Non-Abelian black hole solutions in supergravity

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Abstract. In this contribution we shall discuss some analytic examples of non-Abelian black holes in a specific $N = 2$ $d = 4$ supergravity theory. Some remarks will be made on a possible non-Abelian version of the attractor mechanism, that works in the Abelian theories.

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The class of supersymmetric solutions in (ungauged) $N = 2$ $d = 4$ supergravity theories that can give rise to black hole spacetimes is completely characterised [1]: the most general of such solutions is parametrised by an even number of real functions that are harmonic on $\mathbb{R}^3$ and to which we shall refer as *seed functions*. Given these seed functions there exists an algorithm to express all the physical fields in terms of them. Unsurprisingly, not every choice for the seed functions leads to a regular black hole spacetime, but the necessary and sufficient conditions that allow for a static, asymptotically flat, spherically symmetric black hole are known (see *e.g.* [2]). Apart from the absence of NUT charge and the positivity of the ADM mass, the only remaining condition one has to impose is that the *near horizon geometry*, *i.e.* the geometry that is obtained by zooming in on the horizon, is that of an $aDS_2 \times S^2$ geometry and the entropy of the black hole is then easily read off by using the area law.

The surprising part of the above story is that the entropy of the resulting black hole depends only on the electric and magnetic charges of the various Maxwell fields, and not on the asymptotic values of the scalar fields. As befitting uncharged scalar fields, they are constant on the horizon, but their horizon values are also only given in terms of the electric and magnetic charges of the Maxwell fields. This fact goes under the name of *attractor mechanism* [3] and is fundamental to the successful matching of the entropy calculated in supergravity with the microscopical one in string theory [4].

For definiteness, we shall consider a specific model, namely the $N = 2$ $d = 4$ supergravity model based on the special geometry $SU(1,3)/U(3)$. The bosonic field content of this model consists of the metric, 4 gauge fields A^Λ ($\Lambda = 0,1,2,3$) and 3 complex scalar fields Z^i ($i = 1,2,3$). Choosing an $SO(3)$-gauging, 3 of the vector fields, A^i, constitute the gauge fields, leaving A^0 as an Abelian field; the scalars transform as a triplet under the $SO(3)$. The bosonic action for the model is

$$\int_4 \sqrt{g} \left[\tfrac{1}{2}R + \mathscr{G}_{i\bar{j}} \mathtt{D}_a Z^i \mathtt{D}^a \overline{Z}^{\bar{j}} - V(Z,\overline{Z}) + \mathrm{Im}(\mathscr{N})_{\Lambda\Sigma} F^\Lambda_{ab} F^{\Sigma ab} - \mathrm{Re}(\mathscr{N})_{\Lambda\Sigma} F^\Lambda_{ab} (\star F)^{\Sigma ab} \right], \quad (1)$$

and the field strengths and the covariant derivatives are given by

$$F^0 = dA^0 \ , \ F^i = dA^i + \tfrac{g}{2}\varepsilon_{jk}{}^i A^j \wedge A^k \ , \ \mathtt{D}Z^i = dZ^i + g\, \varepsilon_{jk}{}^i A^j Z^k \ , \quad (2)$$

where g is the coupling constant. As the metric $\mathscr{G}$ is Kähler it can be derived from a Kähler potential, $\mathscr{K}$, which for the chosen model reads $e^{-\mathscr{K}} = 1 - |Z|^2$. Observe that the Kähler potential imposes the constraint $0 \leq |Z|^2 \leq 1$, but it can be shown that a BH solution automatically satisfies this bound [2]. The explicit form of the complex matrix $\mathscr{N}$ and the potential V, which by construction is positive semi-definite, can be written down, but as they will not be used we shall refrain from writing them down.

