Non-singlet baryons in gauge/gravity duality

Yolanda Lozano (U. Oviedo)

ITS Lisboa, April 2012

Motivation:

Non-singlet baryons are predicted in N=4 SYM
by the AdS/CFT correspondence.
Do they exist also in more realistic gauge theories?

How:

Analyze the holographic description in various backgrounds with reduced supersymmetries and/or confining

- Reduced SUSY: $AdS_5 \times Y_5$, Lunin-Maldacena β deformed, Frolov multi-β deformed

- Confining: Maldacena-Nuñez

Results:

- Non-singlet baryons exist in all these backgrounds
- Same number of quarks in all $AdS_5 \times Y_5$ Einstein manifolds with 5-form flux, independent of SUSY
- More restricted number of quarks in MN
- Stable against fluctuations
- Non-singlet baryons at finite 't Hooft coupling

(Based on arXiv:1203.6817, D. Giataganas, Y.L., M. Picos, K. Siampos)

Outline

1. Introduction
    1.1. Gauge/gravity (holographic) correspondence
    1.2. D-Branes
    1.3. Less supersymmetric extensions
    1.4. Baryons and AdS/CFT
2. The baryon vertex in $AdS_5 \times S^5$
3. Gauge/gravity calculation of the energy
4. Reduce the number of fundamental strings
    4.1. The classical solution
    4.2. Stability analysis
5. Baryons at finite 't Hooft coupling
6. Conclusions

Outline

1.1. Gauge/gravity (holographic) correspondence

The gauge/gravity correspondence states that some gauge theories in flat space at strong (weak) coupling are dual to weakly (strongly) coupled string theories with one extra dimension.

Possibility to apply string theory techniques to strongly coupled systems (quark-gluon plasma, condensed matter systems).

Possibility to learn something about quantum gravity by studying low dimensional systems with a holographic description (concrete realization of QG and spacetime as emergent phenomena).

Original proposal by Maldacena:

Equivalence between Type IIB string theory on $AdS_5 \times S^5$ and 4 dim $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills theory

AdS_5 : anti-de Sitter space in 5 dim (maximally symmetric solution of the Einstein eqs. with a negative cosmological constant)

Original proposal by Maldacena:

Equivalence between Type IIB string theory on $AdS_5 \times S^5$ and 4 dim $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills theory

AdS_5 : anti-de Sitter space in 5 dim (maximally symmetric solution of the Einstein eqs. with a negative cosmological constant)

The symmetry group of 5 dim anti-de Sitter space matches precisely with the group of conformal symmetries of $\mathcal{N} = 4$ SYM $\Rightarrow$ AdS/CFT

Original proposal by Maldacena:

Equivalence between Type IIB string theory on $AdS_5 \times S^5$ and 4 dim $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills theory

AdS_5 : anti-de Sitter space in 5 dim (maximally symmetric solution of the Einstein eqs. with a negative cosmological constant)

The symmetry group of 5 dim anti-de Sitter space matches precisely with the group of conformal symmetries of $\mathcal{N} = 4$ SYM $\Rightarrow$ AdS/CFT

Objects carrying the representations of the symmetry group can also be matched

Starting point: Study of N coincident D3-branes in Type IIB in the large N limit, based on two dual descriptions:

- As solution of the classical eqs. of motion of ST/SUGRA
- As a D-brane system

Starting point: Study of N coincident D3-branes in Type IIB in the large N limit, based on two dual descriptions:

- As solution of the classical eqs. of motion of ST/SUGRA
- As a D-brane system

$AdS_5 \times S^5$ emerges as the near horizon geometry of the D3-brane solution. The radius is given by $R^4 = 4\pi g_s N \, l_s^4$

Starting point: Study of N coincident D3-branes in Type IIB in the large N limit, based on two dual descriptions:

- As solution of the classical eqs. of motion of ST/SUGRA
- As a D-brane system

$AdS_5 \times S^5$ emerges as the near horizon geometry of the D3-brane solution. The radius is given by $R^4 = 4\pi g_s N\, l_s^4$

The U(N) gauge theory living on the branes becomes 4 dim $\mathcal{N} = 4$ SYM, with $g_{YM}^2 = g_s$:

UV finite, conformally invariant

Topological large N expansion with 't Hooft parameter: $\lambda = g_{YM}^2 N$

Starting point: Study of N coincident D3-branes in Type IIB in the large N limit, based on two dual descriptions:

- As solution of the classical eqs. of motion of ST/SUGRA
- As a D-brane system

$AdS_5 \times S^5$ emerges as the near horizon geometry of the D3-brane solution. The radius is given by $R^4 = 4\pi g_s N\, l_s^4$

The U(N) gauge theory living on the branes becomes 4 dim $\mathcal{N} = 4$ SYM, with $g_{YM}^2 = g_s$:

UV finite, conformally invariant

Topological large N expansion with 't Hooft parameter:

$\lambda = g_{YM}^2 N$

Through the correspondence: $R^4 = 4\pi\lambda\, l_s^4$

$\lambda = g_s N \qquad R^4 = 4\pi\lambda\, l_s^4$ implies

$\lambda = g_s N \qquad R^4 = 4\pi\lambda\, l_s^4$ implies

$N \to \infty$, finite $\lambda \Leftrightarrow g_s \to 0 \Rightarrow$ Planar limit $\leftrightarrow$ Classical limit of IIB ST

$\lambda = g_s N \qquad R^4 = 4\pi\lambda\, l_s^4$ implies

$N \to \infty$, finite $\lambda \;\Leftrightarrow\; g_s \to 0 \Rightarrow$ Planar limit $\leftrightarrow$ Classical limit of IIB ST

$\lambda \to \infty \;\Leftrightarrow\; l_s \to 0 \;\Rightarrow$ Strong 't Hooft coupling limit $\leftrightarrow$ SUGRA limit

$\lambda = g_s N \qquad R^4 = 4\pi\lambda\, l_s^4$ implies

$N \to \infty$, finite $\lambda \;\Leftrightarrow\; g_s \to 0 \Rightarrow$ Planar limit $\leftrightarrow$ Classical limit of IIB ST

$\lambda \to \infty \;\Leftrightarrow\; l_s \to 0 \;\Rightarrow$ Strong 't Hooft coupling limit $\leftrightarrow$ SUGRA limit

$\Rightarrow$ Strong-weak coupling duality

$\lambda = g_s N \qquad R^4 = 4\pi\lambda\, l_s^4$ implies

$N \to \infty$, finite $\lambda \Leftrightarrow g_s \to 0 \Rightarrow$ Planar limit $\leftrightarrow$ Classical limit of IIB ST

$\lambda \to \infty \Leftrightarrow l_s \to 0 \Rightarrow$ Strong 't Hooft coupling limit $\leftrightarrow$ SUGRA limit

$\Rightarrow$ Strong-weak coupling duality

For each field in the 5 dim bulk $\leftrightarrow$ Operator in the dual CFT

Hard..

Easier for certain operators due to their symmetries.

1.2. D-branes

The fundamental string (perturbative state of the spectrum) occurs as a solitonic solution, electric source for B_2

A class of p-branes, Dp-branes, are the electric sources for C_{p+1}. They are non-perturbative.

