Hints on 5d Fixed Point Theories from Non-Abelian T-duality

Yolanda Lozano (U. Oviedo)

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Universidad de Oviedo

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  One of them is in fact dual to an AdS_6 SUGRA background
  $\rightarrow$ Study them using AdS/CFT
- This AdS_6 background seems to be on the other hand quite unique
- Use non-Abelian T-duality to generate a new AdS_6 solution which may possibly lead to a new 5d fixed point theory through AdS/CFT

Y.L., E. O Colgain, D. Rodriguez-Gomez, K. Sfetsos, PRL (2013)
Y.L., E. O Colgain, D. Rodriguez-Gomez, arXiv:1311.4842

Outline

1. Introduction

1.1. Gauge/gravity (holographic) correspondence

The gauge/gravity correspondence states that some gauge theories in flat space at strong (weak) coupling are dual to weakly (strongly) coupled string theories with one extra dimension.

Possibility to apply string theory techniques to strongly coupled systems (quark-gluon plasma, condensed matter systems).

Possibility to learn something about quantum gravity by studying low dimensional systems with a holographic description (concrete realization of QG and spacetime as emergent phenomena).

Original proposal by Maldacena:

Equivalence between Type IIB string theory on $AdS_5 \times S^5$ and 4 dim $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills theory

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Objects carrying the representations of the symmetry group can also be matched

Starting point: Study of N coincident D3-branes in Type IIB in the large N limit, based on two dual descriptions:

- As solution of the classical eqs. of motion of ST/SUGRA
- As a D-brane system

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UV finite, conformally invariant

Topological large N expansion with 't Hooft parameter:

$\lambda = g_{YM}^2 N$

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$\lambda \to \infty \;\Leftrightarrow\; l_s \to 0 \;\Rightarrow$ Strong ‘t Hooft coupling limit $\leftrightarrow$ SUGRA limit

$\Rightarrow$ Strong-weak coupling duality

1.2. D-branes

The fundamental string (perturbative state of the spectrum) occurs as a solitonic solution, electric source for B_2

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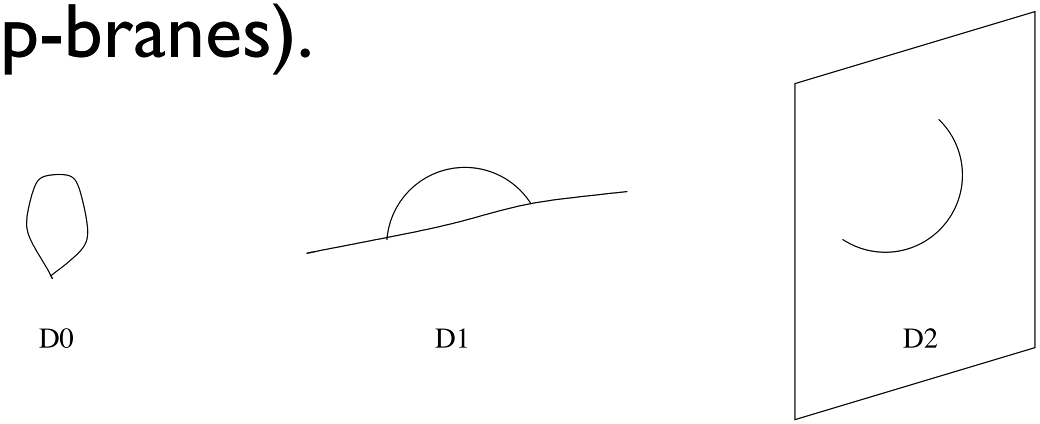
They are also (p+1)-dimensional hypersurfaces on which open strings can end $\Rightarrow$

Dynamics determined at weak coupling by open string perturbation theory $\Rightarrow$ U(1) gauge theory (U(N) for a system of Dp-branes).



