NON-SINGLET BARYONS IN GAUGE/GRAVITY DUALITY

Yolanda Lozano (U. Oviedo)

Branes and Symmetries:
A Scientific Commemoration of Laurent

Solvay Institutes, 5th November 2012

I will report on some recent work that explores the prediction of non-singlet baryons in gauge theories with a gravity dual

But first:

Scientific Commemoration of
Laurent

Colleague, collaborator,
but mostly,
a very good friend

I first met Laurent
in the Summer School
in Cargèse in 1995

Quite unexpectedly
this was the origin of
a very strong
friendship

Strengthened during
my postdoc in Utrecht

This friendship continued to grow in spite of the physical distance (London vs Santiago de Chile, Utrecht vs London, Geneva vs Milano, Oviedo vs Brussels)

We made the most of meetings and invited seminars. Remarkable ones were:

Kolymbari, Postdam, Cambridge, Geneva, London, Granada, Groningen.......................

Oviedo 2005

His visit to Oviedo right after his wedding

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Many more personal visits followed, with Barbara, and then Aitana and then Manuela

And a strong friendship with Barbara (and my future daughters in law) was also born

Scientifically Laurent and I worked together for a year on different aspects of non-BPS and brane-antibrane systems

1) Branes from unstable systems of branes
By Laurent Houart, Yolanda Lozano.
hep-th/0011285.
Fortsch.Phys. 49 (2001) 543-550.

2) Brane descent relations in M theory
By Laurent Houart, Yolanda Lozano.
hep-th/0001170.
Phys.Lett. B479 (2000) 299-307.

3) S duality and brane descent relations
By Laurent Houart, Yolanda Lozano.
hep-th/9911173.
JHEP 0003 (2000) 031.

4) Type II branes from brane - anti-brane in M theory
By Laurent Houart, Yolanda Lozano.
hep-th/9910266.
Nucl.Phys. B575 (2000) 195-210.

Non-singlet baryons in gauge/ gravity duality

Motivation:

Non-singlet baryons are predicted in N=4 SYM by the AdS/CFT correspondence. Their interpretation in the CFT is unclear.

Do they also exist in more realistic theories?

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How:

Analyze the holographic description in various backgrounds with reduced supersymmetries and/or confining

- Reduced SUSY: $AdS_5 \times Y_5$, Lunin-Maldacena β deformed, Frolov multi-β deformed

- Confining: Maldacena-Nuñez

Results:

- Non-singlet baryons exist in all these backgrounds
- Same number of quarks in all $AdS_5 \times Y_5$ Einstein manifolds with 5-form flux, independent of SUSY
- More restricted number of quarks in MN
- Stable against fluctuations
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(Based on arXiv:1203.6817, D. Giataganas, Y.L., M. Picos, K. Siampos, JHEP)

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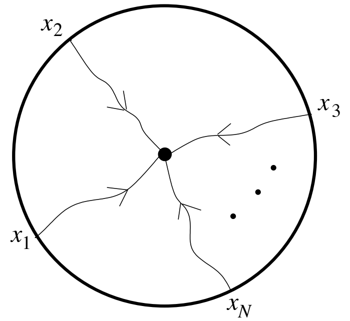

