# Duality in String Theory

Yolanda Lozano (U. Oviedo)

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Duality: New insight into the non-perturbative regimes of both String Theory and QFT

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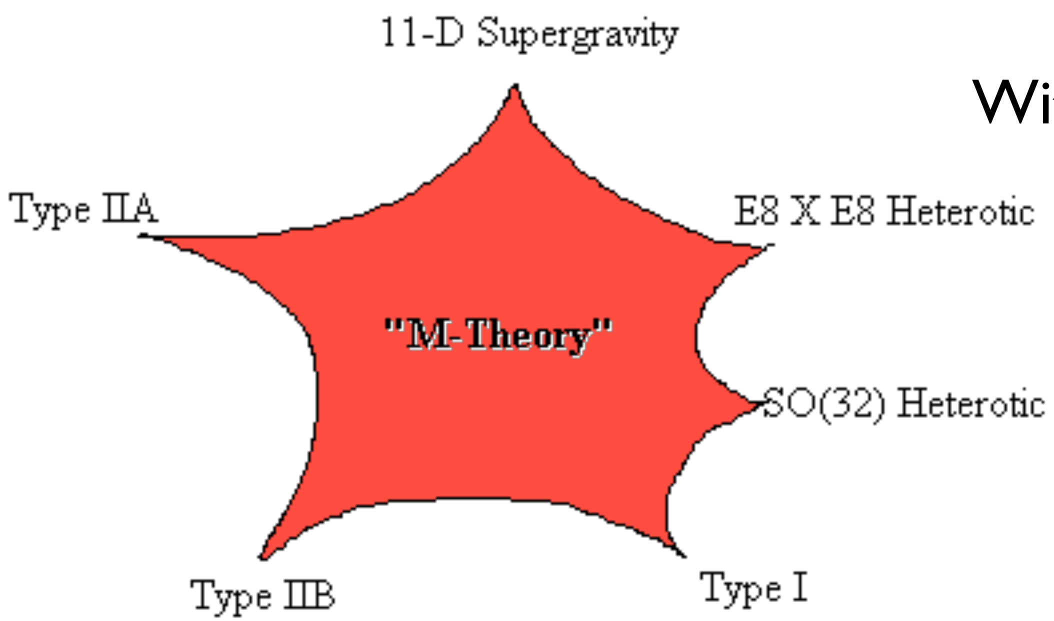
- In String Theory: T- and S-dualities




Witten'95

M-theory: Fully non-perturbative formulation of superstring theory. Unifies gravity with particle physics interactions in a unique way.

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Gauge/gravity duality (Maldacena'97):

Some gauge theories at strong (weak) coupling are dual to weakly (strongly) coupled string theories with one extra dimension

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Holographic duality

One of the most innovative ideas in Theoretical Physics in the recent past:

Possibility to apply string theory techniques to strongly coupled systems (quark-gluon plasma, condensed matter systems).

Possibility to learn something about quantum gravity by studying low dimensional systems with a holographic description (concrete realization of QG and space-time as emergent phenomena).

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## ◆ T-duality:

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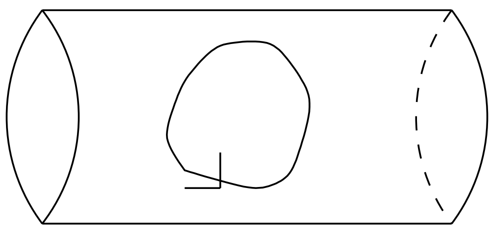
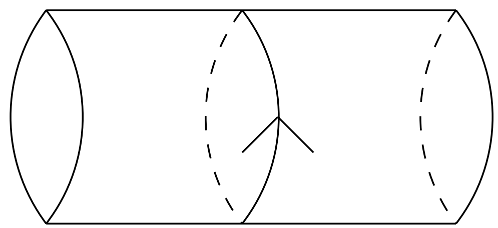
In toroidal compactifications: $x \to x + 2\pi n R$



Momentum states:

$$p = \frac{m}{R}$$



Winding states:

$$p = \frac{nR}{\alpha'}$$

$\alpha' = l_s^2\,;\ l_s$ : string length  ( $l_s \to 0$ Field Theory limit)

$$\boxed{\begin{array}{l} R \to \dfrac{\alpha'}{R} \\ m \leftrightarrow n \end{array}} :$$

Symmetry of the bosonic string, maps Type IIA and Type IIB and Het SO(32) and Het $E_8 \times E_8$

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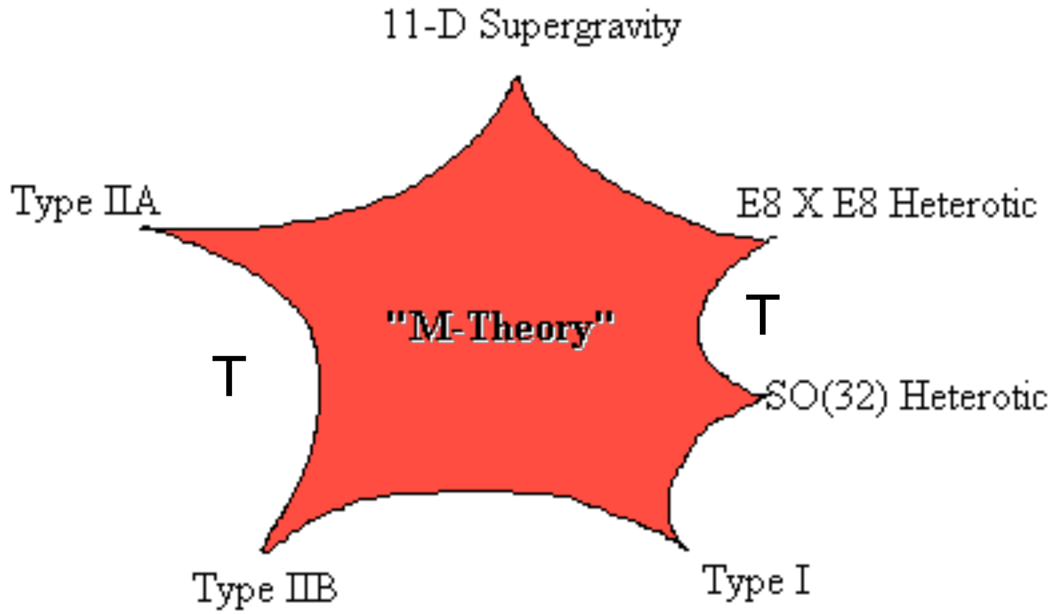

