Giant Gravitons and gauge/gravity duality

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- Motivation: Giant gravitons are useful states in which to test the AdS/CFT correspondence $\Rightarrow$ Use them to test the novel AdS_4/CFT_3 duality of ABJM

- How: Construct giant graviton solutions in the $AdS_4 \times S^7/\mathbb{Z}_k$ and $AdS_4 \times CP^3$ dual backgrounds, work out the operator mapping, emergence of geometry..

- Results:
  - M5-brane giant graviton in $AdS_4 \times S^7/\mathbb{Z}_k$
  - New SUSY NS5-brane in $AdS_4 \times CP^3$
  - Partial results on giant gravitons in $AdS_4 \times CP^3$

(Based on arXiv:1107.5475 [hep-th], JHEP, with M. Herrero and M. Picos) (Also, work in progress with J. Murugan and A. Prinsloo, from U. Cape Town)

Outline

1. Introduction

1.1. Gauge/gravity (holographic) correspondence

1.2. D-Branes

1.3. Giant gravitons and AdS/CFT

2. GST giant gravitons

3. Giant gravitons in ABJM

3.1. A few words about AdS_4/CFT_3

3.2. Giant graviton solutions

3.3. An M5-brane giant graviton in $AdS_4 \times S^7/\mathbb{Z}_k$

3.4. Giant gravitons in Type IIA?

4. Giant gravitons as fuzzy manifolds

5. Conclusions

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1.1. Gauge/gravity (holographic) correspondence

The gauge/gravity correspondence states that some gauge theories in flat space at strong (weak) coupling are dual to weakly (strongly) coupled string theories with one extra dimension.

Possibility to apply string theory techniques to strongly coupled systems (quark-gluon plasma, condensed matter systems).

Possibility to learn something about quantum gravity by studying low dimensional systems with a holographic description (concrete realization of QG and spacetime as emergent phenomena).

Original proposal by Maldacena:

Equivalence between Type IIB string theory on $AdS_5 \times S^5$ and 4 dim $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills theory

AdS_5 : anti-de Sitter space in 5 dim (maximally symmetric solution of the Einstein eqs. with a negative cosmological constant)

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Objects carrying the representations of the symmetry group can also be matched

Starting point: Study of N coincident D3-branes in Type IIB in the large N limit, based on two dual descriptions:

- As solution of the classical eqs. of motion of ST/SUGRA
- As a D-brane system

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The U(N) gauge theory living on the branes becomes 4 dim $\mathcal{N} = 4$ SYM, with $g_{YM}^2 = g_s$:

UV finite, conformally invariant

Topological large N expansion with ‘t Hooft parameter: $\lambda = g_{YM}^2 N$

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For each field in the 5 dim bulk $\leftrightarrow$ Operator in the dual CFT

Hard..

Easier for certain operators due to their symmetries.

1.2. D-branes

The fundamental string (perturbative state of the spectrum) occurs as a solitonic solution, electric source for B_2