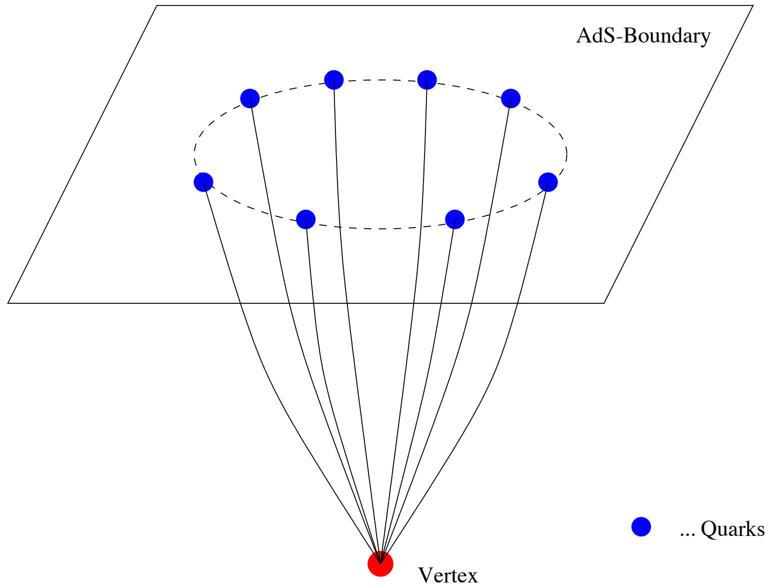

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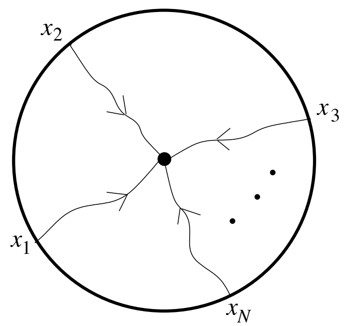

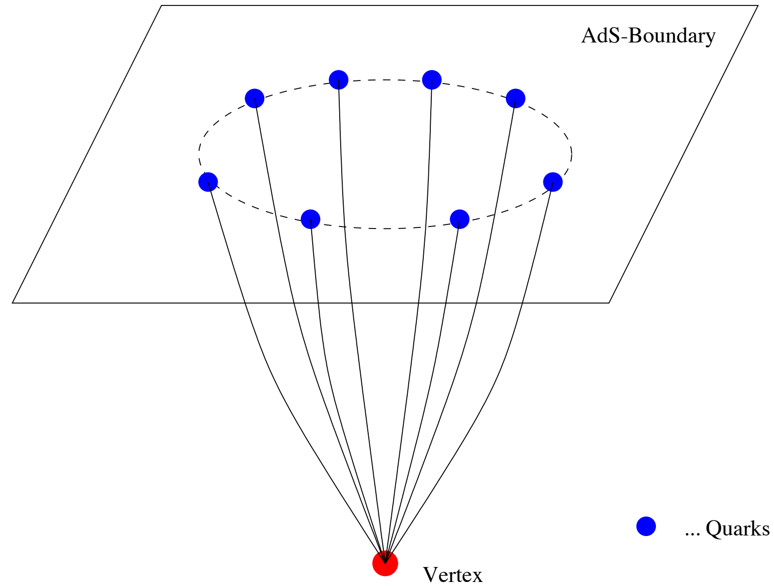

Baryon vertex in the gravity side: D5-brane wrapped on the 5-sphere (Witten'98):

$$S_{CS} = 2\pi T_5 \int_{\mathbb{R}\times S^5} P[F_5] \wedge A = N \int_{\mathbb{R}} dt A_t$$

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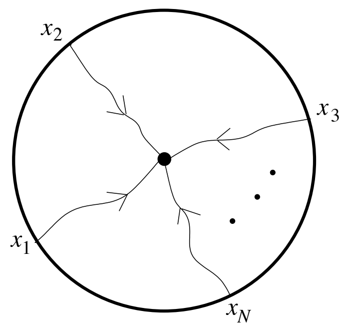

