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PERMANENT SEMINAR ON EFFICIENCY AND PRODUCTIVITY

THE DYNAMICS OF EFFICIENCY IMPROVING INPUT ALLOCATION AND REORGANIZATION COSTS[♠]

Spiro E. Stefanou, Onelack Choi and Jeffrey Stokes[*]

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Abstract

The theory of production addresses rational producers seeking to minimize production costs given their target outputs. Achieving allocative efficiency requires input reallocation and neoclassical theory assumes there are no transition costs associated with this reallocation. This paper relaxes the assumption that decision-making units have the ability to reorganize their activities instantaneously and costlessly to arrive at an efficient input allocation. A dynamic cost minimization problem is presented to identify the dynamics of input transitions. The theory of gradual transition toward efficient input allocation which we develop is applied to agricultural banking institutions in the United States (US).

Keywords: Allocative efficiency, dynamic cost minimization, agricultural banks.

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1. Introduction

The theory of production addresses rational producers seeking to minimize production costs given their target outputs. With a convex production technology, achieving allocative efficiency is guaranteed once the first order conditions of cost minimization or profit maximization are satisfied. Achieving allocative efficiency requires input reallocation and the neoclassical theory assumes there are no transition costs associated with this reallocation. This paper relaxes the assumption that decision-making units have the ability to reorganize their activities instantaneously and costlessly to arrive at an efficient input allocation. Input reallocation focuses on the decision maker moving along an isoquant to find the cost minimizing input combination. In the context of inefficiency, the decision maker may not even be on the isoquant (i.e., technically inefficient) leaving the reallocation decision to involve moving toward the frontier of the input requirement set and finding the optimal point on that frontier simultaneously.

Each point on an isoquant is a technique of a production process (or technology) and the smooth isoquant results in the presence of an infinite number of techniques to achieve a given output level. Changing the input bundle as the firm reorients the techniques of production to enhance efficiencies can cause the firm to incur costs associated with dealing with the internal inertia of reallocation. The firm may incur higher monitoring costs or other costs associated with reorganizing the production process. This is revealed by the presence of transition costs associated with reallocating inputs. Thus, the assumption of costless reallocation is relaxed and the standard cost minimization framework is modified accordingly.

The traditional approach to examining input allocation decision making is to identify the degree of inefficiency by using production frontier estimation methods and then examining the relative magnitude of the technical and allocative inefficiency levels. This involves following a ray from the origin to identify technical efficiency first (Farrell 1957). Upon identifying technical efficiency, allocative efficiency can be evaluated as the measured deviation of the technically efficient input bundle from the cost minimizing input bundle. This approach inevitably involves identifying the target location on the production frontier from which one can evaluate the technical inefficiency of the firm. The traditional order of the efficiency decomposition is to evaluate the degree of technical inefficiency first and then define the allocative efficiency as the residual. Bogetoft and Färe (1999) oppose this traditional order of efficiency decomposition by proposing a reverse Farrell approach. They discuss how to measure allocative efficiency without presuming technical efficiency where reallocating resources within a hierarchy or a market is easier than changing production procedures.

Transition costs are associated with efficiency improving input reallocation decisions. A transition to reduce a firm's inefficiencies requires reorganizing the production process. This can involve transition costs associated with reorganizing activities which are based on the two properties: 1) costs associated with implementing efficiency improving input re-allocations at a rapid rate per unit of time; and, 2) these transition costs increase rapidly with the absolute rate of transition and are so rapid that the inefficient firm never attempts to achieve a full shift in its efficient position at any given moment. With the transition of input use toward the fully efficient input bundle being gradual, the control variable is defined as the change in input levels, while the stock variable is defined as the input level upon entering the decision period. In this context, static approaches to production efficiency analysis are expected to be incomplete in addressing how firms become efficient.

The notion of optimality in neoclassical economics is based on a frictionless world absent of transaction costs. Zero transaction costs arise when all relevant information is available costlessly, and the decision maker exhibits perfect rationality with instantaneous access to all available information. However, positive transaction costs are inevitable in actual production processes, and a human decision maker has bounded rationality and an imperfect ability to promptly adjust (Williamson, 1985). That is, the decision maker's ability to recognize all available options and compare each option to others at a given point in time is limited. In the presence of positive transaction costs and bounded rationality, transition costs can arise whenever input reallocation decisions are made. For example, a decision maker may want to explore and imitate "best practice" firms in the industry to improve inefficiencies of his firm. Each exploratory and imitating activity can be costly and require associated costs in the form of search, learning and reorganization costs.

Some authors attempt to link bounded rationality to emotional and cognitive factors. Conlisk (1996) depicts bounded rationality as mainly concerned with cognitive problems while Kaufman (1999) suggests that economic rationality is bounded not only by cognitive limitations but also by emotional considerations.

