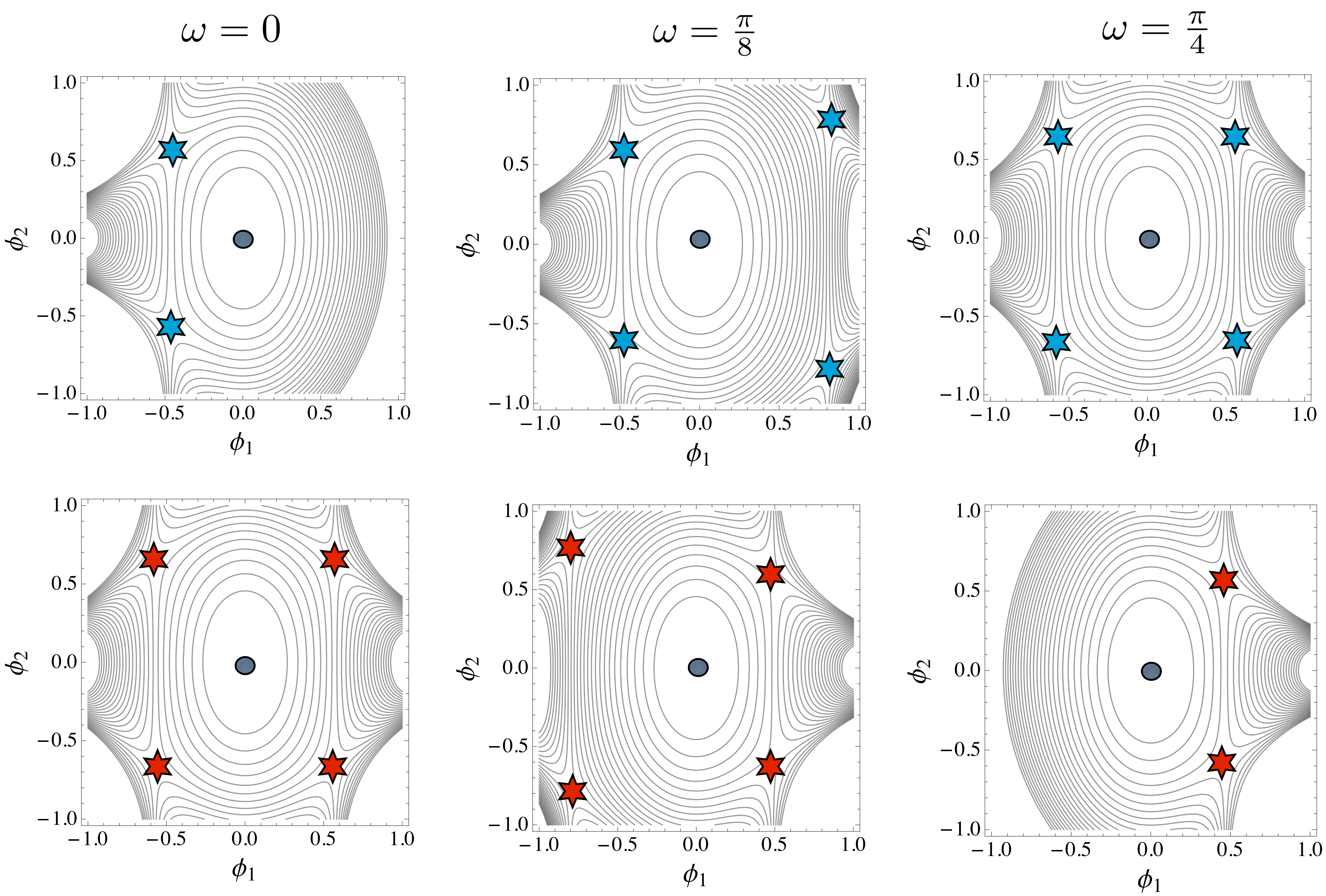
> NO ω-dependence at all !!

ii) spinorial (upper) & conjugate (lower) embeddings : $8_{s/c} = (1,1) + (3,1) + (1,1) + (1,3)$



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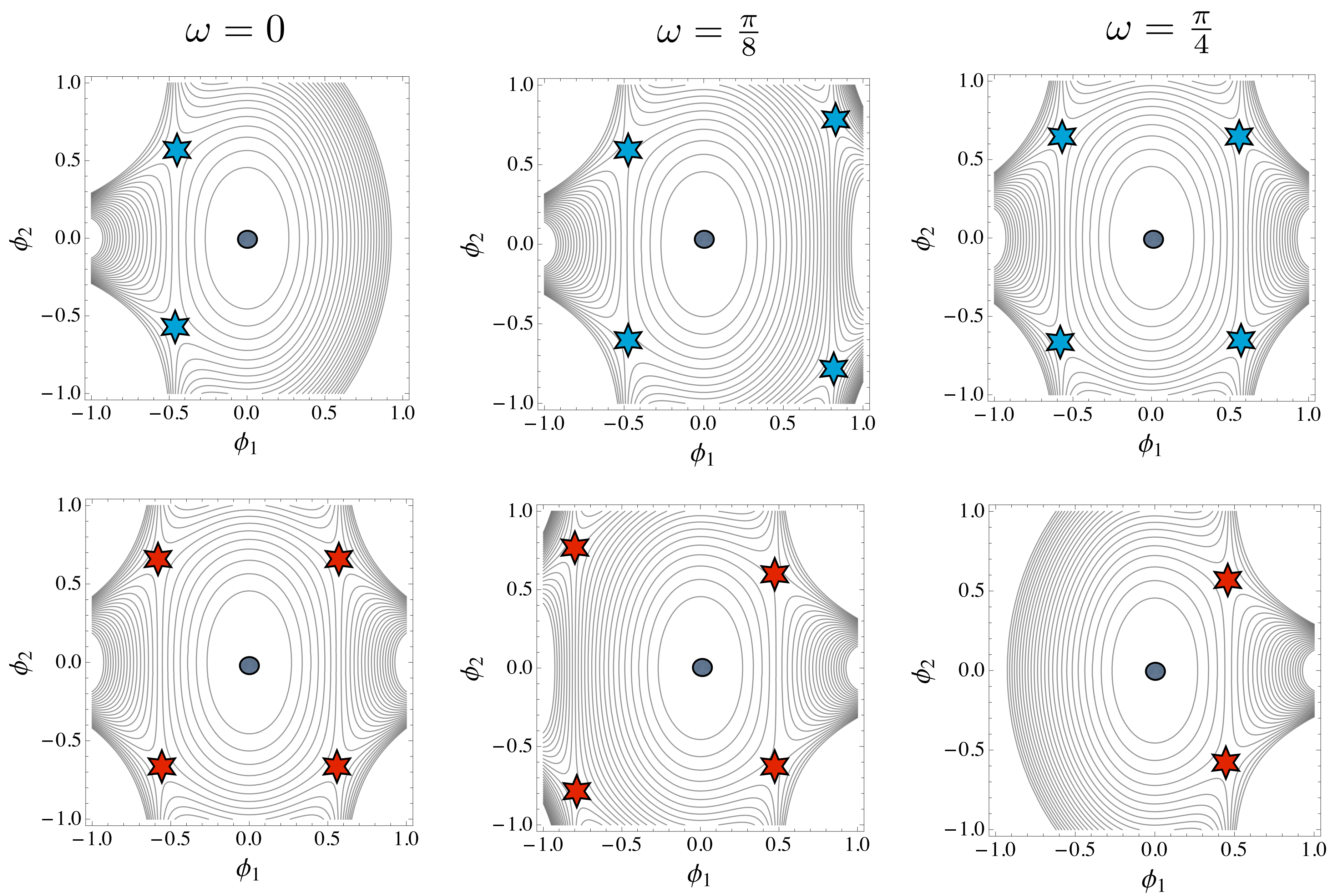

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✶ (red star)	$SO(4)_c$	$\mathcal{N}=0$	×

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Hope : "Sit on top" of a critical point and travel with it to see what happens...

Answer : It wants to migrate to a different theory (gauging) ⇒ the fermion masses can be used to monitoring its story !!

[Borghese, A.G & Roest '12, '13]

Fermion mass terms as solution trackers

Tracking solutions using fermion masses

Going to the origin : If a critical point is found at $\phi = \phi_0$ with a residual symmetry G_0 , it can always be brought to $\phi_0 = 0$ via an E_7-transformation

[Dibitetto, A.G & Roest'11]
[Dall'Agata & Inverso '11]
[Kodama & Nozawa '12]

Applicability : After going to the origin, the quantities in the theory, e.g. fermi masses, will adopt a form compatible with G_0

[Borghese, A.G & Roest '12, '13]

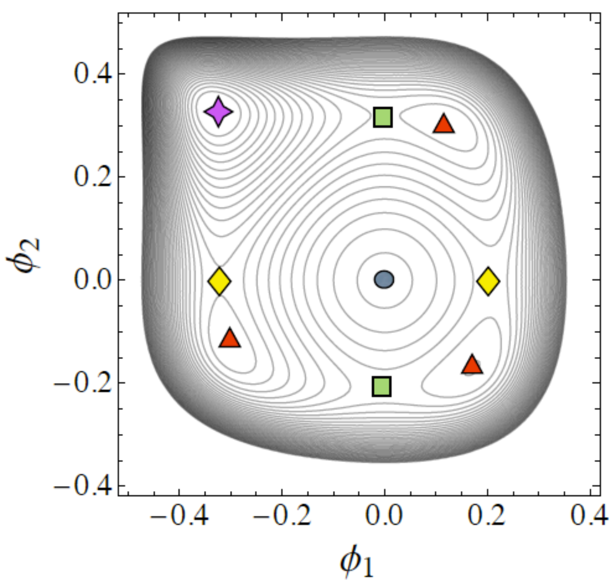
$$\mathcal{L}_{\text{fermi}} = \frac{\sqrt{2}}{2} \boxed{\mathcal{A}_{\mathcal{I}\mathcal{J}} \, \overline{\psi}_{\mu}{}^{\mathcal{I}} \, \gamma^{\mu\nu} \, \psi_{\nu}{}^{\mathcal{J}}} + \frac{1}{6} \boxed{\mathcal{A}_{\mathcal{I}}{}^{\mathcal{J}\mathcal{K}\mathcal{L}} \, \overline{\psi}_{\mu}{}^{\mathcal{I}} \, \gamma^{\mu} \, \chi_{\mathcal{J}\mathcal{K}\mathcal{L}}} + \boxed{\mathcal{A}^{\mathcal{I}\mathcal{J}\mathcal{K},\mathcal{L}\mathcal{M}\mathcal{N}} \, \overline{\chi}_{\mathcal{I}\mathcal{J}\mathcal{K}} \, \chi_{\mathcal{L}\mathcal{M}\mathcal{N}}}$$

gravitino-gravitino mass

gravitino-dilatino mass

dilatino-dilatino mass (dependent)

Example 1 : Critical point with $G_0 = G_2$ (✦)



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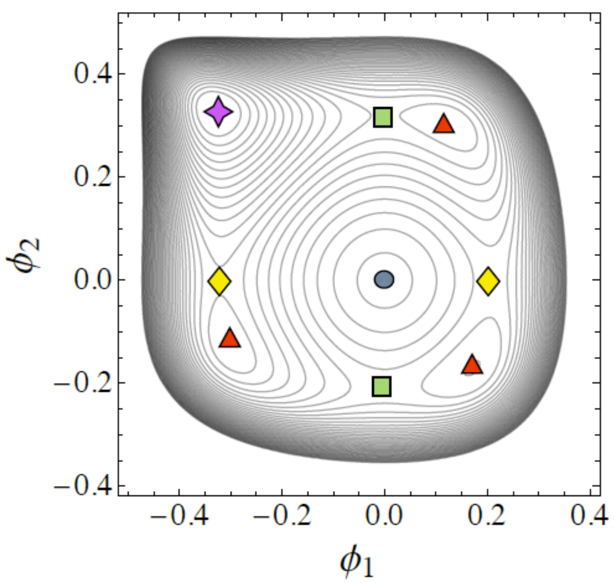