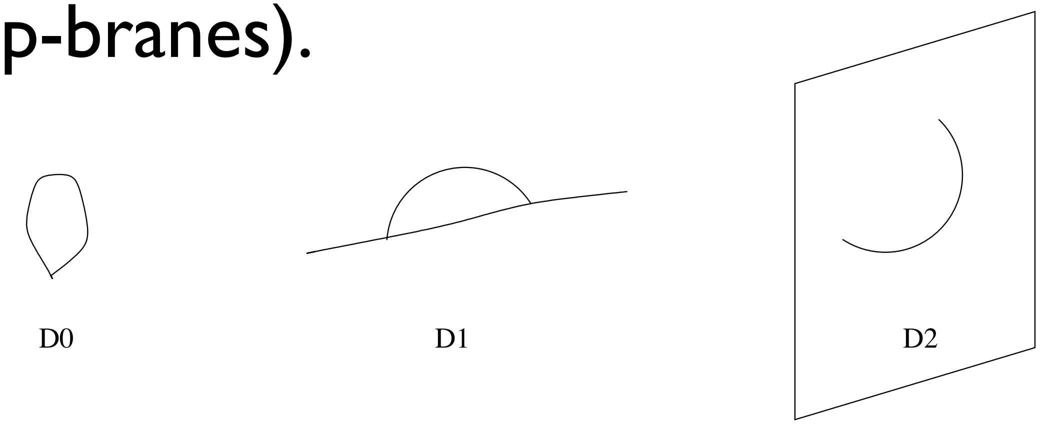
1.2. D-branes

The fundamental string (perturbative state of the spectrum) occurs as a solitonic solution, electric source for B_2

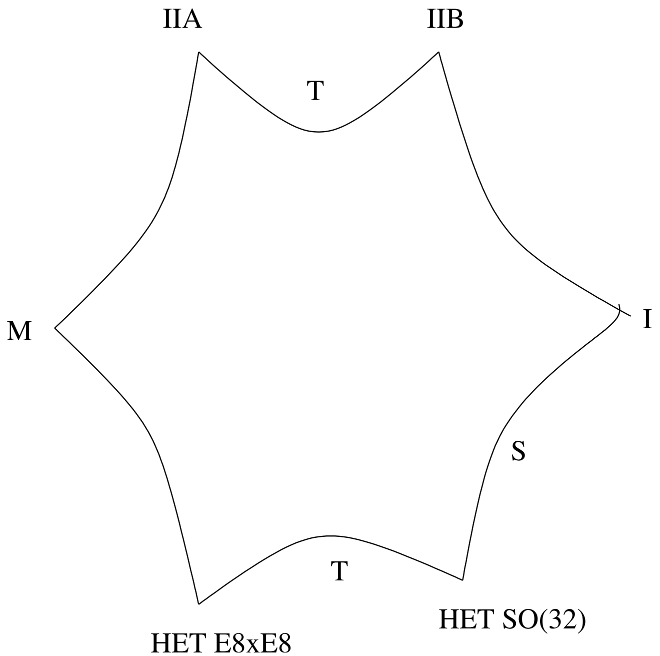
A class of p-branes, Dp-branes, are the electric sources for C_{p+1}. They are non-perturbative.

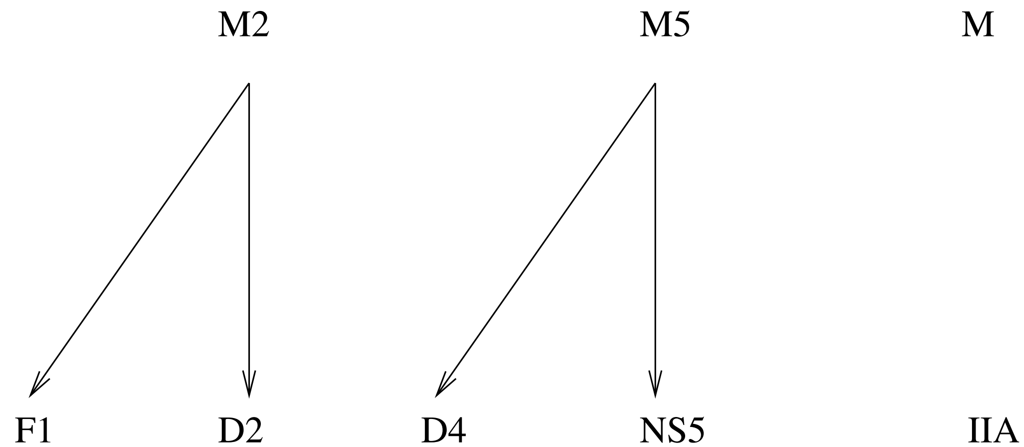
They are also (p+1)-dimensional hypersurfaces on which fundamental strings can end $\Rightarrow$

Dynamics determined at weak coupling by open string perturbation theory $\Rightarrow$ U(1) gauge theory (U(N) for a system of Dp-branes).

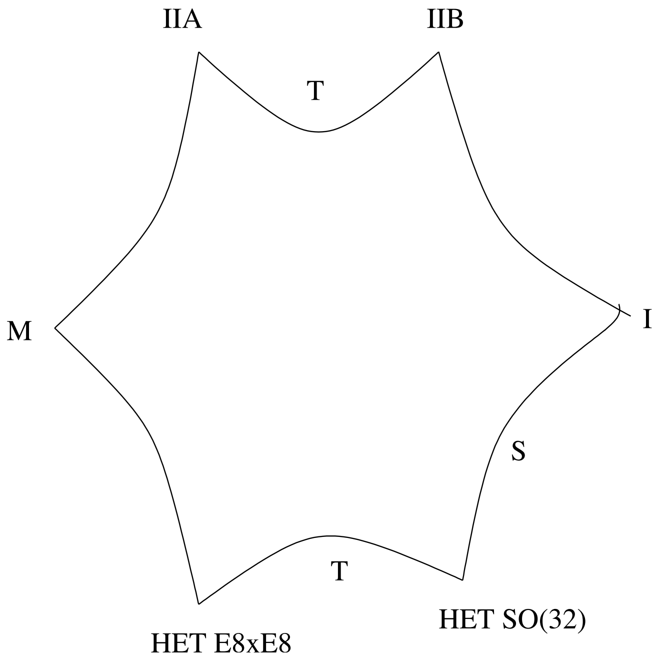


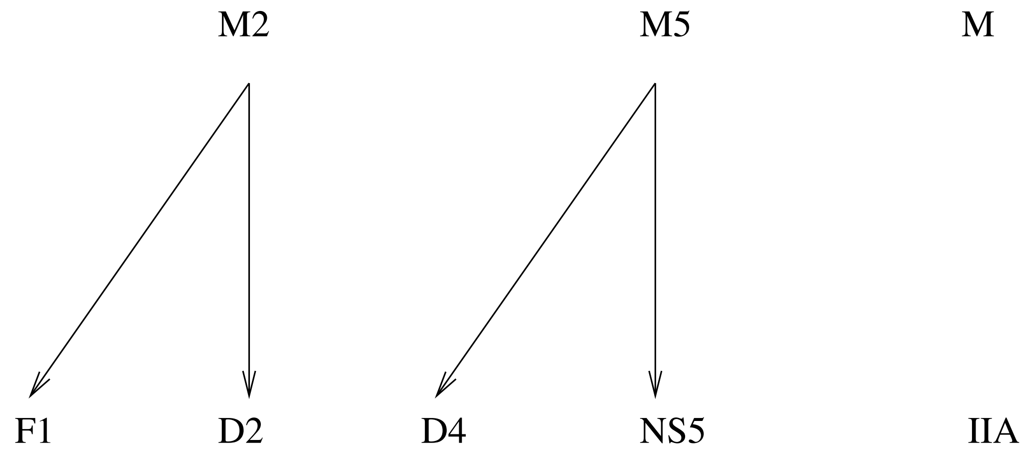
D-branes are related to other branes through the web of dualities that relate the different string theories in 10 dim:





D-branes are related to other branes through the web of dualities that relate the different string theories in 10 dim:





N coincident M2-branes $\rightarrow$ M-theory on $AdS_4 \times S^7$ dual to the 3 dim $\mathcal{N} = 8$ gauge theory living on the M2-branes

N coincident M5-branes $\rightarrow$ M-theory on $AdS_7 \times S^4$ dual to the 6 dim (2,0) field theory living on the M5-branes

Recently, M2-branes on an orbifold $\rightarrow$ M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ dual to the $\mathcal{N}=6$ Chern-Simons matter theory living on the M2: ABJM model

(Aharony, Bergman, Jafferis, Maldacena'08)

