

# Non-singlet baryons in gauge/ gravity duality

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Non-singlet baryons are predicted in N=4 SYM by the AdS/CFT correspondence.

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## How:

Analyze the holographic description in various backgrounds with reduced supersymmetries and/or confining

- Reduced SUSY:  $AdS_5 \times Y_5$ , Lunin-Maldacena  $\beta$  deformed, Frolov multi- $\beta$  deformed
- Confining: Maldacena-Nuñez

## Results:

- Non-singlet baryons exist in all these backgrounds
- Same number of quarks in all  $AdS_5 \times Y_5$  Einstein manifolds with 5-form flux, independent of SUSY
- More restricted number of quarks in MN
- Stable against fluctuations
- Non-singlet baryons at finite 't Hooft coupling

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(Based on arXiv:1203.6817, D. Giataganas, Y.L., M. Picos, K. Siampos, JHEP)

# I. The baryon vertex in $AdS_5 \times S^5$

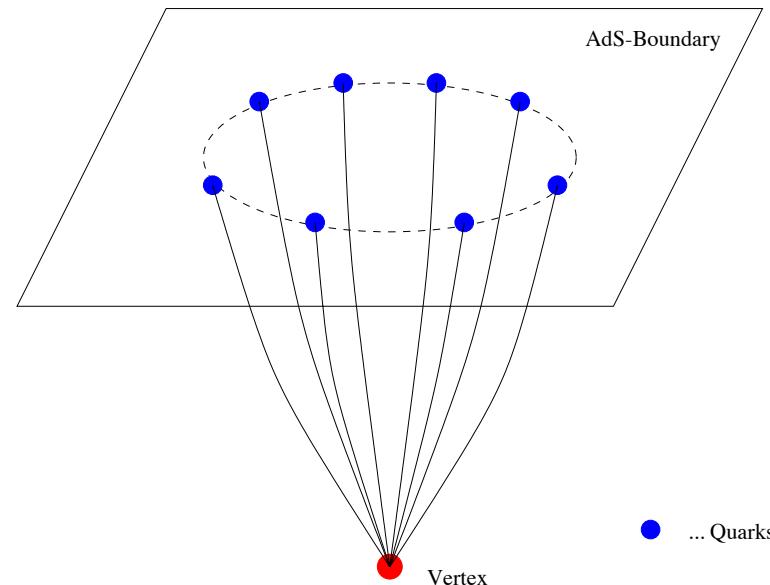
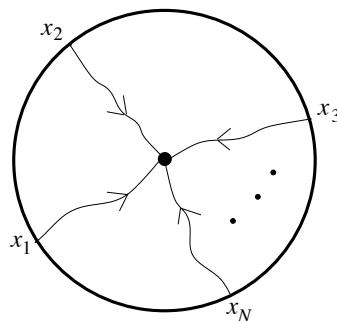
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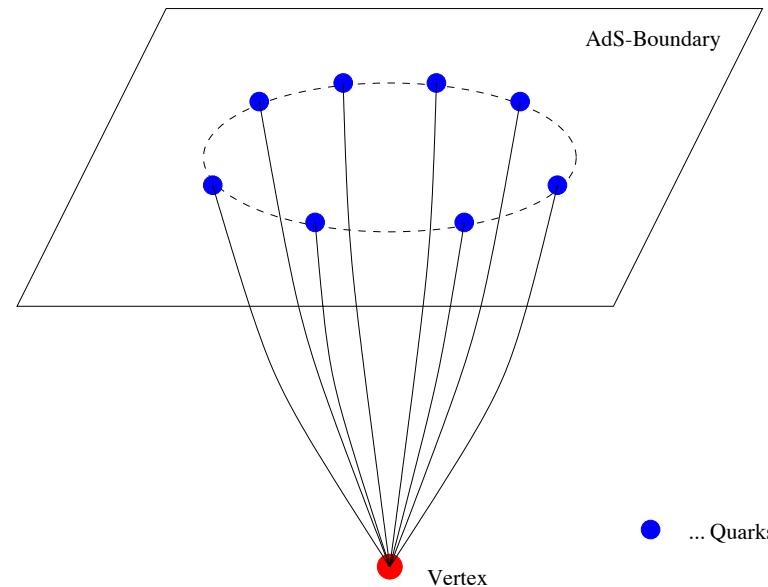
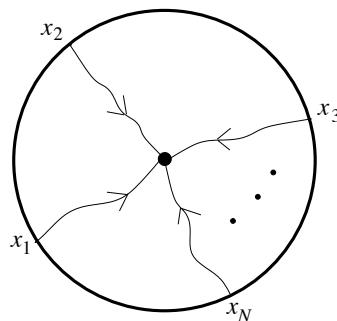
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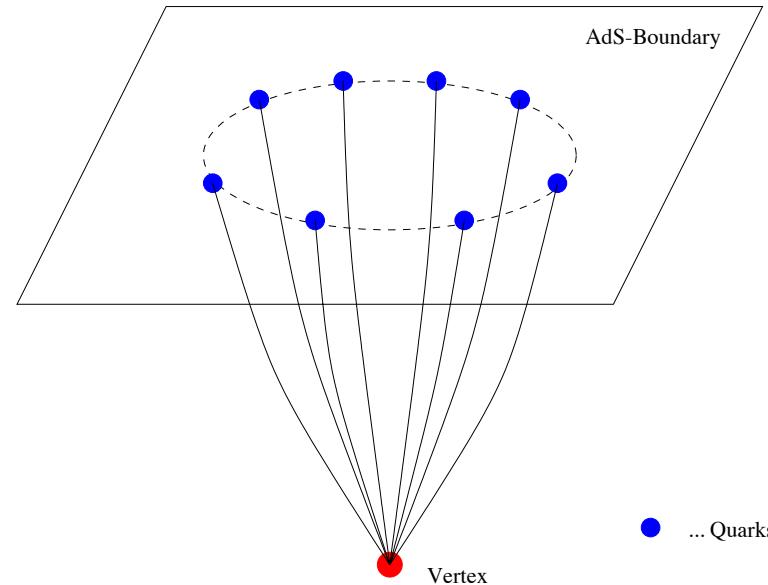
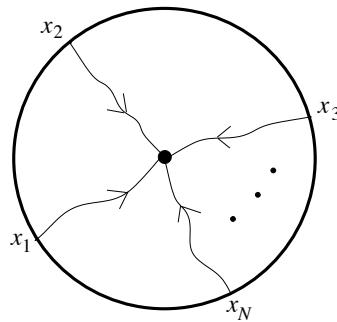
Baryon vertex in the gravity side: D5-brane wrapped on the 5-sphere (Witten'98):