- Pattern of fermi masses :

$$I = 1 \oplus m$$

i) gravitino-gravitino mass $\mathcal{A}^{IJ}(\phi_0)$ ⇒ $\mathcal{A}^{11} = \alpha_1 \;,\; \mathcal{A}^{mn} = \alpha_2\, \delta^{mn}$

ii) gravitino-dilatino mass $\mathcal{A}_I{}^{JKL}(\phi_0)$ ⇒ $\mathcal{A}_1{}^{mnp} = \beta_1\, \kappa^{mnp} \;,\; \mathcal{A}_m{}^{1np} = \beta_2\, \kappa_m{}^{np}$

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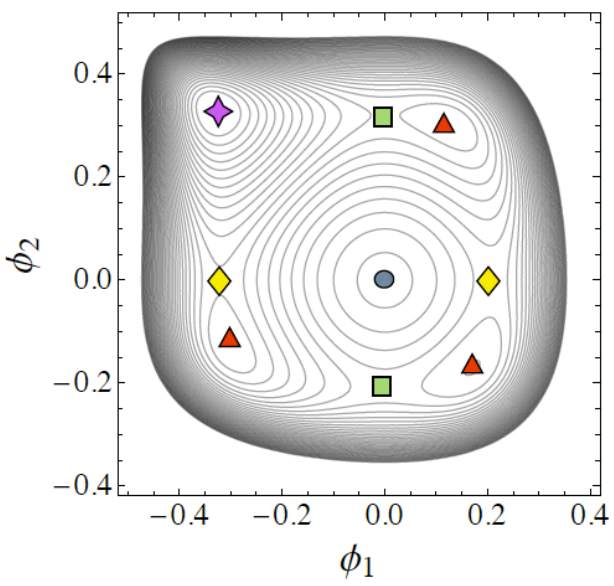

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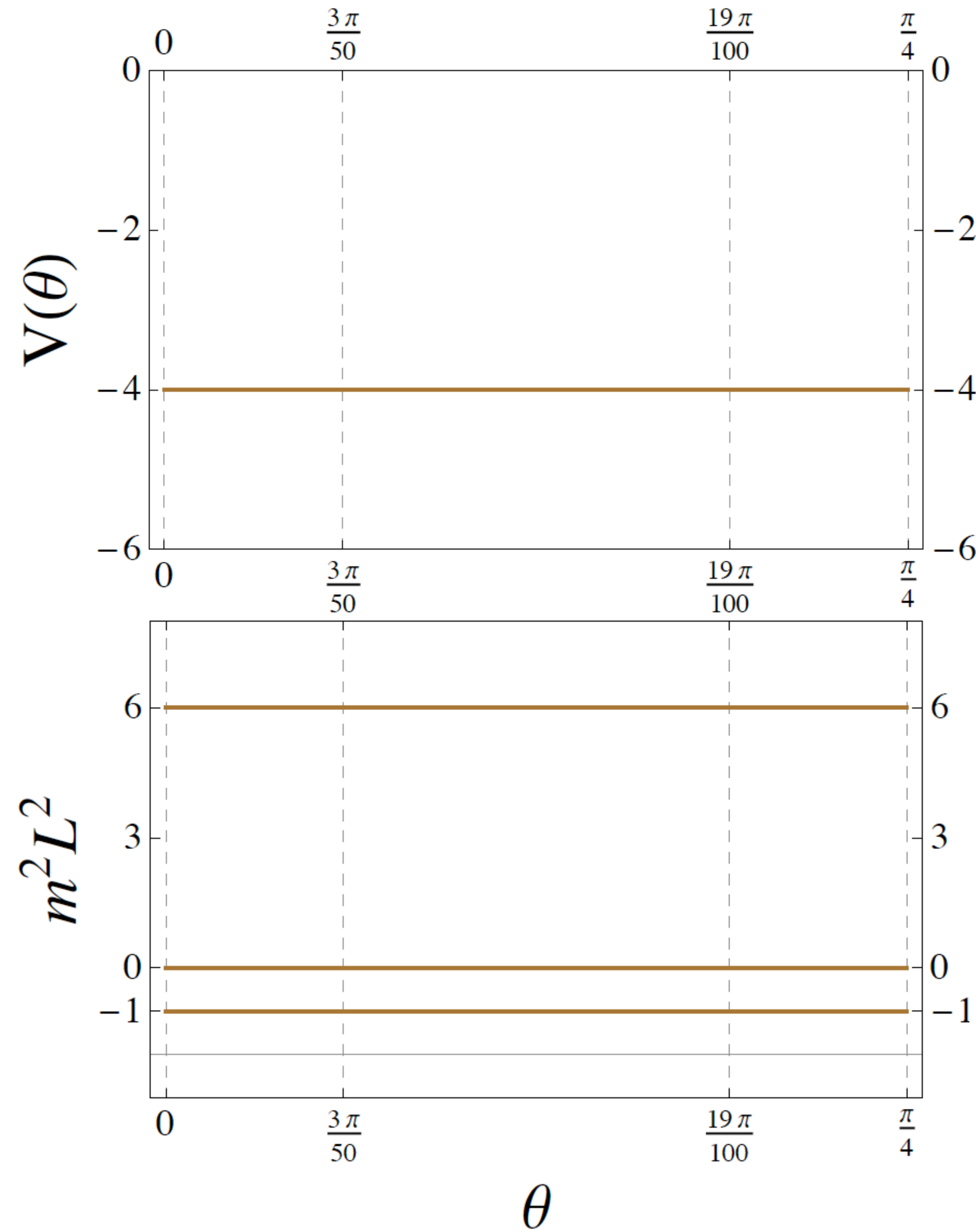
Solving QC & EOM : One-parameter family of theories compatible with $G_0 = G_2$

$$\alpha_1(\theta) \, , \, \alpha_2(\theta) \, , \, \beta_1(\theta) \, , \, \beta_2(\theta)$$

A boring excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = G_2$ can be tracked



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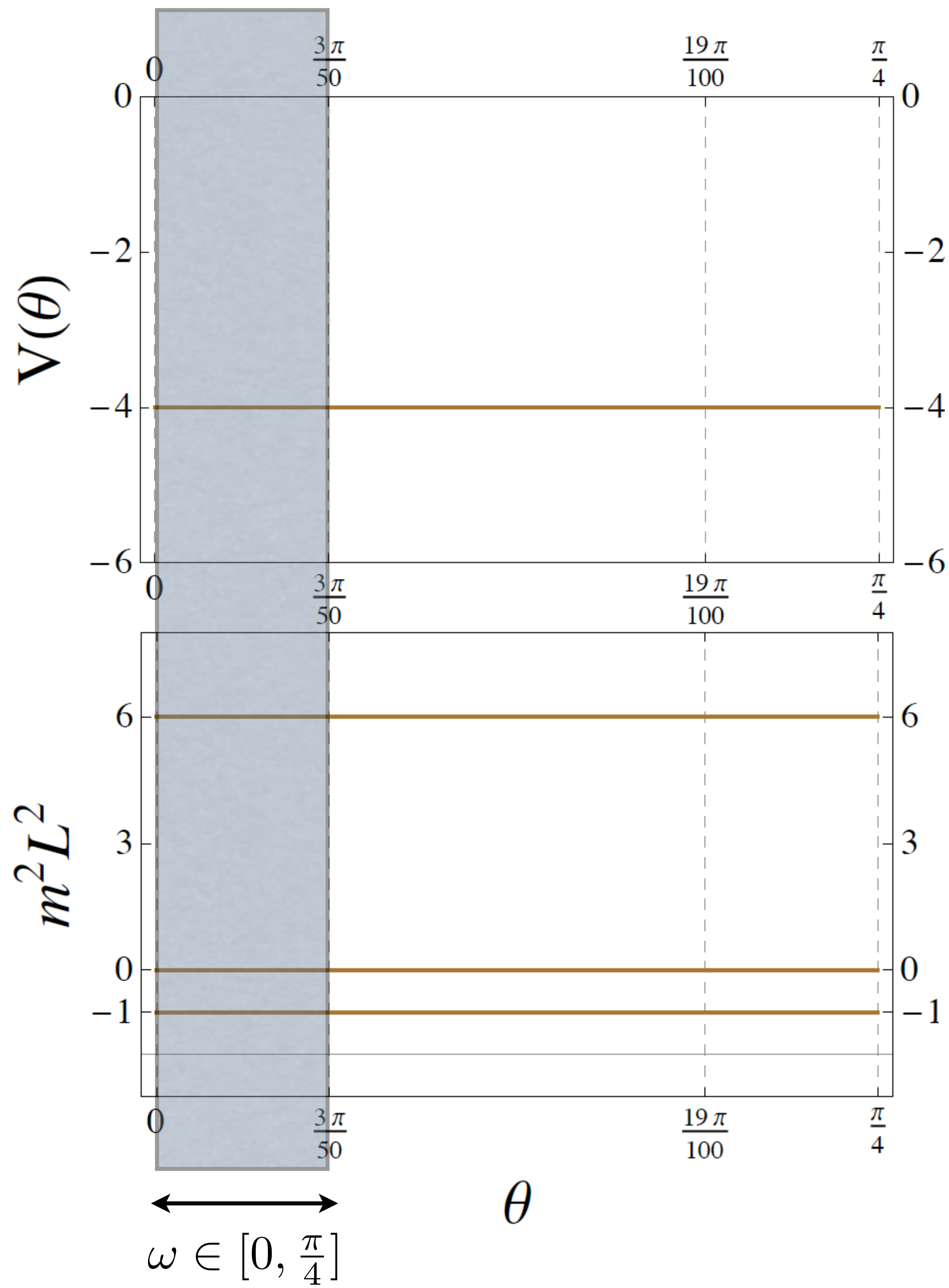
[stable AdS_4 solutions]



