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The Relationship Between Technical Efficiency and Farm Size

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THE RELATIONSHIP BETWEEN TECHNICAL EFFICIENCY AND FARM SIZE[1]

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Abstract

This paper analyzes the relationship between technical efficiency and size in the framework provided by a production model. Technical efficiency is introduced in the model as a parameter, as a result of which it affects both the input demand and output supply of a profit maximizing producer. This framework allows us to study the theoretical relationship between technical efficiency and size. An empirical application explores this relationship using panel data of dairy farms in Spain.

Key words: technical efficiency, size, dairy farms, panel data.

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1. Introduction

The study of the relationship between farm size and efficiency has been a traditional field of research in agricultural economics. In developed countries the conventional wisdom is that small farms are inefficient. This belief is sometimes based on the fact that there is a tendency for the gradual disappearance of small farms (Stigler, 1958). However, the evidence found in empirical studies has not been conclusive with regard to which are more efficient, small or big farms.

This is an important issue with considerable agricultural policy implications (Barrett, 1996). For example, the existence of a negative relationship between efficiency and size makes a strong case for redistributive land reforms. However, if it is concluded that there is no relationship between size and efficiency, then the size of a farm becomes relevant only as far as total farm income is concerned, not for the efficiency of resource allocation.

This is an interesting question when assessing economic policies in developing countries or even in problematic areas of otherwise developed countries. The European Union, for instance, shows concern for specific areas with farms that are apparently too small to compete in markets that are becoming less regulated and more competitive. In this regard, farm policy decisions across the EU suggest that policy makers firmly believe that the increase in farm size is the right solution for a number of ills ranging from low efficiency to farm household welfare. Therefore, the existence of a relationship between efficiency and size and the nature of this relationship (in particular, the direction of causality) are of crucial importance when analyzing a whole set of farm policies worldwide.

Some empirical literature exists which analyzes the relationship between technical efficiency (TE) and size (Bagi, 1982, Page, 1984). However, these studies do not contain an analytical framework, and the empirical results are therefore difficult to interpret and of little practical use for policy or management purposes.[2]

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[2] A different strand of literature examines the relationship between economic efficiency and size by looking at the shape of the long run average cost curve (Vlastuin *et al.*, 1982; Moschini, 1988).

In this paper we revisit this important topic in an attempt to bridge the gap between empirical applications and economic theory using two simple production models. In the first model size is a function of technical efficiency, while the second model allows technical efficiency to be a function of size. The models provide the basis for studying the theoretical relationship between technical efficiency and size as well as assessing the best approach for empirical analysis. Our empirical application uses panel data of 85 Spanish dairy farms for the years 1987-1991.

The structure of the paper is as follows. Section 2 reviews the literature on TE and size. In Section 3 we present the production model. The empirical model is contained in Section 4 and the data is described in Section 5. Section 6 contains the econometric estimation, and in Section 7 we offer some conclusions.

2. Technical efficiency and size. A review of the literature

Many studies have analyzed the relationship between size and technical efficiency, although some problems arise with the concepts used. Following the classical definition of Farrell (1957), a firm is considered to be technically efficient if it obtains the maximum attainable output given the amount of inputs used and the technology. However, efficiency is sometimes used in the literature synonymously with average productivity. On the other hand, the concept of size is not a clear-cut one (Lund and Price, 1998; Shalit and Sankar, 1987). While total output seems to be a reasonable measure of size, most studies have employed a single (quasi-fixed) input, such as land or number of cows.

The first wave of research dealing with this issue looked at the relationship between productivity (measured as output per acre) and size (measured by acres of land) in Indian agriculture. Sen (1962) found an inverse relationship between farm size and yields per acre, giving rise to a set of follow-up papers that tried to confirm his results or to study related issues such as the impact of technical progress on the productivity of small and large farms (Deolikar, 1981).