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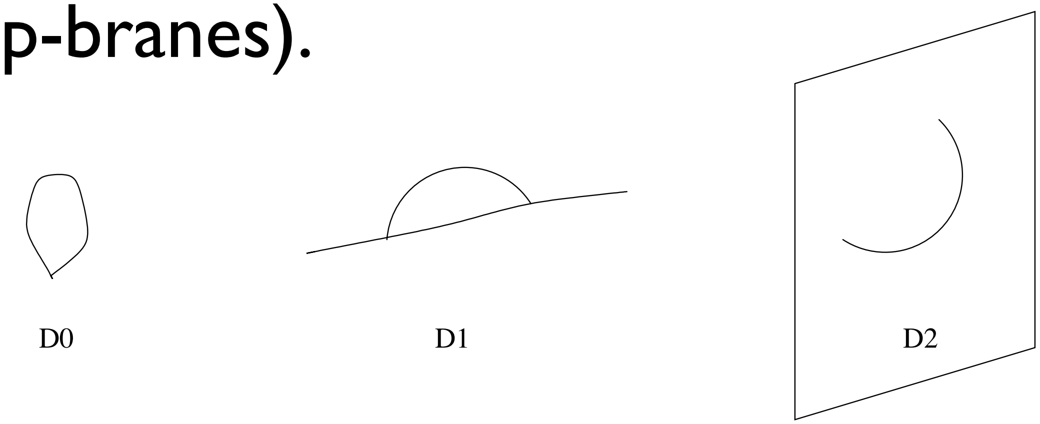
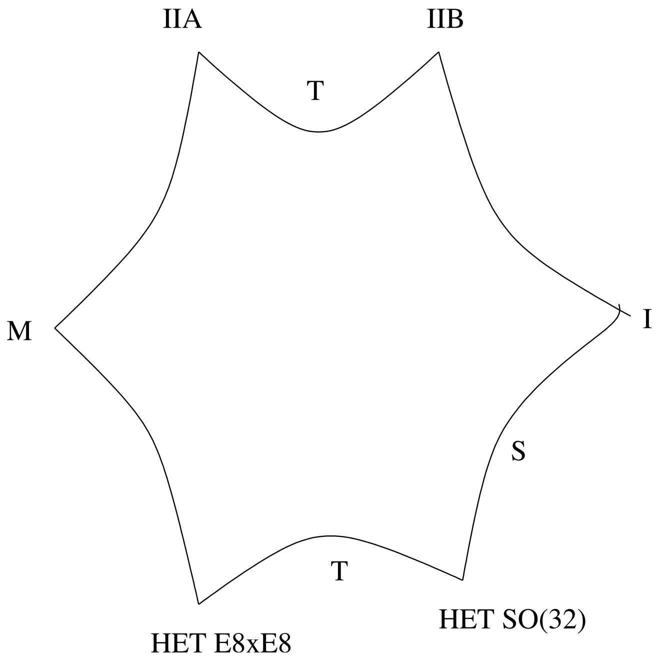
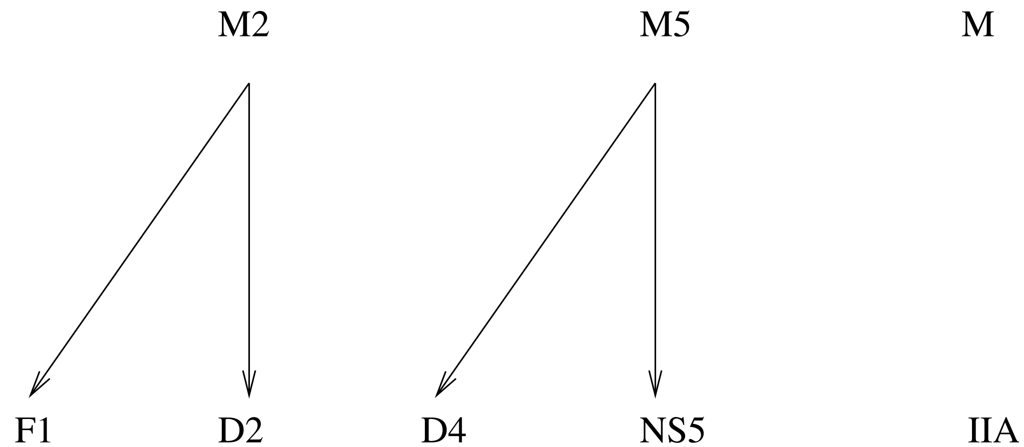
They are also (p+1)-dimensional hypersurfaces on which fundamental strings can end $\Rightarrow$

Dynamics determined at weak coupling by open string perturbation theory $\Rightarrow$ U(1) gauge theory (U(N) for a system of Dp-branes).