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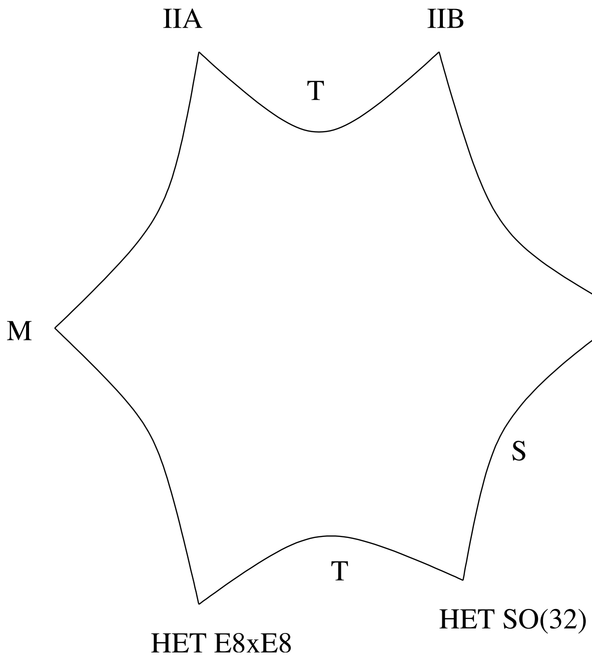

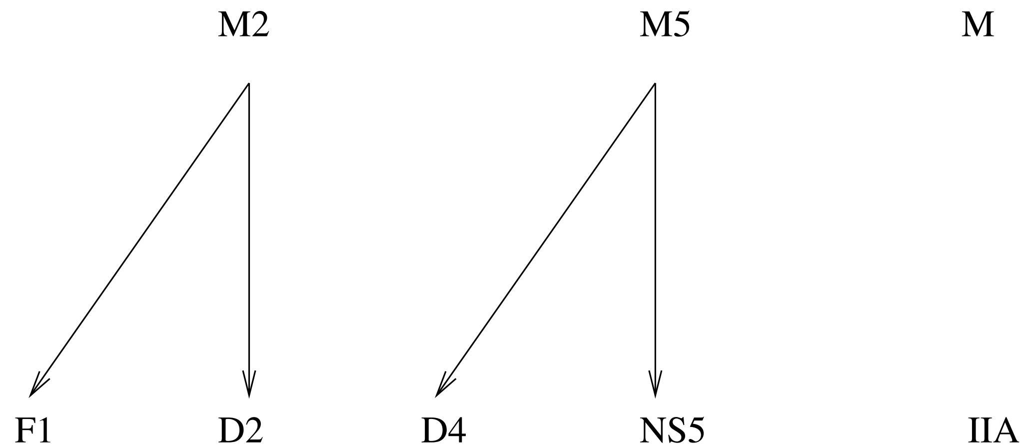

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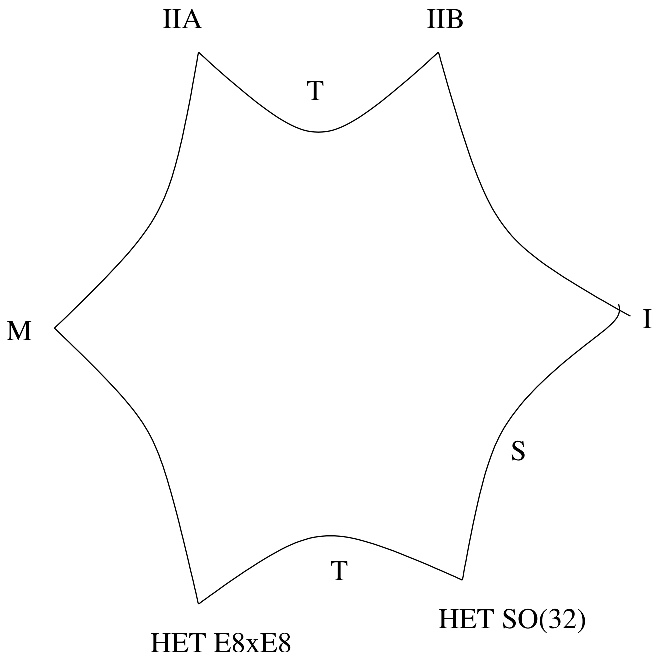