Other papers estimate production functions by ordinary least squares, including a dummy variable for size, and test for the significance of the coefficient on this variable. For example, Bagi (1982) estimated a Cobb-Douglas production function for three groups of Indian farms, including a size dummy (based on land) both additively and interactively with the rest of the inputs. He found that, given the level of inputs, small farms produced more output than large farms.[3]

Another group of papers is based on the notion of production frontiers. As a first step, a TE index is calculated using either econometric or linear programming methods. In the second step, a regression of the estimated TE index is run against a set of variables, including size. This second step can be traced back to Timmer (1971) and has been pervasive in this literature. For example, Page (1984) calculated TE for four Indian manufacturing industries. In the second stage he included a size dummy variable (based on number of employees), but he failed to find conclusive relationships for most industries.[4]

Kalaitzandonakes *et al*. (1992) fail to find robust results when analyzing the relationship between TE and size, with the results changing depending on the method used to estimate TE. For this reason, they use a latent variable model where some TE indices are used to obtain the "true" (latent) TE. They report a positive and significant relationship between the "true" TE and size.

Some recent papers (Yuengert, 1993) provide another refinement in the analysis of TE and size. In particular they use stochastic frontiers with heteroskedastic disturbances for the TE error component, a modeling which allows the estimated TE to depend on firm size.

The last set of papers are also based on the notion of frontiers but they use panel data in the first step estimation, allowing for time varying TE. For example, Ahmad and Bravo-Ureta (1995) found a negative correlation between herd size and TE, even though the rate of change of TE was positively correlated to herd size.

In summary, we find that previous papers in the literature carry out empirical analyses with little reference to the underlying economic model. As a consequence, the direction of causality (TE to size or viceversa) is not clear and empirical results are sometimes

[3] The paper by Lau and Yotopoulos (1971) can be considered as an early example of this approach using a profit function.
[4] In this sense Kalaitzandonakes *et al.* (1992) argue that farm size can act as a "catchall" variable which captures the effect of non included variables that can have an effect on efficiency, such as age, education, experience in farming.

difficult to interpret. These are important shortcomings of the literature because farm growth policies are based on two implicit assumptions. First, there is a positive relationship between efficiency and size. Second, there is a clear direction of causality from size to efficiency. Otherwise, efficiency can not be improved by increasing size. For this reason, in the next section we analyze both directions of causality (size to TE vs TE to size) in a simple production model providing a basic framework for empirical analysis of this important issue.

3. Technical efficiency and size in a production model

This section relies in two closely related microeconomic models of production where the level of technical efficiency is introduced as a parameter in the production function. This approach is based on previous work by Lau and Yotopoulos (1971) and therefore differs from the main body of literature which, following Aigner and Chu (1968) and Aigner, Lovell and Schmidt (1977), models TE as part of the random disturbance term in a production function.[5] The main advantage of our approach is that having TE specifically parameterized in the production function allows for the analysis of the role that TE plays in the production decisions of firms.

We now present the two models. In the first, firm size is a function of TE, while in the second model the direction of causality is reversed so that TE is considered to be a function of size.

Model 1: Size as a function of TE

The starting point of this model is a technology represented by a production function with a single fixed and variable input.[6]

$$y_i = A_i f(z_i, x_i) \tag{1}$$

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[5] Atkinson and Cornwell (1993, 1994, 1998) are recent examples of papers that model TE as a parameter using a dual approach.

[6] We believe that this simple model allows us to describe the main features of the relationship between TE and size. As is shown in the Appendix, it is not hard to get similar results with several inputs under mild assumptions about the role of technical efficiency in the production function.

where subscript i denotes firms, y_i is production, f denotes a quasi-concave production function, z_i is a fixed input, x_i is a variable input, and A_i are firm-specific parameters that capture the technical efficiency of each firm.

The issue that we want to study is the differences in production choices between two firms with different levels of TE but otherwise facing the same variable input prices, fixed inputs, and technology. The analytically equivalent question is what happens in this model to producers' choices when there is an increase in TE. In order to address these questions, we assume that producers maximize profits subject to a technological constraint. Since TE appears in the production function, we expect it to show up in the input demand and output supply functions. The comparative statics analysis will provide an answer to these questions.

In a competitive industry let the short-run profit function for firm i be:

$$\pi_i = pA_i f(z_i, x_i) - wx_i \tag{2}$$

where p is output price and w represents the variable input price.