D-branes are related to other branes through the web of dualities that relate the different string theories in 10 dim:





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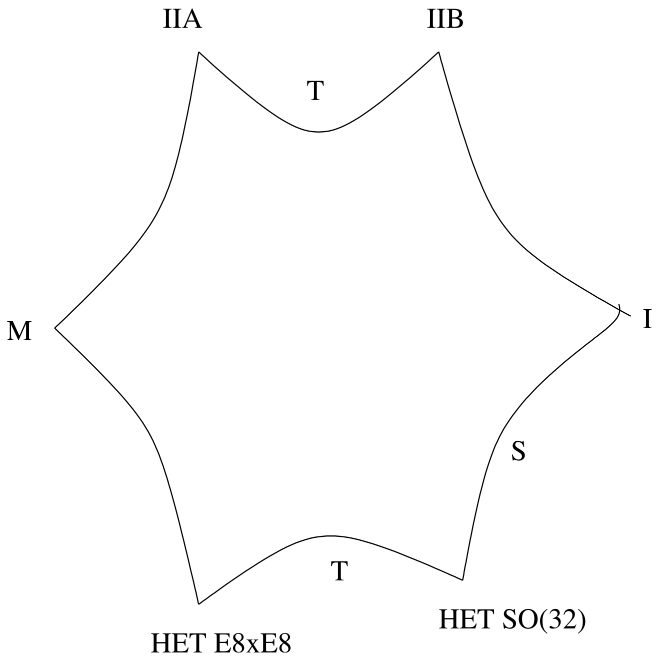

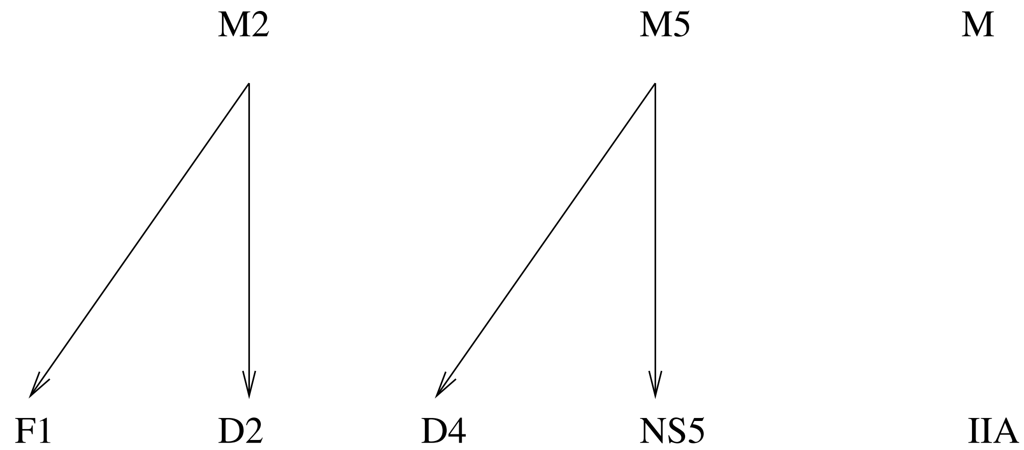

N coincident M2-branes $\rightarrow$ M-theory on $AdS_4 \times S^7$ dual to the 3 dim $\mathcal{N}=8$ gauge theory living on the M2-branes

N coincident M5-branes $\rightarrow$ M-theory on $AdS_7 \times S^4$ dual to the 6 dim (2,0) field theory living on the M5-branes

Recently, M2-branes on an orbifold $\rightarrow$ M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ dual to the $\mathcal{N}=6$ Chern-Simons matter theory living on the M2: ABJM model

(Aharony, Bergman, Jafferis, Maldacena'08)

New AdS/CFT pair in which to test the gauge/gravity correspondence

3 dimensions $\rightarrow$ Applications in condensed matter

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New AdS/CFT pair in which to test the gauge/gravity correspondence

3 dimensions $\rightarrow$ Applications in condensed matter

Other examples with less SUSY, confining, etc

1.3. Giant gravitons and AdS/CFT

Giant gravitons have proven very useful in the context of the AdS/CFT correspondence:

- Matching D-brane configs. in string/M-theory with gauge invariant operators in the dual CFT
- Explicit realization of the stringy exclusion principle
- Emergent geometry (global and local geometries of the dual branes encoded in the gauge theory)

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Family of chiral primary operators	Graviton states with angular momentum in S^n
Maximum weight	Upper bound on the angular momentum