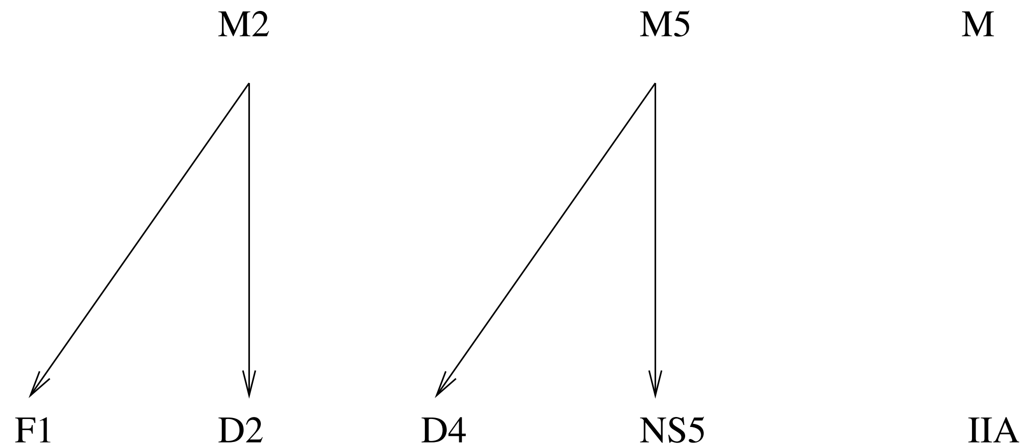

N coincident M2-branes $\rightarrow$ M-theory on $AdS_4 \times S^7$ dual to the 3 dim $\mathcal{N} = 8$ gauge theory living on the M2-branes

N coincident M5-branes $\rightarrow$ M-theory on $AdS_7 \times S^4$ dual to the 6 dim (2,0) field theory living on the M5-branes

M2-branes on an orbifold $\rightarrow$ M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ dual to the ABJM model

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D4-D8 system $\rightarrow$ Type I' string theory on a warped $AdS_6 \times S^4$ dual to the 5d fixed point theory obtained in the infinite bare coupling limit of $Sp(N)$ gauge theory with $N_f < 8$ fundamental hypermultiplets and one antisymmetric

(Brandhuber, Oz'99)

(Subject of this talk)

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Other examples with less SUSY, confining, etc

2. 5d fixed point theories

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- Suitably choosing the gauge group and matter content they can be at fixed points, where they can exhibit interesting phenomena, such as exceptional global symmetry groups

(Seiberg'96;
Intriligator, Morrison, Seiberg'97)

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New class of 5d fixed point theories?

Y.L., O Colgain, Rodríguez-Gómez'13

3. The D4-D8 system

(Brandhuber, Oz'99)

5d SUSY fixed points with E_{N_f+1} global symmetry can be obtained in the infinite bare coupling limit of N=2 SUSY gauge theories with gauge group Sp(N), $N_f < 8$ fundamental hypermultiplets and one antisymmetric hypermultiplet

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D8-brane background metric:

$$H_8(z) = c + 16\frac{z}{l_s} - \sum_{i=1}^{16}\frac{|z-z_i|}{l_s} - \sum_{i=1}^{16}\frac{|z+z_i|}{l_s}$$

