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## **Estimation of a Panel Data Model with Parametric Temporal Variation in Individual Effects**

**Chirok Han, Luis Orea y Peter Schmidt**



**Departamento de Economía**



**Universidad de Oviedo**

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**ESTIMATION OF A PANEL DATA MODEL WITH PARAMETRIC  
TEMPORAL VARIATION IN INDIVIDUAL EFFECTS**

**Chirok Han\*, Luis Orea<sup>†</sup> and Peter Schmidt\***

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**Abstract**

This paper is an extension of Ahn, Lee and Schmidt (2001) to allow a parametric function for time-varying coefficients on the individual effects. It is shown that the main results of Ahn, Lee and Schmidt (2001) hold for our model too. Least squares is consistent, given white noise errors, but less efficient than a GMM estimator. An application of the GMM estimators to the measurement of cost efficiency of Spanish banks is also included. The empirical study shows the consequences of increasing the number of assumptions made regarding the error term. The GMM estimates, especially for private banks, cast doubt on the normality assumption supporting the traditional MLE frontier models.

**Keywords:** panel data, individual effects, temporal variation, GMM, cost efficiency, banks.

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\* Victoria University, Wellington, New Zealand.

<sup>†</sup> Universidad de Oviedo, Spain.

\* Michigan State University, USA.

# 1 Introduction

In this paper we consider the model:

$$y_{it} = X'_{it}\beta + Z'_i\gamma + \lambda_t(\theta)\alpha_i + \epsilon_{it}, \quad i = 1, \dots, N, \quad t = 1, \dots, T. \quad (1.1)$$

We treat  $T$  as fixed, so that “asymptotic” means as  $N \rightarrow \infty$ . The distinctive feature of the model is the interaction between the time-varying parametric function  $\lambda_t(\theta)$  and the individual effect  $\alpha_i$ . We consider the case that the  $\alpha_i$  are “fixed effects,” as will be discussed in more detail below. In this case estimation may be non-trivial due to the “incidental parameters problem” that the number of  $\alpha$ 's grows with sample size; see, for example, Chamberlain (1980).

Models of this form have been proposed and used in the literature on frontier production functions (measurement of the efficiency of production). For example, Kumbhakar (1990) proposed the case that  $\lambda_t(\theta) = [1 + \exp(\theta_1 t + \theta_2 t^2)]^{-1}$ , and Battese and Coelli (1992) proposed the case that  $\lambda_t(\theta) = \exp(-\theta(t - T))$ . Both of these papers considered random effects models in which  $\alpha_i$  is independent of  $X$  and  $Z$ . In fact, both of these papers proposed specific (truncated normal) distributions for the  $\alpha_i$ , with estimation by maximum likelihood. The aim of the present paper is to provide a fixed-effects treatment of models of this type.

There is also a literature on the case that the  $\lambda_t$  themselves are treated as parameters. That is, the model becomes:

$$y_{it} = X'_{it}\beta + Z'_i\gamma + \lambda_t\alpha_i + \epsilon_{it}, \quad i = 1, \dots, N, \quad t = 1, \dots, T. \quad (1.2)$$

This corresponds to using a set of dummy variables for time rather than a parametric function  $\lambda_t(\theta)$ , and now  $\lambda_t\alpha_i$  is just the product of fixed time and individual effects. This model has been considered by Kiefer (1980), Holtz-Eakin, Newey and Rosen (1988), Lee (1991), Chamberlain (1992), Lee and Schmidt (1993) and Ahn, Lee and Schmidt (2001), among others. Lee (1991) and Lee and Schmidt (1993) have applied this model to the frontier production function problem, in

order to avoid having to assume a specific parametric function  $\lambda_t(\theta)$ . Another motivation for the model is that a fixed-effects version allows one to control for unobservables (e.g. macro events) that are the same for each individual, but to which different individuals may react differently.

Ahn, Lee and Schmidt (2001) establish some interesting results for the estimation of model (1.2). A generalized method of moments (GMM) estimator of the type considered by Holtz-Eakin, Newey and Rosen (1988) is consistent given exogeneity assumptions on the regressors  $X$  and  $Z$ . Least squares applied to (1.2), treating the  $\alpha_i$  as fixed parameters, is consistent provided that the regressors are strictly exogenous and that the errors  $\epsilon_{it}$  are white noise. The requirement of white noise errors for consistency of least squares is unusual, and is a reflection of the incidental parameters problem. Furthermore, if the errors are white noise, then a GMM estimator that incorporates the white noise assumption dominates least squares, in the sense of being asymptotically more efficient. This is also a somewhat unusual result, since in the usual linear model with normal errors, the moment conditions implied by the white noise assumption would not add to the efficiency of estimation.

The results of Ahn, Lee and Schmidt apply only to the case that the  $\lambda_t$  are unrestricted, and therefore do not apply to the model (1.1). However, in this paper we show that essentially the same results do hold for the model (1.1). This enables us to use a parametric function  $\lambda_t(\theta)$ , and to test the validity of this assumption, while maintaining only weak assumptions on the  $\alpha_i$ . This may be very useful, especially in the frontier production function setting. Applications using unrestricted  $\lambda_t$  have yielded temporal patterns of efficiency that seem unreasonably variable and in need of smoothing, which a parametric function can accomplish.

The plan of the paper is as follows. Section 2 restates the model and lists our assumptions. Section 3 considers GMM estimation under basic exogeneity assumptions, while Section 4 considers GMM when we add the conditions implied by white noise errors. Section 5 considers least squares estimation and the sense in which it is dominated by GMM. In Section 6, this methodology

is applied to the measurement of cost efficiency of Spanish banks. Finally, Section 7 contains some concluding remarks.

## 2 The model and assumptions

The model is given in equation (1.1) above. We can rewrite it in matrix form, as follows. Let  $y_i = (y_{i1}, \dots, y_{iT})'$ ,  $X_i = (X_{i1}, \dots, X_{iT})'$ , and  $\epsilon_i = (\epsilon_{i1}, \dots, \epsilon_{iT})'$ . Thus  $y_i$  is  $T \times 1$ ,  $X_i$  is  $T \times K$ ,  $\epsilon_i$  is  $T \times 1$ ,  $\beta$  is  $K \times 1$ ,  $\gamma$  is  $g \times 1$ , and  $\alpha_i$  is a scalar. (In this paper, all the vectors are column vectors, and the data matrices are “vertically tall.”) Define a function  $\lambda : \Theta \rightarrow \mathbb{R}^T$ , where  $\Theta$  is a compact subset of  $\mathbb{R}^p$ , such that  $\lambda(\theta) = (\lambda_1(\theta), \dots, \lambda_T(\theta))'$ . Note that  $T$  is fixed. In matrix form, our model is:

$$y_i = X_i\beta + 1_T Z_i' \gamma + \lambda(\theta)\alpha_i + \epsilon_i, \quad i = 1, \dots, N. \quad (2.1)$$

$\lambda(\theta)$  must be normalized in some way such as  $\lambda(\theta)'\lambda(\theta) \equiv 1$  or  $\lambda_1(\theta) \equiv 1$ , to rule out trivial failure of identification arising from  $\lambda(\theta) = 0$  or scalar multiplications of  $\lambda(\theta)$ . Here we choose the normalization  $\lambda_1(\theta) \equiv 1$ .

Let  $W_i = (X'_{i1}, \dots, X'_{iT}, Z'_i)'$ . We make the following “orthogonality” and “covariance” assumptions.

**Assumption 1 (Orthogonality).**  $E(W'_i, \alpha_i)'\epsilon'_i = 0$ .

**Assumption 2 (Covariance).**  $E\epsilon_i\epsilon'_i = \sigma_\epsilon^2 I_T$ .

Assumption 1 says that  $\epsilon_{it}$  is uncorrelated with  $\alpha_i$ ,  $Z_i$ , and  $X_{i1}, \dots, X_{iT}$ , and therefore contains an assumption of strict exogeneity of the regressors. Note that it does not restrict the correlation between  $\alpha_i$  and  $[Z_i, X_{i1}, \dots, X_{iT}]$ , so that we are in the fixed-effects framework. Assumption 2 asserts that the errors are white noise.

We also assume the following regularity conditions.

**Assumption 3 (Regularity).**

- (i)  $(W_i', \alpha_i, \epsilon_i)'$  is independently and identically distributed over  $i$ ;
- (ii)  $\epsilon_i$  has finite fourth moment, and  $E\epsilon_i = 0$ ;
- (iii)  $(W_i', \alpha_i)'$  has finite nonsingular second moment matrix;
- (iv)  $EW_i(Z_i', \alpha_i)$  is of full column rank;
- (v)  $\lambda(\theta)$  is twice continuously differentiable in  $\theta$ .

The first four of these conditions correspond to assumptions (BA.1)–(BA.4) of Ahn, Lee and Schmidt (2001), who give some explanation. Condition (v) is new, and self-explanatory.

### 3 GMM under the Orthogonality Assumption

Let  $u_{it} = u_{it}(\beta, \gamma) = y_{it} - X_{it}'\beta - Z_{it}'\gamma$ , and  $u_i = u_i(\beta, \gamma) = (u_{i1}, \dots, u_{iT})'$ . Since  $u_{it} = \lambda_t(\theta)\alpha_i + \epsilon_{it}$ , it follows that  $u_{it} - \lambda_t(\theta)u_{i1} = \epsilon_{it} - \lambda_t(\theta)\epsilon_{i1}$ , which does not depend on  $\alpha_i$ . This is a sort of generalized within transformation to remove the individual effects. The Orthogonality Assumption (Assumption 1) then implies the following moment conditions:

$$EW_i[u_{it}(\beta, \gamma) - \lambda_t(\theta)u_{i1}(\beta, \gamma)] = 0, \quad t = 2, \dots, T. \quad (3.1)$$

These moment conditions can be written in matrix form, as follows. Define  $G(\theta) = [-\lambda_*(\theta), I_{T-1}]'$ , where  $\lambda_* = (\lambda_2, \dots, \lambda_T)'$ . The generalized within transformation corresponds to multiplication by  $G(\theta)'$ , and the moment conditions (3.1) can equivalently be written as follows:

$$Eb_{1i}(\beta, \gamma, \theta) = E[G(\theta)'u_i(\beta, \gamma) \otimes W_i] = 0. \quad (3.2)$$

(This corresponds to equation (7) of Ahn, Lee and Schmidt (2001), but looks slightly different because our  $W_i$  is a column vector whereas theirs is a row vector.) This is a set of  $(T - 1)(TK + g)$  moment conditions.