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Stringy exclusion principle (Maldacena, Strominger'98)

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Gravitons expanding into a sphere in the S^n part (giant gravitons) with radius proportional to the angular momentum (Mc.Greevy, Susskind and Toumbas'00)

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In $AdS_m \times S^n$

$$ds^2 = ds^2_{AdS_m} + L^2(d\theta^2 + \cos^2\theta d\phi^2 + \sin^2\theta d\Omega^2_{n-2})$$

take $\theta = \text{const}\,,\ \phi = \phi(\tau)\,,\ r = 0$

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Spherical (n-2)-brane with radius $L\sin\theta$ orbiting the S^n in the ϕ direction

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The equilibrium radius of the spherical brane is then determined by $H'(\theta) = 0$:

$\theta = 0 \rightarrow H = P_\phi / L$: point-like graviton

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$R = L \sin\theta \leq L$ implies that $P_\phi \leq N \Rightarrow$ Upper bound on the angular momentum

Also:

- $M = P_\phi / L\,,\ Q = P_\phi / L\ \Rightarrow$ BPS
- $\dot{\phi} = 1/L$, as the point-like graviton
- Preserves same SUSYs than point-like graviton:

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SUSY algebra:

$$\{Q_\alpha, Q_\beta\} = P_0 + \Gamma^{012} Z_{12} \qquad \text{(membrane)}$$

$$\{Q_\alpha, Q_\beta\} = P_0 + \Gamma^{0i} P_i \qquad \text{(grav. wave)}$$

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$P^0 = P_i \Rightarrow \{Q_\alpha, Q_\beta\} = P_0\,(1+\Gamma^{0i}) \;\Rightarrow$ SUSYs preserved by a gravitational wave propagating on ϕ

Dual giant gravitons

Gravitons can also expand into a sphere on the AdS part (dual giant gravitons)

(Grisaru, Myers, Tafjord'00; Hashimoto, Hirano, Itzakhi'00)

Radius proportional to the angular momentum, but AdS non-compact $\Rightarrow$ No upper bound on the angular momentum

Realization of the stringy exclusion principle?

Giant gravitons dual to subdeterminant operators

(Balasubramanian, Berkooz, Naqvi, Strassler'01)

Dual giant gravitons dual to semiclassical coherent states

(Hashimoto, Hirano, Itzakhi'00)

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- Type IIA (M-theory) is a good description when $N^{1/5} << k$ ($N^{1/5} >> k$)

- Like AdS_5/CFT_4 it is a strong weak coupling duality, with 't Hooft coupling $\lambda = N/k$:
  - The string background describes the 't Hooft limit of the theory: $N, k \to \infty$ with $\lambda = N/k$ fixed
  - IIA weakly curved when $k << N$ (large 't Hooft coupling)

3.2. Giant gravitons solutions

D2-brane spherical dual giant graviton (Nishioka, Takayanagi'08) (Hamilton, Murugan, Prinsloo, Strydom'09)

D4-brane giant graviton expanding in the CP^3 (Herrero, Y.L., Picos'12; Giovannoni, Murugan, Prinsloo'12)

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Giant gravitons, but also new supersymmetric states by reducing on the orbifold:

- Toroidal D2-brane
  (Nishioka, Takayanagi'08)

- Spherical D2-brane with D0-brane charge
  (Nishioka, Takayanagi'08)

- NS5-brane with D0-brane charge
  (Herrero, Picos, Y.L.'12)

3.3. An M5-brane giant graviton in $AdS_4 \times S^7/\mathbb{Z}_k$

$S^7/\mathbb{Z}_k$: $S^1/\mathbb{Z}_k$ bundle over the CP^3

$$ds^2_{S^7/\mathbb{Z}_k} = \left(\frac{1}{k}d\tau + \mathcal{A}\right)^2 + ds^2_{CP^3}$$