The FOC for profit maximization can be written as:

$$\frac{\partial \pi_i}{\partial x_i} = pA_i f'(z_i, x_i) - w = 0 \tag{3}$$

Solving in (3) for x, one obtains the input demand function:

$$x_i = x\left(z_i, \frac{pA_i}{w}\right) \tag{4}$$

which shows that the demand for inputs depends not only on input prices and fixed inputs, but also on technical efficiency.

Differentiating the first order condition in (3) the relationship between technical efficiency and input demand can be written as:

$$\frac{\partial x_i}{\partial A_i} = -\frac{f'(z_i, x_i)}{A_i f''(z_i, x_i)} > 0 \tag{5}$$

The derivative in (5) is positive because of the assumption of quasi-concavity for f.

Substituting (4) in (1), we obtain the output supply function:

$$y_i = y\left(z_i, \frac{pA_i}{w}\right) \tag{6}$$

Now, differentiating equation (1) we have that:

$$\frac{\partial y_i}{\partial A_i} = f(z_i, x_i) + f'(z_i, x_i)\frac{\partial x_i}{\partial A_i} > 0 \tag{7}$$

Therefore, expression (7) shows how in a simple production model there is a positive effect of TE on size (measured by output).

In summary, this model shows that more efficient producers buy more inputs (5) and use them better (1); therefore, they produce more output (7). If this model were the data generating process of observed data, a positive relationship would be found between TE and size, measured by output.

Jovanovic (1982) finds similar results in a model with exogenous technical efficiency affecting costs.[7] Lundvall and Battese (2000) look for empirical evidence regarding the hypothesis developed by Jovanovic (1982). Using the empirical model in Battese and Coelli (1995), they find a positive relationship between TE and size in a sample of manufacturing firms in Kenya.

The theoretical result in (7) has a clear pro-market flavor: efficient producers do a good job and end-up running a large operation, with the implication that there is no need for or advantage to be gained from farm growth policies. At the same time, the result in (7) could be interpreted as an argument in favor of policies directed at improving the quality of management. In summary, this does not seem to be the underlying model in a number of farm growth policies found around the world. In fact, if farm growth is a policy objective then the underlying hypothesis must be that large farms can do things better than small farms. We explore this issues analytically in model 2.

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[7] Some extensions of Jovanovic's approach can be found in Hopenhayn (1992) and Ericson and Pakes (1995).

Model 2: TE as a function of size

Now, we entertain the common hypothesis that TE is related with size in the following sense:[8]

$$A_i = g(z_i, x_i) \tag{8}$$

In equation (8), TE changes with the level of operation as measured by input use. In this way, we try to introduce in the model the notion that things can be done differently at different levels of operation. For example, large size can force farms to use completely different management techniques or open opportunities for factor specialization. Alternatively, some learning can take place in the growth process. A point worth some discussion is the expected sign for g'. If technical inefficiency is due to lack of knowledge of the technology, then the question is what will happen to a producer when he/she tries to manage a larger firm. Then, ceteris paribus the level of management, one would expect that farmers will perform worse some tasks at a greater size of operation. If this is the case, technical efficiency will decrease with size and, therefore, g' should be negative.[9]

Substituting (8) in (1), the production model becomes:

$$y_i = g(z_i, x_i) f(z_i, x_i) \tag{9}$$

The elasticity of size in this model can be written as:

$$\varepsilon_i = x_i \left(\frac{g'(z_i, x_i)}{g(z_i, x_i)} + \frac{f'(z_i, x_i)}{f(z_i, x_i)} \right) \tag{10}$$

Thus, in this model the effect of TE on size (g) is intertwined with the scale effect.[10] At this point, we face an identification problem in that it is difficult to separate two different effects: technological returns to scale (measured by f'/f) and the fact that TE changes with size (measured by g'/g). The identification of these effects relies on assumptions

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[8] A similar model is widely used in endogenous growth theory (Romer, 1994). In this literature, technical change is a function of economic activity when they try to model learning by doing processes.

[9] The decrease in TE due to an increase in size is sometimes explained in terms of a principal-agent model. It is increasingly difficult to supervise the performance of the work force and/or to devise incentive schemes that lead to an efficient use of resources. However, this model is not of great relevance to our empirical setting of family farms with little hired labor.

about the functional form of f and g and, as a result, identification is only as good as the assumptions we make (Manski, 1995).