Some further analysis is needed to establish that (3.2) contains *all* of the moment conditions implied by the Orthogonality Assumption. Let  $\Sigma_{WW} = EW_iW_i'$ ,  $\Sigma_{W\alpha} = EW_i\alpha_i$ , and  $\sigma_\alpha^2 = E\alpha_i^2$ . Given the model (2.1), the Orthogonality Assumption holds if and only if the following moment conditions hold:

$$E[u_i(\beta, \gamma) \otimes W_i - \lambda(\theta) \otimes \Sigma_{W\alpha}] = 0. \quad (3.3)$$

We could use these moment conditions as the basis for GMM estimation. Alternatively, we can remove the parameter  $\Sigma_{W\alpha}$  by applying a nonsingular linear transformation to (3.3) in such a way that the transformed set of moment conditions is separated into two subsets, where the first subset does not contain  $\Sigma_{W\alpha}$  and the second subset is exactly identified for  $\Sigma_{W\alpha}$ , given  $(\beta, \gamma, \theta)$ . The following transformation accomplishes this.

$$E \begin{bmatrix} G' \otimes I_d \\ \lambda' \otimes I_d \end{bmatrix} [u_i \otimes W_i - \lambda \otimes \Sigma_{W\alpha}] = 0 \quad (3.4)$$

where  $d \equiv TK + g$  for notational simplicity; similarly,  $G$ ,  $\lambda$  and  $u_i$  are shortened expressions for  $G(\theta)$ ,  $\lambda(\theta)$  and  $u_i(\beta, \gamma)$ . This is a nonsingular transformation, since  $(G, \lambda)$  is nonsingular, and therefore GMM based on (3.4) is asymptotically equivalent to GMM based on (3.3). Now split (3.4) into its two parts:

$$E(G' u_i \otimes W_i) = 0 \quad (3.5)$$

$$E(\lambda' u_i) W_i - (\lambda' \lambda) \Sigma_{W\alpha} = 0. \quad (3.6)$$

Here (3.6) is exactly identified for  $\Sigma_{W\alpha}$ , given  $\beta$ ,  $\gamma$  and  $\theta$ , in the sense that the number of moment conditions in (3.6) is the same as the dimension of  $\Sigma_{W\alpha}$ . Also  $\Sigma_{W\alpha}$  does not appear in (3.5). It follows (e.g., Ahn and Schmidt (1995), Theorem 1) that the GMM estimates of  $\beta$ ,  $\gamma$  and  $\theta$  from (3.5) alone are the same as the GMM estimates of  $\beta$ ,  $\gamma$  and  $\theta$  if we use both (3.5) and (3.6), and estimate the full set of parameters  $(\beta, \gamma, \theta, \Sigma_{W\alpha})$ . But (3.5) is the same as (3.2), which establishes that (3.2) contains all the useful information about  $\beta$ ,  $\gamma$  and  $\theta$  implied by the Orthogonality Assumption.

Let  $\bar{b}_1(\beta, \gamma, \theta) = N^{-1} \sum_{i=1}^N b_{1i}(\beta, \gamma, \theta)$ . Then the optimal GMM estimator  $\hat{\beta}$ ,  $\hat{\gamma}$ , and  $\hat{\theta}$  based on the Orthogonality Assumption solves the problem

$$\min_{\beta, \gamma, \theta} N \bar{b}_1(\beta, \gamma, \theta)' V_{11}^{-1} \bar{b}_1(\beta, \gamma, \theta) \quad (3.7)$$

where  $V_{11} = E b_{1i} b_{1i}'$  evaluated at the true parameters. As usual,  $V_{11}$  can be replaced by any consistent estimate. A standard estimate would be

$$\hat{V}_{11} = \frac{1}{N} \sum_{i=1}^N b_{1i}(\tilde{\beta}, \tilde{\gamma}, \tilde{\theta}) b_{1i}(\tilde{\beta}, \tilde{\gamma}, \tilde{\theta})' \quad (3.8)$$

where  $(\tilde{\beta}, \tilde{\gamma}, \tilde{\theta})$  is an initial consistent estimate of  $(\beta, \gamma, \theta)$  such as GMM using identity weighting matrix. Under certain regularity conditions (Hansen (1982), Assumption 3) the resulting GMM estimator is  $\sqrt{N}$ -consistent and asymptotically normal.

To express the asymptotic variance of the GMM estimator analytically, we need a little more notation. Let  $S_X$  be the  $T(TK + g) \times K$  selection matrix such that  $X_i = (I_T \otimes W_i)' S_X$ , and let  $S_Z$  be the  $T(TK + g) \times g$  selection matrix such that  $1_T Z_i' = (I_T \otimes W_i)' S_Z$ .  $S_X$  and  $S_Z$  have the following forms:

$$S_X = (I_K \ O \ \cdots \ O \ O_{K \times g} \ \vdots \ O \ I_K \ \cdots \ O \ O_{K \times g} \ \vdots \ \cdots \ \vdots \ O \ O \ \cdots \ I_K \ O_{K \times g})' \quad (3.9)$$

$$S_Z = (O_{g \times K} \ \cdots \ O_{g \times K} \ I_g \ \vdots \ \cdots \ \vdots \ O_{g \times K} \ \cdots \ O_{g \times K} \ I_g)' = 1_T \otimes (O_{g \times TK}, I_g)' \quad (3.10)$$

where  $O$ 's without dimension subscript stand for  $O_{K \times K}$ . Define  $\Lambda_* = \partial \lambda_*(\theta_0) / \partial \theta'$ . The variance of the asymptotic distribution of the GMM estimates of  $\beta$ ,  $\gamma$  and  $\theta$  equals  $(B_1' V_{11}^{-1} B_1)^{-1}$  where  $V_{11} = E b_{1i} b_{1i}'$  as above and

$$B_1 = [(G \otimes \Sigma_{WW})' S_X, (G \otimes \Sigma_{WW})' S_Z, \Lambda_* \otimes \Sigma_{W\alpha}]. \quad (3.11)$$

This result can be obtained either by direct calculation, or by applying the chain rule to  $B_1$  calculated in Ahn, Lee and Schmidt (2001, p. 251). This asymptotic variance form is obtained from the Orthogonality Assumption only and does not need any further assumption.



A practical problem with this GMM procedure is that it is based on a rather large set of moment conditions. Some considerable simplifications are possible if we make the following assumption of no conditional heteroskedasticity (NCH) of  $\epsilon_i$ :

$$E(\epsilon_i \epsilon_i' | W_i) = \Sigma_{\epsilon\epsilon}. \quad (\text{NCH})$$

Under the NCH assumption,

$$V_{11} = E[G(\theta_0)' \epsilon_i \epsilon_i' G(\theta_0) \otimes W_i W_i'] = G(\theta_0)' \Sigma_{\epsilon\epsilon} G(\theta_0) \otimes \Sigma_{WW}. \quad (3.12)$$

$\Sigma_{WW}$  can be consistently estimated by  $\hat{\Sigma}_{WW} = N^{-1} \sum_{i=1}^N W_i W_i'$ . Also, for any sequence  $(\beta_N, \gamma_N)$  that converges in probability to  $(\beta_0, \gamma_0)$ , we have

$$\frac{1}{N} \sum_{i=1}^N u_i(\beta_N, \gamma_N) u_i(\beta_N, \gamma_N)' \xrightarrow{p} \Sigma_{\epsilon\epsilon} + \sigma_\alpha^2 \lambda(\theta_0) \lambda(\theta_0)'. \quad (3.13)$$

Since  $G(\theta)' \lambda(\theta) = 0$ , for any initial consistent estimate  $(\tilde{\beta}, \tilde{\gamma}, \tilde{\theta})$ ,

$$G(\tilde{\theta})' \left( N^{-1} \sum_{i=1}^N u_i(\tilde{\beta}, \tilde{\gamma}) u_i(\tilde{\beta}, \tilde{\gamma})' \right) G(\tilde{\theta}) \quad (3.14)$$

will consistently estimate  $G(\theta_0)' \Sigma_{\epsilon\epsilon} G(\theta_0)$ . Thus it is easy to construct a consistent estimate of  $V_{11}$  as given in (3.12).

In order to consistently estimate the asymptotic variance under NCH, we need to estimate  $\Sigma_{WW}$ ,  $\Sigma_{W\alpha}$ , and  $G' \Sigma_{\epsilon\epsilon} G$ . Estimation of  $\Sigma_{WW}$  and  $G' \Sigma_{\epsilon\epsilon} G$  was discussed above. We can obtain an estimate of  $\Sigma_{W\alpha}$  from the GMM problem (3.4). A direct algebraic calculation gives us that

$$\hat{\Sigma}_{W\alpha} = \frac{1}{N} \sum_{i=1}^N W_i \frac{\hat{\lambda}' \hat{u}_i}{\hat{\lambda}' \hat{\lambda}} - \frac{1}{N} \sum_{i=1}^N W_i [\widehat{\lambda' \Sigma_{\epsilon\epsilon} G} (\widehat{G' \Sigma_{\epsilon\epsilon} G})^{-1} \hat{G}' \hat{u}_i] / (\hat{\lambda}' \hat{\lambda}) \quad (3.15)$$

where  $\hat{u}_i = u_i(\hat{\beta}, \hat{\gamma})$ ,  $\hat{\lambda} = \lambda(\hat{\theta})$ ,  $\hat{G} = G(\hat{\theta})$ , and  $\widehat{\lambda' \Sigma_{\epsilon\epsilon} G}$  is a consistent estimate of  $\lambda' \Sigma_{\epsilon\epsilon} G$ , one possibility of which is  $N^{-1} \sum_{i=1}^N \hat{\lambda}' \hat{u}_i \hat{u}_i' \hat{G}$ .