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M5-brane wrapping an $S^5 \subset S^7/\mathbb{Z}_k$ and propagating on τ

$$ds^2_{M5} = -dt^2 + R^2\left[\frac{1}{k^2}d\tau^2 + \frac{2}{k}\sin^2\mu\,(d\chi + A) + \sin^2\mu\, ds^2_{S^5}\right]$$

$$ds^2_{S^5} = (d\chi + A)^2 + ds^2_{CP^2}$$

$$C_6 = \frac{R^6}{k}\sin^6\mu\, d\chi \wedge d\tau \wedge d\mathrm{Vol}(CP^2)$$

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M5 wrapped on χ , isometric $\Rightarrow$ Use the action for $M5_W$

M2-branes ending on this brane must also be wrapped on the isometric direction $\Rightarrow$ Self-dual 2-form replaced by a vector

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$$S_{DBI} = -T_4 \int_{M_5} k \sqrt{|\det(\mathcal{G} + k^{-1}\mathcal{F})|}$$

$$S_{CS} = T_4 \int_{M_5} \left[i_k C_6 + \frac{1}{2} \frac{k_1}{k^2} \wedge \mathcal{F} \wedge \mathcal{F} + \dots \right]$$

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k^μ : Killing vector pointing on the isometric direction

$\mathcal{G}_{\mu\nu} = g_{\mu\nu} - k^{-2} k_\mu k_\nu$: reduced metric

$i_k C_6$: interior product

$k^{-2} k_1 = g_{\mu\chi}/g_{\chi\chi}\, dx^\mu$: Momentum operator on χ

Substituting for our background:

$$H = \frac{k}{R} P_\tau \sqrt{1 + \tan^2 \mu \left(1 - \frac{N}{P_\tau} \sin^4 \mu\right)^2} = H(\mu)$$

Minimum energy solution:

$\mu = 0 \quad \rightarrow$ Point-like graviton

$\sin \mu = \left(\frac{P_\tau}{N}\right)^{1/4} \rightarrow$ Giant graviton. For this sol. $P_\tau \leq N$

For both $E = \frac{k}{R} P_\tau$

In Type IIA: Reduce along $\tau \;\Rightarrow\; P_\tau$ charge becomes D0-brane charge, and the M5 a NS5

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$$ds^2_{NS5} = -dt^2 + L^2 \sin^2\mu \Big[\cos^2\mu (d\chi + A)^2 + ds^2_{CP^2}\Big]$$

Also C_1, C_5

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Quite non-trivial state from the NS5-brane point of view

In Type IIA: Reduce along $\tau \Rightarrow P_\tau$ charge becomes D0-brane charge, and the M5 a NS5

$\Rightarrow$ New SUSY state: NS5 brane wrapping a twisted 5-sphere inside the CP^3, with D0 charge

$$ds^2_{NS5} = -dt^2 + L^2 \sin^2\mu \left[\cos^2\mu (d\chi + A)^2 + ds^2_{CP^2}\right]$$

Also C_1, C_5

Quite non-trivial state from the NS5-brane point of view

Action for a wrapped NS5:

$$S_{CS} = T_4 \int i_k C_5 \wedge dc_0$$

(c_0 associated to D0-branes)

$$S_{DBI} = -T_4 \int d^5\xi \, e^{-2\phi} \sqrt{k^2 + e^{2\phi}(i_k C_1)^2} \sqrt{|\det\Big(\mathcal{G} + \frac{2^{2\phi} k^2}{k^2 + e^{2\phi}(i_k C_1)^2} \mathcal{F}_1^2\Big)|}$$

$$\mathcal{F}_1 = dc_0 + P[C_1]$$

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c_0 **cyclic** $\Rightarrow$ **Conserved conjugate momentum**

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Minimum energy solution:

$\mu = 0 \quad \rightarrow$ **Point-like D0**

$\sin\mu = \left(\frac{M}{N}\right)^{1/4} \rightarrow$ **Giant D0.** **For this sol.** $M \leq N$

For both $E = \frac{k}{L} M$

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New realization of the stringy exclusion principle

In the maximal case $\sin\mu = 1 \;\Rightarrow\; M = N \;\Rightarrow\; E = \frac{k}{L} N$