Gauge coupling of the D4:

$$\frac{1}{g^2} = \frac{H_8}{l_s}$$

In the field theory limit $l_s \to 0$ + gauge coupling constant

$\Rightarrow \phi = \dfrac{z}{l_s^2}$ constant $\Rightarrow$ Region near $z = 0$

$$\frac{1}{g^2} = \frac{1}{g_{cl}^2} + 16\phi - \sum_{i=1}^{16} |\phi - m_i| - \sum_{i=1}^{16} |\phi + m_i|$$

with $m_i = \dfrac{z_i}{l_s^2}$. Taking N_f massless hypermultiplets:

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Seiberg'96: The theory at $g_{cl} = \infty$ with $N_f < 8$ massless hypermultiplets is a fixed point

Near horizon geometry: Fibration of AdS_6 over half-S^4

$$ds^2 = \frac{W^2\, L^2}{4}\left[9\, ds^2(AdS_6) + 4\, ds^2(S^4)\right] \qquad \theta \in [0, \frac{\pi}{2}]$$

$$F_4 = 5\, L^4\, W^{-2}\, \sin^3\theta\, d\theta \wedge \mathrm{Vol}(S^3)$$

$$m = \frac{8 - N_f}{2\pi l_s}$$

$$e^{-\phi} = \frac{3\, L}{2\, W^5}\,, \qquad W = (m\, \cos\theta)^{-\frac{1}{6}}$$

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- $SO(2,5) \leftrightarrow$ Conformal symmetry
- Boundary at $\theta = \frac{\pi}{2} \leftrightarrow$ Location of the $O8^-$ fixed plane

3.1. A unique SUSY solution?

Passias'12: Analyze the constraints imposed by SUSY on the geometry and fluxes of $AdS_6 \times M_4$ warped backgrounds in massive IIA

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We will see: New AdS_6 solution in Type IIB through non-Abelian T-duality

4. The AdS_6 non-Abelian T-dual

Back in the 90's:

Abelian T-duality for ST in a curved background (Buscher'88; Rocek, Verlinde'91)

$$S = \frac{1}{4\pi\alpha'} \int \left(g_{\mu\nu}\, dX^\mu \wedge *dX^\nu + B_{\mu\nu}\, dX^\mu \wedge dX^\nu \right) + \frac{1}{4\pi} \int R^{(2)} \phi$$

In the presence of an Abelian isometry: $\delta X^\mu = \epsilon\, k^\mu$ /

$$\mathcal{L}_k g = 0\,,\ \mathcal{L}_k B = d\omega\,,\ i_k d\phi = 0$$

i) Go to adapted coordinates: $X^\mu = \{\theta, X^\alpha\}$ such that

$\theta \to \theta + \epsilon$ and $\partial_\theta(\text{backgrounds}) = 0$

ii) Gauge the isometry: $d\theta \rightarrow D\theta = d\theta + A$

A non-dynamical gauge field / $\delta A = -d\epsilon$

iii) Add a Lagrange multiplier term: $\tilde{\theta}\, dA$, such that

$\int \mathcal{D}\tilde{\theta} \;\rightarrow\; dA = 0 \;\Rightarrow\; A$ exact

(in a topologically trivial worldsheet)

+ fix the gauge: $A = 0 \;\rightarrow$ Original theory

iv) Integrate the gauge field

+ fix the gauge: $\theta = 0 \;\rightarrow$ Dual sigma model:

$\{\theta, X^\alpha\} \rightarrow \{\tilde{\theta}, X^\alpha\}$ and

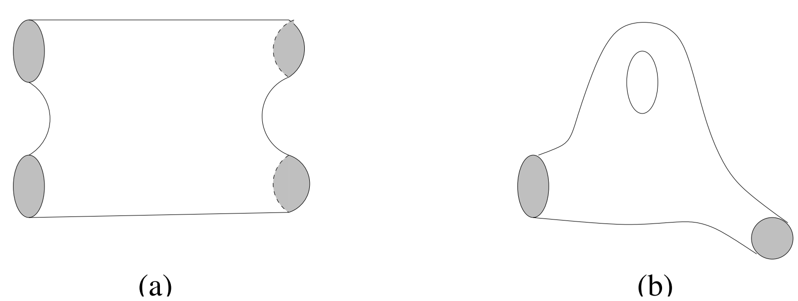
$$\tilde{g}_{00} = \frac{1}{g_{00}}; \quad \tilde{g}_{0\alpha} = \frac{B_{0\alpha}}{g_{00}}; \quad \tilde{g}_{\alpha\beta} = g_{\alpha\beta} - \frac{g_{0\alpha}g_{0\beta} - B_{0\alpha}B_{0\beta}}{g_{00}}$$