Finally, under the NCH assumption, the set of moment conditions (3.2) can be converted into an *exactly identified* set of moment conditions that yield an asymptotically equivalent GMM estimate. Specifically, we can replace the moment conditions  $E b_{1i} = 0$  by the moment conditions

$EB_1'V_{11}^{-1}b_{1i} = 0$ . Routine calculation using the forms of  $B_1$ ,  $V_{11}$  and  $b_{1i}$  yields the explicit expression:

$$EX_i'G(G'\Sigma_{\epsilon\epsilon}G)^{-1}G'u_i = 0 \quad (3.16a)$$

$$EZ_i1_T'G(G'\Sigma_{\epsilon\epsilon}G)^{-1}G'u_i = 0 \quad (3.16b)$$

$$E\Sigma_{W\alpha}'\Sigma_{WW}^{-1}W_i \cdot \Lambda'_*(G'\Sigma_{\epsilon\epsilon}G)^{-1}G'u_i = 0. \quad (3.16c)$$

These three sets of moment conditions respectively correspond to (21a), (21b), and (21c) of Ahn, Lee and Schmidt (2001, p. 229). We can replace the nuisance parameters  $\Sigma_{\epsilon\epsilon}$ ,  $\Sigma_{W\alpha}$  and  $\Sigma_{WW}$  by consistent estimates, as given above (based on some initial consistent GMM estimates of  $\beta$ ,  $\gamma$  and  $\theta$ ). The point of this simplification is that we have drastically reduced the set of moment conditions: there are  $(T-1)(TK+g)$  moment conditions in  $b_{1i}$  (equation (3.2)) but only  $K+g+p$  moment conditions in (3.16).

We note that this is a stronger result than the corresponding result (Proposition 1, p. 229) of Ahn, Lee and Schmidt (2001). In order to reach essentially the same conclusion on the reduction of the number of moment conditions, they impose the assumption that  $\epsilon_i$  is independent of  $(W_i, \alpha_i)$ , a much stronger assumption than our NCH assumption.

## 4 GMM under the Orthogonality and Covariance Assumptions

In this section we continue to maintain the Orthogonality Assumption (Assumption 1), but now we add the Covariance Assumption (Assumption 2), which asserts that  $E\epsilon_i\epsilon_i' = \sigma_\epsilon^2 I_T$ .

Clearly the Covariance Assumption holds if and only if

$$E(u_i u_i') = \sigma_\alpha^2 \lambda \lambda' + \sigma_\epsilon^2 I_T. \quad (4.1)$$

Condition (4.1) contains  $T(T+1)/2$  distinct moment conditions. It also contains the two nuisance parameters  $\sigma_\alpha^2$  and  $\sigma_\epsilon^2$ , and so it should imply  $T(T+1)/2 - 2$  moment conditions for the estimation

of  $\beta$ ,  $\gamma$  and  $\theta$ . These are in addition to the moment conditions (3.2) implied by the Orthogonality Assumption.

To write these moment conditions explicitly, we need to define some notation. Let  $H = \text{diag}(H_2, H_3, \dots, H_T)$ , with  $H_t$  equal to the  $T \times (T - t)$  matrix of the last  $T - t$  columns (the  $(t + 1)$ th through  $T$ th columns) of  $I_T$  for  $t < T$ , and with  $H_T$  equal to a  $T \times (T - 2)$  matrix of the second through  $(T - 1)$ -th columns of  $I_T$ .<sup>1</sup> Then we can write the distinct moment conditions implied by the Orthogonality and Covariance Assumptions as follows:

$$Eb_{1i} = E(G'u_i \otimes W_i) = 0 \quad (4.2a)$$

$$Eb_{2i} = EH'(G'u_i \otimes u_i) = 0 \quad (4.2b)$$

$$Eb_{3i} = E\left[G'u_i \otimes \frac{\lambda'u_i}{\lambda'\lambda}\right] = 0. \quad (4.2c)$$

(In these expressions,  $G$  is short for  $G(\theta)$ ,  $\lambda$  is short for  $\lambda(\theta)$ , and  $u_i$  is short for  $u_i(\beta, \gamma)$ .)

The moment conditions  $b_{1i}$  in (4.2a) are exactly the same as those in (3.2) of the previous section, and follow from the Orthogonality Assumption.

The moment conditions  $b_{2i}$  in (4.2b) correspond to those in equation (12) of Ahn, Lee and Schmidt (2001). Note that it is not the case that  $E(G'u_i \otimes u_i) = 0$ . Rather, looking at a typical element of this product, we have  $E(u_{it} - \lambda_t u_{i1})u_{is}$ , which equals zero for  $s \neq t$  and  $s \neq 1$ . The selection matrix  $H'$  picks out the logically distinct products of expectation zero, the number of which equals  $T(T - 1)/2 - 1$ . The selection matrix  $H$  plays the same role as the definition of the matrices  $U_{it}^\circ$  plays in Ahn, Lee and Schmidt (2001). We note that the moment conditions  $b_{2i}$  follow from the non-autocorrelation of the  $\epsilon_{it}$ ; homoskedasticity would not be needed.

The  $(T - 1)$  moment conditions in  $b_{3i}$  in (4.2c) correspond to those in equation (13) of Ahn, Lee and Schmidt (2001). They assert that, for  $t = 2, \dots, T$ ,  $E(u_{it} - \lambda_t u_{i1})(\sum_{s=1}^T \lambda_s u_{is}) = 0$ , and

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<sup>1</sup>For any matrix  $B$  with  $T$  rows,  $H'_t B$  selects the last  $T - t$  rows of  $B$  for  $t < T$ , and  $H'_T B$  selects the second through  $(T - 1)$ -th rows of  $B$ . For any matrix  $B$  with  $T$  columns,  $BH_t$  selects the last  $T - t$  columns of  $B$  for  $t < T$ , and  $BH_T$  selects the second through  $(T - 1)$ -th columns of  $B$ .

their validity depends on both the non-autocorrelation and the homoskedasticity of the  $\epsilon_{it}$ .

Some further analysis may be useful to establish that (4.2b) and (4.2c) represent all of the useful implications of the Covariance Assumption. We begin with the implication (4.1) of the Covariance Assumption, which we rewrite as

$$E(u_i \otimes u_i) = \sigma_\alpha^2(\lambda \otimes \lambda) + \sigma_\epsilon^2 \text{vec} I_T. \quad (4.3)$$

Now, let  $S$  be the  $T^2 \times T(T+1)/2$  selection matrix such that, for a  $T \times 1$  vector  $u$ ,  $\text{vech}(uu') = S'(u \otimes u)$ , where “vech” is the vector of distinct elements. Then

$$ES'(u \otimes u) = S'[\sigma_\alpha^2(\lambda \otimes \lambda) + \sigma_\epsilon^2 \text{vec} I_T] \quad (4.4)$$

contains the distinct moment conditions.

Now we transform the moment conditions (4.4) by multiplying them by a nonsingular matrix, in such a way that (i) the first  $T(T+1)/2 - 2$  transformed moment conditions are those given in (4.2b) and (4.2c); and (ii) the last two moment conditions are exactly identified for the nuisance parameters ( $\sigma_\alpha^2$  and  $\sigma_\epsilon^2$ ), given the other parameters. This will imply that the last two moment conditions are redundant for the estimation of  $\beta$ ,  $\gamma$  and  $\theta$ , and thus that (4.2b) and (4.2c) contain all of the useful information implied by the Covariance Assumption for estimation of  $\beta$ ,  $\gamma$  and  $\theta$ .

To exhibit the transformation, let  $G_t$  be the  $(t-1)$ th column of  $G$ ; let  $e_t^*$  equal the  $t$ th column of  $I_{T-2}$  and  $e_T$  equal the last column of  $I_T$ ; and define

$$(H_T^{**})' = [-\lambda_T H_T', e_1^* e_T', \dots, e_{T-2}^* e_T', O_{(T-2) \times T}]. \quad (4.5)$$

( $H_T$  was defined above.) Then

$$[G_2 \otimes H_2, \dots, G_{T-1} \otimes H_{T-1}, H_T^{**}]' S \cdot S'(u_i \otimes u_i) = H'(G' \otimes I_T)(u_i \otimes u_i), \quad (4.6)$$

which is the same as in  $b_{2i}$  in (4.2b). Also, let  $J_1^* = I_T - \lambda\lambda'$  and  $J_t^*$ ,  $t = 2, \dots, T$ , is equal to  $\text{diag}\{O_{t \times t}, \lambda_t I_{T-t}\}$  plus a  $T \times T$  matrix with zero elements except for the  $t$ th row which is  $\lambda'$ .

Then

$$H'_1[J_1^*, \dots, J_T^*]S \cdot S'(u_i \otimes u_i) = (\lambda' \otimes G')(u_i \otimes u_i), \quad (4.7)$$

which is equal to  $b_{3i}$  in (4.2c).