$\Rightarrow$ Bound state of N dimonopoles, with energy $E_{D0} = k/L$
or k dibaryons, with energy $E_{D4} = N/L$

Realization of the duality of Young Tableaux with N rows and k columns ($\leftrightarrow$ instability realized in the string theory side in terms of a NS5 instanton that turns the k D4 into N D0)
(ABJM)

3.4. Giant gravitons in Type IIA?

- M5 with magnetic flux to induce momentum on χ, through

$$S_{CS} = \frac{T_4}{2} \int \frac{k_1}{k^2} \wedge F \wedge F \qquad ds^2_{S^5} = (d\chi + A)^2 + ds^2_{CP^2}$$

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In Type IIA: NS5 with D0 charge and momentum. Giant grav. only in the maximal case

- M5 wrapped on $S^1/\mathbb{Z}_k$ and moving on the S^1 :

In Type IIA: D4-brane wrapped on the squashed CP^2

$$ds^2_{D4} = -dt^2 + L^2 \sin^2 \mu \left[\cos^2 \mu (d\chi + A)^2 + ds^2_{CP^2} \right]$$

and moving on χ

Introduce an electric flux (to undo the squashing):

$E_i = L \sin\mu \cos\mu \, A_i$

$C_5 \neq 0$

$$H = \frac{P_\chi}{L \sin\mu} \sqrt{1 + \tan^2\mu \Big(1 - \frac{N}{P_\chi} \sin^4\mu \Big)^2}$$

Ground state: $\sin\mu = 1 \;\Rightarrow\; P_\chi = N$ **and** $H = \dfrac{N}{L}$

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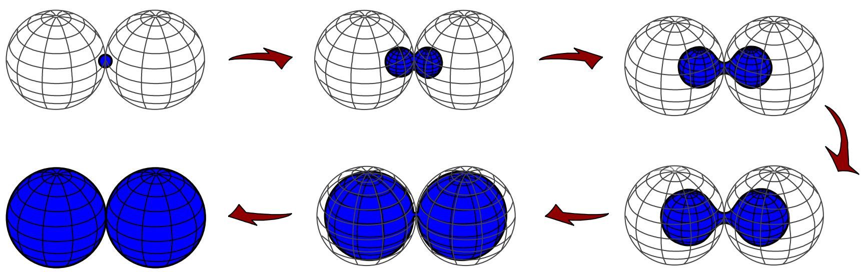
No giant graviton away from the maximal case

The giant graviton in this space is a more complicated solution (Giovannoni, Murugan, Prinsloo'12) :

Different ansatz: CP^3 as a U(1) bundle over $S^2 \times S^2$ with an extra coordinate that controls the sizes of the two S^2

At maximal size the giant factorizes into two dibaryons:



Our ansatz: Select a CP^2 submanifold.

Outline

1. Introduction

1.1. Gauge/gravity (holographic) correspondence

1.2. D-Branes

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4. Giant gravitons as fuzzy manifolds

Giant graviton: p-brane with S^p or $CP^{p/2}$ topology with angular momentum $\Rightarrow$ Macroscopical $(R >> l_s)$

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Can we give a microscopical description, in terms of expanding gravitons?

4. Giant gravitons as fuzzy manifolds

Giant graviton: p-brane with S^p or $CP^{p/2}$ topology with angular momentum $\Rightarrow$ Macroscopical $(R >> l_s)$

Can we give a microscopical description, in terms of expanding gravitons?

Connect with known examples of expanded brane configs: Myers dielectric effect

4. Giant gravitons as fuzzy manifolds

Giant graviton: p-brane with S^p or $CP^{p/2}$ topology with angular momentum $\Rightarrow$ Macroscopical $(R >> l_s)$

Can we give a microscopical description, in terms of expanding gravitons?