Upon reduction to Type IIA: Type IIA on $AdS_4 \times CP^3$

New AdS/CFT pair in which to test the gauge/gravity correspondence

3 dimensions $\rightarrow$ Applications in condensed matter

Recently, M2-branes on an orbifold $\rightarrow$ M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ dual to the $\mathcal{N}=6$ Chern-Simons matter theory living on the M2: ABJM model

(Aharony, Bergman, Jafferis, Maldacena'08)

Upon reduction to Type IIA: Type IIA on $AdS_4 \times CP^3$

New AdS/CFT pair in which to test the gauge/gravity correspondence

3 dimensions $\rightarrow$ Applications in condensed matter

Other examples with less SUSY, confining, etc

1.3. Less supersymmetric extensions

Type IIB on $AdS_5 \times Y_5$ with Y_5 an Einstein manifold bearing 5-form flux:

1.3. Less supersymmetric extensions

Type IIB on $AdS_5 \times Y_5$ with Y_5 an Einstein manifold bearing 5-form flux:

$Y_5 \leftrightarrow C_6$ with C_6 Ricci-flat such that $ds^2_{C_6} = dr^2 + r^2 ds^2_{Y_5}$

1.3. Less supersymmetric extensions

Type IIB on $AdS_5 \times Y_5$ with Y_5 an Einstein manifold bearing 5-form flux:

$Y_5 \leftrightarrow C_6$ with C_6 Ricci-flat such that $ds^2_{C_6} = dr^2 + r^2 ds^2_{Y_5}$

SUSY preserved $\Leftrightarrow$ Y_5 is Sasaki-Einstein ($\Leftrightarrow$ C_6 is Ricci-flat and Kähler (Calabi-Yau 3-fold))

1.3. Less supersymmetric extensions

Type IIB on $AdS_5 \times Y_5$ with Y_5 an Einstein manifold bearing 5-form flux:

$Y_5 \leftrightarrow C_6$ with C_6 Ricci-flat such that $ds^2_{C_6} = dr^2 + r^2 ds^2_{Y_5}$

SUSY preserved $\Leftrightarrow$ Y_5 is Sasaki-Einstein ($\Leftrightarrow$ C_6 is Ricci-flat and Kähler (Calabi-Yau 3-fold))

Examples: $Y_5 = S^5, \quad C_6 = \mathbb{C}^6$

$$Y_5 = T^{1,1} = \frac{SU(2) \times SU(2)}{U(1)}\,, \qquad C_6 \text{ is the conifold}$$

1.3. Less supersymmetric extensions

Type IIB on $AdS_5 \times Y_5$ with Y_5 an Einstein manifold bearing 5-form flux:

$Y_5 \leftrightarrow C_6$ with C_6 Ricci-flat such that $ds^2_{C_6} = dr^2 + r^2 ds^2_{Y_5}$

SUSY preserved $\Leftrightarrow$ Y_5 is Sasaki-Einstein ($\Leftrightarrow C_6$ is Ricci-flat and Kähler (Calabi-Yau 3-fold))

Examples: $Y_5 = S^5$, $\quad C_6 = \mathbb{C}^6$

$$Y_5 = T^{1,1} = \frac{SU(2) \times SU(2)}{U(1)}\,, \qquad C_6 \text{ is the conifold}$$

$\leftrightarrow$ Dual to N=1 SCFTs arising from a stack of D3-branes sitting at the tip of the CY cone

$T^{1,1}$: Klebanov-Witten model

$Y_5 = Y^{p,q}$, p, q coprime positive integers

(Gauntlett, Martelli, Sparks, Waldram'04)

Include $S^5/\mathbb{Z}_3$, $T^{1,1}/\mathbb{Z}_2$ plus quasi-regular and irregular SE manifolds

New tests of AdS/CFT (central charges of the CFT irrational..)

$Y_5 = Y^{p,q}$, p, q coprime positive integers

(Gauntlett, Martelli, Sparks, Waldram'04)

Include $S^5/\mathbb{Z}_3$, $T^{1,1}/\mathbb{Z}_2$ plus quasi-regular and irregular SE manifolds

New tests of AdS/CFT (central charges of the CFT irrational..)

β deformed Lunin-Maldacena backgrounds:

Exactly marginal deformations of $AdS_5 \times S^5 \rightarrow$ $AdS_5 \times S^5_{\rm deformed}$, dual to a N=1 SCFT exactly marginal deformation of N=4 SYM

Frolov: Natural extension depending on three deformation parameters dual to a N=0 SCFT

Replace AdS by asymptotically AdS (CFT perturbed by relevant operators), add fractional branes,... $\rightarrow$

Confining backgrounds (Klebanov-Strassler, Maldacena-Nuñez)

1.4. Baryons and AdS/CFT

What in gauge theory corresponds to non-perturbative objects, such as D-branes, in string theory?

1.4. Baryons and AdS/CFT

What in gauge theory corresponds to non-perturbative objects, such as D-branes, in string theory?

Euclidean Dp-branes wrapped over p+1 cycles $\leftrightarrow$ Instantons

Dp-branes over p cycles $\leftrightarrow$ Baryons, dibaryons..

Dp-branes over p-2 cycles $\leftrightarrow$ Domain walls

1.4. Baryons and AdS/CFT

What in gauge theory corresponds to non-perturbative objects, such as D-branes, in string theory?

Euclidean Dp-branes wrapped over p+1 cycles $\leftrightarrow$ Instantons

Dp-branes over p cycles $\leftrightarrow$ Baryons, dibaryons..

Dp-branes over p-2 cycles $\leftrightarrow$ Domain walls

In $AdS_5 \times S^5$: D5-brane wrapped on the S^5 $\leftrightarrow$ Baryon

Outline

1. Introduction
   1.1. Gauge/gravity (holographic) correspondence
   1.2. D-Branes
   1.3. Less supersymmetric extensions
   1.4. Baryons and AdS/CFT
2. **The baryon vertex in $AdS_5 \times S^5$**
3. Gauge/gravity calculation of the energy
4. Reduce the number of fundamental strings
   4.1. The classical solution
   4.2. Stability analysis
5. Baryons at finite 't Hooft coupling
6. Conclusions

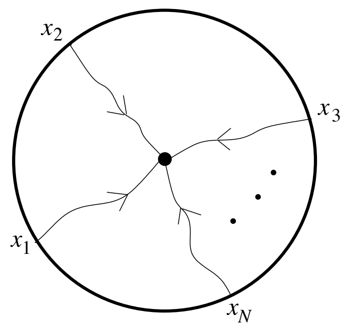
2. The baryon vertex in $AdS_5 \times S^5$

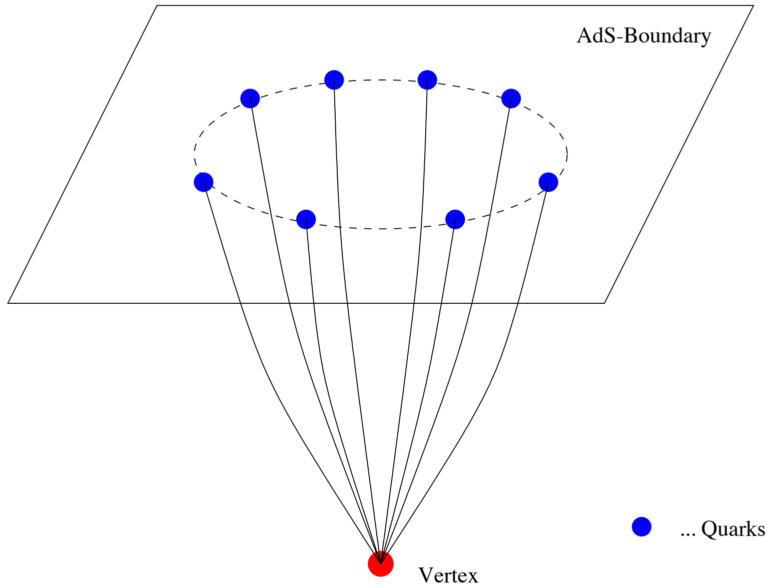
Gauge invariant coupling of N external quarks