$$\tilde{B}_{0\alpha} = \frac{g_{0\alpha}}{g_{00}}; \quad \tilde{B}_{\alpha\beta} = B_{\alpha\beta} - \frac{g_{0\alpha}B_{0\beta} - g_{0\beta}B_{0\alpha}}{g_{00}}$$

$$\tilde{\phi} = \phi - \log g_{00}$$

Buscher's formulae

- Conformally invariant (Rocek, Verlinde'91)
- Involutive transformation: $\tilde{S} \xrightarrow[\tilde{\theta} \to \tilde{\theta} + \epsilon]{} S$
- Arbitrary worldsheets? (symmetry of string perturbation theory):



$\Rightarrow$ Non-trivial topologies + compact isometry orbits

Large gauge transformations: $\oint_\gamma d\epsilon = 2\pi n\,;\ n \in \mathbb{Z}$

To fix them:

Multivalued Lagrange multiplier: $\oint_\gamma d\tilde{\theta} = 2\pi m\,;\ m \in \mathbb{Z}$
such that

$$\int [\text{exact}] \rightarrow dA = 0 \qquad \textbf{+} \qquad \int [\text{harmonic}] \Rightarrow A \text{ exact}$$

$\Rightarrow$ The gauging procedure works for all genera

(Rocek, Verlinde'91)

4.1. Non-Abelian T-duality

(De la Ossa, Quevedo'93)

Non-Abelian continuous isometry: $X^m \to g^m_n X^n \,, g \in G$

i) Gauge it: $dX^m \to DX^m = dX^m + A^m_n X^n$

$A \in$ Lie algebra of G $\quad A \to g(A+d)g^{-1}$

ii) Add a Lagrange multiplier term: $\mathrm{Tr}(\chi F)$

$F = dA - A \wedge A$

$\chi \in$ Lie Algebra of G, $\chi \to g\chi g^{-1}$, such that

$\int \mathcal{D}\chi \;\to\; F = 0 \;\Rightarrow\; A$ exact
(in a topologically trivial worldsheet)

+ fix the gauge: $A = 0 \Rightarrow$ Original theory

iii) Integrate the gauge field + fix the gauge $\rightarrow$ Dual theory

Example: Principal chiral model with group SU(2):

Geometrically: S^3

$$L = Tr(g^{-1}dg \wedge *g^{-1}dg)\,;\;\; g \in SU(2)$$

Invariant under:

$$g \rightarrow h_1\, g\, h_2\,;\;\; h_1, h_2 \in SU(2)$$

Choose: $g \rightarrow hg\,;\;\; h \in SU(2)$

$$\tilde{L} = \frac{1}{1+\chi^2}\Big(\delta_{ij} - \epsilon_{ijk}\chi^k + \chi_i\chi_j\Big)d\chi^i \wedge *d\chi^j$$

Invariant under $\chi \rightarrow h\chi h^{-1}\,;\;\; h \in SU(2)$

- Higher genus generalization? Set to zero $W_\gamma = P\, e^{\oint_\gamma A}$
- Global properties?

  $\chi \in \mathbb{R}^3$: Global completion of $\mathbb{R}^3$?
- Conformal invariance not proved in general
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True symmetry in String Theory?