This identification problem can be easily illustrated with a Cobb-Douglas technology, where:

$$\begin{aligned} &\text{a) } y_i = A_i x_i^{\beta} \\ &\text{b) } A_i = a x_i^{\gamma} \end{aligned} \tag{11}$$

Substituting in (11b) into (11a), the production model becomes

$$y_i = a x_i^{\gamma} x_i^{\beta} = a x_i^{\gamma+\beta} \tag{12}$$

Therefore, unless one is willing to put some more structure in the model, it is impossible to identify γ and β.

This second model is not independent of the first one. That is, in this model not only is TE assumed to be a function of size but size is still a function of TE following the arguments in equations (1) to (8). In fact, if in equation (12) the constant term is assumed to vary with i (a_i), then model 2 subsumes to model 1 and they are indistinguishable. In fact, the effect of size on efficiency shows up in the level of returns to scale. Therefore, at the current level of understanding of the problem, increasing returns to scale are a valid argument for farm growth policies.

4. Empirical model

In this section we analyze the relationship between TE and size by estimating an empirical version of the theoretical model presented above.

The functional form chosen is a Cobb-Douglas production function

$$y_i = A_i z_i^{\delta} x_i^{\beta} \tag{13}$$

where subscript i denotes firms, y is production, z is a single fixed input, x is the only variable input, β is a production parameter, and A_i are firm-specific parameters that

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[10] The elasticity of scale in model (1) can be written as: $\varepsilon_i = x_i \dfrac{f'(x_i)}{f(x_i)}$.

capture the technical efficiency of each firm. We assume that β is positive and less than one, thereby ruling out the presence of increasing or constant returns to scale.

After some manipulation, the output supply function can be expressed in the following form:

$$y_i = A_i^{\frac{1}{1-\beta}} z_i^{\frac{\delta}{1-\beta}} \left[\frac{\beta p}{w}\right]^{\frac{\beta}{1-\beta}} \tag{14}$$

Taking logarithmic derivatives in (14) with respect to the parameter that represents technical efficiency, we have that:

$$\frac{\partial \ln y_i}{\partial \ln A_i} = \frac{1}{1-\beta} \tag{15}$$

Given that β is less than 1, this derivative is always positive. Therefore, expression (15) shows that in a simple model there is a positive effect of TE on size.

The production function in (13) cannot be estimated from cross-section data since the A_i are not identifiable, making the case for panel data. Indeed, there is a convenient similarity between equation (13) and the fixed-effects model for the estimation of TE with panel data, which can be written as:

$$\ln y_{it} = \alpha_i + \lambda_t + \delta \ln z_{it} + \beta \ln x_{it} + \varepsilon_{it} \tag{16}$$

where y_{it} is the output of firm i in period t, z_{it} a vector of fixed inputs and x_{it} the vector of variable inputs. The parameters α_i are the firm effects, while λ_t are the time effects. The term ε_{it} is assumed to be i.i.d. (0, σ^2).

In this model, relative indexes of technical efficiency can be computed from the comparison of the individual effects. In the case of a logarithmic specification, the expression is (Schmidt and Sickles, 1984):

$$TE_i = \exp(\alpha_i - \max \alpha_j) \tag{17}$$

This index takes the value 1 for the firm with the largest individual effect. The remaining firms will show indexes lower than 1, reflecting the existence of unobservables that make them less efficient.

The estimation of TE from the fixed effects of a panel data model has several advantages over the cross section stochastic frontier (Schmidt and Sickles, 1984). The main advantage arising from the stochastic frontier estimated with panel data is that it is not necessary to assume non correlation between input demands and the level of TE. This property is very important in this model, where input demands are theoretically correlated with the level of TE.

After calculation of the TE indexes, and assuming a Cobb-Douglas functional form, the empirical counterpart of expression (14) that relates TE with size can be written as:

$$\ln y_i = a_0 + a_1 \ln TE_i + a_2 \ln p_i + a_3 \ln w_i + a_4 \ln z_i + u_i \tag{18}$$

Note that subscript t has been dropped. In order to avoid a potential bias if the error term in (18) is correlated with TE, the production function in (16) is estimated with just the first four years of the sample. Since TE is time invariant, we estimate equation (18) using only the fifth year of the sample.