$$S_{CS} = 2\pi T_5 \int_{\mathbb{R} \times S^5} P[F_5] \wedge A = N T_{F1} \int_{\mathbb{R}} dt A_t$$

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N charge cancelled by N F-strings ending on the 5-brane

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In  $AdS_5 \times S^5$ :  $5N/8 \leq k \leq N$

In  $AdS_4 \times CP^3$  :  $2N/3 \leq k \leq N$  (Y.L., Picos, Sfetsos, Siampos'11)

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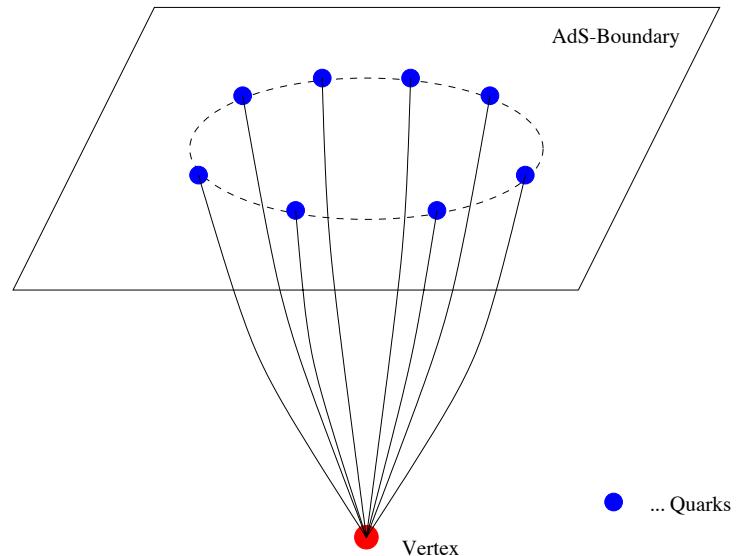
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And at finite 't Hooft coupling?