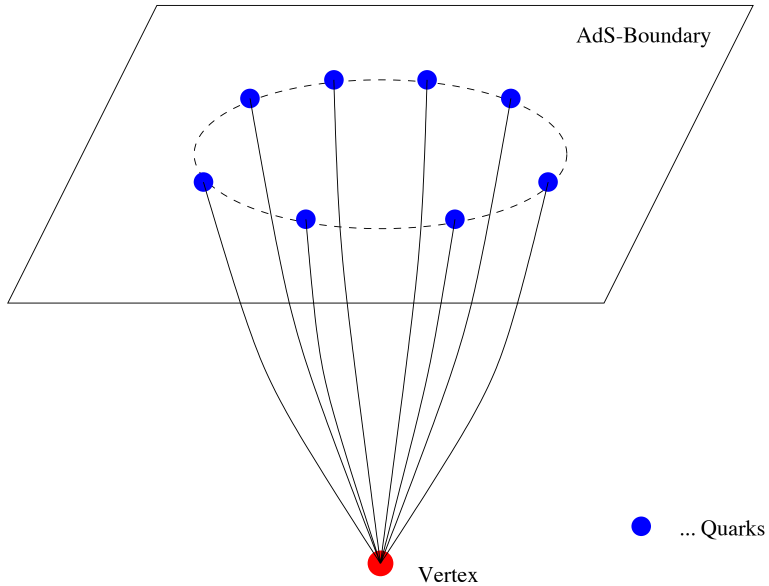

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N charge cancelled by N F-strings ending on the 5-brane

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In $AdS_5 \times S^5$: $5N/8 \leq k \leq N$

In $AdS_4 \times CP^3$: $2N/3 \leq k \leq N$ (Y.L., Picos, Sfetsos, Siampos'11)

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And at finite 't Hooft coupling?

2. Gauge/gravity calculation of the energy

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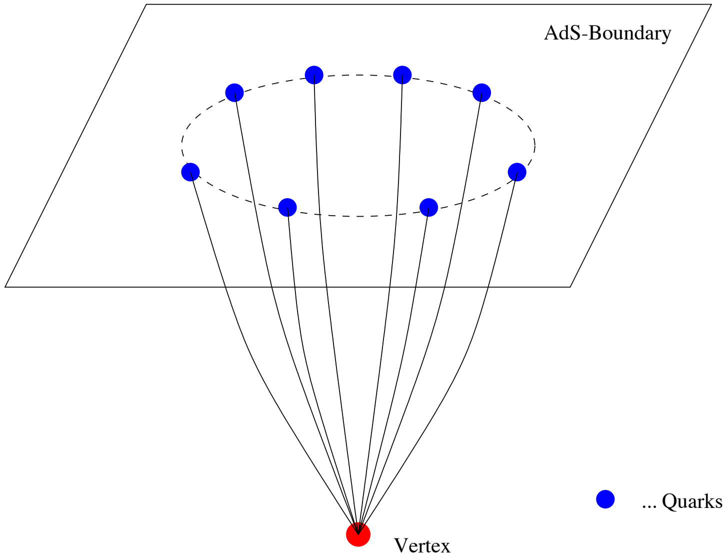

Consider a uniform distribution of strings on an $\mathbb{M}_p$ shell

Non-SUSY but we can ignore the backreaction

In the probe brane approach: $S = S_{Dp} + S_{NF1}$:

$$S_{NF1} = -N\, T_{F1} \int dt\, dr \sqrt{|\mathrm{det} P(G)|}$$

with $\tau = t \quad \sigma = r$ and the AdS direction $\rho = \rho(r)$

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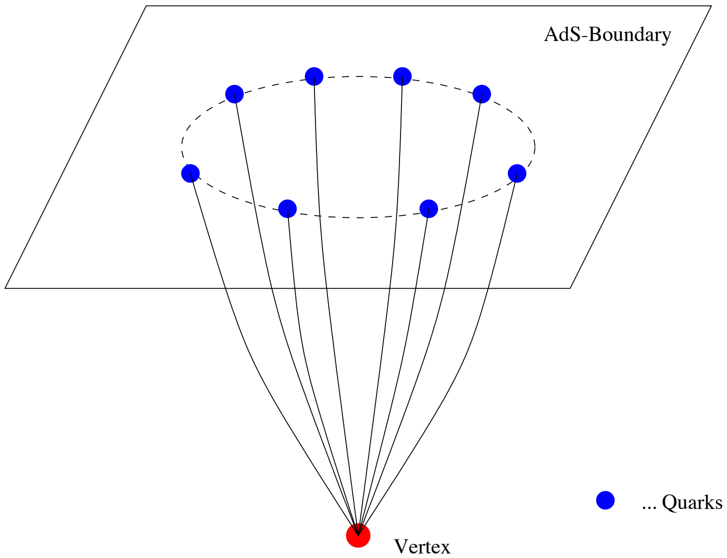