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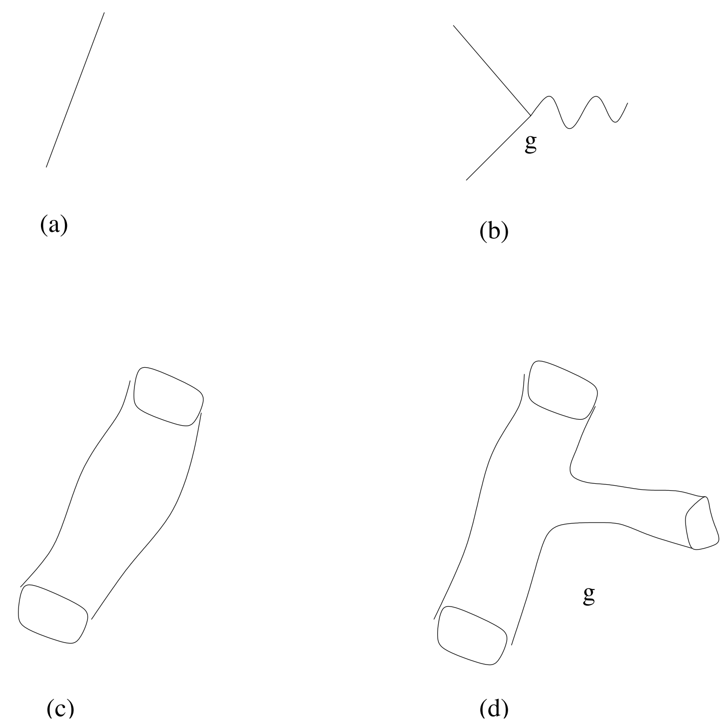
: Symmetry of the bosonic string, maps Type IIA and Type IIB and Het SO(32) and Het $E_8 \times E_8$

$$g_{st} \to \left(\frac{\sqrt{\alpha'}}{R}\right)^d g_{st}$$

$g_{st}$ : String coupling constant

$g_{st} = e^{\langle \phi \rangle}$ ; $\phi =$ dilaton (massless field)



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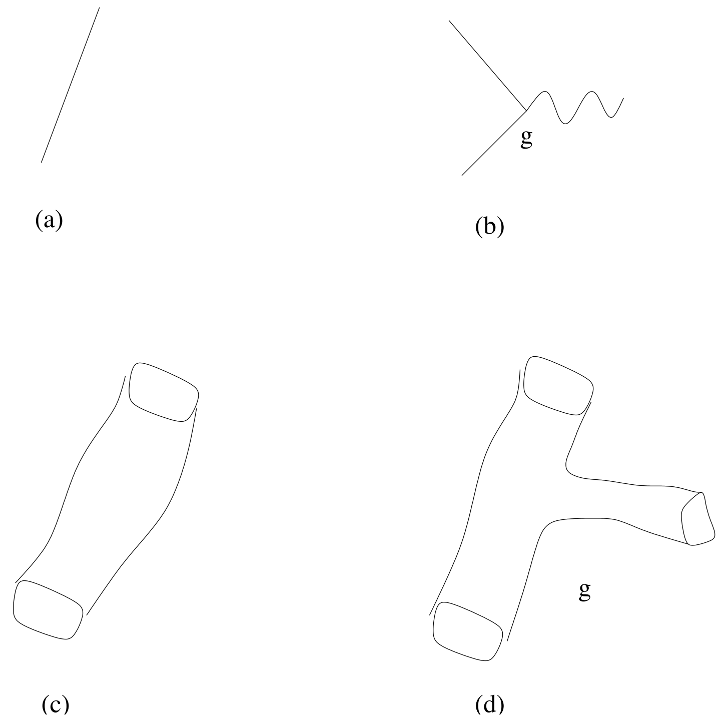


For more general compactifications:
Abelian and non-Abelian T-duality (see later)

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- In a four dimensional Abelian Gauge Theory:
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$$\boxed{g \to \frac{1}{g} \quad + \quad \begin{array}{c} \vec{E} \to \vec{B} \\ \vec{B} \to -\vec{E} \end{array}} \to \quad \text{Symmetry}$$

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In the presence of a $\theta$ term $i\,\theta\, F \wedge F$ :

$$\tau = \frac{\theta}{2\pi} + \frac{i}{g^2} \quad \text{transforms as} \quad \tau \to -\frac{1}{\tau}$$

Together with $\tau \to \tau + 1$ they generate the $SL(2, \mathbb{Z})$ S-duality group

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Look at the BPS states (special class of supersymmetric states with the key property that their mass is completely determined by their charge) $\Rightarrow$ Stable and protected from quantum corrections

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To test a non-perturbative duality relation:

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i) Study their properties perturbatively at weak coupling in one theory
ii) Extrapolate (safely) to strong coupling
iii) Reinterpret in terms of non-perturbative configurations in the dual theory

- In String Theory:

S-duality transformation:

$$g_{st} \to \frac{1}{g_{st}} \quad + \quad \begin{array}{c} B_2 \to C_2 \\ C_2 \to -B_2 \end{array}$$

( $B_2$ : NS-NS 2-form, $C_2$ : RR 2-form)

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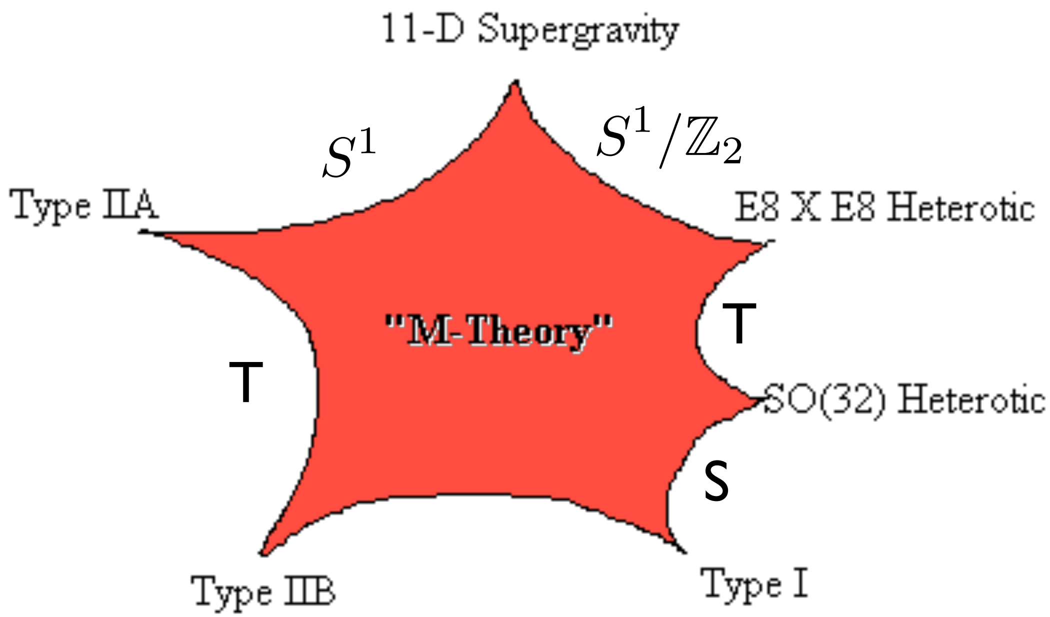
In the presence of a $C_0$ RR field:

$\lambda = C_0 + \frac{i}{g_{st}}$ transforms as $\lambda \to -\frac{1}{\lambda}$

Together with $\lambda \to \lambda + 1$ they generate the $SL(2,\mathbb{Z})$ S-duality group

Symmetry of Type IIB, maps Type I with Heterotic SO(32).
Type IIA and Heterotic $E_8 \times E_8$ give different compactifications of M-theory in the strong coupling limit



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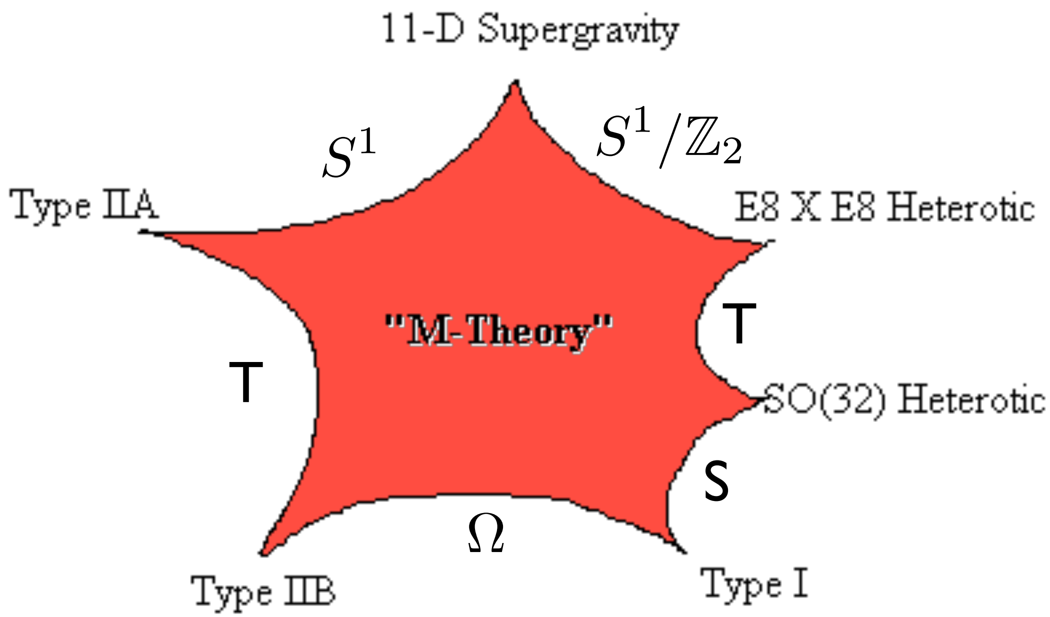

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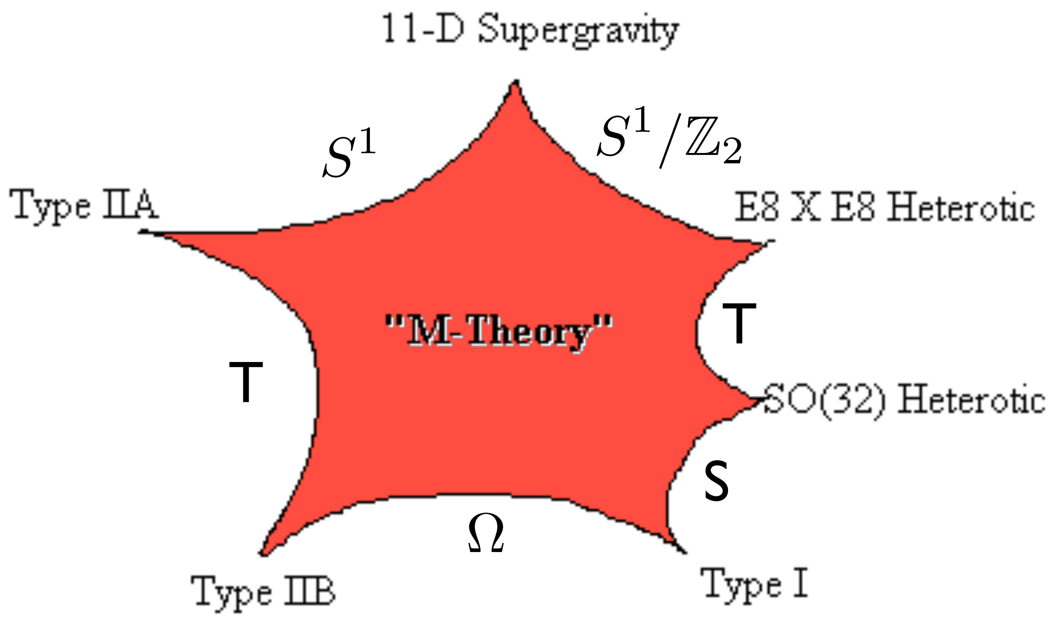


To get deeper into these dualities in String Theory:

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All these branes are BPS objects, satisfying $q = T$:

$$T_{F1} = \frac{1}{l_s^2}\,; \qquad T_{Dp} = \frac{1}{l_s^{p+1}}\frac{1}{g_{st}}\,; \qquad T_{NS5} = \frac{1}{l_s^6}\frac{1}{g_{st}^2}$$

(non-perturbative)