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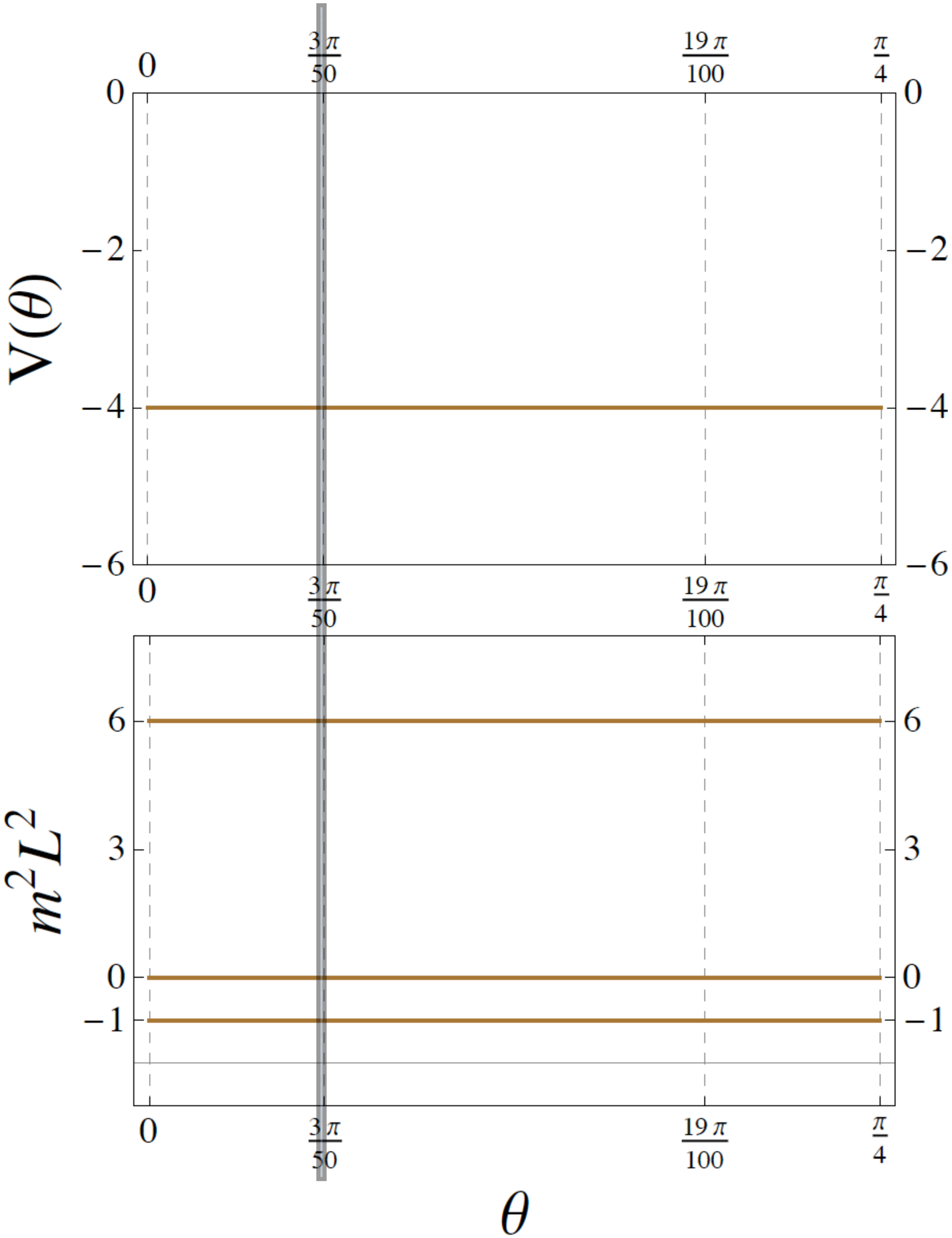

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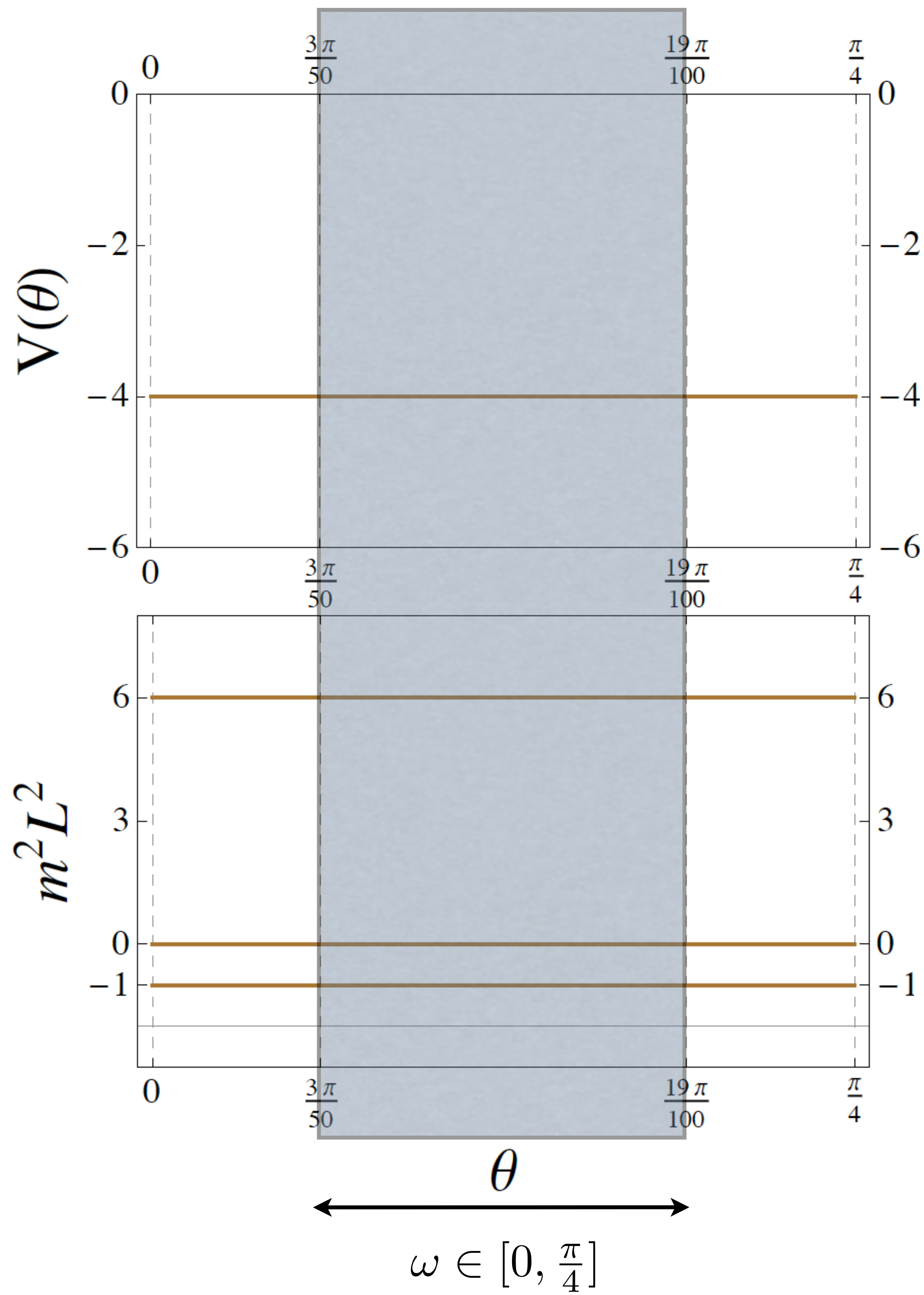

$$iii) \quad \frac{3\pi}{50} < \theta < \frac{19\pi}{100} \quad \rightarrow \quad G = SO(7,1)$$

[stable AdS$_4$ solutions]

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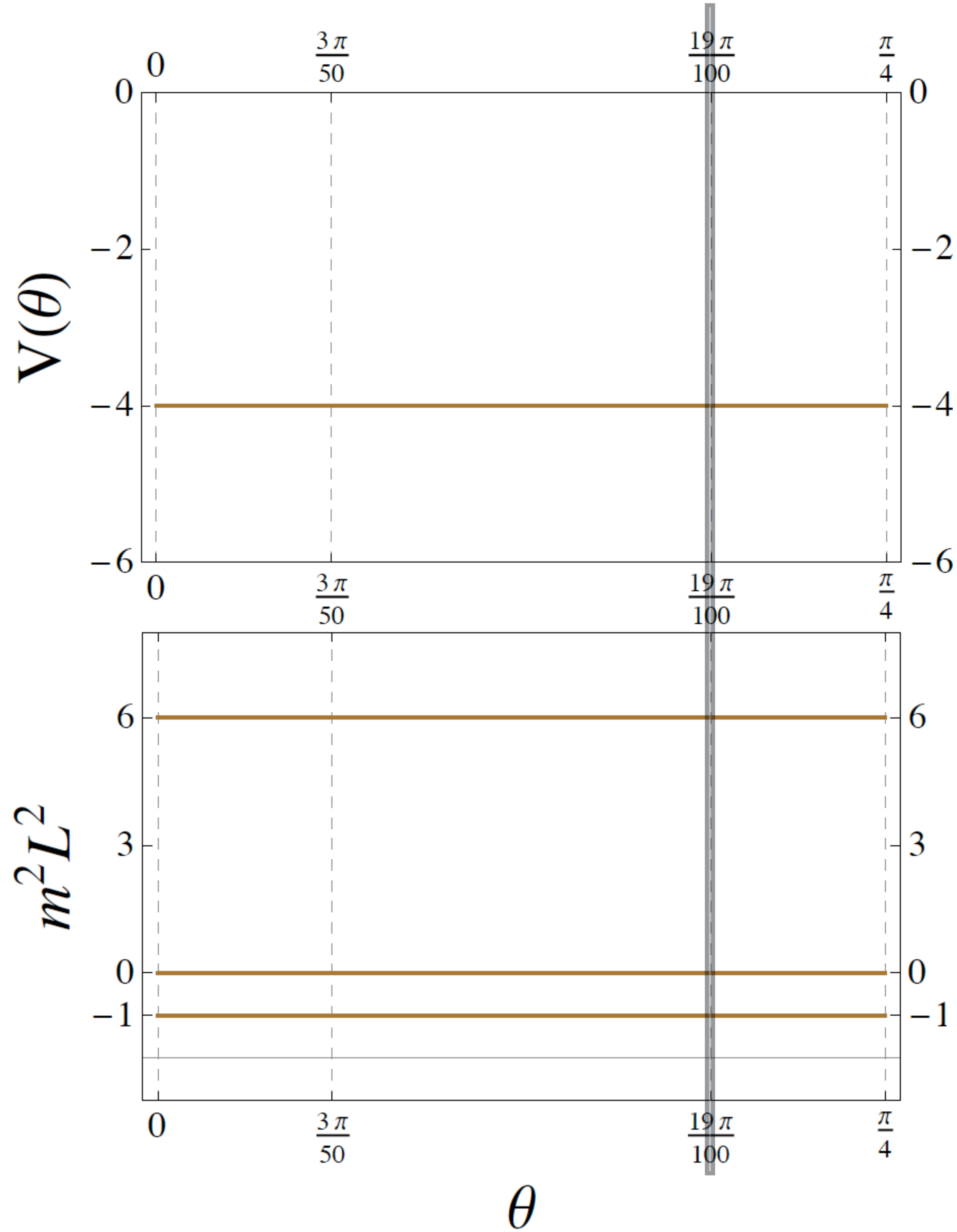