The point of the above argument is that the transformations preceding  $S'(u_i \otimes u_i)$  in (4.6) and (4.7), stacked vertically, construct a  $[T(T+1)/2 - 2] \times T(T+1)/2$  matrix of full row rank, and yield the moment conditions  $b_{2i}$  and  $b_{3i}$ . The remaining two moment conditions that determine the nuisance parameters are

$$E \begin{bmatrix} u_{i1}^2 \\ u_{i2}u_{i1} \end{bmatrix} = \begin{bmatrix} \sigma_\alpha^2 + \sigma_\epsilon^2 \\ \lambda_2 \sigma_\alpha^2 \end{bmatrix} \quad (4.8)$$

and must be linearly independent of the others (since they involve  $\sigma_\alpha^2$  and  $\sigma_\epsilon^2$  while the others do not).

The asymptotic variance of the GMM estimate is complicated because it depends on the moments of  $\epsilon_{it}$  up to fourth order. However, we can simplify things with the following ‘‘conditional independence of the moments up to fourth order’’ (CIM4) assumption:

$$\begin{aligned} &\text{Conditional on } (W_i, \alpha_i), \epsilon_{it} \text{ is independent over } t = 1, 2, \dots, T, \text{ with mean} \\ &\text{zero, and with second, third and fourth moments that do not depend on} \\ &(W_i, \alpha_i) \text{ or on } t. \end{aligned} \quad (\text{CIM4})$$

This is a strong assumption; it implies the Orthogonality Assumption, the Covariance Assumption, the NCH assumption, and more. In Appendix A, we calculate the asymptotic variance matrix of the GMM estimate based on (4.2) under the assumption (CIM4).

Let  $\Lambda = \partial\lambda(\theta_0)/\partial\theta$  and note that  $\Lambda_* = G'\Lambda$ . Given assumption (CIM4), the moment conditions (3.16), which are asymptotically equivalent to (4.2a), can be simplified as follows:

$$EX'_i P_G u_i = 0 \quad (4.9a)$$

$$EZ_i 1'_T P_G u_i = 0 \quad (4.9b)$$

$$E\Sigma'_{W\alpha} \Sigma^{-1}_{WW} W_i \cdot \Lambda' P_G u_i = 0. \quad (4.9c)$$

That is, in place of the large set of moment conditions (4.2a), (4.2b) and (4.2c), we can use the reduced set of moment conditions consisting of (4.9), (4.2b) and (4.2c).

A final simplification arises if, conditional on  $(W_i, \alpha_i)$ ,  $\epsilon_{it}$  is i.i.d. normal. In this case, (4.2b) can be shown to be redundant given (4.2a) and (4.2c). (See Proposition 4 of Ahn, Lee and Schmidt (2001, p. 231).) Hence, in that case, the GMM estimator using the moment conditions (4.9) and (4.2c) is efficient.

## 5 Least Squares

In this section we consider the concentrated least squares (CLS) estimation of the model. We treat the  $\alpha_i$  as parameters to be estimated, so this is a true “fixed effects” treatment. We can consider the following least squares problem:

$$\min_{\beta, \gamma, \theta, \alpha_1, \dots, \alpha_N} N^{-1} \sum_{i=1}^N [y_i - X_i\beta - 1_T Z_i' \gamma - \lambda(\theta)\alpha_i]' [y_i - X_i\beta - 1_T Z_i' \gamma - \lambda(\theta)\alpha_i]. \quad (5.1)$$

Solving for  $\alpha_1, \dots, \alpha_N$  first, we get

$$\alpha_i(\beta, \gamma, \theta) = [\lambda(\theta)' \lambda(\theta)]^{-1} \lambda(\theta)' u_i(\beta, \gamma) \quad i = 1, \dots, N. \quad (5.2)$$

where  $u_i(\beta, \gamma) = y_i - X_i\beta - 1_T Z_i' \gamma$  as before. Then the estimates  $\hat{\beta}_{LS}$ ,  $\hat{\gamma}_{LS}$ , and  $\hat{\theta}_{LS}$  minimizing (5.1) are equal to the minimizers of the *sum of the squared concentrated residuals*

$$\bar{C}(\beta, \gamma, \theta) = N^{-1} \sum_{i=1}^N C_i(\beta, \gamma, \theta) = N^{-1} \sum_{i=1}^N u_i(\beta, \gamma)' M_{\lambda(\theta)} u_i(\beta, \gamma) \quad (5.3)$$

which is obtained by replacing  $\alpha_i$  in (5.1) with (5.2). From the name of (5.3), we call  $\hat{\beta}_{LS}$ ,  $\hat{\gamma}_{LS}$  and  $\hat{\theta}_{LS}$  the *concentrated least squares estimator*.

Since  $G'\lambda = 0$ , we have  $M_\lambda G = G$  and therefore  $M_\lambda = P_G = G(G'G)^{-1}G'$ . So the first order

conditions of the CLS estimation become

$$\begin{bmatrix} \partial \bar{C} / \partial \beta \\ \partial \bar{C} / \partial \gamma \\ \partial \bar{C} / \partial \theta \end{bmatrix} = -\frac{2}{N} \sum_{i=1}^N \begin{bmatrix} X_i' P_G u_i \\ Z_i 1_T' P_G u_i \\ \Lambda' P_G u_i u_i' \lambda (\lambda' \lambda)^{-1} \end{bmatrix} = 0. \quad (5.4)$$

Interpreting (5.4) as sample moment conditions, we can construct the corresponding (exactly identified) implicit population moment conditions:

$$E X_i' P_G u_i = 0 \quad (5.5a)$$

$$E Z_i 1_T' P_G u_i = 0 \quad (5.5b)$$

$$E \Lambda' P_G u_i u_i' \lambda (\lambda' \lambda)^{-1} = 0. \quad (5.5c)$$

That is, the CLS estimator is asymptotically equivalent to the GMM estimator based on (5.5).

The moment conditions (5.5a) and (5.5b) are satisfied under the Orthogonality Assumption. However, this is not true of (5.5c). The moment conditions (5.5c) require the Covariance Assumption to be valid (unless we make very specific and unusual assumptions about the form of  $\lambda$  and its relationship to the error variance matrix). Thus, the consistency of the CLS estimator requires *both* the Orthogonality Assumption *and* the Covariance Assumption. This is a rather striking result, since the consistency of least squares does not usually require restrictions on the second moments of the errors, and is a reflection of the incidental parameters problem.

We would generally believe that least squares should be efficient when the errors are i.i.d. normal. However, similarly to the result in Ahn, Lee and Schmidt (2001), this is not true in the present case. The efficient GMM estimator under the Orthogonality and Covariance Assumptions uses the moment conditions (4.2), while the CLS estimator uses only a subset of these. This can be seen most explicitly in the case that, conditional on  $(W_i, \alpha_i)$ , the  $\epsilon_{it}$  are i.i.d. normal. Then (4.2b) is redundant and (4.2a) can be replaced by (4.9), so that the efficient GMM estimator is based on (4.9a), (4.9b), (4.9c) and (4.2c). The CLS estimator is based on (5.5a), which is the same as (4.9a);

(5.5b), which is the same as (4.9b); and (5.5c), which is a subset of (4.2c).<sup>2</sup> So the inefficiency of CLS lies in its failure to use the moment conditions (4.9c) and from its failure to use all of the moment conditions in (4.2c). The latter failure did not arise in the Ahn, Lee and Schmidt (2001) analysis (see footnote 2).

In Appendix B, we calculate the asymptotic variance matrix of the CLS estimator, under the “conditional independence of the moments up to fourth order” (CIM4) assumption of Section 4.

## 6 Empirical Application

This section includes an application of the estimators suggested in previous sections to the measurement of cost efficiency. The application uses panel data from Spanish private and savings banks covering the period 1992-1998. In order to allow for changes in cost efficiency over time, the individual effects are modeled in a parametric form as the “inverse” of the exponential time-varying function proposed by Battese and Coelli (1992) in a MLE framework.

### 6.1 The cost frontier model

The technology of banks is modeled using the following translog cost function:

$$\begin{aligned}
\ln C_{it} &= \ln C^*(q_{it}, w_{it}, \gamma, \beta) + \alpha_{it} + \epsilon_{it} \\
&= \left( \gamma + \sum_{j=1}^m \beta_{q_j} \ln q_{jit} + \frac{1}{2} \sum_{j=1}^m \sum_{l=1}^m \beta_{q_j q_l} \ln q_{jit} \ln q_{lit} + \sum_{k=1}^n \beta_{w_k} \ln w_{kit} \right. \\
&\quad \left. + \frac{1}{2} \sum_{k=1}^n \sum_{h=1}^n \beta_{w_k w_h} \ln w_{kit} \ln w_{hit} + \sum_{j=1}^m \sum_{k=1}^n \beta_{q_j w_k} \ln q_{jit} \ln w_{kit} \right) \\
&\quad + \exp(\theta(t-1)) \alpha_i + \epsilon_{it}
\end{aligned} \tag{6.1}$$

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<sup>2</sup>The moment conditions (5.5c) are equivalent to  $E\Lambda'G(G'G)^{-1}b_{3i} = 0$ . When the number of parameters in  $\theta$  is less than  $T-1$ , the transformation  $\Lambda'G(G'G)^{-1}$  loses information. This will be so in most parametric models for  $\lambda(\theta)$ , though it is not true in the model of Ahn, Lee and Schmidt (2001).



where  $C_{it}$  is observed total cost,  $q_{it}$  is a vector of outputs,  $w_{it}$  is a vector of input prices,  $\beta$  is a vector of parameters to be estimated,  $\gamma$  is a scalar to be estimated, and  $\epsilon_{it}$  is the error term. The individual effects are modeled as the product of an exponential time-varying function  $\lambda_t(\theta) = \exp(\theta(t-1))$  and a time-invariant firm effect.