Still, interesting as a solution generating technique

(Sfetsos, Thompson'10)

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In the Abelian case: Reduce to a unique N=2, d=9 SUGRA

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In the Abelian case: Reduce to a unique N=2, d=9 SUGRA

(Bergshoeff, Hull, Ortín'95)

Hassan'99: Implement the relative twist between left and right movers in the bispinor formed by the RR fields:

$$\hat{P} = P\Omega^{-1} \qquad P = \frac{e^{\phi}}{2}\sum_k \frac{1}{k!} F_{\mu_1\dots\mu_k}\Gamma^{\mu_1\dots\mu_k}$$

with $\Omega = \sqrt{g_{00}^{-1}}\,\Gamma_{11}\Gamma^0$

4.2. Non-Abelian T-duality as a solution generating technique

Need to know how the RR fields transform

In the Abelian case: Reduce to a unique N=2, d=9 SUGRA
(Bergshoeff, Hull, Ortín'95)

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Same thing in the non-Abelian case (Sfetsos, Thompson'10)

A concrete case in IIB: (Itsios, Y.L., O Colgain, Sfetsos'12)

$$ds^2 = ds^2(M_7) + e^{2A} ds^2(S^3) \qquad B\,,\Phi$$

$$F_5 = G_2 \wedge \mathrm{Vol}(S^3) - e^{-3A} \star_7 G_2\ ,$$

$$F_3 = G_3 - m\mathrm{Vol}(S^3) \qquad F_1 = G_1$$

The non-Abelian T-dual w.r.t. one of the $SU(2)$ reads:

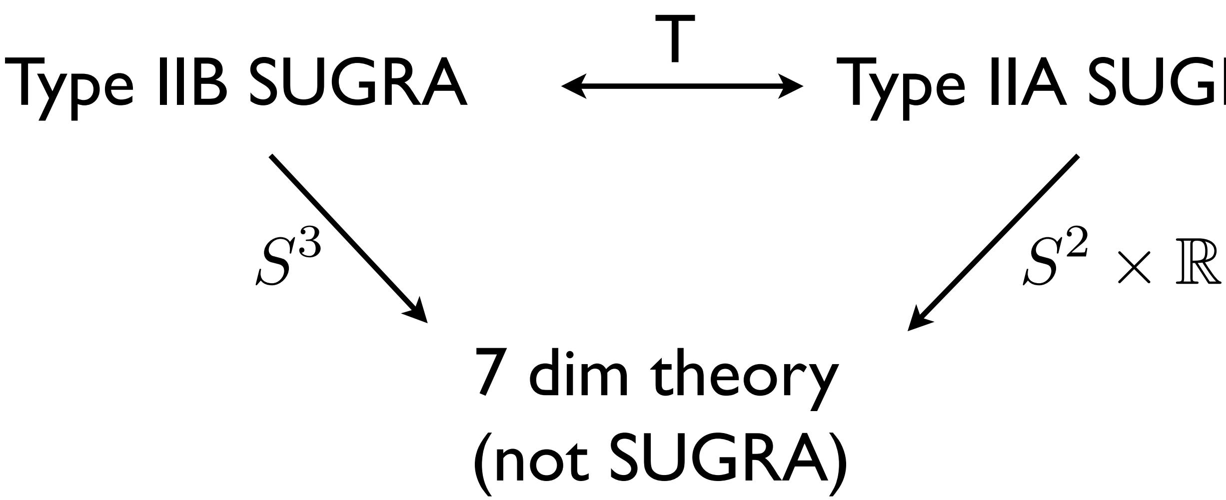
$$d\hat{s}^2 = ds^2(M_7) + e^{-2A}dr^2 + \frac{r^2 e^{2A}}{r^2 + e^{4A}} ds^2(S^2)$$

$$\hat{B} = B + \frac{r^3}{r^2 + e^{4A}} \mathrm{Vol}(S^2) \qquad e^{-2\hat{\Phi}} = e^{-2\Phi} e^{2A}(r^2 + e^{4A})$$