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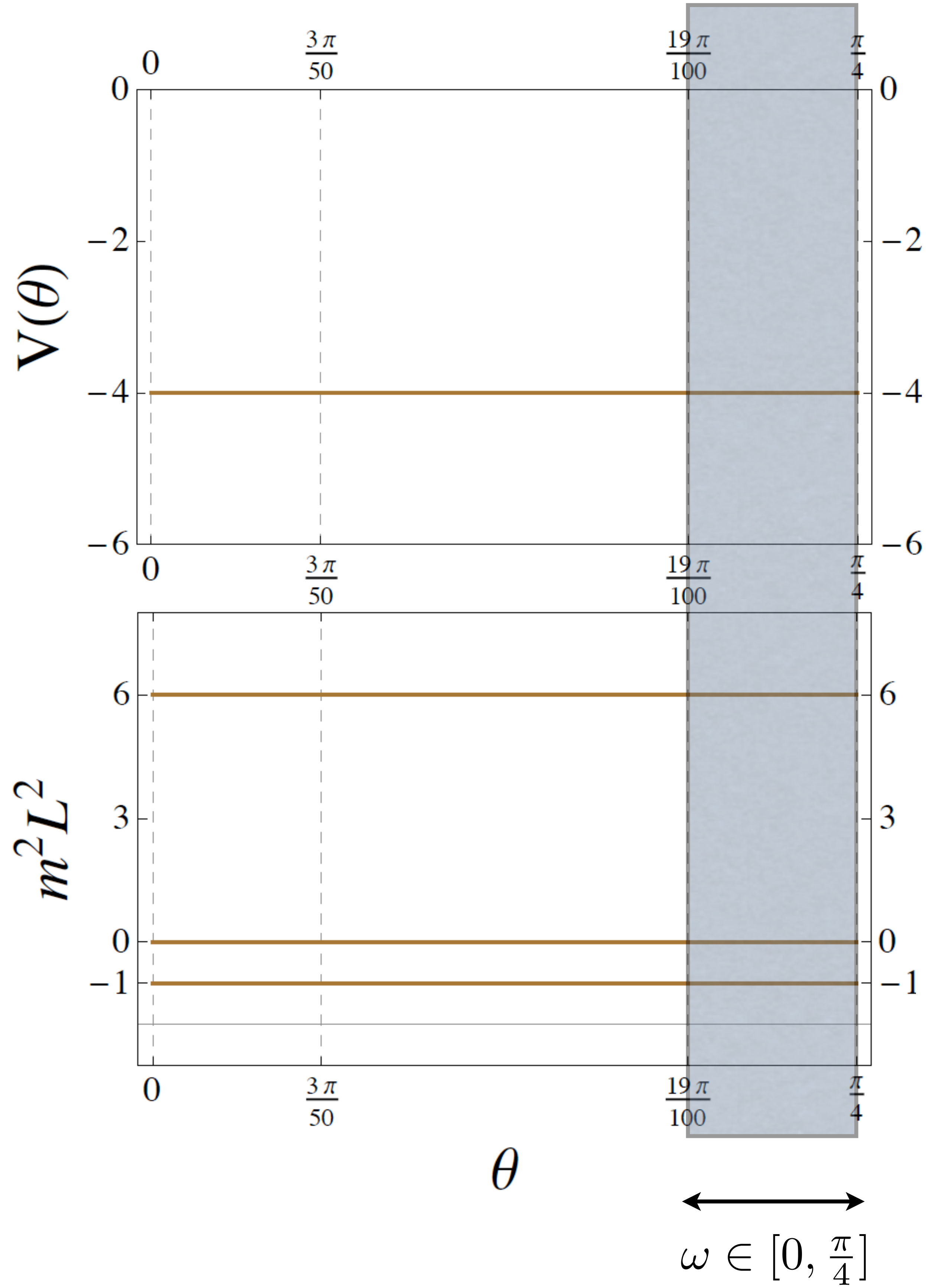

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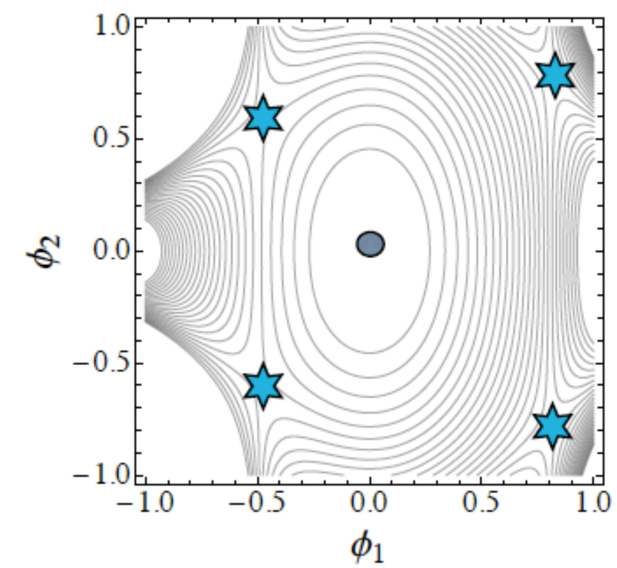
Example 2 : Critical point with $G_0 = D_4 \times SO(4)_s$ (★)



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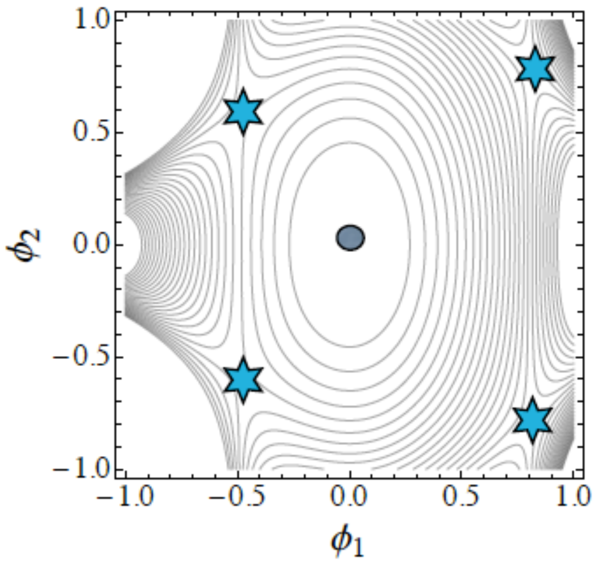

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$$\mathcal{A}_{\hat{i}}{}^{\hat{j}\hat{k}\hat{l}} = -\beta\ \epsilon_{\hat{i}}{}^{\hat{j}\hat{k}\hat{l}} \quad , \quad \mathcal{A}_{\hat{i}}{}^{jk\hat{l}} = -\delta\ \epsilon_{\hat{i}}{}^{jk\hat{l}} + \gamma\ \delta_{\hat{i}}^{[j}\delta^{k]\hat{l}}$$

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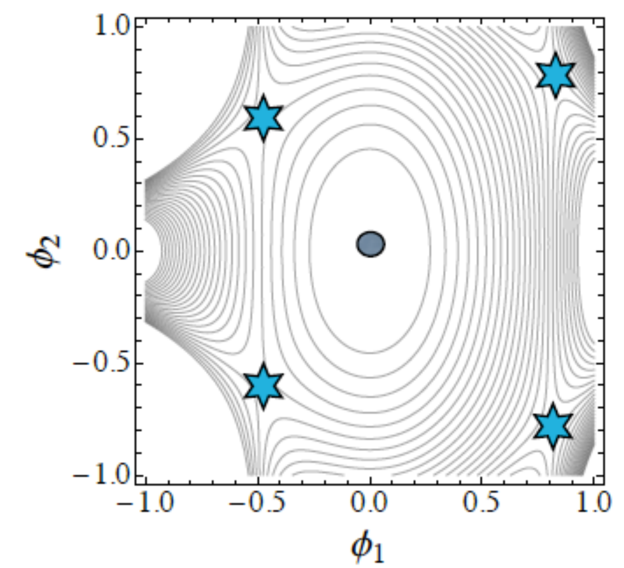

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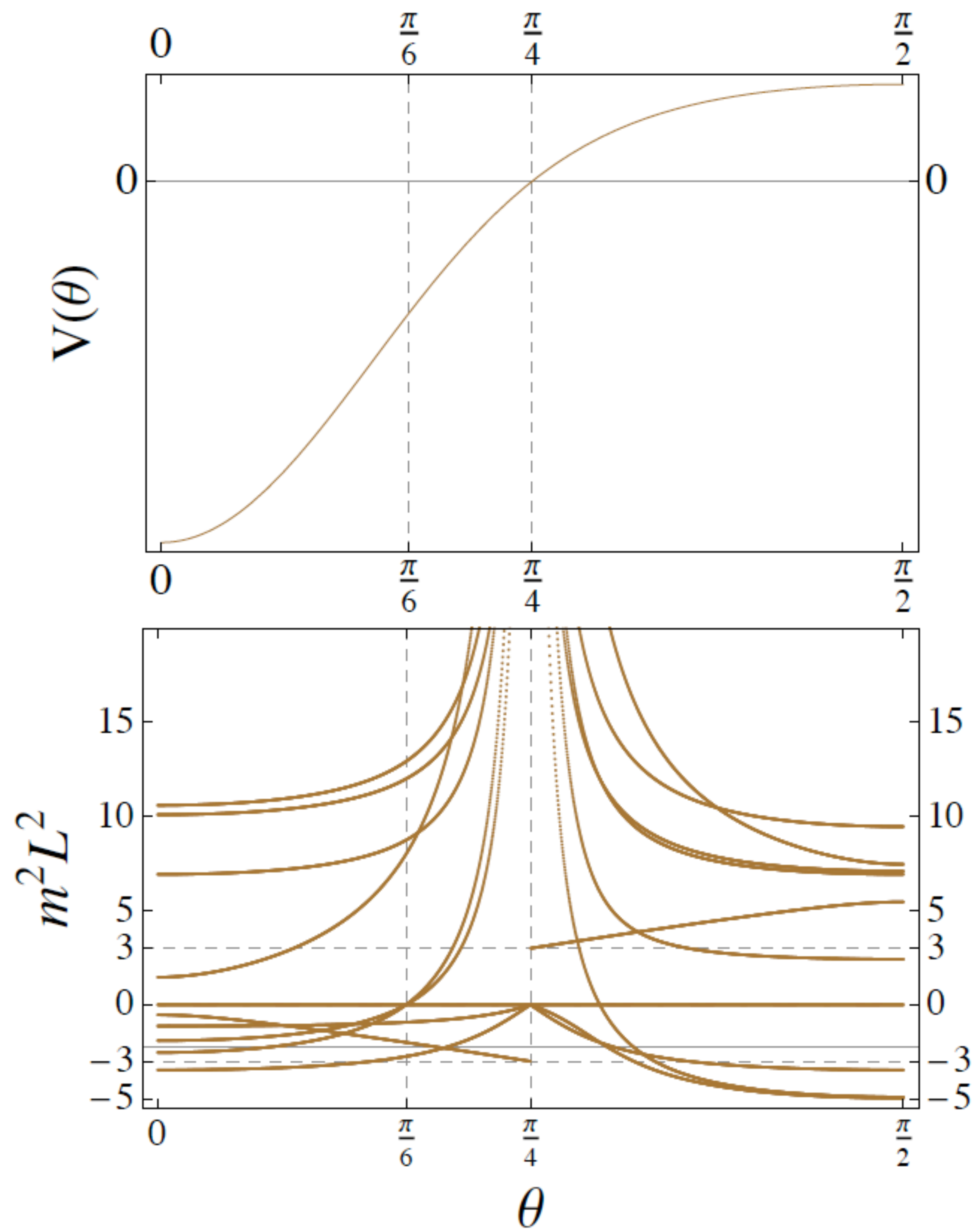