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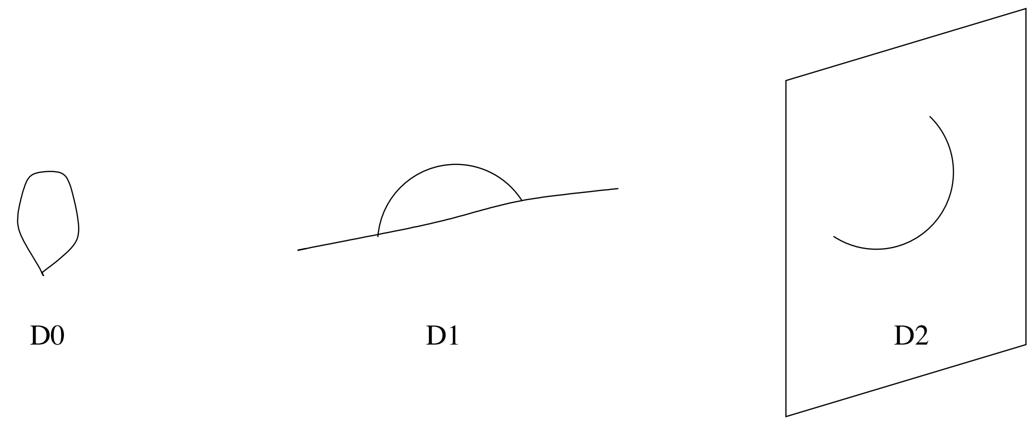

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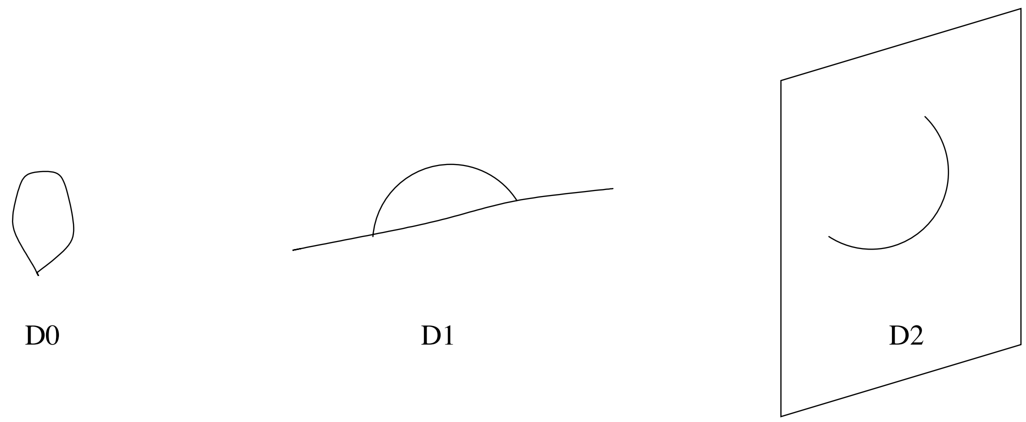


The important implication is that their dynamics can now be determined at weak coupling using open string perturbation theory!

For ex. conformal invariance of the worldsheet theory $\leftrightarrow$ Equations of motion of $S_{\rm eff} = S_{\rm bulk} + S_{\rm D-brane}$ with $S_{\rm D-brane}$ a U(1) gauge theory (U(N) for a system of N D-branes)

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Type I and Heterotic SO(32) contain as well an SO(32) N=1 SYM vector multiplet ($E_8 \times E_8$ for Heterotic $E_8 \times E_8$)

# Now we can go back to S-duality:

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- It relates Type I and Heterotic SO(32):

| $C_2$ | | $B_2$ |
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| D1 | $\to$ | F1 |
| D5 | | NS5 |

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| D1 | $\to$ | F1 |
| D5 | | NS5 |

The strong coupling limit of Type IIA and Heterotic $E_8 \times E_8$ is M-theory (more difficult to see..).

# 3. The gauge/gravity duality

Maldacena'97:

Equivalence between Type IIB string theory on $AdS_5 \times S^5$ and 4 dim $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills theory

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$AdS_5$ : anti-de Sitter space in 5 dim (maximally symmetric solution of the Einstein eqs. with a negative cosmological constant)

First piece of evidence:

The symmetry group of 5 dim anti-de Sitter space matches precisely with the group of conformal symmetries of $\mathcal{N} = 4$ SYM $\Rightarrow$ AdS/CFT

Starting point: Study of N coincident D3-branes in Type IIB in the large N limit, based on two dual descriptions:

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The U(N) gauge theory living on the branes becomes 4 dim $\mathcal{N} = 4$ SYM, with $g_{YM}^2 = g_s$ :

Conformally invariant

Topological large N expansion $\sum_{g=0}^{\infty} N^{2-2g} f_g(\lambda)$ with 't Hooft parameter $\lambda = g_{YM}^2 N$

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Through the correspondence: $R^4 = 4\pi\lambda\, l_s^4$

$\lambda = g_s N \qquad R^4 = 4\pi\lambda\, l_s^4$ imply

$N \to \infty$, finite $\lambda \Leftrightarrow g_s \to 0 \Rightarrow$ Planar limit $\leftrightarrow$ Classical limit of IIB ST

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Many tests:

For each field in the 5 dim bulk $\leftrightarrow$ Operator in the dual CFT
D-branes $\leftrightarrow$ Baryons, domain walls, etc..
...

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$Y_5 \leftrightarrow C_6$ with $C_6$ Ricci-flat such that $ds^2_{C_6} = dr^2 + r^2 ds^2_{Y_5}$

SUSY preserved $\Leftrightarrow$ $Y_5$ is Sasaki-Einstein ( $\Leftrightarrow C_6$ is Ricci-flat and Kähler (Calabi-Yau 3-fold))

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Examples: $Y_5 = S^5$, $\quad C_6 = \mathbb{C}^6$

$$Y_5 = T^{1,1} = \frac{SU(2) \times SU(2)}{U(1)},$$ $C_6$ is the conifold

$Y_5 = Y^{p,q}$ , $p, q$ coprime integers (includes quasi-regular and irregular SE manifolds)

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$\leftrightarrow$ Dual to N=1 SCFTs arising from a stack of D3-branes sitting at the tip of the CY cone

# 4. More on T-duality

4.1. D-branes from open strings
4.2. T-duality for more general backgrounds
4.3. Non-Abelian T-duality

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## 4.1. D-branes from open strings

Closed strings in a toroidal compactification:

Solution of the equations of motion: $\partial_+\partial_- X^i = 0$ **+**
boundary condition $X^i(\tau, \sigma + 2\pi) = X^i(\tau, \sigma) + 2\pi n R$ :

$$X^i(\tau, \sigma) = X_0^i + \alpha' \frac{m}{R} \tau + n R \sigma + ..$$

T-duality transformation:

$$\boxed{\begin{array}{l} R \to \dfrac{\alpha'}{R} \\ m \leftrightarrow n \end{array}} \Leftrightarrow \boxed{\begin{array}{l} \partial_+ X^i \to \partial_+ X^i \\ \partial_- X^i \to -\partial_- X^i \end{array}}$$

($d \to *d$ : Hodge duality)

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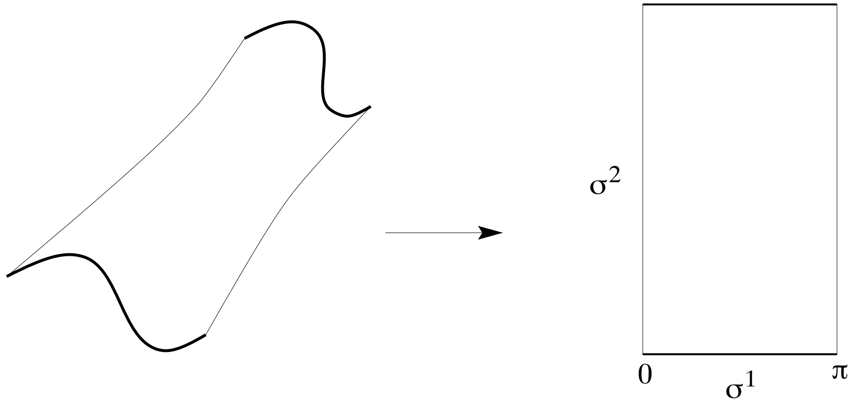
For open strings: $\sigma \in [0, \pi]$




Boundary:

$$\partial\Sigma = \{\sigma = \text{constant}\}$$

Neumann boundary conditions: $\partial_\sigma X^\mu|_{\partial\Sigma} = 0$
(zero momentum flux at the end points)

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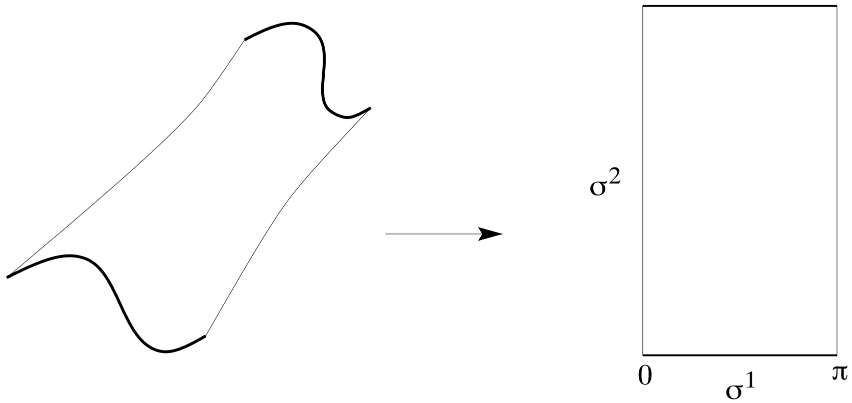


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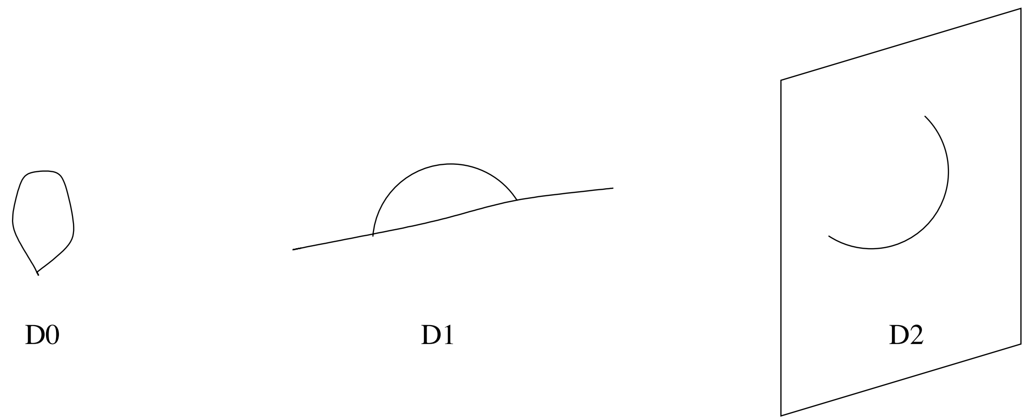
$$\downarrow$$

Dirichlet boundary conditions: $\dot{X}^i|_{\partial\Sigma} = 0\,;\ i = p+1, \ldots, 9$

$$X^\mu = \{X^m, X^i\} \quad m = 0, \ldots, p$$ **non-compact**

The end-points of the strings are fixed on a (p+1) dimensional hypersurface $\equiv$ Dp-brane



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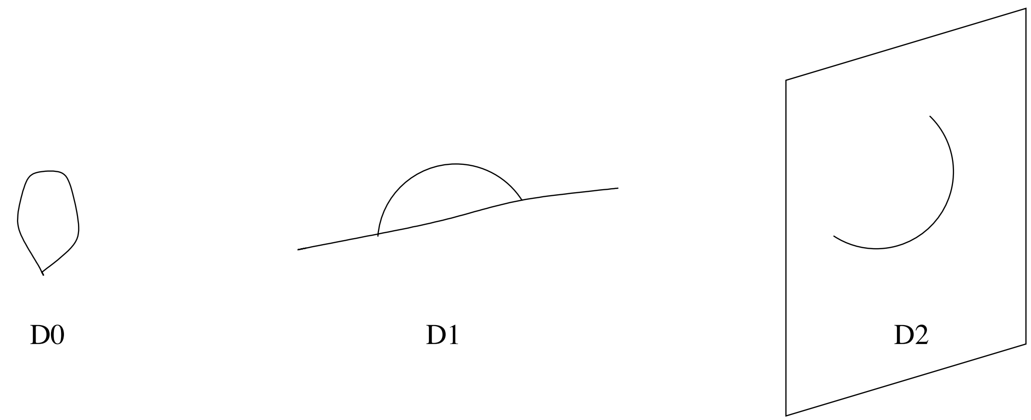