2. The baryon vertex in $AdS_5 \times S^5$

Gauge invariant coupling of N external quarks

Through AdS/CFT external quarks are regarded as endpoints of F-strings in AdS

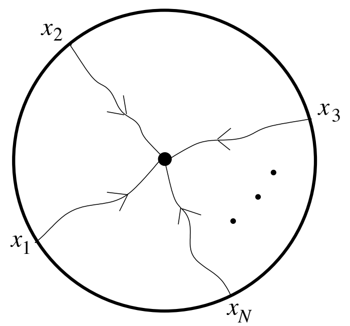


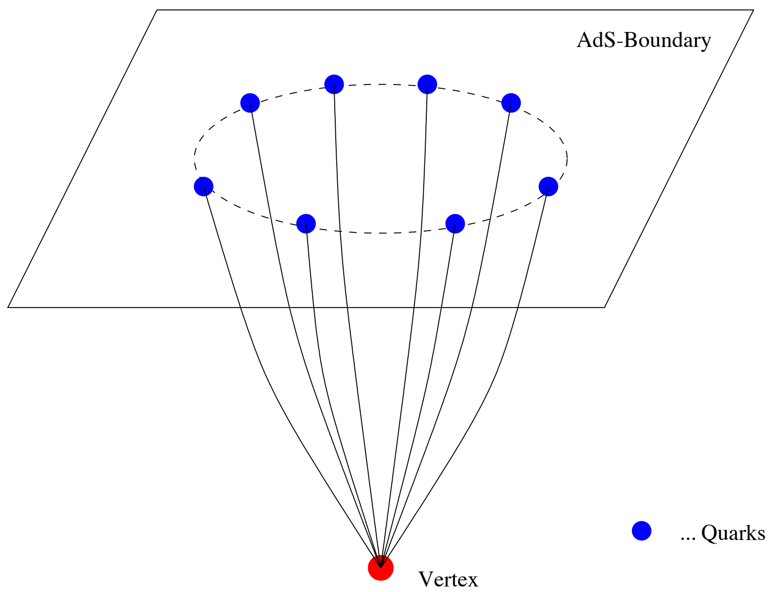


2. The baryon vertex in $AdS_5 \times S^5$

Gauge invariant coupling of N external quarks

Through AdS/CFT external quarks are regarded as endpoints of F-strings in AdS





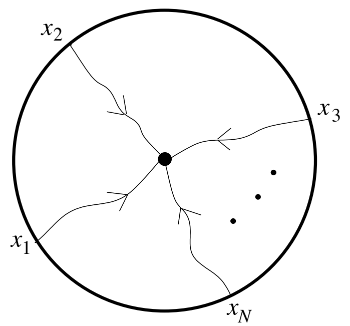
Baryon vertex in the gravity side: D5-brane wrapped on the 5-sphere (Witten'98):

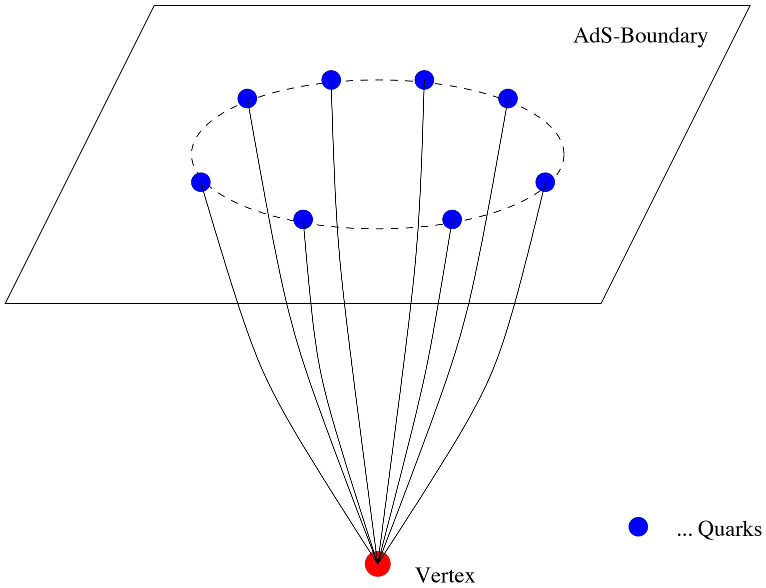
$$S_{CS} = 2\pi\, T_5 \int_{\mathbb{R}\times S^5} P[F_5] \wedge A = N\, T_{F1} \int_{\mathbb{R}} dt A_t$$

2. The baryon vertex in $AdS_5 \times S^5$

Gauge invariant coupling of N external quarks

Through AdS/CFT external quarks are regarded as endpoints of F-strings in AdS





Baryon vertex in the gravity side: D5-brane wrapped on the 5-sphere (Witten'98):

$$S_{CS} = 2\pi \, T_5 \int_{\mathbb{R}\times S^5} P[F_5] \wedge A = N \, T_{F1} \int_{\mathbb{R}} dt A_t$$

N charge cancelled by N F-strings ending on the 5-brane

The F-strings have a unique ground state which is fermionic
(Bachas, Douglas, Green'97)

$\Rightarrow$ D5-brane antisym combination of N fermionic F-strings

$\leftrightarrow$ quarks

The F-strings have a unique ground state which is fermionic
(Bachas, Douglas, Green'97)

$\Rightarrow$ D5-brane antisym combination of N fermionic F-strings
$\leftrightarrow$ quarks

However, within the gauge/gravity correspondence it is possible to construct bound states of l quarks with $l < N$ (non-singlets) (Brandhuber, Itzhaki, Sonnenschein, Yankielowitz'98; Imamura'98)

The F-strings have a unique ground state which is fermionic (Bachas, Douglas, Green'97)

$\Rightarrow$ D5-brane antisym combination of N fermionic F-strings

$\leftrightarrow$ quarks

However, within the gauge/gravity correspondence it is possible to construct bound states of l quarks with $l < N$ (non-singlets) (Brandhuber, Itzhaki, Sonnenschein, Yankielowitz'98; Imamura'98)

In $AdS_5 \times S^5$: $5N/8 \leq l \leq N$

In $AdS_4 \times CP^3$: $2N/3 \leq l \leq N$ (Y.L., Picos, Sfetsos, Siampos'11)

The F-strings have a unique ground state which is fermionic
(Bachas, Douglas, Green'97)

$\Rightarrow$ D5-brane antisym combination of N fermionic F-strings
$\leftrightarrow$ quarks

However, within the gauge/gravity correspondence it is possible to construct bound states of l quarks with $l < N$ (non-singlets) (Brandhuber, Itzhaki, Sonnenschein, Yankielowitz'98; Imamura'98)

In $AdS_5 \times S^5$: $5N/8 \leq l \leq N$

In $AdS_4 \times CP^3$: $2N/3 \leq l \leq N$ (Y.L., Picos, Sfetsos, Siampos'11)

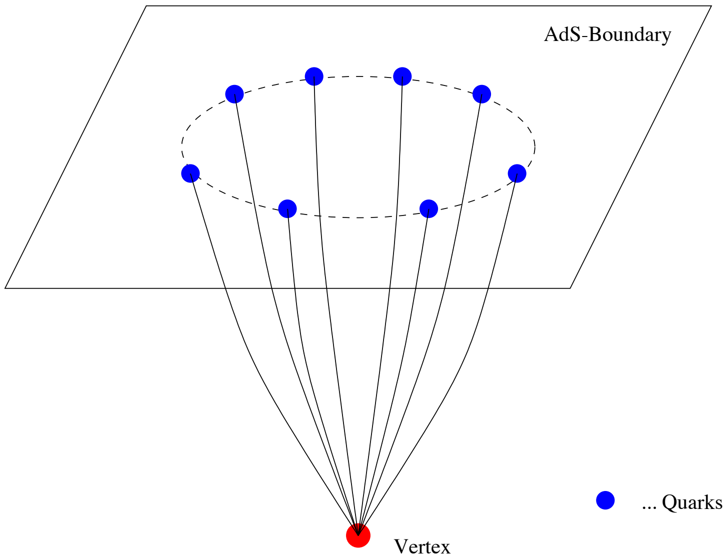
What happens in less supersymmetric and/or confining backgrounds?
And at finite 't Hooft coupling?