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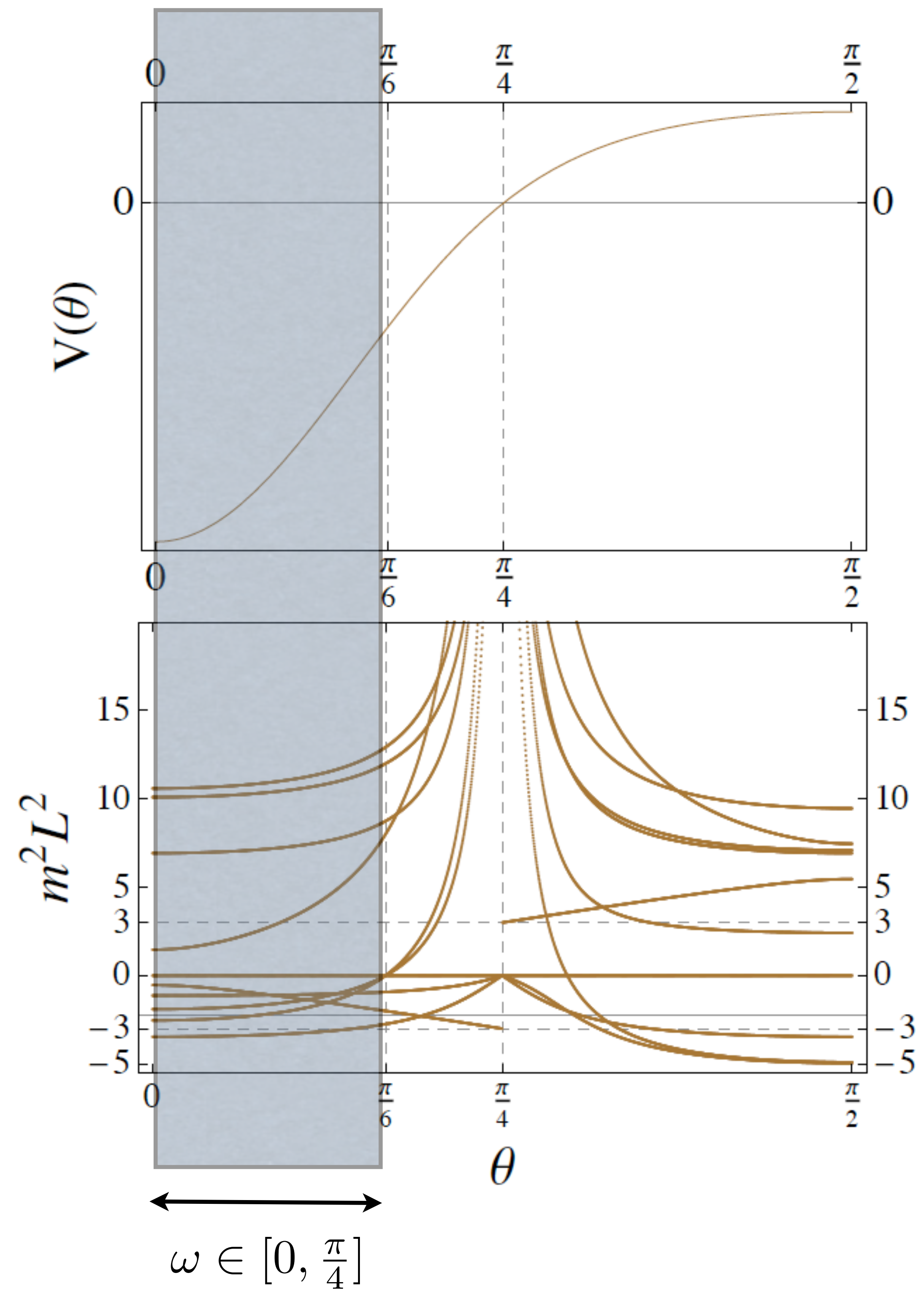
  $i) \quad 0 \leq \theta < \frac{\pi}{6} \quad \rightarrow \quad G = SO(8)$

  [unstable AdS_4 solutions]

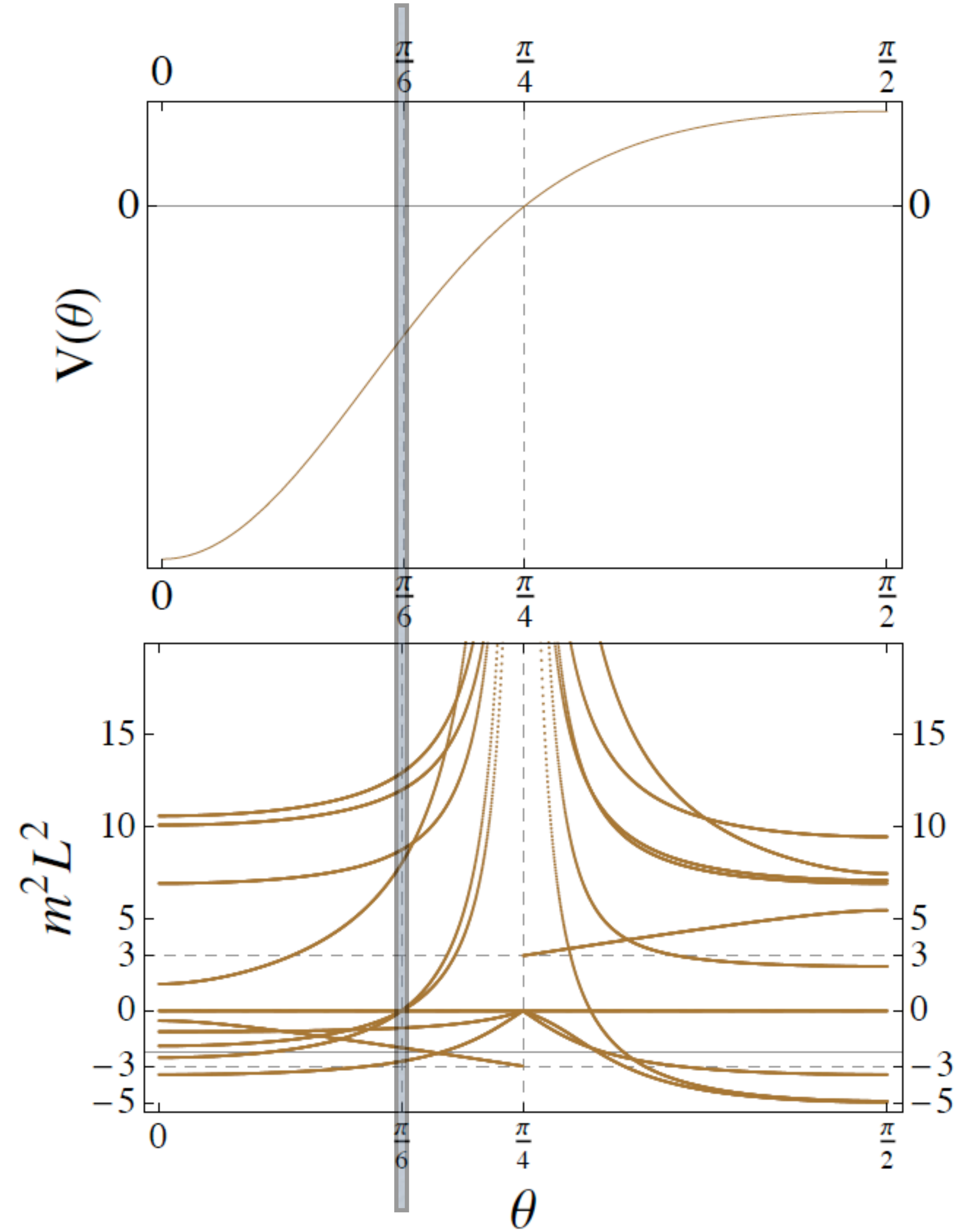


A funny excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = \mathrm{SO}(4)_s$ can be tracked

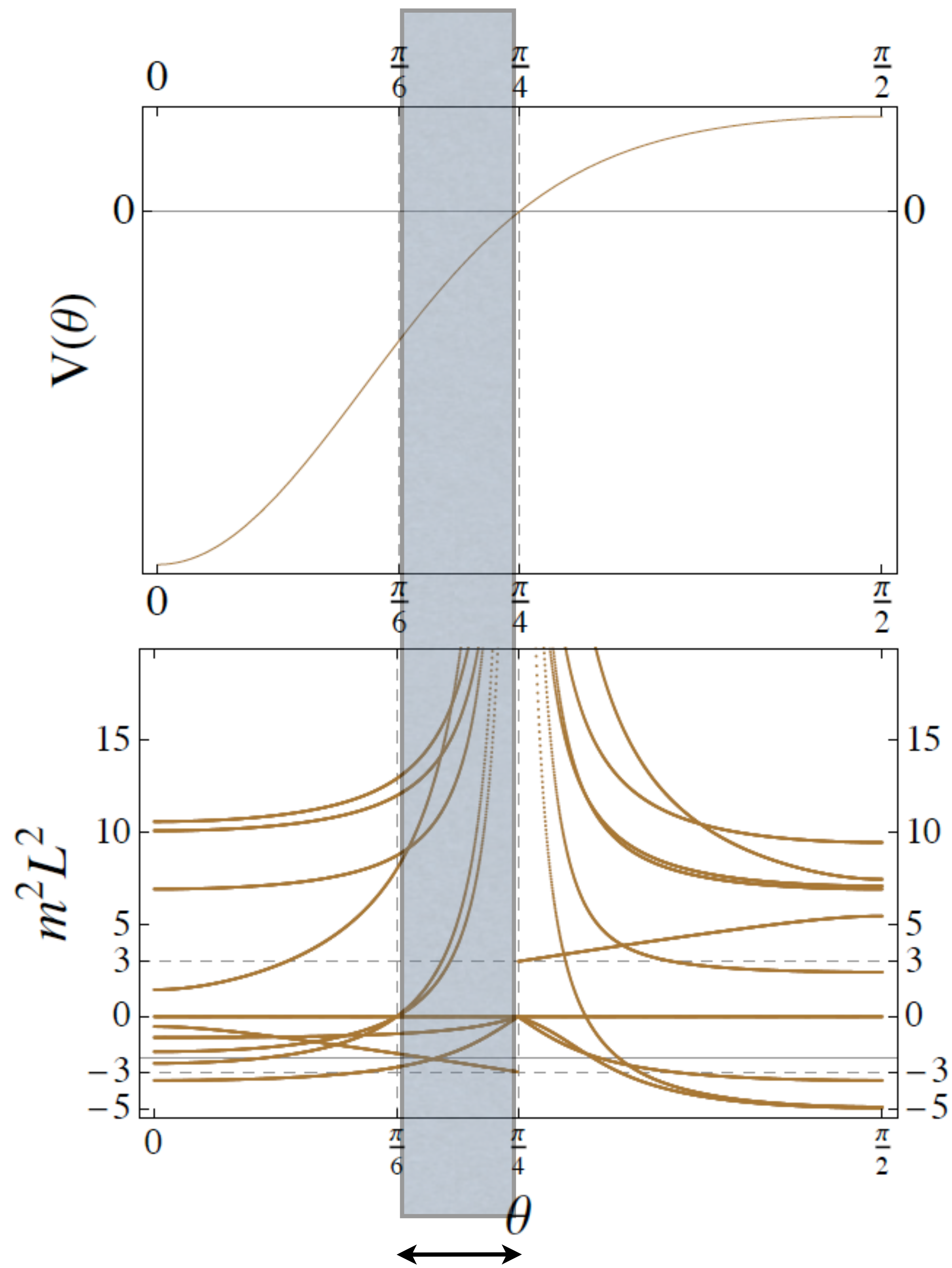
ii) $\theta = \frac{\pi}{6} \;\; \rightarrow \;\; G = SO(2) \times SO(6) \ltimes T^{12}$

[unstable AdS_4 solution]



A funny excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = SO(4)_s$ can be tracked



$iii)\quad \frac{\pi}{6} < \theta < \frac{\pi}{4} \quad \rightarrow \quad G = SO(6,2)$

[unstable AdS_4 solutions]