Connect with known examples of expanded brane configs:
Myers dielectric effect

$\rightarrow$ Giant gravitons as dielectric gravitational waves

(Janssen, Y.L., Rodriguez-Gomez'02-05)

Myers dielectric effect (an example):

n D0's in a constant RR F_4 :

$$\int d\tau \mathrm{STr}(i_X i_X C^{(3)}) = -\frac{1}{3}\int d\tau \mathrm{STr}(X^j X^i X^k) F^{(4)}_{0ijk} + \dots$$

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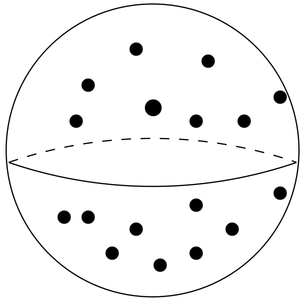
$F^{(4)}_{0ijk} = 2f\epsilon_{ijk} \rightarrow$ Ground state: n D0's expanded into a fuzzy S^2

$$X^i = \frac{f}{2} J^i, \qquad [J^i, J^j] = 2i\epsilon^{ijk} J^k$$



$$\Rightarrow \sum_{i=1}^{3} (X^i)^2 = \left(\frac{f}{2}\right)^2 (n^2 - 1)\mathbb{I} = R^2\,\mathbb{I}$$

C_3 dipole moment $\Rightarrow$ Fuzzy D2-brane

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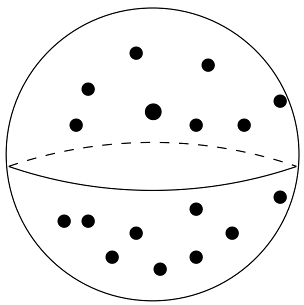

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C_3 dipole moment $\Rightarrow$ Fuzzy D2-brane

Regime of validity: $4\pi R^2/n << l_s^2$

In the context of the gauge/gravity duality: $\lambda << n^2 \Rightarrow$

Finite 't Hooft coupling regime

D0-branes are gravitons in M-theory, moving along the 11th direction:

$$
\begin{tikzcd}
 & M0 \arrow[dl] \arrow[d] & & M2 \arrow[dl] \arrow[d] & & M5 \arrow[dl] \arrow[d] \\
\text{wave} & D0 & F1 & D2 & D4 & NS5
\end{tikzcd}
$$

$\Rightarrow$ Uplift Myers action for N D0-branes

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$\Rightarrow$ Uplift Myers action for N D0-branes

Also, Matrix theory calculation:

- Matrix string theory: (Non-Abelian) Type IIA strings with non-zero light-cone momentum

- Sen-Seiberg limit + static gauge $\rightarrow$ (Non-Abelian) massless particles with spatial momentum (IIA gravitational waves)

Dielectric couplings?: Matrix string theory in a weakly curved background $\rightarrow$ Precise agreement with the linear expansion of Myers action

Dielectric couplings?: Matrix string theory in a weakly curved background $\rightarrow$ Precise agreement with the linear expansion of Myers action

The action for M-theory gravitons

$$S = T_0 \int d\tau \, \mathrm{STr}\{-k^{-1}\sqrt{E_{00} + E_{0i}(Q^{-1} - \mathbb{I})^i_k E^{kj} E_{j0}}\sqrt{\det Q}$$
$$-k^{-2} k_i \partial X^i + i(i_X i_X) C^{(3)} + \frac{1}{2}(i_X i_X)^2 i_k C^{(6)} + \ldots\}$$

k^μ : Killing vector pointing on the direction of propagation (isometric $\leftrightarrow$ momentum eigenstate)

$$E = \mathcal{G} + k^{-1}(i_k C^{(3)}) \,, \qquad Q^i_j = \delta^i_j + ik[X^i, X^k] E_{kj}$$

In the Abelian limit: Legendre transformation:

$$S[\gamma] = -\frac{T_0}{2} \int d\tau \sqrt{|\gamma|}\gamma^{-1}\partial X^\mu \partial X^\nu g_{\mu\nu}$$

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With this action:

Giant	Fuzzy manifold
S^2	Fuzzy S^2
S^3	S^1 over S^2_{fuzzy}
S^1	Fuzzy cylinder
S^5	S^1 over CP^2_{fuzzy}