5. Data

This study uses technical and accounting data from a group of 85 dairy farms located in Northern Spain which are enrolled in a voluntary Record Keeping Program. We have data on these farms for a period of five years (1987-91).

The variables used in the estimation of the production frontier are:

Milk:	Milk production (thousands of liters)
Labor:	Number of man-equivalent units
Cows:	Number of milking cows
Feedstuffs:	Total amount of feedstuffs fed to the dairy cows (tons).[11]
Land:	Hectares of land devoted to pasture and crops.

Table 1 shows some descriptive statistics of the variables. The coefficients of variation are quite large, indicating the existence of an important degree of heterogeneity among the production decisions of the farms in the sample.

[11] Since farms have different replacement rates, feedstuffs have been adjusted to include only concentrates given to milking cows.

Table 1.- Descriptive statistics of the data

Variable	Mean	Coeff. Variation	Min	Max
Milk	100.1	0.52	27.6	386.3
Labor	1.95	0.21	1	3.5
Cows	21.0	0.39	8.1	56.0
Feedstuffs	31.2	0.67	1.1	149.3
Land	11.9	0.32	6.8	53.5

6. Estimation and results

As stated before, equation (16) was estimated using the within-group estimator with data from the first four years in the sample and we use the fifth year for the estimation of the relationship between TE and size in equation (18). The estimate of TE is a random variable correlated with the disturbance of the production function in equation (16), which in turn may be correlated with the random disturbance of the supply equation (18). We try to avoid the potential correlation between the random disturbance of equation (16) and the estimated index of TE by splitting the sample in two periods (4+1).

The results of the estimation of the production function in equation (16) can be seen in Table 2.[12] The elasticities are positive and significantly different from zero at conventional levels of significance, except for the coefficient of labor.[13] The standard errors were computed using a variance-covariance matrix robust to heteroskedasticity of unknown form.

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[12] All models were estimated using LIMDEP 7.0 (Greene, 1995).
[13] The result that labor is not significant is not unusual in production functions using dairy farm data. See, for example, Ahmad and Bravo-Ureta (1995).

Table 2. Estimates of the parameters of the production function (1987-1990)

Variable	Estimate	t-ratio
Labor	0.019	0.57
Cows	0.678	9.27***
Feedstuffs	0.235	5.48***
Land	0.129	1.76*
d88	0.021	1.89**
d89	-0.012	-1.04
d90	0.063	3.89***
$R^2 = 0.97$		

*,**, *** Significantly different from zero at the 0.10, 0.05, 0.01 significance level.

The null hypothesis of no correlation between the individual effects and the regressors was rejected using a Hausman test. This result provides empirical evidence that the input demands are correlated with TE (individual effects), as predicted by our model in equation (5).

Technical efficiency indexes were calculated for each firm using the formula in equation (17), with the average TE found to be 0.75. This index is now used as an explanatory variable in the supply equation (18) and the results of the estimation are shown in Table 3. The number of cows is used as a measure of quasi-fixed inputs. The price of feedstuffs is the price of the variable input.

Table 3.- Relationship between size and technical efficiency (supply function)

Independent variables	Estimate	t-ratio
Constant	5.881	4.039***
TE	0.961	9.011***
Price of milk	0.806	2.067**
Price of feedstuffs	-0.055	-0.712
Cows	1.043	25.21***
$R^2 = 0.95$		

, * Significant at the 0.05, 0.01 level.

The coefficient of TE is positive and significantly different from zero at conventional levels of significance. This result is consistent with the theoretical result of expression

(7). In other words, we confirm the hypothesis that TE and size are positively correlated. The coefficients of the other explanatory variables have the expected sign. However, the coefficient on the price of feedstuffs is not significantly different from zero.

7. Conclusions

This paper analyzes the relationship between TE and size in the framework of a simple production model. In this theoretical model, output supply (a measure of size) is a function of output and input prices, quasi-fixed inputs but it is also related to the level of TE. In our empirical application we find a positive and significant relationship between TE and size when controlling for the effects of output prices, input prices, and quasi-fixed inputs, as suggested by theory.