## 2. Gauge/gravity calculation of the energy

(Brandhuber, Itzhaki, Sonnenschein, Yankielowitz'98; Imamura'98; Maldacena'98)



Consider a uniform distribution of strings on an  $\mathbb{M}_p$  shell  
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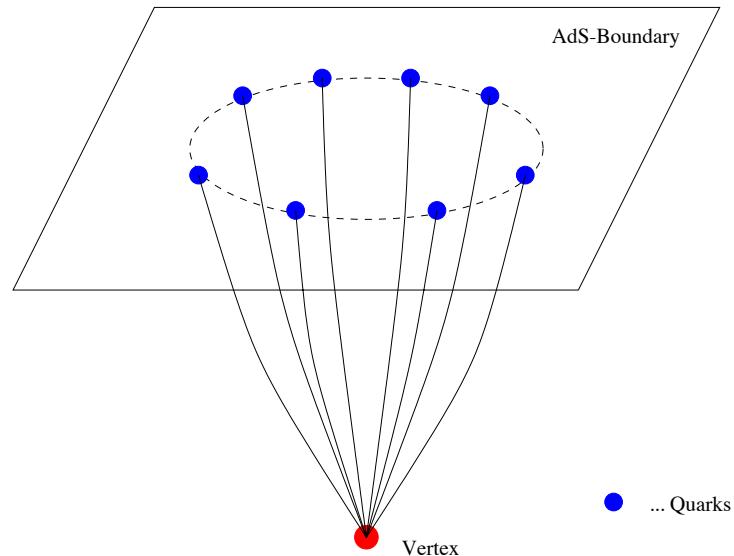
In the probe brane approach:  $S = S_{Dp} + S_{NF1}$ :

$$S_{NF1} = -N T_{F1} \int dt dr \sqrt{|\det P(G)|}$$

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$$S_{Dp} = -T_p \int_{\mathbb{R} \times \mathbb{M}_p} d^{p+1}\xi \sqrt{|\det P(G + 2\pi F - B)|}$$

In  $AdS_5 \times Y_5$ :

$$ds^2 = \frac{\rho^2}{R^2} dx_{1,3}^2 + \frac{R^2}{\rho^2} d\rho^2 + R^2 ds_{Y_5}^2$$

$$R^4 = \frac{4\pi^4 N g_s}{\text{Vol}(Y_5)}, \quad F_5 = 4 R^4 (1 + *) d\text{Vol}(Y_5)$$

**Bulk equation of motion:**  $\frac{\rho^4}{\sqrt{\frac{\rho^4}{R^4} + \rho'^2}} = c$

**Boundary equation of motion:**  $\frac{\rho_0'}{\sqrt{\frac{\rho_0^4}{R^4} + {\rho_0'}^2}} = \frac{T_5 R^4 \text{Vol}(Y_5)}{N T_{F1}}$

**Define**  $\sqrt{1 - \beta^2} = \frac{T_5 R^4 \text{Vol}(Y_5)}{NT_{F1}}$  **with**  $\beta \in [0, 1]$

The two equations can be combined into:

$$\frac{\rho^4}{\sqrt{\frac{\rho^4}{R^4} + \rho'^2}} = \beta \rho_0^2 R^2$$

Integrating: **Size of the configuration:**

$$L = \frac{R^2}{\rho_0} \int_1^\infty dz \frac{\beta}{z^2 \sqrt{z^4 - \beta^2}}$$

**On-shell energy:**

$$E = E_{Dp} + E_{NF1} = NT_{F1} \rho_0 \left( \sqrt{1 - \beta^2} + \int_1^\infty dz \frac{z^2}{\sqrt{z^4 - \beta^2}} \right)$$

## Binding energy:

$$E_{\text{bin}} = NT_{F1}\rho_0 \left( \sqrt{1 - \beta^2} + \int_1^\infty dz \left[ \frac{z^2}{\sqrt{z^4 - \beta^2}} - 1 \right] - 1 \right)$$

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$$E_{\text{bin}} = -f(\beta) \frac{\sqrt{\lambda}}{L}$$

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- ⇒ - The configuration is stable
- $E_{\text{bin}} \sim 1/L$  dictated by conformal invariance
  - $E_{\text{bin}} \sim \sqrt{\lambda}$  non-trivial prediction for the non-perturbative regime of the gauge theory

# Universal behavior for all $AdS_5 \times Y_5$ backgrounds, independent of SUSY

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Confining background?