$$\rho(0) = \rho_0$$
$$\rho(L) = \infty$$

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$$S_{Dp} = -T_p \int_{\mathbb{R}\times\mathbb{M}_p} d^{p+1}\xi \sqrt{|\det P(G + 2\pi F - B)|}$$

In $AdS_5 \times Y_5$:

$$ds^2 = \frac{\rho^2}{R^2} dx_{1,3}^2 + \frac{R^2}{\rho^2} d\rho^2 + R^2 ds_{Y_5}^2$$

$$R^4 = \frac{4\pi^4 N g_s}{\mathrm{Vol}(Y_5)}, \qquad F_5 = 4\, R^4 (1 + *) d\mathrm{Vol}(Y_5)$$

Bulk equation of motion: $$\frac{\rho^4}{\sqrt{\frac{\rho^4}{R^4} + \rho'^2}} = c$$

Boundary equation of motion: $$\frac{\rho_0{}'}{\sqrt{\frac{\rho_0^4}{R^4} + \rho_0{}'^2}} = \frac{T_5\, R^4\, \mathrm{Vol}(Y_5)}{N T_{F1}}$$

Define $\sqrt{1-\beta^2} = \frac{T_5\, R^4\, \mathrm{Vol}(Y_5)}{N T_{F1}}$ with $\beta \in [0,1]$

The two equations can be combined into:

$$\frac{\rho^4}{\sqrt{\frac{\rho^4}{R^4} + \rho'^2}} = \beta\, \rho_0^2\, R^2$$

Integrating: Size of the configuration:

$$\boxed{L = \frac{R^2}{\rho_0} \int_1^\infty dz \frac{\beta}{z^2 \sqrt{z^4 - \beta^2}}}$$

On-shell energy:

$$E = E_{Dp} + E_{NF1} = N T_{F1} \rho_0 \Big(\sqrt{1-\beta^2} + \int_1^\infty dz \frac{z^2}{\sqrt{z^4 - \beta^2}} \Big)$$

Binding energy:

$$E_{\rm bin} = N T_{F1} \rho_0 \Big(\sqrt{1-\beta^2} + \int_1^\infty dz \Big[\frac{z^2}{\sqrt{z^4-\beta^2}} - 1 \Big] - 1 \Big)$$

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As a function of L :

$$\boxed{E_{\rm bin} = -f(\beta)\frac{\sqrt{\lambda}}{L}}$$

with $f(\beta) \geq 0$

$\Rightarrow$ - The configuration is stable

- $E_{\rm bin} \sim 1/L$ dictated by conformal invariance
- $E_{\rm bin} \sim \sqrt{\lambda}$ non-trivial prediction for the non-perturbative regime of the gauge theory

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Confining background?

Maldacena-Nuñez: $E_{\rm bin} \sim L$

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Non-singlets?