The cost equation (6.1) is estimated using the GMM estimators suggested in previous sections. We will denote the GMM estimator based on the moment conditions (3.2) by GMM1. Assuming, in addition, no conditional heteroskedasticity (NCH) we get the GMM2 estimator. The GMM estimator that assumes orthogonality and covariance is denoted by GMM3 and is based on the moment conditions (4.2). If we impose conditional independence of the moments up to the fourth order we get the GMM4 estimator which is obtained using (4.9), (4.2b) and (4.2c). We also consider an estimator that, in addition to GMM4, assumes that the error term is i.i.d. normal.<sup>3</sup> This estimator uses only the moment conditions (4.9) and (4.2c) and we will denote it by GMM5. Thus, as we go from GMM1 to GMM5 we are relying on stronger and stronger assumptions. The concentrated least squares estimator that minimizes the objective function (5.3) will be denoted by CLS. As mentioned above, the CLS uses the same assumptions as GMM3, but the latter is more efficient.

These estimations are also compared with the traditional WITHIN estimator and with the MLE estimator proposed by Battese and Coelli (1992). In the latter case, additional distributional assumptions must be imposed. In particular, the noise term  $\epsilon_{it}$  is assumed to follow a normal distribution with mean zero and variance  $\sigma_\epsilon^2$  and the individual effect  $\alpha_i$  is assumed to come from a non-negative truncated normal distribution with zero mean and variance  $\sigma_\alpha^2$ . Since we impose non-negativity on  $\alpha_i$ , the cost equation (6.1) is equivalent to a stochastic cost frontier where the firm effect  $\alpha_{it}$  is interpreted as an inefficiency term.<sup>4</sup> We will denote this estimator by MLE1 in

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<sup>3</sup>Note that CIM4 does not restrict the distribution of the error term to be symmetric and “bell-shaped”.

<sup>4</sup>For estimation purposes the model above is parameterised in terms of  $\sigma^2$  and  $\pi$ , where  $\sigma^2 = \sigma_\epsilon^2 + \sigma_\alpha^2$  is the overall variance and  $\pi = \sigma_\epsilon^2/\sigma_\alpha^2$  is a useful indicator of the relative importance of both noise and inefficiency variances.

order to distinguish it from a different MLE estimator, which we will consider later.

The WITHIN estimator can be viewed as a restricted version of the GMM1, in that if individual effects are time-invariant, the consistency of both estimators relies exclusively on the moment conditions implied by the orthogonality assumptions. The MLE results rely on stronger assumptions. In particular, MLE makes the “random effects” assumptions that  $\alpha_i$  is independent of the regressors. Moreover, the individual effects  $\alpha_i$  in the MLE model are also restricted to be i.i.d. half-normal (i.e. to be positive) and  $\epsilon_{it}$  to be i.i.d. normal. None of the other estimators uses these assumptions, except that GMM3, GMM4, GMM5 and CLS assume white noise, and GMM5 assumes normality.

The model above (6.1) can also be interpreted as a cost frontier in the GMM and CLS frameworks if the time-varying individual effects  $\alpha_{it}$  are decomposed into a frontier intercept which varies over time ( $\alpha_t$ ) and a non-negative inefficiency term ( $v_{it}$ ). That is:

$$\alpha_{it} = \lambda_t(\theta)\alpha_i = \alpha_t + v_{it} \quad (6.2)$$

Following Cornwell, Schmidt and Sickles (1990) the frontier intercept can be estimated as:

$$\hat{\alpha}_t = \min_i(\hat{\alpha}_{it}) = \lambda_t(\hat{\theta}) \cdot \min_i(\hat{\alpha}_i) \quad (6.3)$$

and the inefficiency term as:

$$\hat{v}_{it} = \lambda_t(\hat{\theta})[\hat{\alpha}_i - \min_i(\hat{\alpha}_i)] = \lambda_t(\hat{\theta})\hat{v}_i \quad (6.4)$$

Since the dependent variable is expressed in natural logs, cost efficiency indexes can be calculated from (6.4) as:

$$\widehat{CE}_{it} = \exp(-\lambda_t(\hat{\theta})[\hat{\alpha}_i - \min_i(\hat{\alpha}_i)]) \quad (6.5)$$

As customary, the efficiency indexes in the WITHIN model can be obtained using the expression (6.5) once  $\theta = 0$  or  $\lambda_t(\theta) = 1$  is imposed. (See Schmidt and Sickles, 1984.)

It is easy to see from expressions (6.4) and (6.5) that cost efficiency compares the performance (individual effect) of a particular firm with a firm located on the frontier (i.e. with the minimum effect). Since cost efficiency is a relative concept, the average efficiency index is thus related to the estimated variance  $\sigma_\alpha^2$ : the higher the variance of the individual effects, the smaller the average efficiency.

Note, however, that adjusting the GMM specification to be a frontier yields a model with a frontier intercept that varies over time (i.e. with technical change) which does not appear in MLE1. For this reason, we also estimate a MLE estimator with a time-varying intercept which we will denote MLE2. In this model, the time-varying parametric function  $\lambda_t$  appears twice: first, multiplying a non-negative individual effect ( $v_i$ ) which is assumed to be distributed as a half-normal, and second, multiplying a constant that is the minimum alpha value obtained using GMM5.<sup>5</sup> That is:

$$\ln C_{it} = \ln C^*(q_{it}, w_{it}, \gamma, \beta) + \lambda_t(\theta) [\min_i(\hat{\alpha}_i) + v_i] + \epsilon_{it}, \quad v_i \geq 0. \quad (6.6)$$

## 6.2 Data

The application uses yearly data from Spanish saving and private banks. The number of banks decreased over the last ten years due to mergers and acquisitions. These mergers took place especially among saving banks and mainly in the early 1990s. In order to work with a balanced panel data, we use data from 38 private banks and 50 savings banks over the period 1992–98.

The estimations were carried out separately for savings and private banks since they are involved in quite different activities. Savings banks concentrate on retail banking, providing checking, savings and loans service to individuals (especially mortgage loans), whereas private banks are more involved in commercial and industrial loans. Another difference is the fact that savings

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<sup>5</sup>We have selected this estimator because it is the nearest to MLE and more efficient than CLS.

banks are more specialized than other banks in long-term loans, which do not require a continuous monitoring. Since the two groups are likely to have different cost structures and have been regulated in different ways, we analyse the two groups separately rather than pool them.

The variables used in the analysis are defined in the same way for both groups of banks. We follow the majority of the literature and apply the intermediation approach proposed by Sealey and Lindley (1977) which treats deposits as inputs and loans as outputs. We include three types of outputs and three types of inputs. The outputs are: Loans to banks and other profitable assets ( $q_1$ ); Loans to firms and households ( $q_2$ ); and noninterest income ( $q_3$ ). Using noninterest income goes beyond the intermediation approach as commonly modeled. We include it in an attempt to capture off-balance-sheet activities such as securitization, brokerage services, management financial assets for their customers or mutual funds, which are becoming increasingly important in Spanish banks. This way of measuring nontraditional banking activities is not fully satisfactory (i.e. we cannot distinguish between variations due to changes in volumes and variations due to changes in prices, and noninterest income is partly generated from traditional activities such as fees from service charges on deposits or credits rather than nontraditional activities). Since comprehensive information about the amount of off-balance-sheet services is not available, we prefer to describe them in an approximate way.

The inputs are: Borrowed money, including demand, time and saving deposits, deposits from non-banks, securities sold under agreements to repurchase, and other borrowed money ( $x_1$ ); Labor, measured by total number of employees ( $x_2$ ); and Physical Capital, measured by the value of fixed assets in the balance sheet ( $x_3$ ). All the input prices,  $w_i$  ( $i = 1, 2, 3$ ), were calculated in a straightforward way by dividing nominal expenses by input quantities. Accordingly, total cost includes both interest and operating expenses.

### 6.3 Empirical results

The parameter estimates of the cost frontiers for savings banks and private banks are presented in Table 1 and Table 2 respectively.<sup>6</sup> Since the variables have been normalized by dividing by the sample geometric mean, the first order coefficients can be interpreted as the elasticities evaluated at that point. The standard homogeneity of degree one in input prices is imposed by normalizing cost and input prices using the price of physical capital as a numeraire.

Since all elasticities are positive at the geometric mean, the estimated cost frontiers are increasing at this point in their variables. These results confirm (positive) monotonicity of both cost frontiers. Returns to scale can be estimated as one minus the scale elasticity (i.e. the sum of each output cost elasticity). At the sample mean, the scale elasticity is only a function of the first-order output parameters. Over the whole estimations, the sum of these parameters is smaller than one for both savings and private banks. These results indicate the existence of increasing returns to scale as found in many past analyses of Spanish banks.

Note that we can clearly distinguish two groups of estimators for private banks in terms of the scale elasticity values. This value ranges from 0.756 to 0.860 using WITHIN and GMM up to GMM4, whereas it rises over 0.925 using GMM5 and MLE estimators. The scale elasticity values for the savings banks are more homogeneous. However, they tend to increase as we go from GMM1 to MLE. In particular, the scale elasticity is less than 0.857 using the WITHIN, GMM1 and GMM3 estimators, whereas it is over 0.90 using other estimators.

The estimated parameters in cost frontier (6.1) can be used to calculate individual indexes of cost efficiency. These indexes are obtained using expression (6.5), except for the MLE estimator where we follow Battese and Coelli (1992). This paper allows the computation of estimates of the individual technical inefficiencies from the estimation of a production function. Here, that model

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<sup>6</sup>The  $\theta$  estimates are reported in Table 3 and Table 4 due to they are mainly related with the results regarding the efficiency indices.

is modified to adjust for estimation of cost efficiency.

Tables 3 and 4 report some results for cost efficiency for private and savings banks respectively. These tables also provide the estimated  $\theta$  value, which allows us to assess variations in cost efficiency over time.<sup>7</sup> The cost efficiency indexes increase or decrease over time on the basis of the sign of the  $\theta$ 's. If this parameter is positive (negative), efficiency decreases (increases) and the differences among firms increase (decrease) due to the exponential functional form of  $\lambda_t(\theta)$ .