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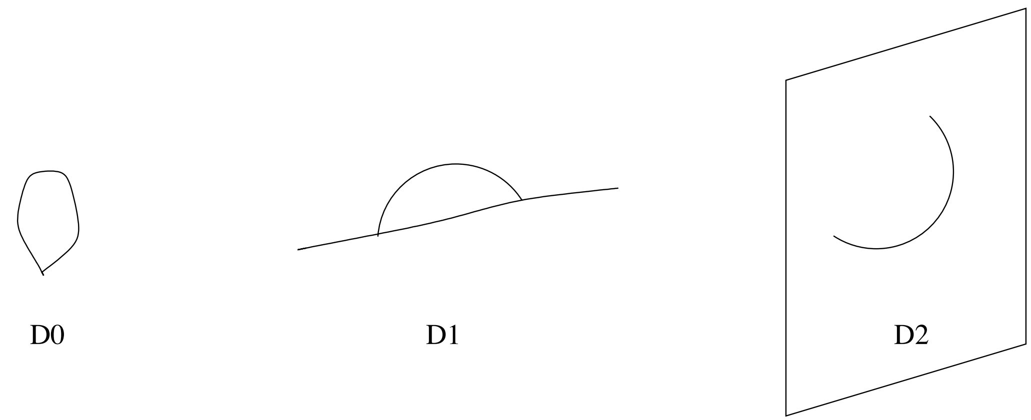


Open strings $\xrightarrow{\text{T-duality}}$ Dp-branes

Dynamical due to the open strings attached

# 4.2. T-duality for more general backgrounds

(Buscher'88; Rocek, Verlinde'92)

The fundamental string couples to the metric, NS-NS 2-form and dilaton through a non-linear sigma-model:

$$S = \frac{1}{4\pi\alpha'} \int \Big( g_{\mu\nu}\, dX^\mu \wedge *dX^\nu + B_{\mu\nu}\, dX^\mu \wedge dX^\nu \Big) + \frac{1}{4\pi} \int R^{(2)} \phi$$

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In the presence of an Abelian isometry: $\delta X^\mu = \epsilon\, k^\mu$ /

$$\mathcal{L}_k g = 0\,,\ \mathcal{L}_k B = d\omega\,,\ i_k d\phi = 0$$

i) Go to adapted coordinates: $X^\mu = \{\theta, X^\alpha\}$ such that

$\theta \to \theta + \epsilon$ and $\partial_\theta(\mathrm{backgrounds}) = 0$

ii) Gauge the isometry: $d\theta \to D\theta = d\theta + A$

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iv) Integrate the gauge field

+ fix the gauge: $\theta = 0 \;\to$ Dual sigma model:

$\{\theta, X^\alpha\} \to \{\tilde{\theta}, X^\alpha\}$ and

$$\tilde{g}_{00} = \frac{1}{g_{00}}; \quad \tilde{g}_{0\alpha} = \frac{B_{0\alpha}}{g_{00}}; \quad \tilde{g}_{\alpha\beta} = g_{\alpha\beta} - \frac{g_{0\alpha}g_{0\beta} - B_{0\alpha}B_{0\beta}}{g_{00}}$$

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$$\tilde{\phi} = \phi - \log g_{00}$$

Buscher's formulae

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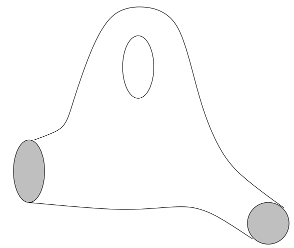
- Conformally invariant (to first order in $l_s^2$)
- Involutive transformation: $\tilde{S} \xrightarrow[\tilde{\theta} \to \tilde{\theta} + \epsilon]{} S$
- $(M, \tilde{M})$ different geometries and topologies

For ex. $M = S^3\,; \quad \tilde{M} = S^2 \times S^1$

(Alvarez, Alvarez-Gaumé, Barbón, Y.L.'93)

- Arbitrary worldsheets? (symmetry of string perturbation theory):


(a)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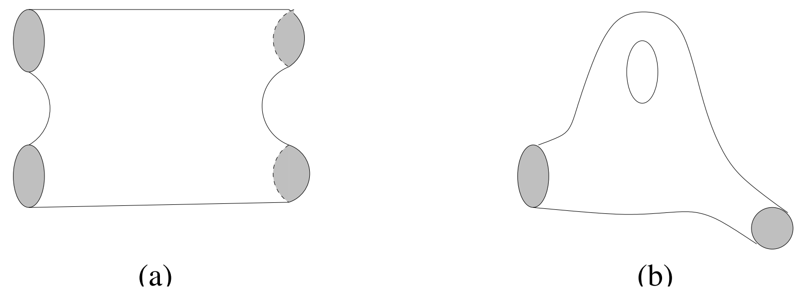


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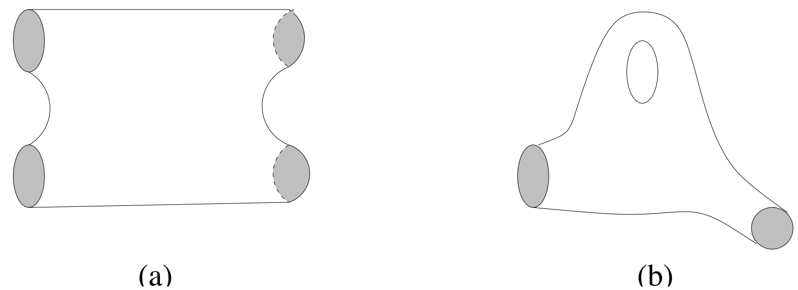


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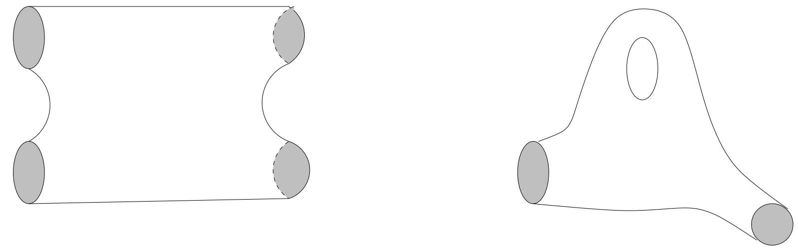