From the policy point of view, an important contribution of the paper is to show that, other things being equal, farmers with more management skills may be found running larger farm operations. In this case, size is only the reflection of good management. However, the reverse argument (which is common in farm policy) that good management practices can be generated just by increasing the size of the operation, has been found not to be amenable to simple empirical analysis. Indeed, our analysis leads us towards working in the direction of previous papers in the literature that analyze the efficiency-size issue in terms of returns to scale.

References

Ahmad, M. and Bravo-Ureta, B. (1995), "An Econometric Analysis of Dairy Output Growth", *American Journal of Agricultural Economics*, 77(4), 914-921.

Aigner, D.J. and S.F. Chu, (1968) "On Estimating the Industry Production Function", *American Economic Review*, 58, 826-839.

Aigner, D.J., C.A.K. Lovell and P.J. Schmidt, (1977), "Formulation and Estimation of Stochastic Frontier Production Function Models", *Journal of Econometrics*, 6, 21-37.

Atkinson S.E. and C. Cornwell, (1993), "Estimation of Technical Efficiency with Panel Data: A Dual Approach", *Journal of Econometrics*, 59, 257-262.

Atkinson S.E. and C. Cornwell, (1994), "Estimation of Output and Input Technical Efficiency Using a Flexible Functional Form and Panel Data", *International Economic Review*, 35, 245-256.

Atkinson S.E. and C. Cornwell, (1998), "Profit versus Cost Frontier Estimation of Price and Technical Inefficiency: A Parametric Approach with Panel Data", *Southern Economic J.*, 64(3), 753-764.

Bagi, F.S., (1982), "Relationship Between Farm Size and Technical Efficiency in West Tennessee Agriculture", *Southern Journal of Agricultural Economics*, 14, 139-44.

Barrett, C., (1996), "On Price Risk and the Inverse Farm Size-Productivity Relationship", *Journal of Development Economics*, 51, 193-215.

Battese, G.E. and T. Coelli, (1995), "A Model for Technical Efficiency Effects in a Stochastic Frontier Production Model for Panel Data", *Empirical Economics*, 20(2), 325-332.

Deolikar, A.B., (1981), "The Inverse Relationship between Productivity and Farm Size: A Test Using Regional Data from India", *American Journal of Agricultural Economics*, 275-9.

Ericson, R. and A. Pakes, (1995), "Markov Perfect Industry Dynamics: A Framework for Empirical Work", *Review of Economic Studies*, 62, 53-82.

Farrell, M.J., (1957), "The Measurement of Productive Efficiency", *Journal of the Royal Statistical Society*, Series A General, Part III, 253-281.

Greene, W., (1995), *LIMDEP version 7.0 User's Manual*, Econometric Software, Inc.

Hopenhayn, H.A., (1992), "Entry, Exit and Firm Dynamics in Long Run Equilibrium", *Econometrica*, 60, 1127-11150.

Jovanovic, B., (1982), "Selection and the Evolution of Industries", *Econometrica*, 50, 649-670.

Kalaitzandonakes, N., S. Wu and J. Ma, (1992), "The Relationship between Technical Efficiency and Size Revisited", *Canadian Journal of Agricultural Economics*, 40, 427-442.

Lau, L.J. and P.A. Yotopoulos, (1971), "A Test for Relative Efficiency and Application to Indian Agriculture", *American Economic Review*, 61, 94-109.

Lund, P. and R. Price, (1998), "The Measurement of Average Farm Size", *Journal of Agricultural Economics*, 49(1), 100-110.

Lundvall, K. and G.E. Battese, (2000), "Firm Size, Age and Efficiency: Evidence from Kenyan Manufacturing Firms", *Journal of Development Studies*, 36(3), 146-163.

Manski, C.F., (1995), *Identification Problems in the Social Sciences*, Harvard University Press.

Moschini, G., (1988), "The Cost Structure of Ontario Dairy Farms: A Microeconometric Analysis", *Canadian Journal of Agricultural Economics*, 36, 187-206.

Page, J.M. Jr., (1984), "Firm Size and Technical Efficiency", *J. of Development Economics*, 16, 129-152.

Romer, P.M., (1994), "The Origins of Endogenous Growth", *Journal of Economic Perspectives*, 1, 3-22.

Schmidt, P. and R. Sickles, (1984), "Production Frontiers and Panel Data", *Journal of Business and Economic Statistics*, 2, 367-374.