The private banks' average efficiency using the WITHIN estimator is quite similar to that obtained using GMM1. This is reasonable because the consistency of both estimators relies on the orthogonality assumption, and we cannot reject time-invariant efficiency using GMM1. The average efficiency from using GMM2 and GMM3 is slightly smaller, but again it is not possible to reject the null hypothesis that  $\theta = 0$ . Time-invariant efficiency is also not rejected using the GMM4 estimator, but now private banks' average efficiency returns to the initial values found using WITHIN and GMM1.

However, average and time-path efficiency change a lot when the error term is assumed to be normal.<sup>8</sup> The efficiency level at  $t = 1$  using the GMM5 estimator is 92% but it decreases strongly over time. Unlike the previous estimators, this means that changes in efficiency over the whole period are now statistically significant. The results from the CLS and MLE estimators, which also assume  $\epsilon_{it}$  to be i.i.d. normal, are quite similar to GMM5.<sup>9</sup>

A quick glance at the correlation coefficients in Table 3 seems to confirm the existence of a breaking point at GMM4, giving rise to two subsets of estimators. This table shows that correlations between efficiency indices from using estimators belonging their own group are rather high

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<sup>7</sup>Except, obviously, for the WITHIN estimator where time-invariance is imposed on the individual effects.

<sup>8</sup>Using the GMM4 estimates, the skewness coefficient  $\kappa_3/\sigma_\epsilon^2$  and the degree of excess  $\kappa_4/\sigma_\epsilon^2$  take the values  $-0.018$  and  $12.42$  respectively. For normal distributions both measures must be zero. Hence, the GMM4 estimates question the normal distribution assumption used in subsequent estimators.

<sup>9</sup>Note that the estimated  $\theta_{MLE}$  is quite similar to that found using GMM5 or CLS once the MLE estimator is adjusted in order to include a time-varying frontier intercept.

(always over 90%). In contrast, correlations between any estimators belonging to opposite groups range from 24% to 70%.

The same subset of estimators can be also appreciated in Figure 1 where all individual efficiency indices are graphed according to bank size. Moreover, this figure shows a negative correlation between efficiency and size.<sup>10</sup> This correlation disappears when normality assumptions are added.

On the other hand, an notable feature of Table 4 is that results for savings banks are, in general, much more homogeneous than those found for private banks. For instance, the average efficiency at  $t = 1$  is over 80% and it seems to decrease over time. Whatever the model, the estimated  $\theta$  value is positive and statistically different from zero.<sup>11</sup> Thus we can conclude that time-varying efficiency models will provide more accurate estimates of savings banks' efficiency than the standard WITHIN model.

## 7 Conclusion

In this paper we have considered a panel data model with parametrically time-varying coefficients on the individual effects. Following Ahn, Lee and Schmidt (2001), we have enumerated the moment conditions implied by alternative sets of assumptions on the model. We have shown explicitly that our sets of moment conditions capture all of the useful information contained in our assumptions, so that the corresponding GMM estimators exploit these assumptions efficiently.

We have also considered concentrated least squares estimation. Here the incidental parameters problem is relevant because we are treating the fixed effects as parameters to be estimated. An

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<sup>10</sup>This negative correlation might indicate that big banks are involved in activities not accounted for by the variables available.

<sup>11</sup>Note that we cannot reject that  $\theta = 0$  using the MLE1 estimator, which does not include a time-varying frontier intercept (or technical change). Thus, the difference between  $\theta_{MLE1}$  and  $\theta_{MLE2}$  can be used as an indicator of the biases caused by omitting the effect of technical change on bank's costs.

interesting result is that the consistency of the least squares estimator requires both exogeneity assumptions and the assumption that the errors are white noise. Furthermore, given the white noise assumption, the least squares estimator is inefficient, because it fails to exploit all of the moment conditions that are available.

We show how the GMM estimation problem can be simplified under some additional assumptions, including the assumption of no conditional heteroskedasticity and a stronger conditional independence assumption. Under these assumptions we also give explicit expressions for the variance matrices of the GMM and least squares estimators.

Finally, we apply the proposed GMM estimations to the measurement of cost efficiency of Spanish banks over the period 1992–1998. The results seem to suggest, especially for private banks, the non-fulfilment of the normality assumption on the error term. This questions the validity of traditional MLE models, which are based on a normality assumption, when examining the efficiency of Spanish banks.



# APPENDIX

In this Appendix we derive the asymptotic variances of the efficient GMM estimator and the CLS estimator. We make the “conditional independence of the moments up to fourth order” (CIM4) assumption of Section 4.

## A The asymptotic variance of the GMM estimator

Under the Orthogonality and Covariance Assumptions, the moment conditions we have are  $b_{1i} = G'u_i \otimes W_i$ ,  $b_{2i} = H'(G'u_i \otimes u_i)$ , and  $b_{3i} = (\lambda'\lambda)^{-1}\lambda'u_i \otimes G'u_i$ . Let  $\delta = (\beta', \gamma', \theta)'$ . Let  $B_j = -E(\partial b_{ji}/\partial \delta)$  for  $j = 1, 2, 3$ , evaluated at the true parameters. Let  $V_{jk} = Eb_{ji}b'_{ki}$  for  $j, k = 1, 2, 3$ , evaluated at the true parameters. Define  $\kappa_3 = E\epsilon_{it}^3/\sigma_\epsilon^2$  and  $\kappa_4 = E(\epsilon_{it}^4 - 3\sigma_\epsilon^4)/\sigma_\epsilon^2$ . Let  $\mu_W = EW_i$ ;  $\Phi = \Phi(\theta) = \lambda_*\lambda'_* + \text{diag}(\lambda_2, \dots, \lambda_T)$ ; and  $\Phi_* = \lambda_*\lambda'_* + \text{diag}(\lambda_2^2, \dots, \lambda_T^2)$ , where  $\lambda_* = (\lambda_2, \dots, \lambda_T)'$ . After some algebra, we get

$$V_{11} = \sigma_\epsilon^2(G'G \otimes \Sigma_{WW}) \quad (\text{A.1})$$

$$V_{12} = \sigma_\epsilon^2(G'G \otimes \Sigma_{W\alpha}\lambda')H \quad (\text{A.2})$$

$$V_{13} = \sigma_\epsilon^2\left[G'G \otimes \Sigma_{W\alpha} + \frac{\kappa_3}{\lambda'\lambda}(\Phi \otimes \mu_W)\right] \quad (\text{A.3})$$

$$V_{22} = \sigma_\epsilon^2 H'[G'G \otimes (\sigma_\alpha^2 \lambda \lambda' + \sigma_\epsilon^2 I_T)]H \quad (\text{A.4})$$

$$V_{23} = \sigma_\epsilon^2 H' \left\{ \left[ \left( \sigma_\alpha^2 + \frac{\sigma_\epsilon^2}{\lambda'\lambda} \right) G'G + \frac{\kappa_3}{\lambda'\lambda} \mu_\alpha \Phi \right] \otimes \lambda \right\} \quad (\text{A.5})$$

$$V_{33} = \sigma_\epsilon^2 \left\{ \left( \sigma_\alpha^2 + \frac{\sigma_\epsilon^2}{\lambda'\lambda} \right) G'G + 2 \frac{\kappa_3}{\lambda'\lambda} \mu_\alpha \Phi + \frac{\kappa_4}{(\lambda'\lambda)^2} \Phi_* \right\} \quad (\text{A.6})$$

and

$$B_1 = [(G \otimes \Sigma_{WW})'S_X, (G \otimes \Sigma_{WW})'S_Z, \Lambda_* \otimes \Sigma_{W\alpha}] \quad (\text{A.7})$$

$$B_2 = H'(I_{T-1} \otimes \lambda)[(G \otimes \Sigma_{W\alpha})'S_X, (G \otimes \Sigma_{W\alpha})'S_Z, \sigma_\alpha^2 \Lambda_*] \quad (\text{A.8})$$

$$B_3 = [(G \otimes \Sigma_{W\alpha})'S_X, (G \otimes \Sigma_{W\alpha})'S_Z, \sigma_\alpha^2 \Lambda_*]. \quad (\text{A.9})$$

With these results, the variance-covariance of the GMM estimator is

$$\text{cov}\sqrt{N}(\hat{\delta} - \delta) = \left[ (B'_1, B'_2, B'_3) \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V'_{12} & V_{22} & V_{23} \\ V'_{13} & V'_{23} & V_{33} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} \right]^{-1}. \quad (\text{A.10})$$

## B The asymptotic variance of the CLS estimator

By the standard Taylor series expansion technique, we find that the asymptotic variance will be equal to  $A_0 B_0^{-1} A_0$  where

$$A_0 = E \frac{\partial^2 C_i}{\partial \delta \partial \delta'}, \quad \text{and} \quad B_0 = E \frac{\partial C_i}{\partial \delta} \frac{\partial C_i}{\partial \delta'} \quad (\text{B.1})$$

evaluated at the true parameter. Let us calculate each of them. Let  $\Lambda = \partial \lambda(\theta_0) / \partial \theta' = (0_{p \times 1}, \Lambda'_*)'$ .