Outline

1. Introduction
    1.1. Gauge/gravity (holographic) correspondence
    1.2. D-Branes
    1.3. Less supersymmetric extensions
    1.4. Baryons and AdS/CFT
2. The baryon vertex in
3. **Gauge/gravity calculation of the energy**
4. Reduce the number of fundamental strings
    4.1. The classical solution
    4.2. Stability analysis
5. Baryons at finite ‘t Hooft coupling
6. Conclusions

3. Gauge/gravity calculation of the energy

(Brandhuber, Itzhaki, Sonnenschein, Yankielowitz'98; Imamura'98; Maldacena'98)



$$\rho(0) = \rho_0$$
$$\rho(L) = \infty$$

Consider a uniform distribution of strings on an $\mathbb{M}_p$ shell

Non-SUSY but we can ignore the backreaction

In the probe brane approach: $S = S_{Dp} + S_{NF1}$:

$$S_{NF1} = -N\, T_{F1} \int dt\, dr \sqrt{|\mathrm{det} P(G)|}$$

with $\tau = t \quad \sigma = r$ and the AdS direction $\rho = \rho(r)$

$$S_{Dp} = -T_p \int_{\mathbb{R}\times\mathbb{M}_p} d^{p+1}\xi \sqrt{|\det P(G + 2\pi F - B)|}$$

In $AdS_5 \times Y_5$:

$$ds^2 = \frac{\rho^2}{R^2} dx_{1,3}^2 + \frac{R^2}{\rho^2} d\rho^2 + R^2 ds_{Y_5}^2$$

$$R^4 = \frac{4\pi^4 N g_s}{\text{Vol}(Y_5)}, \qquad F_5 = 4\, R^4 (1 + *) d\text{Vol}(Y_5)$$

Bulk equation of motion: $$\frac{\rho^4}{\sqrt{\frac{\rho^4}{R^4} + \rho'^2}} = c$$

Boundary equation of motion: $$\frac{\rho_0{}'}{\sqrt{\frac{\rho_0^4}{R^4} + \rho_0{}'^2}} = \frac{T_5\, R^4\, \text{Vol}(Y_5)}{N T_{F1}}$$

Define $\sqrt{1-\beta^2} = \dfrac{T_5\, R^4\, \mathrm{Vol}(Y_5)}{N T_{F1}}$ with $\beta \in [0,1]$

The two equations can be combined into:

$$\frac{\rho^4}{\sqrt{\frac{\rho^4}{R^4} + \rho'^2}} = \beta\, \rho_0^2\, R^2$$

Integrating: Size of the configuration:

$$L = \frac{R^2}{\rho_0} \int_1^\infty dz \frac{\beta}{z^2 \sqrt{z^4 - \beta^2}}$$

On-shell energy:

$$E = E_{Dp} + E_{NF1} = N T_{F1} \rho_0 \Big(\sqrt{1-\beta^2} + \int_1^\infty dz \frac{z^2}{\sqrt{z^4 - \beta^2}} \Big)$$

Binding energy:

$$E_{\rm bin} = NT_{F1}\rho_0\Big(\sqrt{1-\beta^2} + \int_1^\infty dz\Big[\frac{z^2}{\sqrt{z^4-\beta^2}} - 1\Big] - 1\Big)$$

where we have substracted the energy of the constituents (when the brane is located in $\rho_0 = 0$ the strings become radial and correspond to free quarks)

Binding energy:

$$E_{\rm bin} = NT_{F1}\rho_0\Big(\sqrt{1-\beta^2} + \int_1^\infty dz\Big[\frac{z^2}{\sqrt{z^4-\beta^2}} - 1\Big] - 1\Big)$$

where we have substracted the energy of the constituents (when the brane is located in $\rho_0 = 0$ the strings become radial and correspond to free quarks)

As a function of L:

$$E_{\rm bin} = -f(\beta)\frac{\sqrt{\lambda}}{L} \qquad \text{with} \qquad f(\beta) \geq 0$$

$\Rightarrow$ - The configuration is stable

- $E_{\rm bin} \sim 1/L$ dictated by conformal invariance
- $E_{\rm bin} \sim \sqrt{\lambda}$ non-trivial prediction for the non-perturbative regime of the gauge theory

Universal behavior for all $AdS_5 \times Y_5$ backgrounds, independent of SUSY

Universal behavior for all $AdS_5 \times Y_5$ backgrounds, independent of SUSY

Same thing in beta deformed LM backgrounds and multi beta deformed Frolov backgrounds (non-SUSY)

$\rightarrow$ Based on conformality

Universal behavior for all $AdS_5 \times Y_5$ backgrounds, independent of SUSY

Same thing in beta deformed LM backgrounds and multi beta deformed Frolov backgrounds (non-SUSY)

$\rightarrow$ Based on conformality

Confining background?

Maldacena-Nuñez: $E_{\rm bin} \sim L$

(Loewy, Sonnenschein'01)

Universal behavior for all $AdS_5 \times Y_5$ backgrounds, independent of SUSY

Same thing in beta deformed LM backgrounds and multi beta deformed Frolov backgrounds (non-SUSY)

$\rightarrow$ Based on conformality

Confining background?

Maldacena-Nuñez: $E_{\rm bin} \sim L$

(Loewy, Sonnenschein'01)

In $AdS_4 \times CP^3$: $E_{\rm bin} = -f(\beta)\frac{\sqrt{\lambda}}{L}$ with a different $f(\beta)$

(Y.L., Picos, Sfetsos, Siampos'11)

Universal behavior for all $AdS_5 \times Y_5$ backgrounds, independent of SUSY

Same thing in beta deformed LM backgrounds and multi beta deformed Frolov backgrounds (non-SUSY)

$\rightarrow$ Based on conformality

Confining background?

Maldacena-Nuñez: $E_{\rm bin} \sim L$

(Loewy, Sonnenschein'01)

In $AdS_4 \times CP^3$: $E_{\rm bin} = -f(\beta)\dfrac{\sqrt{\lambda}}{L}$ with a different $f(\beta)$

(Y.L., Picos, Sfetsos, Siampos'11)

Non-singlets?

Outline

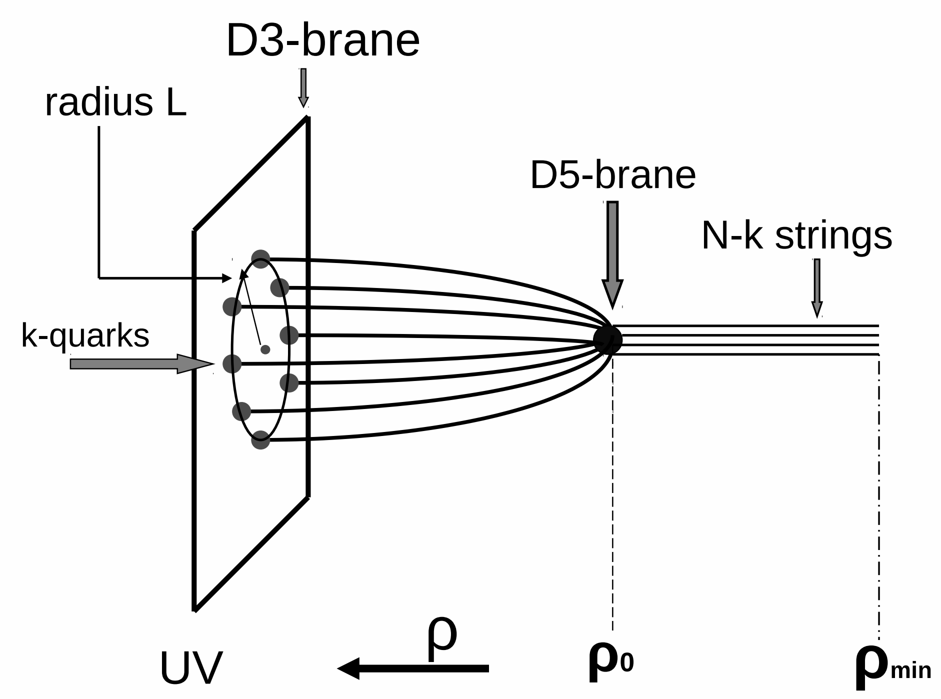
4. Reduce the number of fundamental strings