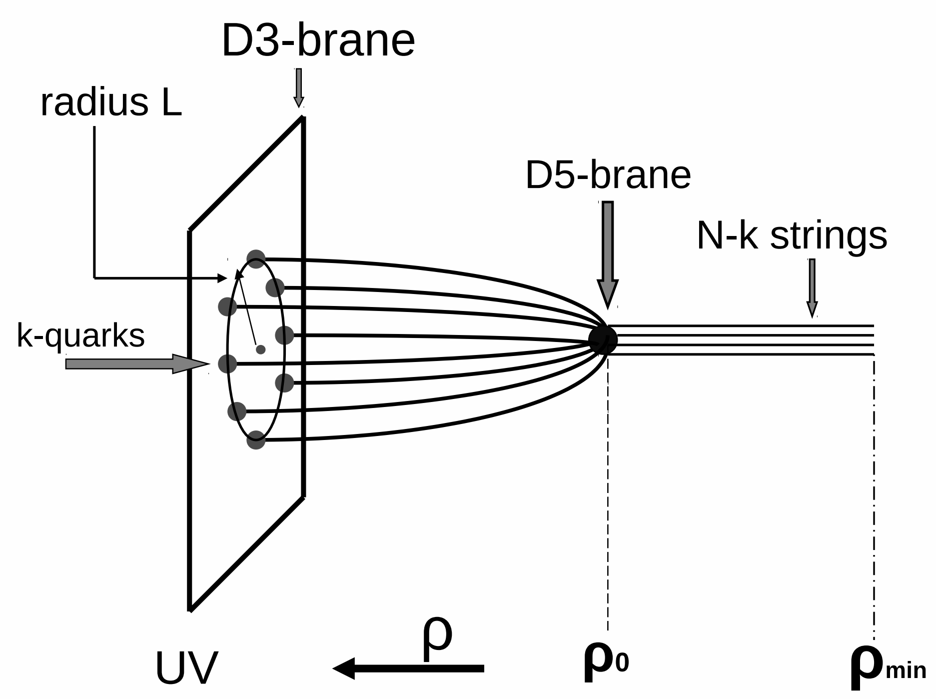
3. Reduce the number of quarks

In $AdS_5 \times S^5$: Baryon vertex classical solutions with number of quarks $5N/8 \leq k \leq N$ (non-singlet)

(Brandhuber, Itzhaki, Sonnenschein, Yankielowitz'98; Imamura'98)

Stable against fluctuations for $0.813\, N \leq k \leq N$

(Sfetsos, Siampos'08)



3.1. The classical solution

The boundary equation of motion changes:

$$\frac{\rho_0'}{\sqrt{\frac{\rho_0^4}{R^4} + {\rho_0'}^2}} = \frac{T_5\, R^4\, \mathrm{Vol}(Y_5)}{k\, T_{F1}} + \frac{N-k}{k} \leq 1$$

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$\Rightarrow$ Non-singlet states in non-SUSY or confining backgrounds

3.2. Stability analysis

Important in establishing the physical parameter space
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Ansatz for the fluctuations (for the strings):

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Expand the Nambu-Goto action to quadratic order and study the zero mode problem $\leftrightarrow$ Critical curve in the parametric space separating the stable and unstable regions

Stability reduced to an eigenvalue problem of the general Sturm-Liouville type

Instabilities emerge from longitudinal fluctuations of the strings

For $AdS_5 \times Y_5$ and beta deformed:

Bound for the number of F-strings coming from stability:

$$k \geq \frac{N}{1+\gamma_c}(1+\sqrt{1-\beta^2}) \qquad \gamma_c = 0.538$$

More restrictive than the bound imposed by the existence of a classical solution:

$$k \geq \frac{N}{2}(1+\sqrt{1-\beta^2})$$

For MN: Same bound for stability and existence of classical sol.

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Can we reach the finite 't Hooft coupling region?

5. Baryons at finite ‘t Hooft coupling

Generalize the baryon vertex adding a magnetic flux.

A non-trivial flux adds lower dim brane charges $\rightarrow$
Complementary description of the baryon in terms of
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- Complementary at finite n. Should agree at large n

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The bound for the number of quarks is modified ($(k, \mathcal{N})$ parameter space bounded by the values for which the baryon vertex reduces to free quarks):

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Similar micro analysis in MN in terms of D1's expanding into a fuzzy S^2

6. Conclusions

- Non-singlet baryons are predicted by gauge/gravity duality in less supersymmetric and/or confining backgrounds

- They are stable against fluctuations

- At finite ‘t Hooft coupling:

  Microscopical description of the vertex

  Complete this analysis with α' corrections to the NG action of the strings (or microscopic spike), α' corrections to the background

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Thanks!