$B_0$  is the same as in Ahn, Lee and Schmidt (2001, p. 253). Let  $\Psi = G(G'G)^{-1} \Phi \cdot (G'G)^{-1} G'$ ;

$\Psi_* = G(G'G)^{-1} \Phi_* (G'G)^{-1} G'$ ; and  $\mu_\alpha = E \alpha_i$ . Then

$$E \frac{\partial C_i}{\partial \beta} \frac{\partial C_i}{\partial \beta'} = 4\sigma_\epsilon^2 S'_X (P_G \otimes \Sigma_{WW}) S_X \quad (\text{B.2})$$

$$E \frac{\partial C_i}{\partial \beta} \frac{\partial C_i}{\partial \gamma'} = 4\sigma_\epsilon^2 S'_X (P_G \otimes \Sigma_{WW}) S_Z \quad (\text{B.3})$$

$$E \frac{\partial C_i}{\partial \beta} \frac{\partial C_i}{\partial \theta'} = 4\sigma_\epsilon^2 S'_X \left[ P_G \otimes \Sigma_{W\alpha} + \frac{\kappa_3}{\lambda' \lambda} (\Psi \otimes \mu_W) \right] \Lambda \quad (\text{B.4})$$

$$E \frac{\partial C_i}{\partial \gamma} \frac{\partial C_i}{\partial \gamma'} = 4\sigma_\epsilon^2 S'_Z (P_G \otimes \Sigma_{WW}) S_Z \quad (\text{B.5})$$

$$E \frac{\partial C_i}{\partial \gamma} \frac{\partial C_i}{\partial \theta'} = 4\sigma_\epsilon^2 S'_Z \left[ P_G \otimes \Sigma_{W\alpha} + \frac{\kappa_3}{\lambda' \lambda} (\Psi \otimes \mu_W) \right] \Lambda \quad (\text{B.6})$$

$$E \frac{\partial C_i}{\partial \theta} \frac{\partial C_i}{\partial \theta'} = 4\sigma_\epsilon^2 \Lambda' \left\{ \left( \sigma_\alpha^2 + \frac{\sigma_\epsilon^2}{\lambda' \lambda} \right) P_G + 2 \frac{\kappa_3}{\lambda' \lambda} \mu_\alpha \Psi + \frac{\kappa_4}{(\lambda' \lambda)^2} \Psi_* \right\} \Lambda. \quad (\text{B.7})$$

$A_0$  is obtained from the following.

$$E \frac{\partial^2 C_i}{\partial \beta \partial \delta'} = 2[S'_X (P_G \otimes \Sigma_{WW}) S_X, S'_X (P_G \otimes \Sigma_{WW}) S_Z, S'_X (P_G \otimes \Sigma_{W\alpha}) \Lambda] \quad (\text{B.8})$$

$$E \frac{\partial^2 C_i}{\partial \gamma \partial \delta'} = 2[S'_Z (P_G \otimes \Sigma_{WW}) S_X, S'_Z (P_G \otimes \Sigma_{WW}) S_Z, S'_Z (P_G \otimes \Sigma_{W\alpha}) \Lambda] \quad (\text{B.9})$$

$$E \frac{\partial^2 C_i}{\partial \theta \partial \delta'} = 2[\Lambda' (P_G \otimes \Sigma'_{W\alpha}) S_X, \Lambda' (P_G \otimes \Sigma'_{W\alpha}) S_Z, \sigma_\alpha^2 \Lambda' P_G \Lambda]. \quad (\text{B.10})$$

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**Table 1.** Estimated coefficients. Private Banks

	WHITIN		GMM1		GMM2		GMM3		GMM4		GMM5		CLS		MLE1		MLE2	
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
$\ln q_1$	0.311	31.11	0.307	14.40	0.328	40.82	0.225	11.49	0.309	45.81	0.344	40.58	0.346	39.86	0.365	48.71	0.355	54.29
$\ln q_2$	0.412	27.85	0.421	14.69	0.420	18.40	0.359	12.46	0.422	42.43	0.448	27.39	0.455	27.47	0.442	31.96	0.457	35.64
$\ln q_3$	0.074	3.15	0.043	0.64	0.054	2.54	0.172	3.81	0.081	5.13	0.138	7.36	0.123	6.43	0.135	8.63	0.126	8.66
$\ln w_1$	0.714	83.05	0.651	35.17	0.660	66.82	0.707	22.65	0.714	66.36	0.688	45.06	0.677	43.66	0.776	79.66	0.666	54.85
$\ln w_2$	0.256	16.52	0.326	10.78	0.303	16.78	0.273	6.90	0.249	18.27	0.288	14.64	0.293	14.60	0.207	15.04	0.309	19.19
$.5(\ln q_1)^2$	0.190	13.33	0.165	5.85	0.165	13.35	0.203	7.35	0.186	19.39	0.179	12.14	0.184	12.28	0.168	12.53	0.169	13.63
$.5(\ln q_2)^2$	0.046	1.07	0.107	0.97	0.200	3.81	-0.088	-1.04	-0.001	-0.05	0.192	3.74	0.117	2.26	0.082	1.83	0.120	2.82
$.5(\ln q_3)^2$	0.134	2.43	0.160	1.61	0.131	2.55	0.375	3.49	0.061	1.65	0.176	3.09	0.105	1.82	0.124	2.36	0.104	2.22
$.5(\ln w_1)^2$	0.150	4.12	-0.017	-0.33	0.062	1.70	-0.115	-1.56	0.094	3.71	0.111	2.97	0.093	2.44	0.145	4.29	0.100	3.21
$.5(\ln w_2)^2$	-0.051	-0.77	-0.283	-2.31	-0.052	-0.86	-0.417	-3.16	-0.199	-4.38	0.001	0.01	-0.031	-0.44	0.021	0.35	-0.018	-0.30
$\ln q_1 \ln q_2$	-0.063	-3.08	-0.038	-0.89	-0.114	-5.32	0.036	0.90	-0.064	-4.68	-0.080	-3.70	-0.085	-3.88	-0.078	-3.82	-0.083	-4.85
$\ln q_1 \ln q_3$	-0.122	-5.20	-0.092	-2.05	-0.026	-1.19	-0.199	-4.32	-0.111	-6.99	-0.074	-2.96	-0.065	-2.55	-0.078	-3.47	-0.059	-3.12
$\ln q_1 \ln w_1$	0.054	4.18	0.075	3.43	0.065	5.32	0.049	1.82	0.066	7.28	0.084	6.51	0.095	7.28	0.074	5.97	0.090	8.46
$\ln q_1 \ln w_2$	-0.049	-2.28	-0.148	-3.78	-0.103	-5.74	-0.147	-3.42	-0.083	-5.57	-0.094	-4.46	-0.113	-5.29	-0.080	-4.09	-0.106	-5.93
$\ln q_2 \ln q_3$	0.009	0.21	-0.089	-1.04	-0.087	-1.97	-0.048	-0.61	0.068	2.51	-0.109	-2.34	-0.040	-0.86	-0.019	-0.47	-0.043	-1.12
$\ln q_2 \ln w_1$	-0.004	-0.17	-0.097	-2.17	0.018	0.66	-0.149	-2.99	0.010	0.59	0.000	-0.01	-0.026	-0.97	-0.016	-0.70	-0.024	-1.02
$\ln q_2 \ln w_2$	0.087	2.18	0.243	3.11	0.025	0.61	0.255	3.28	0.056	2.10	0.043	0.96	0.070	1.56	0.076	2.04	0.073	1.93
$\ln q_3 \ln w_1$	-0.038	-1.55	0.004	0.10	-0.071	-2.64	0.099	2.09	-0.063	-3.77	-0.054	-2.04	-0.045	-1.70	-0.037	-1.60	-0.053	-2.45
$\ln q_3 \ln w_2$	-0.015	-0.32	-0.048	-0.66	0.102	2.27	-0.029	-0.33	0.052	1.72	0.043	0.88	0.043	0.86	0.017	0.40	0.045	1.09
$\ln w_1 \ln w_2$	-0.047	-1.16	0.118	2.04	0.006	0.14	0.245	2.97	0.040	1.42	-0.038	-0.87	-0.015	-0.33	-0.074	-1.94	-0.025	-0.67
Intercept			8.769	11.14	9.417	38.55	10.473	4.69	9.758	33.86	9.825	546.00	9.834	506.44	9.687	772.09	9.856	701.07
$\sigma_\alpha^2$							0.5812		0.0317		0.0037		0.0046		0.0054		0.0059	
$\sigma_\epsilon^2$							0.0042		0.0005		0.0011		0.0011		0.0011		0.0008	
Obj. Func.			21.445				18.832		26.513		10.077		0.0045		464.69		502.97	