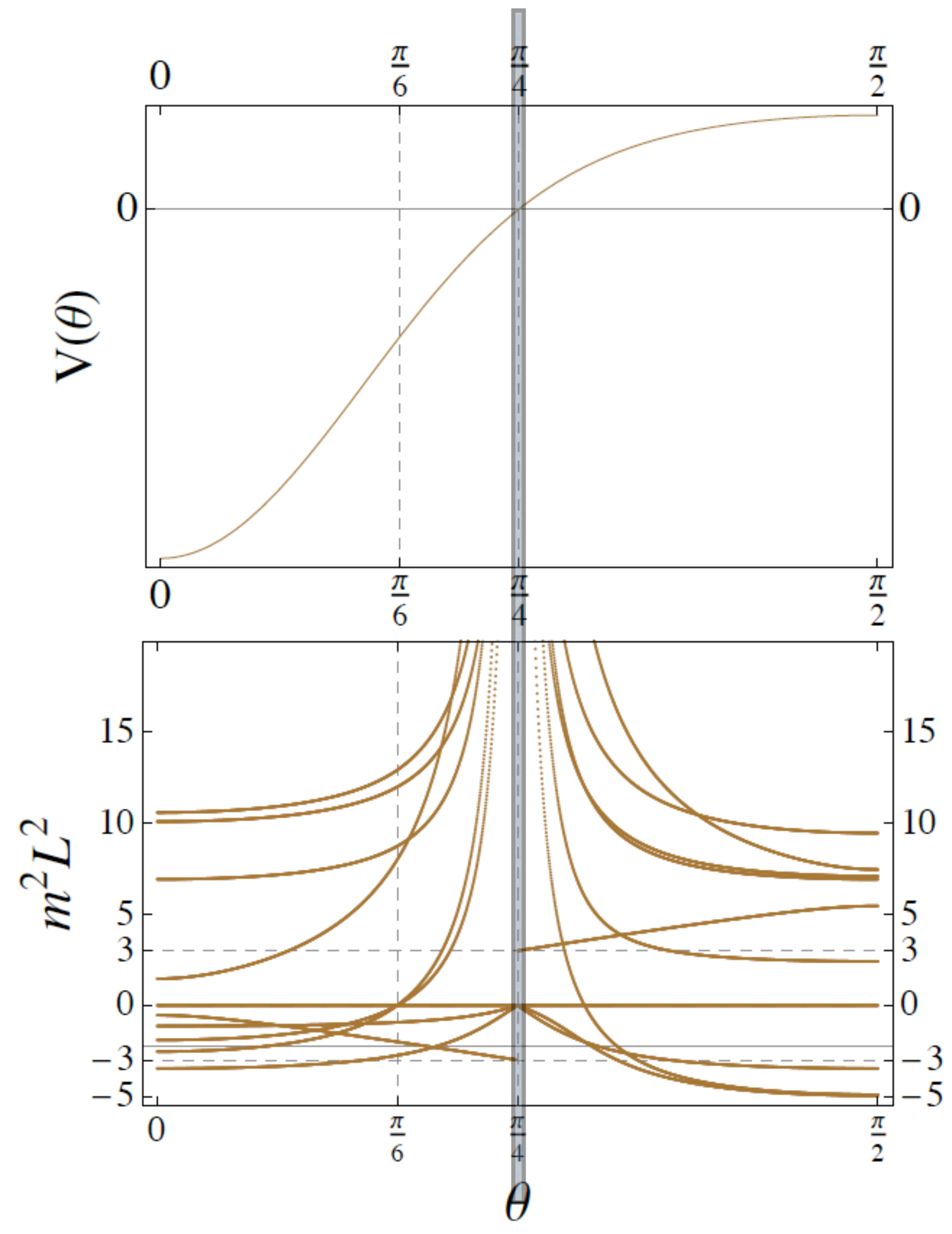
$\omega \in [0, \frac{\pi}{4}]$

A funny excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = \mathrm{SO}(4)_s$ can be tracked

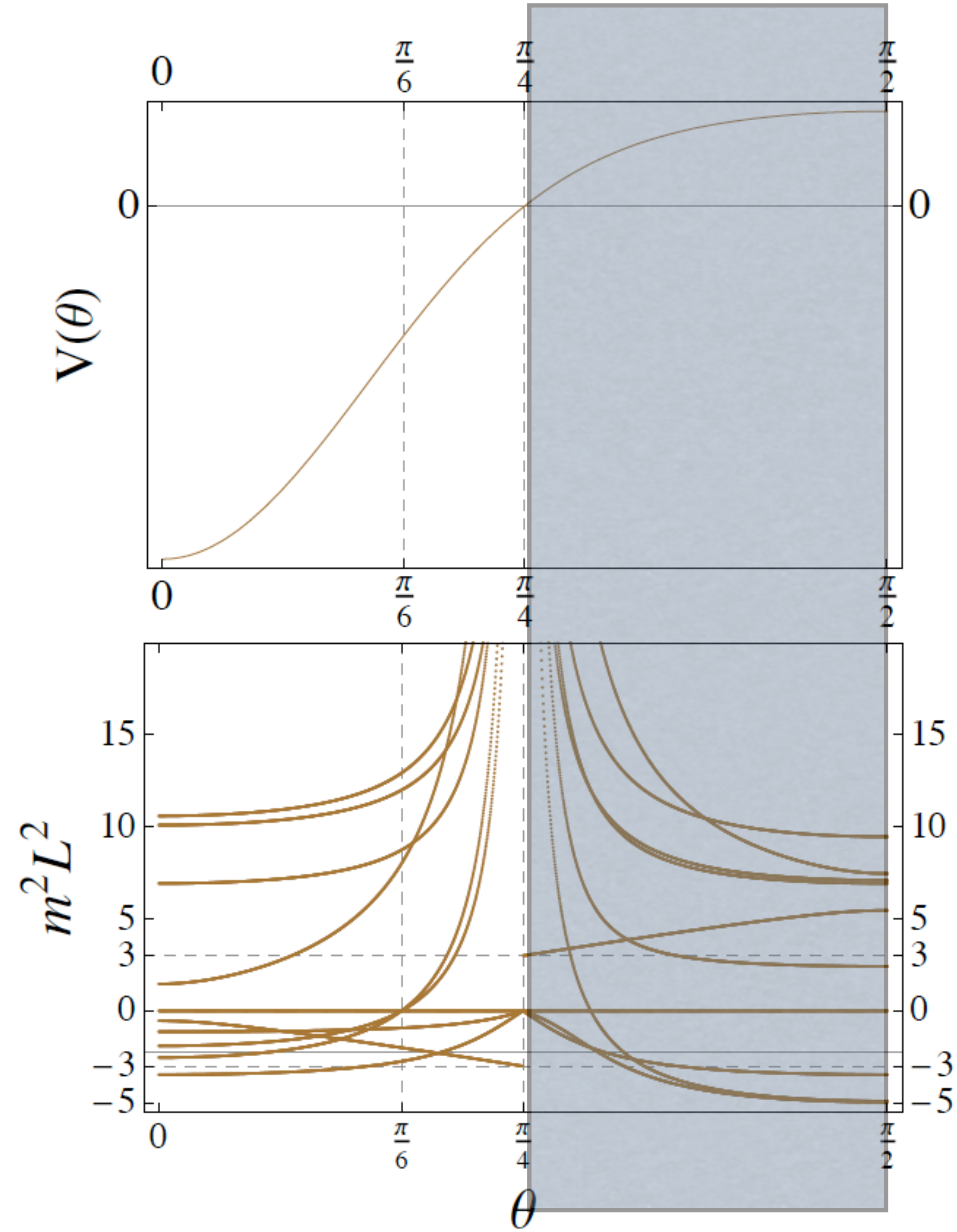
$iv) \quad \theta = \frac{\pi}{4} \quad \rightarrow \quad G = SO(3,1)^2 \ltimes T^{16}$

[Mkw solution with flat directions]



A funny excursion through theories (gaugings)

- The whole story of a solution preserving $G_0 = SO(4)_s$ can be tracked



$\omega \in [0, \frac{\pi}{4}]$

$v) \quad \frac{\pi}{4} < \theta \leq \frac{\pi}{2} \quad \rightarrow \quad G = SO(4,4)$

[dS_4 solutions with tachyon dilution]

[Dall'Agata & Inverso '12]

The SU(3) invariant sector & Domain-walls

The SU(3) invariant sector

- R-symmetry branching : $\mathbf{8} \rightarrow \mathbf{1} + \mathbf{1} + \mathbf{3} + \bar{\mathbf{3}}$ ⇒ $\mathcal{N} = 2$ SUSY

gravitini : $\psi_\mu^I \rightarrow \psi_\mu^1 \;,\; \psi_\mu^{\hat{1}} \;,\; \psi_\mu^a \;,\; \psi_\mu^{\hat{a}}$

[Warner '83]

- Scalars fields : $\mathbf{70} \rightarrow \mathbf{1}\ (\times 6) + \text{non-singlets}$ ⇒ 6 real scalars

scalars :

$$
\begin{aligned}
\varpi &= \lambda\, e^{i\alpha} \,, \\
\varpi_1 &= \lambda' \left(e^{i\phi} \cos\theta \cos\psi - e^{-i\phi} \sin\theta \sin\psi \right) \,, \\
\varpi_2 &= \lambda' \left(e^{i\phi} \cos\theta \sin\psi + e^{-i\phi} \sin\theta \cos\psi \right) \,,
\end{aligned}
$$

$\lambda \,,\, \alpha \,,\, \lambda' \,,\, \phi \,,\, \theta \,,\, \psi$

- $\mathcal{N} = 2$ supergravity coupled with 1 vector + 1 hyper

$$
\mathcal{M}_{scalar} = \underbrace{\frac{\mathrm{SL}(2)}{\mathrm{SO}(2)}}_{(\lambda\,,\,\alpha)} \times \underbrace{\frac{\mathrm{SU}(2,1)}{\mathrm{SU}(2)\times \mathrm{U}(1)}}_{(\lambda'\,,\,\phi\,,\,\theta\,,\,\psi)}
$$

$\mathcal{N}=2$ superpotentials & scalar potential

[Ahn & Woo '00]
[A.G , in progress...]