The M5-brane in $AdS_4 \times S^7/\mathbb{Z}_k$ microscopically

- $CP^{p/2}$: coset manifold $SU(\frac{p}{2}+1)/U(\frac{p}{2})$

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- $CP^{p/2}$: coset manifold $SU(\frac{p}{2}+1)/U(\frac{p}{2})$. Submanifold of $\mathbb{R}^{\frac{p^2}{4}+p}$ determined by the constraints:

$$\sum_{i=1}^{\frac{p^2}{4}+p} (x^i)^2 = 1\,, \qquad \sum_{j,k=1}^{\frac{p^2}{4}+p} d^{ijk} x^j x^k = \frac{\frac{p}{2}-1}{\sqrt{\frac{p}{4}(\frac{p}{2}+1)}} x^i$$

$\rightarrow p$ dimensional manifold

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Fubini-Study metric of the $CP^{\frac{p}{2}}$ given by

$$ds^2_{CP^{\frac{p}{2}}} = \frac{p}{4(\frac{p}{2}+1)} \sum_{i=1}^{\frac{p^2}{4}+p} (dx^i)^2$$

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- Matrix level definition $\leftrightarrow$ Fuzzy $CP^{\frac{p}{2}}$:

$X^i = \frac{1}{\sqrt{C_n}} T^i$, T^i : generators of $SU(\frac{p}{2}+1)$ in the $(m,0)$ irrep

Substituting in the action for M-theory gravitons:

$$H(\mu) = \frac{k}{R}\,P_\tau\sqrt{1+\tan^2\mu\left(1-\frac{2nN}{P_\tau(m^2+3m)}\sin^4\mu\right)^2}$$

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Given that $n = \frac{(m+1)(m+2)}{2}$ when $m \to \infty$ we reproduce the macroscopical result:

$$H = \frac{k}{R} P_\tau \sqrt{1 + \tan^2\mu \Big(1 - \frac{N}{P_\tau} \sin^4\mu\Big)^2} = H(\mu)$$

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1.2. D-Branes

1.3. Giant gravitons and AdS/CFT

2. GST giant gravitons

3. Giant gravitons in ABJM

3.1. A few words about AdS_4/CFT_3

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5. Conclusions

M5-brane giant graviton in $AdS_4 \times S^7/\mathbb{Z}_k$:

$\rightarrow$ New NS5-brane with non-trivial geometry, satisfying the stringy exclusion principle and realizing on the gravity side the duality of Young tableaux

D4-brane giant graviton in $AdS_4 \times CP^3$ only in the maximal case:

$\rightarrow$ Dibaryon: Limiting case of a D4-brane with momentum wrapping a squashed CP^2

Microscopical description:

$\rightarrow$ These configurations exist also at finite 't Hooft coupling (useful if they are not BPS)

Thanks!

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In the parameterization of the CP^3 as a U(1) bundle over $S^2 \times S^2$:

$$ds^2_{CP^3} = d\zeta^2 + \frac{1}{4}\Big[\cos^2\zeta\,\sin^2\zeta\,(d\psi + A_1 + A_2)^2 + \cos^2\zeta\,ds^2_{S_1^2} + \sin^2\zeta\,ds^2_{S_2^2}\Big]$$

take ζ constant and a NS5-brane wrapped on $S^1_\psi \times S^2_1 \times S^2_2$ with D0-charge M :

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$\zeta = 0 \quad \rightarrow \quad$ 2-sphere of radius L

$\sin 2\zeta = \sqrt{\frac{2M}{N}} \quad \rightarrow \quad$ NS5 wrapped on the CP^3 with constant ζ

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Related to the symmetry of Young tableaux

Realization of the stringy exclusion principle?

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Related to the symmetry of Young tableaux

Realization of the stringy exclusion principle?

In 11 dim:

$\zeta = 0$ is not a point-like graviton, but a squashed S^2, with arbitrary angular momentum

$\sin 2\zeta = \sqrt{\frac{2P_\tau}{N}}$ is an M5 wrapped on $S^1_\psi \times S^2_1 \times S^2_2$

with an angular momentum $P_\tau \leq \frac{N}{2}$, which should be an implication of the stringy exclusion principle