The sources of inefficiencies are related to managerial ability, factor fixities, regulations, characteristics of capital, and quality or environmental attributes that can prevent a firm from attaining full efficiency in a given time period. There may be residual sources of inefficiencies as well. Peters and Waterman (1982) note out that successful learning activity within a firm can reduce the possibility of disagreement and can be an attribute of a successful organization.

The economic consequences of inefficiencies vary with the environment in which firms are operating. If the market is competitive and if there is no government intervention to support inefficient firms, competition will eventually expel an inefficient firm from the market. However, a firm's survival over time does not depend on its current efficiency, but on its ability to make efficient decisions over time, which necessarily involves growth. Productivity growth can be decomposed into a scale effect, an allocative efficiency gain/loss effect measured as the impact of the difference between the observed input cost share and the efficiency input cost share, a technical efficiency gain/loss effect measured by the shifting in the cost function, and a classical exogenously-driven technical change effect (Bauer 1990; Lovell 1996). Clearly, faster improvement toward an efficient input allocation will result in greater productivity growth. However, efficiency changes have been treated as exogenous in the literature. An attempt to endogenize efficiency changes can also contribute to the theory and methods for analyzing growth and its causes.

A dynamic cost minimization problem is presented to identify the dynamics of input transitions. Transition costs are the key factors leading to the dynamic specification of the cost minimization framework since these costs contribute to the flow of costs over time, which inevitably involve dynamic relationships. A firm seeking to improve its efficiencies will incur transition costs as long as the benefits from these transitions are greater than the associated costs.

The specific form of cost function is identified that will be substituted into the dynamic cost minimization problem. In the presence of both technical and allocative inefficiencies, the static cost function should account for these inefficiencies. Thus, the cost function is specified as a function of efficiency related parameters as well as input prices and the output. This theory of gradual transition toward efficient input allocation is developed and applied to agricultural banking institutions in the United States (US).

2. Model of Input Transition

The presence of both allocative and technical inefficiency is reflected in the behavioral cost function $C^b(w^*, y, h)$ where $w_i^* = w_i x_i$, *î* is the allocative inefficiency parameter for the *ith* input, w_i is the observed *ith* input price, *y* is the output target, and *ç* reflects technical inefficiency. Formally,

$$C^b(w^*, y, h) = \min_{x^b}\left\{ w^{*\prime} x + l \left[y - f\left(\frac{x}{h}\right)\right]\right\} \quad (1)$$

implying

$$w_i x_i = w_i^* = l^b \frac{1}{h} f'\left(\frac{x^b}{h}\right) \quad (2)$$

and *ç* scales the observed input bundle x^b such that

$$y = f\left(\frac{x^b}{h}\right) \quad (3)$$

If the *ith* input is allocatively efficient, $î_i$ *= 1.* The values of $î_i$ *< 1 (> 1)* imply that the decision maker is allocated more (less) of input *i* compared to the cost minimizing allocation. The superscripts *b* reflects the behavioral value function. The value function $C^b(w^*, y, h)$ reflects the input use $x^b(w^*, y, h)$, which embodies the inefficiency levels implied by $î_i$ and *ç*. The movement toward a fully efficient cost minimizing input allocation involves the incentives to change $î_i$ and *ç* where changes in $î_i$ and *ç* lead to lower costs.

Input Allocation Transitions

The proposition is that input allocation changes toward a fully efficient bundle necessarily do not occur instantly. These transitions involve reorganization and restructuring of the firm's activities and suggest the presence of costs associated with changing the input bundles.

Looking at transition costs involves looking at changes in input bundles, which requires a fixed starting point. The initial input decision, x_0, is assumed to be fixed and the objective is to select the next period input bundle recognizing the transition cost depends on the magnitude of the input bundle change. The transition cost, *Ø*, depends on the changes in the input bundle defined as

$$\Delta_i(t) = x^b_{i,t-1} - x^b_{i,t} \tag{4}$$

where $x^b_{i,t-1}$ is the initial input *i* decision at time *t-1* and $x^b_{i,t}$ is the present period input *i* decision. The gradual movement toward the fully cost efficient input allocation occurs when the transition costs present a particular direction and curvature. Define the transition cost function as the relationship between the cost associated with the transition and the magnitude of the transition, $\Psi(\Delta)$. The first proposition is that $\Psi(\Delta) > 0$ for $\Delta \gtrless 0$. Figure 1 presents three possible forms for $\Psi(\Delta)$. For illustrative purposes, consider a two-period problem where *Ä** is the optimal input change necessary to achieve the fully cost efficient allocation for input *i*. With a transition cost of the form *A-A'*, a gradual transition strategy of, say, *½Ä**, in each of the two periods leads to the same total transition costs as making an immediate transition of magnitude *Ä** (i.e., *a = 2b*), even the absence of discounting. With discounting, gradual transition entails a greater cost of transition over immediate and full transition. Thus, linear transition costs cannot lead to gradual transition.