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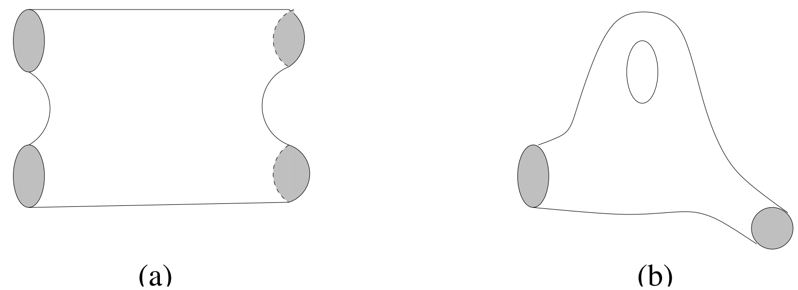


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$\Rightarrow$ The gauging procedure works for all genera

(Alvarez, Alvarez-Gaumé, Barbón, Y.L.'93)

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In this set-up:

$\partial_+ X^i \to \partial_+ X^i$

$\partial_- X^i \to -\partial_- X^i$

is generalized to

$$\boxed{\begin{aligned} \partial_+\tilde{\theta} &= g_{00}\,\partial_+\theta + (g_{0\alpha} - B_{0\alpha})\,\partial_+ X^\alpha \\ \partial_-\tilde{\theta} &= -(g_{00}\,\partial_-\theta + (g_{0\alpha} + B_{0\alpha})\,\partial_- X^\alpha) \end{aligned}}$$

For the open strings:

Neumann boundary conditions are generalized to

$$g_{00}\theta' + g_{0\alpha}X^{\alpha\prime} - B_{0\alpha}\dot{X}^{\alpha} = 0$$

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Suitable for the study of boundary conditions (also fermions)

Quantum mechanically: $H\, e^{i\mathcal{F}} = \tilde{H}\, e^{i\mathcal{F}}$ implies:

$$\psi_k[\tilde{\theta}] = N(k) \int \mathcal{D}\theta(\sigma)\, e^{i\mathcal{F}[\theta,\tilde{\theta}]}\, \phi_k[\theta(\sigma)]$$

In our case:

$$\mathcal{F} = \frac{1}{2} \oint_{S^1} d\sigma(\theta'\tilde{\theta} - \theta\tilde{\theta}')$$

- Derive global properties
- Arbitrary genus Riemann surfaces

# 4.3. Non-Abelian T-duality

(De la Ossa, Quevedo'93; Alvarez, Alvarez-Gaume, Y.L.'95)

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$A \in$ Lie algebra of $G$ $\quad A \to g(A+d)g^{-1}$

ii) Add a Lagrange multiplier term: $\mathrm{Tr}(\chi F)$

$F = dA - A \wedge A$

$\chi \in$ Lie Algebra of $G$, $\chi \to g \chi g^{-1}$, such that

$\int \mathcal{D}\chi \ \to \ F = 0 \ \Rightarrow \ A$ exact
(in a topologically trivial worldsheet)

+ fix the gauge: $A = 0 \Rightarrow$ Original theory

iii) Integrate the gauge field + fix the gauge $\rightarrow$ Dual theory

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General properties:

- Loss of symmetries in the dual $\Rightarrow$ Non-involutive
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Example: Principal chiral model with group SU(2):

Geometrically: $S^3$

$$\boxed{L = Tr(g^{-1}dg \wedge *g^{-1}dg)\,;\;\; g \in SU(2)}$$

Invariant under:

$$g \rightarrow h_1\, g\, h_2\,;\;\; h_1, h_2 \in SU(2)$$

Choose: $\quad g \rightarrow hg\,;\;\; h \in SU(2)$

$$\boxed{\tilde{L} = \frac{1}{1+\chi^2}\Big(\delta_{ij} - \epsilon_{ijk}\chi^k + \chi_i\chi_j\Big)d\chi^i \wedge *d\chi^j}$$

Invariant under $\chi \to h\chi h^{-1}\,;\; h \in SU(2)$

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True symmetry in String Theory?

Partial results about its realization as a canonical transformation: (Y.L.'95)

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For principal chiral models:

$$L = Tr(g^{-1}dg \wedge *g^{-1}dg)\,; \;\; g \in G$$

Use Maurer-Cartan forms: $\boxed{g^{-1}dg \equiv \Omega_a(\theta)d\theta^a}$

Then:

$$H = \frac{1}{4}\omega^{ak}\omega^{bk}\Pi_a\Pi_b + \Omega_a^k\Omega_b^k\theta^{a\prime}\theta^{b\prime}$$

$$\Omega_a = \Omega_a^k T^k, \qquad \omega^{ak}(\theta)\Omega_a^i(\theta) = \delta^{ki}$$

# The generating functional

$$\mathcal{F}[\chi,\theta] = \oint_{S^1} d\sigma\, \mathrm{Tr}\left(\chi\, \Omega_a(\theta)\partial_\sigma \theta^a\right)$$

generates the canonical transformation that gives the dual:

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In this approach:

- Involutive
- Generalize to arbitrary genus Riemann surfaces
- Global properties

Interesting as a solution generating technique (Sfetsos, Thompson'10; Y.L., O Colgain, Sfetsos, Thompson'11; Itsios, Y.L., O Colgain, Sfetsos'12):

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$$ds^2 = ds^2(AdS_5) + ds^2(S^5)$$

$$ds^2(S^5) = 4(d\theta^2 + \sin^2\theta d\phi^2) + \cos^2\theta\, ds^2(S^3)$$

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Dual Type IIA background:

$$d\tilde{s}^2 = ds^2(AdS_5) + 4(d\theta^2 + \sin^2\theta d\phi^2) + \frac{dr^2}{\cos^2\theta} + \frac{r^2\cos^2\theta}{\cos^4\theta + r^2} ds^2(S^2)$$