Superpotentials = mass terms for the two SU(3)-singlet gravitini ψ_μ^1 and $\psi_\mu^{\hat{1}}$

$$W_1 = e^{i\omega}\,\mathcal{A}_+^{11} + e^{-i\omega}\,\mathcal{A}_-^{11} \qquad \text{or} \qquad W_{\hat{1}} = e^{i\omega}\,\mathcal{A}_+^{\hat{1}\hat{1}} + e^{-i\omega}\,\mathcal{A}_-^{\hat{1}\hat{1}}$$

Fermi mass terms:

$$\begin{aligned}
\mathcal{A}_+^{11} &= \tfrac{3}{2}e^{i(2\alpha+2\phi)}\cosh(\lambda)\sinh^2(\lambda)\sinh^2(2\lambda') + \cosh^3(\lambda)\,f(\lambda',\phi)\\
\mathcal{A}_-^{11} &= \tfrac{3}{2}e^{i(\alpha+2\phi)}\sinh(\lambda)\cosh^2(\lambda)\sinh^2(2\lambda') + e^{3i\alpha}\,\sinh^3(\lambda)\,f(\lambda',\phi)
\end{aligned}$$

where $f(\lambda',\phi) = \cosh^4(\lambda') + e^{4i\phi}\sinh^4(\lambda')$ and with $\mathcal{A}_\pm^{\hat{1}\hat{1}}$ obtained by $\phi \to -\phi$

Scalar potential:

$$\begin{aligned}
V(\lambda,\alpha,\lambda',\phi) &= g^2\left[\frac{2}{3}\,|\partial_\lambda W|^2 + \frac{1}{2}\,|\partial_{\lambda'} W|^2 - 6\,|W|^2\right]\\
&= g^2\left[\frac{2}{3}\,(\partial_\lambda|W|)^2 + \frac{2}{3\cosh^2(\lambda)\sinh^2(\lambda)}\,(\partial_\alpha|W|)^2\right.\\
&+ \left.\frac{1}{2}\,(\partial_{\lambda'}|W|)^2 + \frac{1}{2\cosh^2(\lambda')\sinh^2(\lambda')}\,(\partial_\phi|W|)^2 - 6\,|W|^2\right]
\end{aligned}$$

$W = W_1$ or $W_{\hat{1}}$

$$
\begin{aligned}
V(\lambda,\alpha,\lambda',\phi) \ &= \\
= \ & \frac{g^2}{128\,(c^2+1)}\Big[4\,\Big((c^2+1)\cosh(6\lambda)\sinh^2(2\lambda')\,(19\cosh(4\lambda')+21)\\
- \ & 4\sinh(2\lambda)\Big(2\sinh^2(2\lambda)\cos(4\phi)\sinh^4(2\lambda')\Big((c^2-1)\cos(3\alpha)-2c\sin(3\alpha)\Big)\\
+ \ & \sinh^2(2\lambda')\Big(3(c^2-1)\cos(\alpha)\Big(\cosh(4\lambda)(3\cosh(4\lambda')+2\cos(2\phi)+3)\\
+ \ & \cosh(4\lambda')-6\cos(2\phi)-7\Big)+\sinh^2(2\lambda)\,(\cosh(4\lambda')+3)\Big((c^2-1)\cos(3\alpha)-2c\sin(3\alpha)\Big)\\
+ \ & 6\,c\,\sin(\alpha)\Big(\cosh(4\lambda')-2\,(\cosh(4\lambda)-3)\cos(2\phi)-7\Big)\Big)\\
+ \ & 3\sinh^2(4\lambda')\Big(3c\sin(\alpha)\cosh(4\lambda)-(c^2+1)\cos(2\alpha)\sinh(4\lambda)\cos(2\phi)\Big)\Big)\Big)\\
+ \ & 32\,(c^2+1)\cosh^3(2\lambda)\cos(4\phi)\sinh^4(2\lambda')\\
+ \ & 3\,(c^2+1)\cosh(2\lambda)\,\Big(3\,(\cosh(8\lambda')-45)-124\cosh(4\lambda')\Big)\\
- \ & 192\,\sinh(2\lambda)\cosh^2(2\lambda)\cos(2\phi)\sinh^2(2\lambda')\cosh(4\lambda')\Big((c^2-1)\cos(\alpha)-2c\sin(\alpha)\Big)\Big]\,.
\end{aligned}
$$

$$
\boxed{\omega = \mathrm{Arg}(1+ic)}
$$

11d lifting and domain-walls at $\omega = 0$

[Warner '83 '84]
[Bobev, Halmagyi, Pilch & Warner '10]

- Reduction of 11d supergravity on $AdS_4 \times S^7$ with a round, squashed, stretched or warped 7-sphere (SE_7) and 4-form flux

[Nicolai & Pilch '12]

SUSY	Symmetry	Cosm. constant	Stability
$\mathcal{N}=8$	SO(8)	$-6\ (\times 1)$	✓
$\mathcal{N}=2$	$SU(3)\times U(1)$	$-\frac{9}{2}\sqrt{3}\ (\times 1)$	✓
$\mathcal{N}=1$	G_2	$-\frac{216}{25}\sqrt{\frac{2}{5}\sqrt{3}}\ (\times 2)$	✓
$\mathcal{N}=0$	SO(7)	$-2\sqrt{5\sqrt{5}}\ (\times 1)$	×
		$-\frac{25}{8}\sqrt{5}\ (\times 2)$	×
$\mathcal{N}=0$	SU(4)	$-8\ (\times 1)$	×

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$\mathcal{N}=8$	SO(8)	$-6\ (\times 1)$	✓	[Freund & Rubin '80] [Englert '82]
$\mathcal{N}=2$	$\mathrm{SU}(3)\times\mathrm{U}(1)$	$-\frac{9}{2}\sqrt{3}\ (\times 1)$	✓	[Corrado, Pilch & Warner '01]
$\mathcal{N}=1$	G_2	$-\frac{216}{25}\sqrt{\frac{2}{5}\sqrt{3}}\ (\times 2)$	✓	[de Wit, Nicolai & Warner '85]
$\mathcal{N}=0$	SO(7)	$-2\sqrt{5\sqrt{5}}\ (\times 1)$	×	[Englert '82] [de Wit Nicolai '84]
		$-\frac{25}{8}\sqrt{5}\ (\times 2)$	×	
$\mathcal{N}=0$	SU(4)	$-8\ (\times 1)$	×	[Pope & Warner '85]

11d lifting and domain-walls at $\omega = 0$

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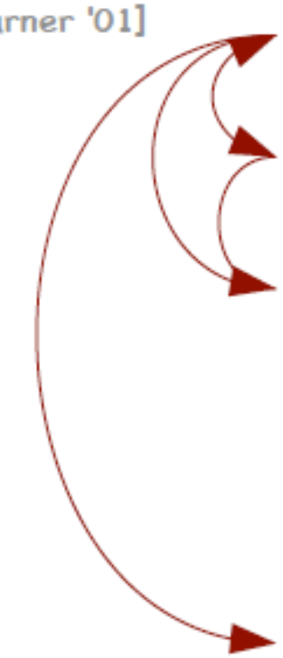
[Jafferis, Klebanov, Pufu & Safdi '11]

[Donos & Gaunlett '11]
[Bobev, Halmagyi, Pilch & Warner '09]
[Corrado, Pilch & Warner '01]
[Ahn & Woo '00]

Domain walls & RG-flows



[Gauntlett, Sonner & Wiseman '09]

SUSY	Symmetry	Cosm. constant	Stability	**Lifting to 11d**
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$\mathcal{N}=0$	$SO(7)$	$-2\sqrt{5\sqrt{5}}\ (\times 1)$	×	[Englert '82] [de Wit Nicolai '84]
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11d lifting and domain-walls at $\omega = 0$

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$\mathcal{N}=0$	$SO(7)$	$-2\sqrt{5\sqrt{5}}\ (\times 1)$	×	[Englert '82] [de Wit Nicolai '84]
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- AdS/CMT applications : Holographic superconductivity

[Gauntlett, Sonner & Wiseman '09]
[Donos & Gauntlett '11]

Critical points at $\omega \neq 0$ [with purely electric counterpart]

[Borghese, Dibitetto, A.G , Roest & Varela '13]

<table>
<tr><th>SUSY</th><th>G_0</th><th>V_0</th><th>$|W_1|$</th><th>$|W_{\hat{1}}|$</th><th>λ_0</th><th>α_0</th><th>λ'_0</th><th>ϕ_0</th><th>Stability</th></tr>
<tr><td>$\mathcal{N}=8$</td><td>SO(8)</td><td>-6</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>✓</td></tr>
<tr><td rowspan="3">$\mathcal{N}=2$</td><td rowspan="3">SU(3) × U(1)</td><td rowspan="3">-8.354</td><td rowspan="3">1.180</td><td rowspan="3">1.180</td><td rowspan="3">0.315</td><td>0.171π</td><td rowspan="3">0.375</td><td>$\pm\frac{\pi}{2}$</td><td rowspan="3">✓</td></tr>
<tr><td rowspan="2">1.329π</td><td>0</td></tr>
<tr><td>π</td></tr>
<tr><td rowspan="8">$\mathcal{N}=1$</td><td rowspan="8">G_2</td><td rowspan="8">-7.943</td><td rowspan="4">1.151*</td><td rowspan="4">1.409</td><td rowspan="8">0.329</td><td rowspan="2">0.373π</td><td rowspan="8">0.329</td><td>0.373π</td><td rowspan="8">✓</td></tr>
<tr><td>1.373π</td></tr>
<tr><td rowspan="2">1.127π</td><td>1.127π</td></tr>
<tr><td>0.127π</td></tr>
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<tr><td>-0.127π</td></tr>
<tr><td rowspan="6">$\mathcal{N}=0$</td><td rowspan="6">SO(7)</td><td rowspan="3">-6.748</td><td rowspan="3">1.232</td><td rowspan="3">1.232</td><td rowspan="3">0.210</td><td rowspan="2">0</td><td rowspan="3">0.210</td><td>0</td><td rowspan="6">×</td></tr>
<tr><td>π</td></tr>
<tr><td>$-\frac{\pi}{2}$</td><td>$\pm\frac{\pi}{2}$</td></tr>
<tr><td rowspan="3">-7.771</td><td rowspan="3">1.322</td><td rowspan="3">1.322</td><td rowspan="3">0.320</td><td rowspan="2">π</td><td rowspan="3">0.320</td><td>0</td></tr>
<tr><td>π</td></tr>
<tr><td>$\frac{\pi}{2}$</td><td>$\pm\frac{\pi}{2}$</td></tr>
<tr><td rowspan="3">$\mathcal{N}=0$</td><td rowspan="3">SU(4)</td><td rowspan="3">-8.581</td><td rowspan="3">1.553</td><td rowspan="3">1.553</td><td rowspan="3">0.115</td><td rowspan="2">π</td><td rowspan="3">0.488</td><td>0</td><td rowspan="3">×</td></tr>
<tr><td>π</td></tr>
<tr><td>$\frac{\pi}{2}$</td><td>$\pm\frac{\pi}{2}$</td></tr>
</table>

* Example at $\omega = \pi/8$

Critical points at $\omega \neq 0$ [without purely electric counterpart]

[Borghese, Dibitetto, A.G , Roest & Varela '13]

SUSY	G_0	V_0	$\lvert W_1 \rvert$	$\lvert W_{\bar{1}} \rvert$	λ_0	α_0	λ'_0	ϕ_0	Stability
$\mathcal{N}=1$	G_2	-7.040	1.083*	1.327	0.242	$-\frac{\pi}{4}$	0.242	$-\frac{\pi}{4}$	✓
								$\frac{3\pi}{4}$	
			1.327	1.083*				$\frac{\pi}{4}$	
								$-\frac{3\pi}{4}$	
$\mathcal{N}=1$	SU(3)	-10.392	1.316*	2.632	0.275	$\frac{3\pi}{4}$	0.573	$\frac{\pi}{4}$	✓
								$-\frac{3\pi}{4}$	
			2.632	1.316*				$-\frac{\pi}{4}$	
								$\frac{3\pi}{4}$	
$\mathcal{N}=0$	G_2	-10.170	2.762	1.595	0.467	$\frac{3\pi}{4}$	0.467	$\frac{3\pi}{4}$	✓
								$-\frac{\pi}{4}$	
			1.595	2.762	0.467	$\frac{3\pi}{4}$	0.467	$-\frac{3\pi}{4}$	
								$\frac{\pi}{4}$	
$\mathcal{N}=0$	SU(3)	-10.237	2.747	1.467	0.400	0.702π	0.512	0.785π	?
								1.785π	
						0.798π		-0.285π	
								-1.285π	
			1.467	2.747		0.702π		-0.785π	
								-1.785π	
						0.798π		0.285π	
								1.285π	

* Example at $\omega = \pi/8$

Critical points at $\omega \neq 0$ [without purely electric counterpart]

[A.G , in progress...]