$$\hat{F}_0 = m \qquad \hat{F}_2 = \frac{mr^3}{r^2 + e^{4A}} \mathrm{vol}(S^2) + r dr \wedge G_1 - G_2$$

$$\hat{F}_4 = \frac{r^2 e^{4A}}{r^2 + e^{4A}} G_1 \wedge dr \wedge \mathrm{vol}(S^2) - \frac{r^3}{r^2 + e^{4A}} G_2 \wedge \mathrm{vol}(S^2) + r dr \wedge G_3 + \ldots$$

- Dual background satisfies the equations of motion
- SUSY: Generically broken by a half
- Do the original and non-Abelian transformed backgrounds reduce to the same SUGRA in (10-dimG) dimensions?



Not clear..

4.3. The AdS_6 non-Abelian T-dual

- Take the $AdS_6 \times S^4$ background

$$ds^2 = \frac{W^2\, L^2}{4}\left[9\, ds^2(AdS_6) + 4\, ds^2(S^4)\right]$$

$$F_4 = 5\, L^4\, W^{-2}\, \sin^3\theta\, d\theta \wedge \mathrm{Vol}(S^3)$$

$$e^{-\phi} = \frac{3\, L}{2\, W^5}\,, \qquad W = (m\, \cos\theta)^{-\frac{1}{6}}$$

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$$ds^2 = \frac{W^2 L^2}{4} \left[9\, ds^2(AdS_6) + 4\, d\theta^2 \right] + e^{-2A}\, dr^2 + \frac{r^2\, e^{2A}}{r^2 + e^{4A}}\, ds^2(S^2)$$

$$B_2 = \frac{r^3}{r^2 + e^{4A}}\, \text{Vol}(S^2) \qquad e^{-\phi} = \frac{3\,L}{2\,W^5}\, e^A \sqrt{r^2 + e^{4A}}$$

$$F_1 = -G_1 - m\, r\, dr \qquad F_3 = \frac{r^2}{r^2 + e^{4A}} \left[-r\, G_1 + m\, e^{4A}\, dr \right] \wedge \text{Vol}(S^2)$$

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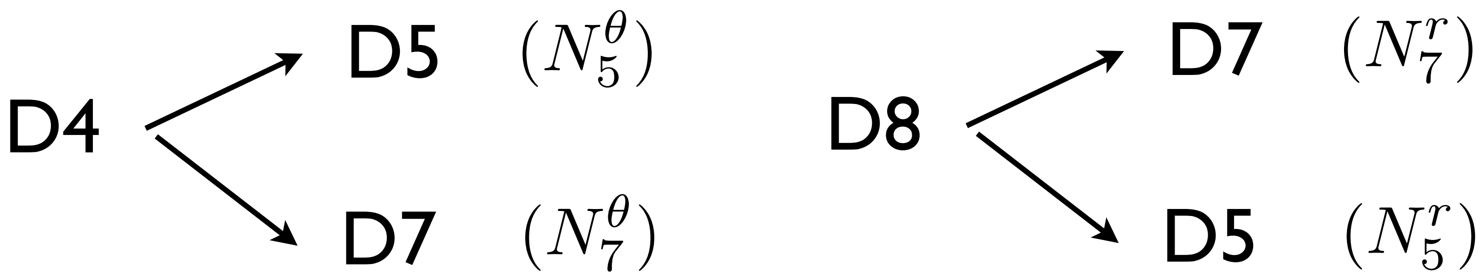
Then, non-Abelian T-duals of KW, KT, KS

(Itsios, Nuñez, Sfetsos, Thompson'12)

5. Hints on the 5d dual CFT

i) Quantization of charges:



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D4 → D5 (N_5^θ), D7 (N_7^θ)

D8 → D7 (N_7^r), D5 (N_5^r)