$$F_2\,, F_4\,, F_6\,, F_8 \neq 0$$

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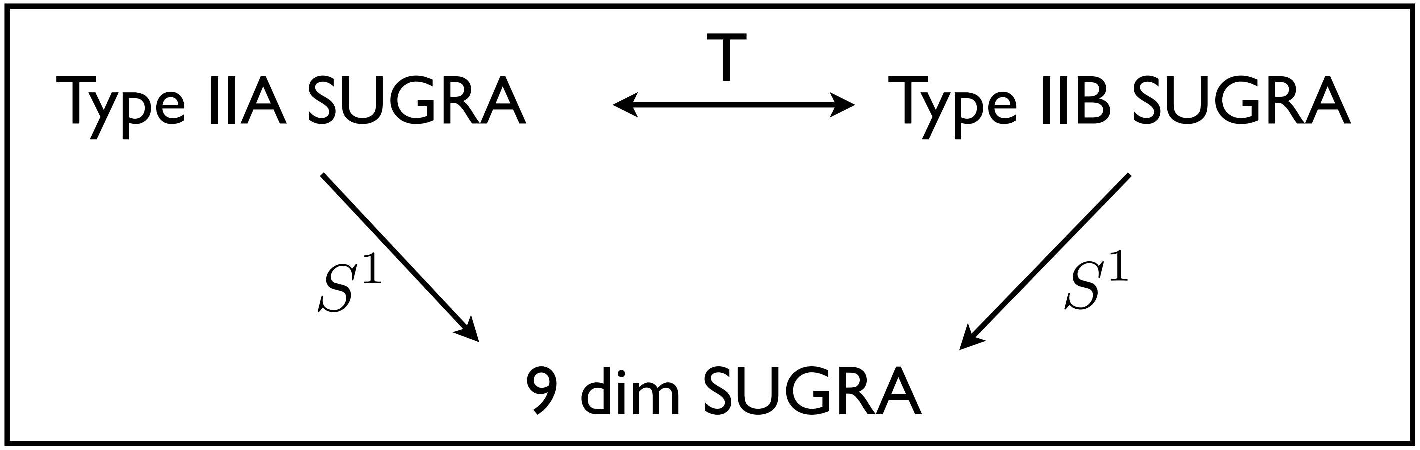

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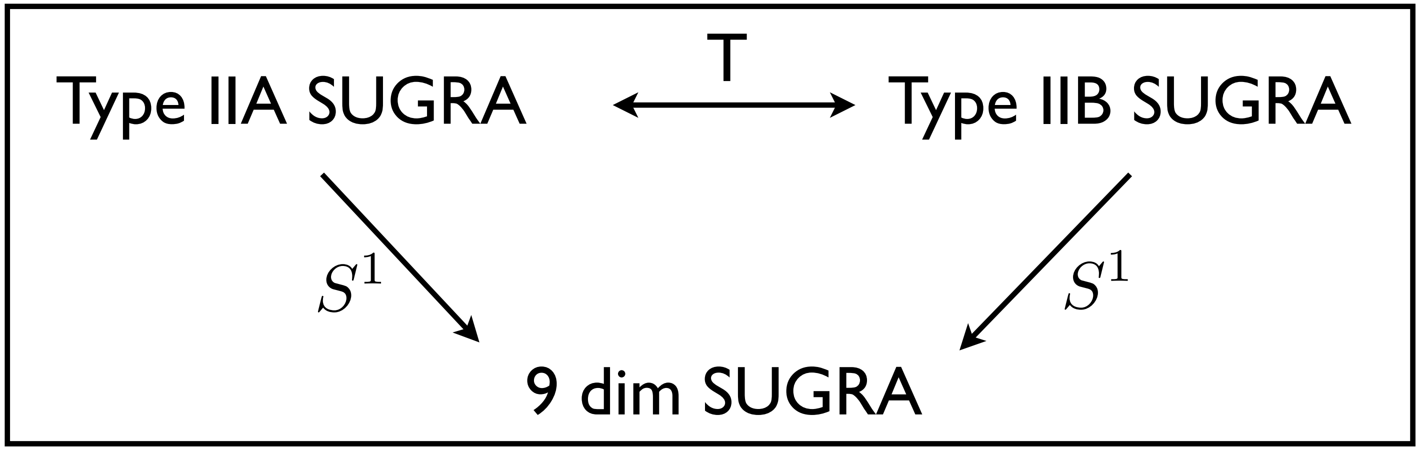
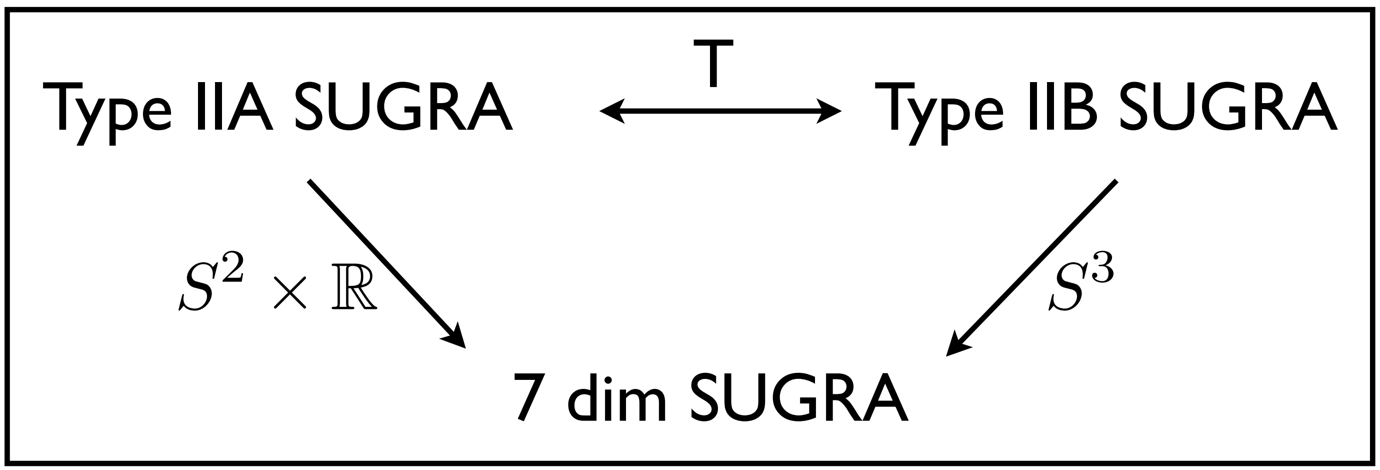



**Non-Abelian**



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Look at the worldvolume of D-branes

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(Y.L.'95; 96)

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# Thanks!

## Towards the formulation of M-theory

(See M.P. García del Moral's talk)

## Extensions and applications of gauge/gravity duality

Recent developments:

**Double field formalism:** Reformulate ten dim SUGRA such that the T-duality group is manifest:

Combine string fields and their T-duals: $dX^I = (dX^\mu, d\tilde{X}^\mu)$

Expand the tangent space from $T\Lambda^1(M)$ to $T\Lambda^1(M) \oplus T\Lambda^{*1}(M)$

Metric on this generalized space transforms under $O(d,d)$

$\rightarrow$ **Doubled geometry**

Mathematics of doubled geometry being developed

Towards the formulation of M-theory

(See M.P. García del Moral's talk)

Extensions and applications of gauge/gravity duality

Thanks!