SUSY	G_0	V_0	$\lvert W_1 \rvert$	$\lvert W_{\hat{1}} \rvert$	λ_0	α_0	λ_0'	ϕ_0	Stability
$\mathcal{N}=1$	G_2	-7.040	1.083*	1.327	0.242	$-\frac{\pi}{4}$	0.242	$-\frac{\pi}{4}$	✓
								$\frac{3\pi}{4}$	
			1.327	1.083*				$\frac{\pi}{4}$	
								$-\frac{3\pi}{4}$	
$\mathcal{N}=1$	SU(3)	-10.392	1.316*	2.632	0.275	$\frac{3\pi}{4}$	0.573	$\frac{\pi}{4}$	✓
								$-\frac{3\pi}{4}$	
			2.632	1.316*				$-\frac{\pi}{4}$	
								$\frac{3\pi}{4}$	
$\mathcal{N}=0$	G_2	-10.170	2.762	1.595	0.467	$\frac{3\pi}{4}$	0.467	$\frac{3\pi}{4}$	✓
								$-\frac{\pi}{4}$	
			1.595	2.762	0.467	$\frac{3\pi}{4}$	0.467	$-\frac{3\pi}{4}$	
								$\frac{\pi}{4}$	
$\mathcal{N}=0$	SU(3)	-10.237	2.747	1.467	0.400	0.702π	0.512	0.785π	STABLE
								1.785π	
						0.798π		-0.285π	
								-1.285π	
			1.467	2.747		0.702π		-0.785π	
								-1.785π	
						0.798π		0.285π	
								1.285π	

STABLE

1) ω-dependent!
2) non-susy
3) fully stable

* Example at $\omega = \pi/8$

Dyonic domain-walls ?

- Domain-wall ansatz : $S_{scalar} = \int d^4x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}K_{ij}(\partial_\mu\Sigma^i)(\partial^\mu\Sigma^j) - V(\Sigma^i)\right)$

$$ds^2 = e^{2A(z)}\,\eta_{\alpha\beta}\,dx^\alpha dx^\beta \;+\; dz^2 \qquad \text{with} \qquad \eta_{\alpha\beta} = \text{diag}(-1,+1,+1)$$

where $\Sigma^i = (\lambda, \alpha, \lambda', \phi)$ and $K_{ij} = \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & \frac{3}{2}\sinh^2(2\lambda) & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 2\sinh^2(2\lambda') \end{pmatrix}$

- Energy per unit of transverse area

[Skenderis & Townsend '99]

$$\begin{aligned} E_{DW}(A,\Sigma^i) &= -\tfrac{1}{a}\,S_{DW}(A,\Sigma^i) \\ &= \tfrac{1}{2}\int_{-\infty}^{\infty} dz\, e^{3A}\left[-6\,(\partial_z A)^2 + K_{ij}(\partial_z\Sigma^i)(\partial_z\Sigma^j) + 2\,V(\Sigma^i)\right] \end{aligned}$$

[A.G , in progress...]

- First order flow-equations :

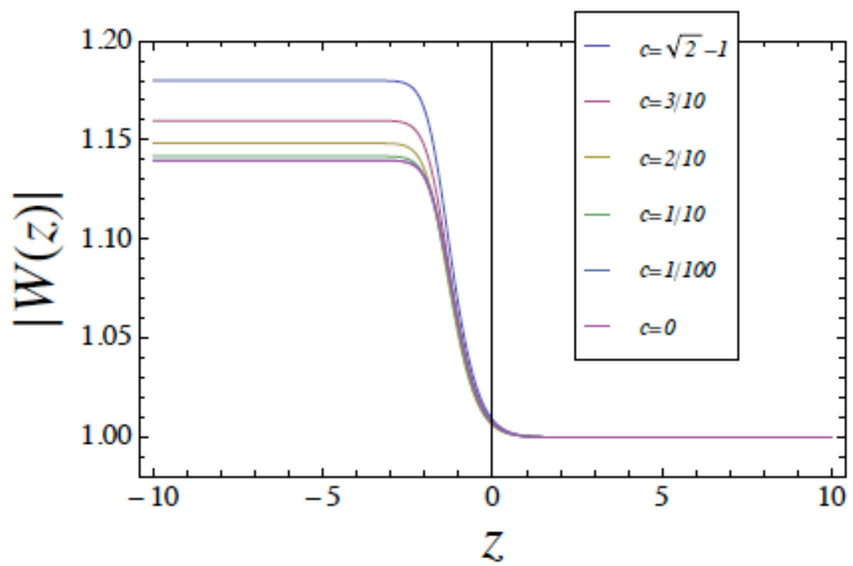
$$\partial_z A = \mp g\sqrt{2}\,|W| \quad ,$$

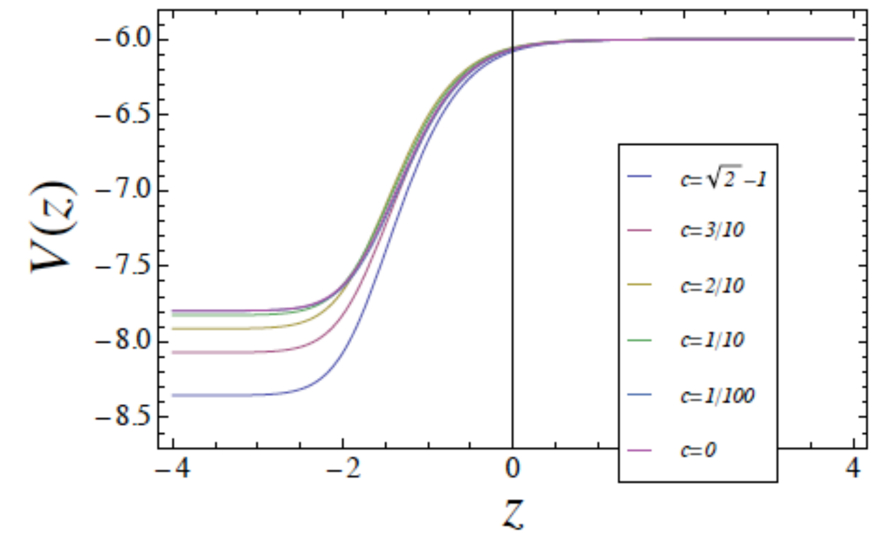
$$\begin{aligned} \partial_z\lambda &= \pm g\,\frac{\sqrt{2}}{3}\,\partial_\lambda|W| \quad , \quad & \partial_z\alpha &= \pm g\,\frac{\sqrt{2}}{3\cosh^2(\lambda)\sinh^2(\lambda)}\,\partial_\alpha|W| \quad , \\ \partial_z\lambda' &= \pm g\,\frac{1}{2\sqrt{2}}\,\partial_{\lambda'}|W| \quad , \quad & \partial_z\phi &= \pm g\,\frac{\sqrt{2}}{3\cosh^2(\lambda')\sinh^2(\lambda')}\,\partial_\phi|W| \quad . \end{aligned}$$

Some numerical BPS results

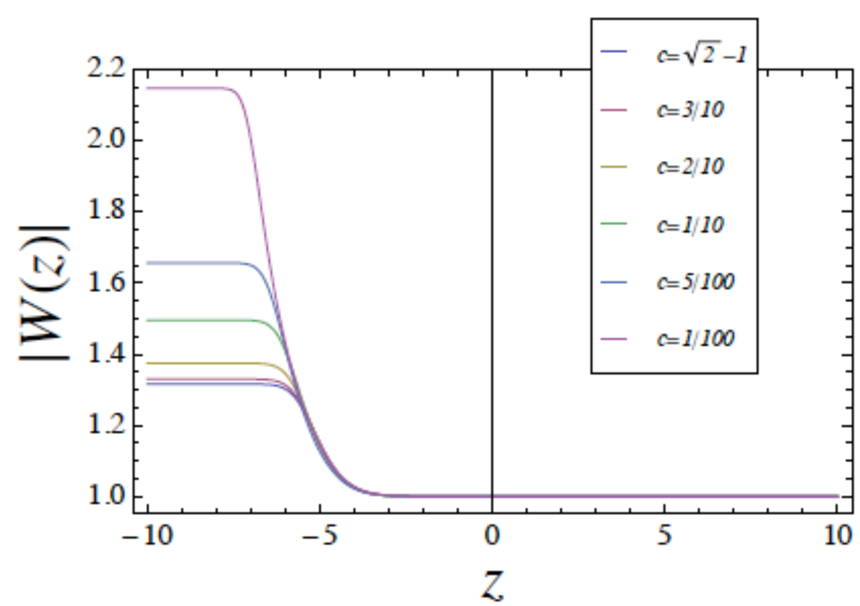
[A.G , in progress...]

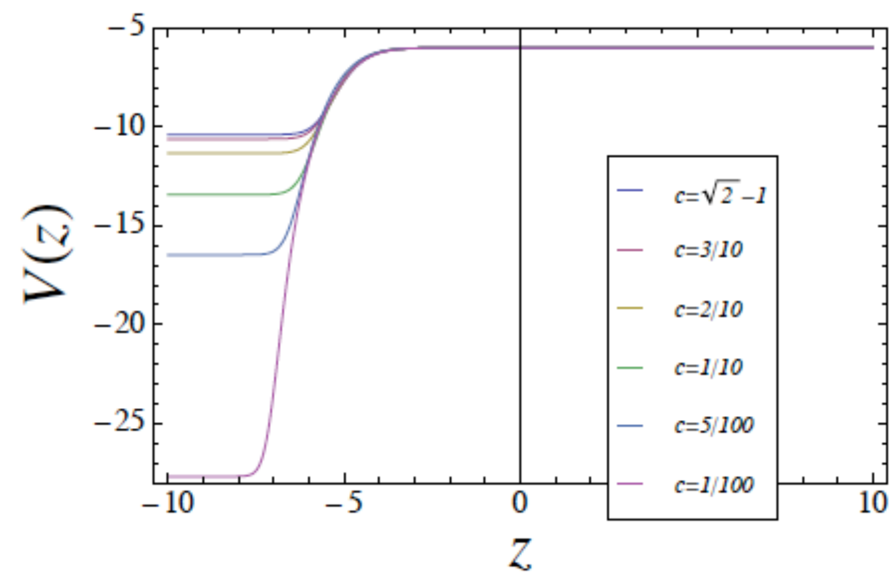
- DW with purely electric counterpart: N=8 SO(8) @ UV to N=2 SU(3)xU(1) @IR





- Genuine DW without purely electric counterpart: N=8 SO(8) @ UV to N=1 SU(3) @IR





Lifting to 11d supergravity?? , What about dual RG flows ?? , ...

Final remarks

- Electromagnetic U(1) rotations pick up a physically relevant direction in the space of the embedding tensor deformations and provide new vacua of $\mathcal{N}=8$ supergravity

- Small residual symmetry groups like SO(4) & SU(3) show ω-dependent mass spectra. Triality restores $\frac{\pi}{4}$-periodicity.

- Critical points running away at $\omega = n\,\frac{\pi}{4}$ in one theory, show up in another. The entire story of a solution can be tracked by computing fermi masses in the GTTO approach

- Tachyon dilution around AdS/Mkw/dS transitions ⇨ stable dS in extended SUGRA ?

- Dyonic domain-walls can be constructed ⇨ dual RG flow interpretation ?

- All these solutions can be obtained as $Z_2 \times Z_2$ Type II orientifolds with non-geometric fluxes (both electric and magnetic). Democratic lifting to M-theory including vectors from A_3 and A_6 ? And oxidation to DFT ?

[de Wit & Nicolai '13]

[Blumenhagen, Gao, Herschmann & Shukla '13]

Thanks for your attention !!