N_5^θ and N_5^r depend on the large gauge transf. of B_2:

$$B_2 = \left(\frac{r^3}{r^2+e^{4A}} - n\pi\right)\text{Vol}(S^2) \;\; / \;\; b = \frac{1}{4\pi^2}\int_{S^2} B_2 \in [0,1]$$

(which seems, on the other hand, to undergo something reminiscent of the cascade in KS), in such a way that they cannot be integers for all (r,θ) unless $n=0$

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Then: Only D7 color branes, $N_5^r \, , N_7^r$ related, $R = \pi$

ii) Probe the Coulomb branch:

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Fluctuations of these branes:

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$$S_{5d\, CS} = \frac{(2\pi)^3}{6} T_5 \int F_1 \int A \wedge F \wedge F = -\frac{N_7^r}{24\, \pi^2} \int A \wedge F \wedge F$$

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D7: Same with $N_7^r \leftrightarrow N_5^r$, D1 $\leftrightarrow$ D3 wrapped on S^2

ii) Baryon-like operators:

Dual to branes wrapped on the internal geometry with a tadpole proportional to the rank of the gauge group

In the D4-D8 background: D4-brane with N charge, projected out by the orbifold

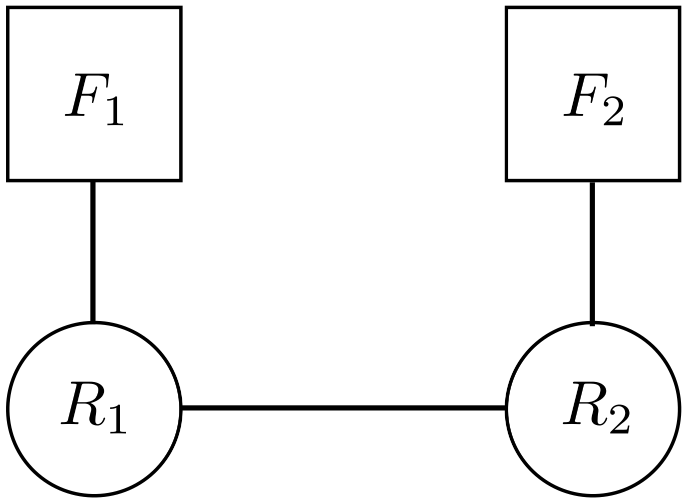
In the non-Abelian dual: D1-brane with N_7^θ charge plus D3-brane (wrapped on S^2) with N_5^θ charge

Projected out by the dual orbifold

In any case they inform about the ranks of the dual gauge groups

iv) Putting it all together:

We seem to have two gauge groups with ranks N_7^θ , N_5^θ
and flavor symmetries N_5^r , N_7^r



N_5^θ actually zero, such that the background is globally well defined

Manifestation in the CFT of a perfectly regular background terminating at a point?

6. Conclusions and open issues

- Could a clear prescription for global properties lead to a regular background for arbitrary large gauge transformations, with non depleted gauge groups in the dual CFT?

  Depletion of the gauge group reminiscent of the cascade or Higgsing sequence? (Aharony'01)

  Non-Abelian T-dual as an effective description?

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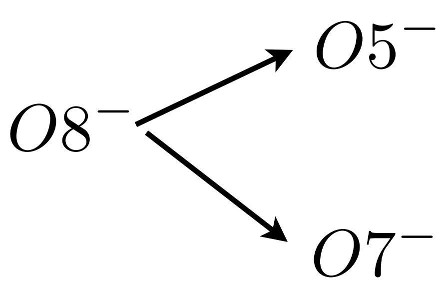
Non-Abelian T-dual as an effective description?

- Nature of the dual gauge groups: What is the orientifold projection in the dual theory?



$I_\theta \Omega \rightarrow I_\theta I_\chi \Omega$:

Dual Op^- located at $\theta = \frac{\pi}{2}, r = 0$

D5-D7 system?

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Thanks!