In $AdS_5 \times S^5$: Baryon vertex classical solutions with number of quarks $5N/8 \leq k \leq N$ (non-singlet)

(Brandhuber, Itzhaki, Sonnenschein, Yankielowitz'98; Imamura'98)

Stable against fluctuations for $0.813\,N \leq k \leq N$

(Sfetsos, Siampos'08)



4.1. The classical solution

The boundary equation of motion changes:

$$\frac{\rho_0'}{\sqrt{\frac{\rho_0^4}{R^4}+{\rho_0'}^2}} = \frac{T_5\, R^4\, \mathrm{Vol}(Y_5)}{k\, T_{F1}} + \frac{N-k}{k} \leq 1$$

$\Rightarrow\ 5N/8 \leq k \leq N.$ For $k = 5N/8$ the baryon reduces to free quarks

Same for all Y_5, independent on SUSY

In fact, same bound for beta and multi beta deformed

Same analysis in Maldacena-Nuñez: $3N/4 \leq k \leq N$

$\Rightarrow$ Non-singlet states in non-SUSY or confining backgrounds

4.2. Stability analysis

Important in establishing the physical parameter space
(Avramis, Sfetsos, Siampos'06-08)

4.2. Stability analysis

Important in establishing the physical parameter space
(Avramis, Sfetsos, Siampos'06-08)

Ansatz for the fluctuations (for the strings):

$$\delta x^\mu(t,\rho) = \delta x^\mu(\rho) e^{-i\omega t} \quad \text{for} \quad x^\mu = r, \theta, \phi$$

4.2. Stability analysis

Important in establishing the physical parameter space
(Avramis, Sfetsos, Siampos'06-08)

Ansatz for the fluctuations (for the strings):

$$\delta x^\mu(t,\rho) = \delta x^\mu(\rho) e^{-i\omega t} \quad \text{for} \quad x^\mu = r, \theta, \phi$$

Expand the Nambu-Goto action to quadratic order and study the zero mode problem $\leftrightarrow$ Critical curve in the parametric space separating the stable and unstable regions

Stability reduced to an eigenvalue problem of the general Sturm-Liouville type

Instabilities emerge from longitudinal fluctuations of the strings

For $AdS_5 \times Y_5$ and beta deformed:

Bound for the number of F-strings coming from stability:

$$k \geq \frac{N}{1+\gamma_c}(1+\sqrt{1-\beta^2}) \qquad \gamma_c = 0.538$$

More restrictive than the bound imposed by the existence of a classical solution:

$$k \geq \frac{N}{2}(1+\sqrt{1-\beta^2})$$

For MN: Same bound for stability and existence of classical sol.

For $AdS_5 \times Y_5$ and beta deformed:

Bound for the number of F-strings coming from stability:

$$k \geq \frac{N}{1+\gamma_c}(1+\sqrt{1-\beta^2}) \qquad \gamma_c = 0.538$$

More restrictive than the bound imposed by the existence of a classical solution:

$$k \geq \frac{N}{2}(1+\sqrt{1-\beta^2})$$

For MN: Same bound for stability and existence of classical sol.

$\Rightarrow$ Non-singlet states exist for $\lambda >> 1$

For $AdS_5 \times Y_5$ and beta deformed:

Bound for the number of F-strings coming from stability:

$$k \geq \frac{N}{1+\gamma_c}(1+\sqrt{1-\beta^2}) \qquad \gamma_c = 0.538$$

More restrictive than the bound imposed by the existence of a classical solution:

$$k \geq \frac{N}{2}(1+\sqrt{1-\beta^2})$$

For MN: Same bound for stability and existence of classical sol.

$\Rightarrow$ Non-singlet states exist for $\lambda >> 1$

Can we reach the finite ‘t Hooft coupling region?

5. Baryons at finite ‘t Hooft coupling

Generalize the baryon vertex adding a magnetic flux.

A non-trivial flux adds lower dim brane charges $\rightarrow$ Complementary description of the baryon in terms of D1-branes expanding by Myers dielectric effect

5. Baryons at finite ‘t Hooft coupling

Generalize the baryon vertex adding a magnetic flux.

A non-trivial flux adds lower dim brane charges $\rightarrow$ Complementary description of the baryon in terms of D1-branes expanding by Myers dielectric effect

- Description in terms of the expanded brane (macroscopical) valid in the sugra limit: $R >> 1 \Leftrightarrow \lambda >> 1$
- Micro when $\dfrac{4\pi R^2}{n} << l_s^2 \Leftrightarrow \lambda << n^2$

5. Baryons at finite ‘t Hooft coupling

Generalize the baryon vertex adding a magnetic flux.

A non-trivial flux adds lower dim brane charges $\rightarrow$ Complementary description of the baryon in terms of D1-branes expanding by Myers dielectric effect

- Description in terms of the expanded brane (macroscopical) valid in the sugra limit: $R >> 1 \Leftrightarrow \lambda >> 1$

- Micro when $\dfrac{4\pi R^2}{n} << l_s^2 \Leftrightarrow \lambda << n^2$

$\Rightarrow$ It allows to explore the region of finite λ

Outline

1. Introduction
   1.1. Gauge/gravity (holographic) correspondence
   1.2. D-Branes
   1.3. Less supersymmetric extensions
   1.4. Baryons and AdS/CFT
2. The baryon vertex in
3. Gauge/gravity calculation of the energy
4. Reduce the number of fundamental strings
   4.1. The classical solution
   4.2. Stability analysis
5. **Baryons at finite ‘t Hooft coupling**
6. Conclusions

5. Baryons at finite 't Hooft coupling

Generalize the baryon vertex adding a magnetic flux.

A non-trivial flux adds lower dim brane charges $\rightarrow$
Complementary description of the baryon in terms of
D1-branes expanding by Myers dielectric effect

5. Baryons at finite 't Hooft coupling

Generalize the baryon vertex adding a magnetic flux.

A non-trivial flux adds lower dim brane charges $\rightarrow$ Complementary description of the baryon in terms of D1-branes expanding by Myers dielectric effect

- Description in terms of the expanded brane (macroscopical) valid in the sugra limit: $R >> 1 \;\Leftrightarrow\; \lambda >> 1$
- Micro when $\dfrac{4\pi R^2}{n} << l_s^2 \;\Leftrightarrow\; \lambda << n^2$

5. Baryons at finite ‘t Hooft coupling

Generalize the baryon vertex adding a magnetic flux.

A non-trivial flux adds lower dim brane charges $\rightarrow$ Complementary description of the baryon in terms of D1-branes expanding by Myers dielectric effect

- Description in terms of the expanded brane (macroscopical) valid in the sugra limit: $R >> 1 \Leftrightarrow \lambda >> 1$

- Micro when $\dfrac{4\pi R^2}{n} << l_s^2 \Leftrightarrow \lambda << n^2$

$\Rightarrow$ It allows to explore the region of finite λ

5. Baryons at finite 't Hooft coupling

Generalize the baryon vertex adding a magnetic flux.