Maldacena-Nuñez:  $E_{\text{bin}} \sim L$   
(Loewy, Sonnenschein'01)

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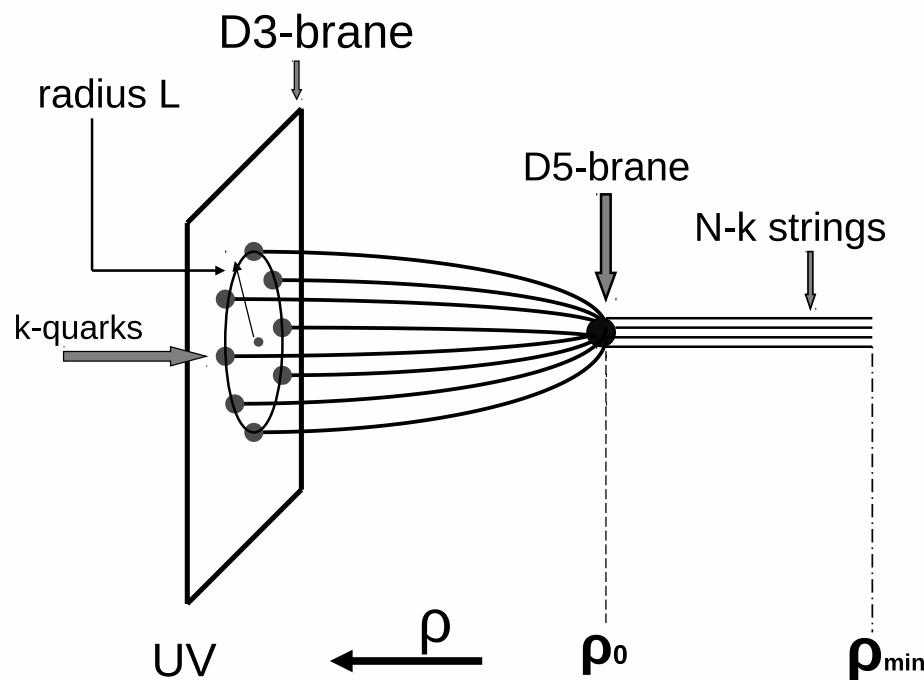
Non-singlets?

### 3. Reduce the number of quarks

In  $AdS_5 \times S^5$  : Baryon vertex classical solutions with number of quarks  $5N/8 \leq k \leq N$  (non-singlet)

(Brandhuber, Itzhaki, Sonnenschein, Yankielowitz'98;  
Imamura'98)

Stable against fluctuations for  $0.813N \leq k \leq N$   
(Sfetsos, Siampos'08)



### 3.I.The classical solution

The boundary equation of motion changes:

$$\frac{\rho'_0}{\sqrt{\frac{\rho_0^4}{R^4} + \rho'_0{}^2}} = \frac{T_5 R^4 \text{Vol}(Y_5)}{k T_{F1}} + \frac{N - k}{k} \leq 1$$

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$\Rightarrow$  Non-singlet states in non-SUSY or confining backgrounds

## 3.2. Stability analysis

Important in establishing the physical parameter space  
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Expand the Nambu-Goto action to quadratic order and study  
the zero mode problem  $\leftrightarrow$  Critical curve in the parametric  
space separating the stable and unstable regions

Stability reduced to an eigenvalue problem of the general  
Sturm-Liouville type

Instabilities emerge from longitudinal fluctuations of the  
strings

For  $AdS_5 \times Y_5$  and beta deformed:

Bound for the number of F-strings coming from stability:

$$k \geq \frac{N}{1 + \gamma_c} (1 + \sqrt{1 - \beta^2}) \quad \gamma_c = 0.538$$

More restrictive than the bound imposed by the existence of a classical solution:

$$k \geq \frac{N}{2} (1 + \sqrt{1 - \beta^2})$$

For MN: Same bound for stability and existence of classical sol.

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Can we reach the finite 't Hooft coupling region?

## 5. Baryons at finite 't Hooft coupling

Generalize the baryon vertex adding a magnetic flux.

A non-trivial flux adds lower dim brane charges →

Complementary description of the baryon in terms of  
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- Description in terms of the expanded brane (macroscopical)  
valid in the sugra limit:  $R \gg 1 \Leftrightarrow \lambda \gg 1$

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- Complementary at finite  $n$ . Should agree at large  $n$

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$$R^4 = \frac{4\pi^4 N g_s}{\text{Vol}(Y^{p,q})}, \quad ds_{Y^{p,q}}^2 = ds^2(M_4) + \left(\frac{1}{3}d\psi + \sigma\right)^2$$

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Take  $F = \mathcal{N}J$  with  $J = \frac{1}{2}d\sigma$  the Kähler form:

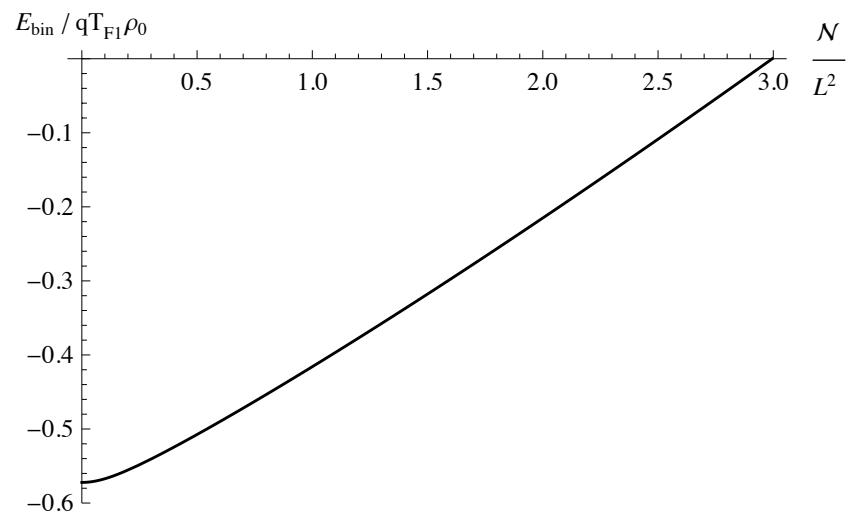
$$\sqrt{1 - \beta^2} = \frac{T_5 R^4 \text{Vol}(Y^{p,q})}{NT_{F1}} \left(1 + \frac{4\pi^2 \mathcal{N}^2}{R^4}\right)$$

$$\beta \leq 1 \Rightarrow \mathcal{N}_{\max} \leftrightarrow \beta = 0$$

## Binding energy:

$$E_{\text{bin}} = NT_{F1}\rho_0 \left( \sqrt{1 - \beta^2} + \int_1^\infty dz \left[ \frac{z^2}{\sqrt{z^4 - \beta^2}} - 1 \right] - 1 \right)$$

- $E_{\text{bin}}$  negative and decreases monotonically with  $\beta$
- $E_{\text{bin}} = 0$  for  $\beta = 0$  (**N free radial strings stretching from  $\rho_0$  to  $\infty$  plus a Dp-brane at  $\rho_0$** )



$$\mathcal{N}_{\max} \sim \sqrt{\lambda}$$

$$E_{\text{bin}} = -f(\beta) \frac{\sqrt{\lambda}}{L}$$

## Non-singlets:

The bound for the number of quarks is modified (  $(k, \mathcal{N})$  parameter space bounded by the values for which the baryon vertex reduces to free quarks):

$$k \geq \frac{5}{8}N + \frac{\mathcal{N}^2}{8\pi^2} \text{Vol}(Y^{p,q})$$

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$k_{\min}$  is maximum for the 5-sphere, the most SUSY case

$$\pi^3 = \text{Vol}(S^5) > \frac{16}{27}\pi^3 = \text{Vol}(T^{1,1}) > \text{Vol}(Y^{2,1}) \approx 0.29\pi^3$$

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Spike solutions non-SUSY for  $T^{1,1}$ ,  $Y^{p,q}$  and MN

(Areán, Crooks, Ramallo'04; Canoura, Edelstein, Pando Zayas, Ramallo, Vaman'05; Imamura'04)

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Important to obtain information at finite ‘t Hooft coupling.

**Micro description:**

DI's expanding into a fuzzy  $M_4$ . Only known for the  $S^5$   
(Janssen, Y.L., Rodriguez-Gomez'06) and the  $T^{1,1}$

## The fuzzy $T^{1,1}$

The  $T^{1,1}$  is a  $U(1)$  bundle over  $S^2 \times S^2 \Rightarrow$  Take the D1's wrapped on the  $U(1)$  fibre and expanding into a  $S_{\text{fuzzy}}^2 \times S_{\text{fuzzy}}^2$

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Substituting in Myers action for D1-branes:

$$S_{DBI} = - \int S \text{Tr} \left\{ e^{-\phi} \sqrt{\det \left( P [E_{\mu\nu} + E_{\mu i} (Q^{-1} - \delta)_j^i E^{jk} E_{k\nu}] \right) \det Q} \right\}$$

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Agreement with  
macro descrip.  
in large m

## The FI in the micro description

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Similar micro analysis in MN in terms of DI's expanding into a fuzzy  $S^2$

## 6. Conclusions

- Non-singlet baryons are predicted by gauge/gravity duality in less supersymmetric and/or confining backgrounds
- They are stable against fluctuations
- At finite 't Hooft coupling:
  - Microscopical description of the vertex
  - Complete this analysis with  $\alpha'$  corrections to the NG action of the strings (or microscopic spike),  $\alpha'$  corrections to the background

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Thanks!