In the ungauged case, *i.e.* when $g = 0$, there are 6 seed functions, denoted $\mathscr{I}^\Lambda$ and $\mathscr{I}_\Lambda$, which in the spherically symmetric case are given by

$$\mathscr{I}^\Lambda = h^\Lambda + \frac{p^\Lambda}{r} \quad , \quad \mathscr{I}_\Lambda = h_\Lambda + \frac{q_\Lambda}{r} \, , \tag{3}$$

where p^Λ and q_Λ are the magnetic and electric charges of the 4 Maxwell fields. As mentioned above, given the seed functions we can generate the complete expressions of the fields, and in this letter, we shall be interested in static BH-like solutions: said staticity imposes the constraint $h_\Lambda p^\Lambda - h^\Lambda q_\Lambda = 0$ which is equivalent to imposing the vanishing of a possible NUT charge. Here we shall solve this constraint by putting $\mathscr{I}_\Lambda = 0$, so that we will be dealing with purely magnetic solutions only. The supergravity fields can then be expressed in terms of the seed functions as[1]

$$\vec{Z} = \frac{\vec{\mathscr{I}}}{\mathscr{I}^0} \; , \; ds^2 = K^{-1}\, dt^2 - K\left[dr^2 + r^2 dS^2\right] \;\longleftarrow\; K = (\mathscr{I})^2 - \vec{\mathscr{I}}^2 \, . \tag{4}$$

where dS^2 stands for the standard metric on the 2-sphere. Without loss of generality we can normalise the solution as to asymptote to ordinary Minkowski space by putting $(h^0)^2 = 1 + \vec{h}^2$. The ADM mass can be seen to be $M = h^0 p^0 - \vec{h}\cdot\vec{p}$ and must be taken to be positive. As one can see, the horizon is located at $r = 0$ and the condition for the solution to describe the geometry outside a regular black hole is the one that the area of the horizon is positive, whence it can be used to calculate the entropy of the BH. By an abuse of language, then, we say that the condition for the existence of a horizon is equivalent to the positivity of the entropy, which in this case reads

$$S_{bh} = (p^0)^2 - \vec{p}^2 > 0 \, . \tag{5}$$

Observe that this entropy, conforming to the attractor mechanism, only depends on the magnetic charges. Furthermore, the expression for the scalars on the horizon reads $\vec{Z} = \vec{p}/p^0$.

The general class of supersymmetric solutions to $N = 2$ $d = 4$ sugra with YM-couplings was obtained in Refs. [5]. For the example at hand the expression of the fields in terms of the seed functions is still given by Eq. (4), but the seed functions are no longer harmonic functions on $\mathbb{R}^3$; instead, they have to satisfy

$$\vec{\partial}^2 \mathscr{I}^0 = 0 \quad , \quad \mathtt{D}_i \mathscr{I}^l = \tfrac{1}{2}\varepsilon_{ijk}\, \mathscr{F}^l_{jk} \, , \tag{6}$$

[1] We shall omit the explicit expression for the Maxwell fields and kindly refer the interested reader to Ref. [5].

where the last equation is the Bogomol'nyi equation for the $SO(3)$ Yang-Mills-Higgs system on $\mathbb{R}^3$ determining the pair $(A_i^l, \mathscr{I}^i)$. The most famous spherically symmetric solution to the Bogomol'nyi equation is of course the 't Hooft-Polyakov monopole, which is characterised by the fact that it is completely regular. This regularity enables one to construct a globally regular, asymptotically flat supergravity solution by taking $\mathscr{I}^0$ to be a suitable constant [5].[2]

The B. equation of course admits more spherically symmetric solutions than just the 'tHP monopole [6], but not all of them have the desired properties to be used as seed solutions for constructing black hole solutions. The question what solutions to the B. equation can be used to build BH solutions was addressed in Ref. [7], and the conclusion is that apart from the 'tHP monopole, there is one family of solutions giving rise to hairy black holes with magnetic colour charge $1/g$, and an isolated solution that is similar to a coloured black hole in that its colour charge vanishes.