A non-trivial flux adds lower dim brane charges $\rightarrow$ Complementary description of the baryon in terms of D1-branes expanding by Myers dielectric effect

- Description in terms of the expanded brane (macroscopical) valid in the sugra limit: $R >> 1 \Leftrightarrow \lambda >> 1$

- Micro when $\frac{4\pi R^2}{n} << l_s^2 \Leftrightarrow \lambda << n^2$

$\Rightarrow$ It allows to explore the region of finite λ

- Complementary at finite n. Should agree at large n

The magnetic flux modifies the dynamics:

The magnetic flux modifies the dynamics:

In $AdS_5 \times Y^{p,q}$:

$$ds^2 = \frac{\rho^2}{R^2} dx_{1,3}^2 + \frac{R^2}{\rho^2} d\rho^2 + R^2 ds_{Y^{p,q}}^2$$

$$R^4 = \frac{4\pi^4 N g_s}{\mathrm{Vol}(Y^{p,q})} \ , \qquad ds_{Y^{p,q}}^2 = ds^2(M_4) + (\frac{1}{3} d\psi + \sigma)^2$$

$k^\mu = \delta_\psi^\mu$: Reeb vector

M_4 : Local Kähler-Einstein metric

The magnetic flux modifies the dynamics:

In $AdS_5 \times Y^{p,q}$:

$$ds^2 = \frac{\rho^2}{R^2} dx_{1,3}^2 + \frac{R^2}{\rho^2} d\rho^2 + R^2 ds^2_{Y^{p,q}}$$

$$R^4 = \frac{4\pi^4 N g_s}{\mathrm{Vol}(Y^{p,q})} \ , \qquad ds^2_{Y^{p,q}} = ds^2(M_4) + (\frac{1}{3} d\psi + \sigma)^2$$

$k^\mu = \delta^\mu_\psi$: Reeb vector

M_4 : Local Kähler-Einstein metric

Take $F = \mathcal{N} J$ with $J = \frac{1}{2} d\sigma$ the Kähler form:

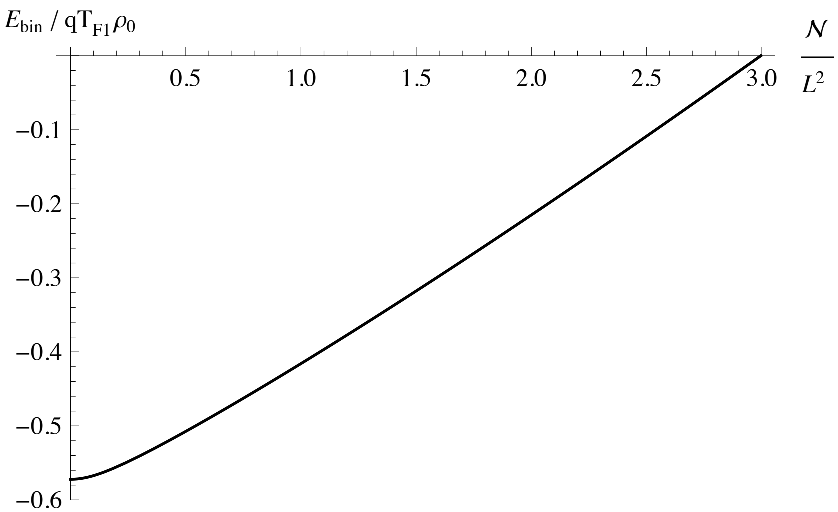
$$\sqrt{1-\beta^2} = \frac{T_5 R^4 \mathrm{Vol}(Y^{p,q})}{N T_{F1}} (1 + \frac{4\pi^2 \mathcal{N}^2}{R^4})$$

$$\beta \leq 1 \Rightarrow \quad \mathcal{N}_{\max} \leftrightarrow \beta = 0$$

Binding energy:

$$E_{\text{bin}} = NT_{F1}\rho_0\left(\sqrt{1-\beta^2} + \int_1^\infty dz\left[\frac{z^2}{\sqrt{z^4-\beta^2}} - 1\right] - 1\right)$$

- E_{bin} negative and decreases monotonically with β
- $E_{\text{bin}} = 0$ for $\beta = 0$ (N free radial strings stretching from ρ_0 to ∞ plus a Dp-brane at ρ_0)



$$\mathcal{N}_{\max} \sim \sqrt{\lambda}$$

$$E_{\text{bin}} = -f(\beta)\frac{\sqrt{\lambda}}{L}$$

Non-singlets:

The bound for the number of quarks is modified ($(k, \mathcal{N})$ parameter space bounded by the values for which the baryon vertex reduces to free quarks):

$$k \geq \frac{5}{8}N + \frac{\mathcal{N}^2}{8\pi^2}\mathrm{Vol}(Y^{p,q})$$

Non-singlets:

The bound for the number of quarks is modified ($(k, \mathcal{N})$ parameter space bounded by the values for which the baryon vertex reduces to free quarks):

$$k \geq \frac{5}{8}N + \frac{\mathcal{N}^2}{8\pi^2}\mathrm{Vol}(Y^{p,q})$$

k_{min} is maximum for the 5-sphere, the most SUSY case

$$\pi^3 = \mathrm{Vol}(S^5) > \frac{16}{27}\pi^3 = \mathrm{Vol}(T^{1,1}) > \mathrm{Vol}(Y^{2,1}) \approx 0.29\pi^3$$

↑ $Y^{p,q}$ with largest volume

Non-singlets:

The bound for the number of quarks is modified ($(k, \mathcal{N})$ parameter space bounded by the values for which the baryon vertex reduces to free quarks):

$$k \geq \frac{5}{8}N + \frac{\mathcal{N}^2}{8\pi^2}\mathrm{Vol}(Y^{p,q})$$

$k_{\min}$ is maximum for the 5-sphere, the most SUSY case

$$\pi^3 = \mathrm{Vol}(S^5) > \frac{16}{27}\pi^3 = \mathrm{Vol}(T^{1,1}) > \mathrm{Vol}(Y^{2,1}) \approx 0.29\pi^3$$

↑ $Y^{p,q}$ with largest volume

In the probe brane approx all configurations are non-SUSY.

Non-singlets:

The bound for the number of quarks is modified ($(k, \mathcal{N})$ parameter space bounded by the values for which the baryon vertex reduces to free quarks):

$$k \geq \frac{5}{8} N + \frac{\mathcal{N}^2}{8\pi^2} \mathrm{Vol}(Y^{p,q})$$

$k_{\min}$ is maximum for the 5-sphere, the most SUSY case

$$\pi^3 = \mathrm{Vol}(S^5) > \frac{16}{27}\pi^3 = \mathrm{Vol}(T^{1,1}) > \mathrm{Vol}(Y^{2,1}) \approx 0.29\pi^3$$

↑ $Y^{p,q}$ with largest volume

In the probe brane approx all configurations are non-SUSY.

Spike solutions non-SUSY for $T^{1,1}$, $Y^{p,q}$ and MN

(Areán, Crooks, Ramallo'04; Canoura, Edelstein, Pando Zayas, Ramallo, Vaman'05; Imamura'04)

Important to obtain information at finite ‘t Hooft coupling.

Important to obtain information at finite 't Hooft coupling.