**Table 2.** Estimated coefficients. Saving Banks

	WHITIN		GMM1		GMM2		GMM3		GMM4		GMM5		CLS		MLE1		MLE2	
	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat	Coef.	t-stat
$\ln q_1$	0.284	22.60	0.314	12.45	0.326	27.06	0.302	15.61	0.320	39.49	0.321	41.32	0.309	44.21	0.316	28.76	0.304	32.16
$\ln q_2$	0.491	24.45	0.423	9.87	0.540	22.36	0.430	12.70	0.548	43.59	0.550	45.19	0.543	47.01	0.592	27.79	0.591	36.04
$\ln q_3$	0.036	2.74	0.120	4.38	0.058	3.71	0.109	5.04	0.062	6.29	0.056	5.98	0.057	7.00	0.032	2.39	0.045	3.87
$\ln w_1$	0.694	61.27	0.634	48.82	0.668	62.82	0.634	32.46	0.685	76.05	0.680	80.58	0.678	91.77	0.754	73.84	0.686	64.8
$\ln w_2$	0.292	21.10	0.328	13.99	0.270	20.02	0.332	14.62	0.299	28.62	0.305	30.90	0.305	35.58	0.245	18.57	0.301	24.70
$.5(\ln q_1)^2$	0.165	6.79	0.191	2.81	0.208	9.68	0.184	4.89	0.183	10.61	0.176	10.74	0.177	12.38	0.181	7.42	0.176	8.10
$.5(\ln q_2)^2$	0.136	2.29	0.029	0.22	0.187	2.89	0.016	0.18	0.103	2.49	0.106	2.70	0.102	2.98	0.122	2.34	0.065	1.27
$.5(\ln q_3)^2$	0.021	0.47	-0.009	-0.09	-0.062	-1.29	-0.055	-0.77	0.041	1.27	0.023	0.73	0.000	0.01	0.052	1.56	0.014	0.36
$.5(\ln w_1)^2$	0.108	4.48	0.140	3.86	0.133	4.19	0.134	3.46	0.179	9.97	0.150	8.79	0.141	9.63	0.140	5.80	0.147	6.69
$.5(\ln w_2)^2$	0.055	1.16	0.218	2.40	0.142	2.95	0.211	2.82	0.082	2.38	0.053	1.62	0.065	2.28	0.053	1.15	0.028	0.67
$\ln q_1 \ln q_2$	-0.119	-4.22	-0.121	-1.72	-0.224	-8.05	-0.132	-3.03	-0.107	-5.31	-0.118	-6.15	-0.126	-7.61	-0.101	-3.41	-0.101	-3.97
$\ln q_1 \ln q_3$	-0.039	-1.72	-0.061	-1.68	0.017	0.79	-0.044	-1.30	-0.083	-5.53	-0.060	-4.12	-0.054	-4.22	-0.079	-3.44	-0.079	-4.14
$\ln q_1 \ln w_1$	0.075	4.36	0.017	0.40	0.061	2.95	0.023	0.82	0.036	2.63	0.038	2.88	0.050	4.65	0.061	3.51	0.048	2.86
$\ln q_1 \ln w_2$	-0.062	-2.44	0.028	0.44	-0.033	-1.24	0.021	0.52	0.030	1.63	0.021	1.20	-0.004	-0.24	-0.046	-1.82	-0.010	-0.45
$\ln q_2 \ln q_3$	-0.003	-0.06	0.043	0.48	0.030	0.63	0.070	0.98	0.011	0.34	0.006	0.18	0.017	0.63	-0.004	-0.10	0.038	0.98
$\ln q_2 \ln w_1$	-0.130	-5.08	-0.035	-0.79	-0.031	-1.03	-0.041	-0.95	-0.056	-2.73	-0.067	-3.41	-0.086	-5.23	-0.142	-5.70	-0.095	-3.85
$\ln q_2 \ln w_2$	0.068	1.77	-0.120	-1.97	-0.039	-0.93	-0.122	-1.96	-0.015	-0.51	0.006	0.20	0.036	1.52	0.082	2.36	0.063	1.99
$\ln q_3 \ln w_1$	0.060	2.63	0.017	0.36	-0.014	-0.56	0.019	0.52	0.039	2.19	0.047	2.75	0.046	3.24	0.082	3.69	0.061	2.70
$\ln q_3 \ln w_2$	0.005	0.15	0.065	1.18	0.045	1.21	0.070	1.29	-0.003	-0.12	-0.012	-0.52	-0.013	-0.63	-0.016	-0.53	-0.026	-0.89
$\ln w_1 \ln w_2$	-0.064	-2.45	-0.125	-2.66	-0.137	-3.90	-0.120	-2.74	-0.155	-7.65	-0.123	-6.34	-0.116	-6.96	-0.103	-3.96	-0.122	-4.99
Intercept			9.968	130.84	9.780	187.90	9.962	96.60	9.745	544.57	9.759	528.06	9.814	391.43	9.504	661.83	9.779	835.35
$\sigma_\alpha^2$							0.0888		0.0102		0.0116		0.0253		0.0195		0.0112	
$\sigma_\epsilon^2$							0.0016		0.0003		0.0003		0.0002		0.0007		0.0006	
Obj. Func.			28.526				27.694		39.247		12.231		0.0032		668.06		706.08	

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**Table 3.** Estimated efficiency levels. Private Banks

Estimator	Average Efficiency			$\theta$	Spearman Rank Correlation Coefficient								
	$t = 1$	$t = 4$	$t = 7$		WITHIN	GMM1	GMM2	GMM3	GMM4	GMM5	CLS	MLE1	MLE2
WITHIN	70.4	70.4	70.4		100								
GMM1	71.3	71.9	72.6	-0.010	99.8	100							
GMM2	65.3	67.7	69.9	-0.030	98.4	98.2	100						
GMM3	60.0	59.6	59.1	0.005	95.5	95.4	93.6	100					
GMM4	70.8	70.4	70.0	0.006	99.8	99.6	98.4	95.4	100				
GMM5	91.9	86.7	78.7	0.177*	58.8	58.4	58.5	50.6	60.6	100			
CLS	91.9	87.1	79.9	0.166*	61.8	60.6	60.5	52.7	62.8	96.6	100		
MLE1	94.6	92.9	90.6	0.099*	33.0	34.2	34.3	24.0	36.7	90.7	89.0	100	
MLE2	93.8	90.5	85.5	0.152*	39.9	38.4	38.6	28.8	40.9	93.8	95.2	94.9	100

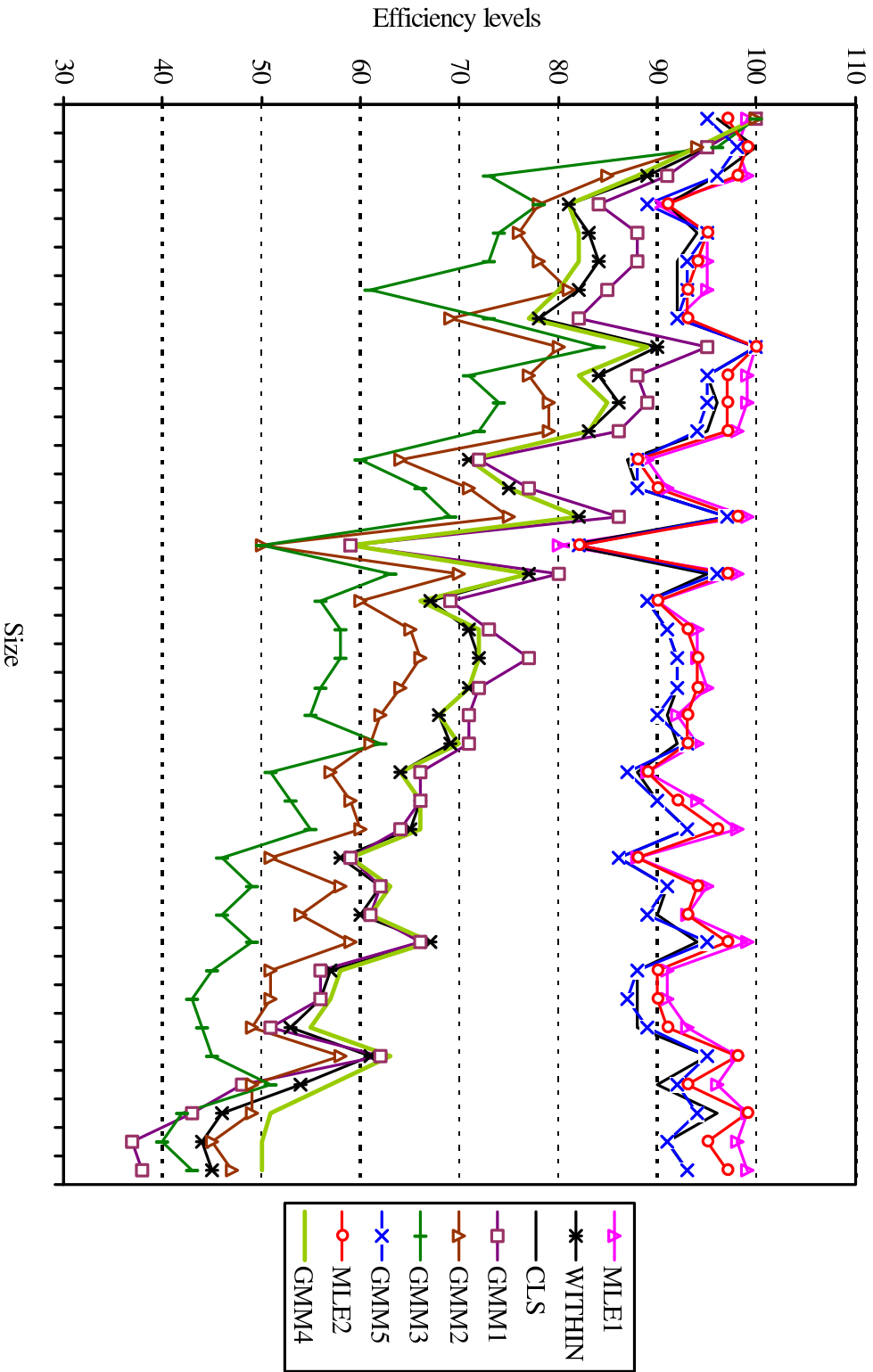
Note: \* indicates that we can reject the null hypothesis  $H_0 : \theta = 0$  at 1% level.

**Table 4.** Estimated efficiency levels. Saving Banks

Estimator	Average Efficiency			$\theta$	Spearman Rank Correlation Coefficient									
	$t = 1$	$t = 4$	$t = 7$		WITHIN	GMM1	GMM2	GMM3	GMM4	GMM5	CLS	MLE1	MLE2	
WITHIN	57.3	57.3	57.3		100									
GMM1	82.5	80.4	78.0	0.045*	86.2	100								
GMM2	86.6	81.5	74.8	0.120*	81.3	92.4	100							
GMM3	82.5	80.5	78.3	0.042*	90.4	98.9	91.8	100						
GMM4	89.8	86.7	82.7	0.098*	71.1	89.8	94.8	86.9	100					
GMM5	89.3	86.2	82.4	0.092*	70.9	90.1	94.4	87.4	99.7	100				
CLS	88.4	86.2	83.7	0.063*	80.5	95.0	96.6	93.6	97.7	97.9	100			
MLE1	89.0	89.0	89.0	0.001	75.1	84.0	92.8	83.2	94.7	94.2	94.3	100		
MLE2	91.5	89.2	86.5	0.083*	68.6	85.4	93.4	83.6	97.9	98.6	96.1	96.2	100	

Note: \* indicates that we can reject the null hypothesis  $H_0 : \theta = 0$  at 1% level.

**Figure 1.** Efficiency indices at first period. Private Banks





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