The sugra fields for the hairy black holes are given by

$$\vec{Z} \;=\; -\frac{\mu}{g}\frac{rH_s(r)}{p^0+h^0 r}\,\vec{n} \;\;,\;\; K \;=\; (h^0+p^0/r)^2 \;-\; \frac{\mu^2}{g^2}\,H_s^2 \;, \tag{7}$$

where $\vec{n}$ is the outward-pointing unit vector, μ is a positive parameter measuring the vacuum expectation value of the 'Higgs' field $g\vec{\mathscr{I}}$ and the function $H_s(r)$ is given by

$$H_s(r) \;=\; \coth(\mu r \,+\, s) \;-\; (\mu r)^{-1} \qquad \text{with } s \geq 0\,. \tag{8}$$

The fundamental characteristic of the above function is that it is a monotonically increasing function on the interval $(0,\infty)$ with asymptotic values $H_s(r\to\infty) = 1$. For $s \neq 0$, the function blows up as r^{-1} around $r = 0$, whereas when $s = 0$ it behaves like $H_s \sim r$ around $r = 0$; in effect, the solution with $s = 0$ corresponds to the 't Hooft-Polyakov monopole.

As in the ungauged case, we can normalise the solution to asymptote to Minkowski space by putting $h^0 = \sqrt{1+\mu^2 g^{-2}}$, after which a small calculation shows that the mass is given by $M = h^0 p^0 + \mu g^{-1}$, and therefore imposes the constraint $h^0 p^0 > 0$ in order for the mass to be positive. The globally regular solution is found by putting $s = 0$ and $p^0 = 0$; in contrast, for any $s \neq 0$ we can build a black hole whenever the 'entropy is positive', which imposes

$$S_{bh} \;=\; (p^0)^2 - \tfrac{1}{g^2} \;>\; 0 \;\;\text{ and then we have }\;\; \vec{Z}|_{hor} \;=\; -\tfrac{1}{gp^0}\,\vec{n}\,. \tag{9}$$

Observe that as in the ungauged case, the near horizon solution is determined in terms of the asymptotic charges only, but that the scalars are not constant over the horizon: they form a hedgehog configuration, which is, however, hardly surprising as we are dealing with charged scalar fields on a 2-sphere. The rôle of the parameter $s \neq 0$ is, seeing that it cannot be expressed in terms of asymptotic data such as the mass, that of a black hole hair and exemplifies the know fact that the no-hair theorems are not valid in

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[2] One immediate question is: Can any monopole solution to the B. equation be embedded into sugra theories as to give rise to a globally regular solution? The answer to this question is negative as a counterexample, to wit an $SU(4)$ monopole in the so-called $\mathbb{Q}$-magic model, can be constructed [5].

EYM theories. The fact that the entropy also doesn't depend on the hair, seems fortunate when thinking about entropy calculations in string theory, but a deeper string theoretical insight into the nature of the hair is, as far as the author is aware, lacking but highly desirable.

The hairy black holes constructed above, seem to indicate that some kind of attractor mechanism is at work in the solutions, but as the last example shows, the story is more complicated as there are solutions whose asymptotic charges vanish: in analogy with the solutions found in EYM theories, we shall call this example a *coloured black hole*, and is given by the expression

$$\vec{Z} = -\frac{\vec{n}}{g(p^0+h^0 r)(1+\lambda r^2)} \ , \ K = (h^0+p^0/r)^2 - \left[gr\left(1+\lambda r^2\right)\right]^{-2} \ \ (\lambda > 0)\ . \quad (10)$$

Normalising the metric as above fixes $|h^0| = 1$ and the mass of the solution becomes $M = h^0 p^0 = |p^0|$, so that the asymptotic data are independent of the parameter λ. In order to construct a black hole, then, we need to have a non-vanishing horizon, which is readily calculated and leads to the same result as in Eq. (9); the resulting spacetime is best interpreted as an extreme Reissner-Nordstrom black hole surrounded by a cloud of glue, whose fall-off is fast enough to not leave an asymptotic imprint but does contribute to the horizon geometries.

The existence of this coloured black hole implies that a non-Abelian version of the attractor mechanism cannot be simply based on asymptotic charges, making the problem none the easier. In this respect it is also worth pointing out that the attractor mechanism also works for a large class of non-supersymmetric solutions, but in that case little to no analytic solutions of non-Abelian BHs are known. Apart from the attractor, the main questions for non-Abelian solutions to supergravity theories involve quantum corrections in that one would like to know if they modify/constrain the hair and whether they are compatible with the globally regular monopole solutions.

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