Micro description:

D1's expanding into a fuzzy M_4. Only known for the S^5 (Janssen, Y.L., Rodriguez-Gomez'06) and the $T^{1,1}$

Myers dielectric effect (an example):

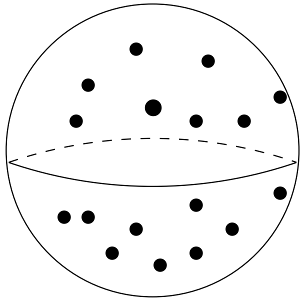
n D0's in a constant RR F_4 :

$$\int d\tau \mathrm{STr}(i_X i_X C^{(3)}) = -\frac{1}{3}\int d\tau \mathrm{STr}(X^j X^i X^k) F^{(4)}_{0ijk} + \dots$$

$F^{(4)}_{0ijk} = 2f\epsilon_{ijk} \rightarrow$ Ground state: n D0's expanded into a fuzzy S^2

$$X^i = \frac{f}{2} J^i, \qquad [J^i, J^j] = 2i\epsilon^{ijk} J^k$$

$$\Rightarrow \sum_{i=1}^3 (X^i)^2 = (\tfrac{f}{2})^2 (n^2 - 1)\mathbb{I} = R^2\, \mathbb{I}$$



C_3 dipole moment $\Rightarrow$ Fuzzy D2-brane

Regime of validity: $4\pi R^2/n << l_s^2$

In the context of the gauge/gravity duality: $\lambda << n^2 \Rightarrow$

Finite 't Hooft coupling regime

The fuzzy $T^{1,1}$

The $T^{1,1}$ is a U(1) bundle over $S^2 \times S^2$ $\Rightarrow$ Take the D1's wrapped on the U(1) fibre and expanding into a $S^2_{\rm fuzzy} \times S^2_{\rm fuzzy}$

The fuzzy $T^{1,1}$

The $T^{1,1}$ is a U(1) bundle over $S^2 \times S^2$ $\Rightarrow$ Take the D1's wrapped on the U(1) fibre and expanding into a $S^2_{\rm fuzzy} \times S^2_{\rm fuzzy}$

Substituting in Myers action for D1-branes:

$$S_{DBI} = -\int \mathrm{STr}\left\{e^{-\phi}\sqrt{|\det\Big(P[E_{\mu\nu} + E_{\mu i}(Q^{-1}-\delta)^i_j E^{jk} E_{k\nu}]\Big)\det Q|}\right\}$$

$$Q^i_j = \delta^i_j + \frac{i}{2\pi}[X^i, X^k]E_{kj}$$

The fuzzy $T^{1,1}$

The $T^{1,1}$ is a U(1) bundle over $S^2 \times S^2$ $\Rightarrow$ Take the D1's wrapped on the U(1) fibre and expanding into a $S^2_{\rm fuzzy} \times S^2_{\rm fuzzy}$

Substituting in Myers action for D1-branes:

$$S_{DBI} = -\int \mathrm{STr}\Big\{e^{-\phi}\sqrt{|\det\Big(P[E_{\mu\nu} + E_{\mu i}(Q^{-1}-\delta)^i_j E^{jk}E_{k\nu}]\Big)\det Q|}\Big\}$$

$$Q^i_j = \delta^i_j + \frac{i}{2\pi}[X^i, X^k]E_{kj}$$

$$E_{nD1} = \frac{N\rho_0}{8\pi}\frac{(m+1)^2}{m(m+2)}\Big(1 + \frac{36\pi^2 m(m+2)}{R^4}\Big)$$

The fuzzy $T^{1,1}$

The $T^{1,1}$ is a U(1) bundle over $S^2 \times S^2 \;\Rightarrow\;$ Take the D1's wrapped on the U(1) fibre and expanding into a $S^2_{\rm fuzzy} \times S^2_{\rm fuzzy}$

Substituting in Myers action for D1-branes:

$$S_{DBI} = -\int \mathrm{STr}\left\{ e^{-\phi} \sqrt{|\det\Big(P[E_{\mu\nu} + E_{\mu i}(Q^{-1} - \delta)^i_j E^{jk} E_{k\nu}]\Big)\det Q|} \right\}$$

$$Q^i_j = \delta^i_j + \frac{i}{2\pi}[X^i, X^k]E_{kj}$$

$$E_{nD1} = \frac{N\rho_0}{8\pi}\,\frac{(m+1)^2}{m(m+2)}\left(1 + \frac{36\pi^2 m(m+2)}{R^4}\right)$$

where $\quad X^i = \dfrac{1}{\sqrt{m(m+2)}} J^i \quad$ for each S^2

The fuzzy $T^{1,1}$

The $T^{1,1}$ is a U(1) bundle over $S^2 \times S^2$ $\Rightarrow$ Take the D1's wrapped on the U(1) fibre and expanding into a $S^2_{\rm fuzzy} \times S^2_{\rm fuzzy}$

Substituting in Myers action for D1-branes:

$$S_{DBI} = -\int \mathrm{STr}\left\{ e^{-\phi} \sqrt{\left|\det\Big(P[E_{\mu\nu} + E_{\mu i}(Q^{-1} - \delta)^i_j E^{jk} E_{k\nu}]\Big) \det Q\right|} \right\}$$

$$Q^i_j = \delta^i_j + \frac{i}{2\pi}[X^i, X^k] E_{kj}$$

$$E_{nD1} = \frac{N\rho_0}{8\pi} \frac{(m+1)^2}{m(m+2)} \left(1 + \frac{36\pi^2 m(m+2)}{R^4}\right)$$

where $X^i = \frac{1}{\sqrt{m(m+2)}} J^i$ for each S^2

Agreement with macro descrip. in large m

The F1 in the micro description

$$S_{CS} = \int \mathrm{STr}\Big\{ P\Big(e^{\frac{i}{2\pi}(i_X i_X)} \sum_q C_q \, e^{-B_2} \Big) e^{2\pi F} \Big\}$$

includes

$$S_{CS} = \frac{1}{\pi} \int dt \, \mathrm{STr}\Big[(i_X i_X)^2 i_k F_5 \Big] A_t$$

The F1 in the micro description

$$S_{CS} = \int \mathrm{STr}\Big\{P\Big(e^{\frac{i}{2\pi}(i_X i_X)}\sum_q C_q\, e^{-B_2}\Big)e^{2\pi F}\Big\}$$

includes

$$S_{CS} = \frac{1}{\pi}\int dt\,\mathrm{STr}\Big[(i_X i_X)^2 i_k F_5\Big]A_t$$

$$= N\frac{(m+1)^2}{m(m+2)}\int dt\, A_t$$

The F1 in the micro description

$$S_{CS} = \int \mathrm{STr}\Big\{P\Big(e^{\frac{i}{2\pi}(i_X i_X)} \sum_q C_q\, e^{-B_2}\Big) e^{2\pi F}\Big\}$$

includes

$$S_{CS} = \frac{1}{\pi} \int dt\, \mathrm{STr}\Big[(i_X i_X)^2 i_k F_5\Big] A_t$$

$$= N \frac{(m+1)^2}{m(m+2)} \int dt\, A_t \quad \to \quad N \int dt\, A_t$$

The F1 in the micro description

$$S_{CS} = \int \mathrm{STr}\Big\{P\Big(e^{\frac{i}{2\pi}(i_X i_X)}\sum_q C_q\, e^{-B_2}\Big)e^{2\pi F}\Big\}$$

includes

$$S_{CS} = \frac{1}{\pi}\int dt\,\mathrm{STr}\Big[(i_X i_X)^2 i_k F_5\Big]A_t$$

$$= N\frac{(m+1)^2}{m(m+2)}\int dt\,A_t \quad \to \quad N\int dt\,A_t$$

Similar micro analysis in MN in terms of D1's expanding into a fuzzy S^2

Outline

1. Introduction
   1.1. Gauge/gravity (holographic) correspondence
   1.2. D-Branes
   1.3. Less supersymmetric extensions
   1.4. Baryons and AdS/CFT
2. The baryon vertex in
3. Gauge/gravity calculation of the energy
4. Reduce the number of fundamental strings
   4.1. The classical solution
   4.2. Stability analysis
5. Baryons at finite ‘t Hooft coupling
6. Conclusions

6. Conclusions

- Non-singlet baryons are predicted by gauge/gravity duality in less supersymmetric and/or confining backgrounds

- They are stable against fluctuations

- At finite 't Hooft coupling:

  Microscopical description of the vertex

  Complete this analysis with α' corrections to the NG action of the strings (or microscopic spike), α' corrections to the background

6. Conclusions

- Non-singlet baryons are predicted by gauge/gravity duality in less supersymmetric and/or confining backgrounds

- They are stable against fluctuations

- At finite 't Hooft coupling:

  Microscopical description of the vertex

  Complete this analysis with α' corrections to the NG action of the strings (or microscopic spike), α' corrections to the background

Thanks!