Sen, A.K., (1962), "An Aspect of Indian Agriculture", *Economic Weekly*, 243-246.

Shalit, S.S. and V. Sankar, (1987), "The Measurement of Farm Size", *Review of Economics and Statistics*, 59, 290-298.

Stigler, G., (1958), "The Economies of Scale", *Journal of Law Economics*, 1, 54-71.

Timmer, C.P., (1971), "Using a Probabilistic Frontier Production Function to Measure Technical Efficiency", *Journal of Political Economy*, 79, 776-794.

Vlastuin, C., D. Lawrence and J. Quiggin, (1982), "Size Economies in Australian Agriculture", *Review of Marketing and Agricultural Economics*, 50, 27-50.

Yuengert, A.M., (1993), "The Measurement of Efficiency in Life Insurance: Estimates of a Mixed Normal-Gamma Error Model", *Journal of Banking and Finance*, 17, 483-496.

APPENDIX

The starting point of this model is a technology represented by a production function.

$$y = f(A, z, x) \tag{A.1}$$

where, y is production, f denotes a quasi-concave production function, z is vector of fixed inputs, x is a vector of variable inputs, and A is a vector of firm-specific parameters that captures the technical efficiency of each firm.

The role of A in the production function is made more explicit with the following two assumptions:

$$\text{a)}\ \frac{\partial f(A, z, x)}{\partial A} > 0 \qquad \text{b)}\ \frac{\partial^2 f(A, z, x)}{\partial x_i \partial A} > 0 \tag{A.2}$$

In words, holding inputs constant, TE increases production (A.2a) and increases the marginal product of inputs (A.2b), Finally, we assume that inputs in the production process are normal (Takayama, 1993, p. 19),

In a competitive industry the profit function for firm i be can be written as:

$$\Pi(A, z, w, p) = \max_{x,y} \{ py - wx \mid y = f(A, z, x) \} \tag{A.3}$$

where p is output price and w is the input price vector.

The associated Lagrangian and FOC for profit maximization can be written as:

$$\begin{aligned}
&L = py - wx + \lambda(f(A, z, x) - y) \\
&\frac{\partial L}{\partial y} = p - \lambda = 0 \\
&\frac{\partial L}{\partial x_i} = -w_i + \lambda \frac{\partial f(A, z, x)}{\partial x_i} = 0 \\
&\frac{\partial L}{\partial \lambda} = f(A, z, x) - y = 0
\end{aligned} \tag{A.4}$$

Using the envelope theorem it is easy to prove that:

$$\frac{\partial \Pi(A, z, w, p)}{\partial A} = p \frac{\partial f(A, z, x)}{\partial A} \tag{A.5}$$

Differentiating (A.5) with respect to output price we have that:

$$\frac{\partial \Pi(A,z,w,p)}{\partial A \partial p} = \frac{\partial f(A,z,x)}{\partial A} + p\sum_i \frac{\partial^2 f(A,z,x)}{\partial A \partial x_i}\frac{\partial x_i}{\partial p} > 0 \qquad \text{(A.6)}$$

The expression in (A.6) is positive using assumptions (A.2a) and (A.2b) plus input normality $\frac{\partial x_i}{\partial p} > 0$ [14].

As a result, using Hotelling's lemma we have that:

$$\frac{\partial y(A,z,w,p)}{\partial A} = \frac{\partial\left(\frac{\partial \Pi(A,z,w,p)}{\partial p}\right)}{\partial A} = \frac{\partial \Pi(A,z,w,p)}{\partial p \partial A} > 0 \qquad \text{(A.7)}$$

Therefore, in a production process with multiple fixed and variable inputs there is a direct relationship between technical efficiency (A) and size measured by output (y),

[14] Input normality is defined in a cost minimizing setting as $\frac{\partial x_i}{\partial y} > 0$ (Takayama, 1993). However, under profit maximization $\frac{\partial y}{\partial p} = \frac{\partial^2 \Pi}{\partial p^2} > 0$ (convexity of profit function). Therefore, $\frac{\partial x_i}{\partial p} = \frac{\partial x_i}{\partial y}\frac{\partial y}{\partial p} > 0$.