Transition cost of the form *B-B'* declines at the margin with increases in the absolute size of the transition; i.e., a declining marginal cost of transition. A gradual transition of *½Ä** in each of the two periods leads to a greater total cost of changing input bundles than an immediate transition (i.e., *a < 2b*). Thus, a declining marginal cost of transition encourages the firm to seek an immediate transition.

Transition costs of the form *C-C'* presents increasing marginal cost of transition. The gradual transition strategy of ½Ä* in each period leads to lower cumulative transition costs compared to the instantaneous transition costs (i.e., *a* > *2d*). The principal difference between *C-C'* and the other forms of transition costs is that the others are characterized by constant or decreasing marginal transition costs.

The introduction of transition costs into a model of firm behavior leads to the gradual transition of input allocations when

$$\Psi'(\Delta) \gtrless 0 \quad for \quad \Delta \gtrless 0 \tag{5}$$

$$\Psi''(\Delta) > 0 \quad for\, all \quad \Delta. \tag{6}$$

The convexity of the transition cost function implies the average cost of transition is increasing in the size of the transition suggesting the presence of diseconomies of greater changes in input transitions.

Transition costs are viewed as internally driven in this context and are likely not readily observable. These costs are the result of an inefficiency reduction, which occurs when more of the excess input use is reduced too quickly. Many of these transition costs can be viewed as learning costs as the decision maker struggles with the reorganization of the techniques of production.

Shadow Cost Function

In this section the dynamic relationship between the cost function and measures of allocative and technical efficiency is derived. The conditional input demand associated with C^b is $x^b(w^*, y, h)$. Differentiating x^b with respect to time yields

$$\dot{x}^b = \dot{w}^{*\prime}\frac{\partial x^b}{\partial w^*} + \dot{y}'\frac{\partial x^b}{\partial y} + \dot{h}'\frac{\partial x^b}{\partial h} \tag{7}$$

Recognizing that $\dot{x}^b = -\Delta$ and $C^b_{w^*w^*} = \dfrac{\partial x^b}{\partial w^*}$, and holding output fixed leads to

$$-\Delta = C^b_{w^*w^*}\dot{w}^* + \dot{h}'\frac{\partial x^b}{\partial h} \qquad (8)$$

which links the dynamics of the input bundle allocation to the dynamics of allocative and technical efficiency levels. The input-oriented technical efficiency presented in (3), x^{*b} is scaled by *ç*[1], implying

$$\frac{\partial x^b}{\partial h} = \frac{x^b}{h}. \qquad (9)$$

Solving for the expression $\dot{h}$ involves differentiating actual costs, $C^a = w'x^b$, with respect to time leading to

$$\dot{C}^a = \dot{w}'x^b + w'\dot{x}^b \qquad (10)$$

where $\dot{x}^b$ is defined in (7). Recognizing the output target is fixed, $\dot{y} = 0$, the actual input price is fixed, $\dot{w} = 0$, and using (9), simplifies (7) to

$$\dot{x}^b = C^b_{w^*w^*}(\dot{wx}) + \frac{x^b}{h}\dot{h} \qquad (11)$$

leading to

$$\dot{C}^a = w'C^b_{w^*w^*}(\dot{wx}) + C^b_{w^*}\frac{\dot{h}}{h} \qquad (12)$$

Equation (12) links the dynamics of actual cost function to the dynamics of technical inefficiency, $\frac{1}{h}$, and allocative inefficiencies, x. If the technical efficiency improves, $\dot{h} < 0$, and allocative efficiencies do not change, the actual cost decreases, $\dot{C}^a < 0$. If allocative

[1] Differentiating the behavioral cost function in (1) with respect to ç leads to $C^b_h = C^b/h$, which further implies that $C^b_y = l^b$ and $C^b_{hy} = \partial l^b/\partial h = l^b/h$. Differentiating (2) with respect to ç leads to $0 = \frac{\partial l^b}{\partial h}\frac{1}{h}f'(\cdot) - \frac{l^b}{h^2}f'(\cdot) + \frac{l^b}{h}f''(\cdot)\left[\frac{\partial x^b}{\partial h} - \frac{x^b}{h^2}\right]$ or $0 = \frac{l^b}{h}f''(\cdot)\left[\frac{\partial x^b}{\partial h}\frac{1}{h} - \frac{x^b}{h^2}\right]$. In the presence of a nonlinear technology, this condition leads to (9).

efficiencies improve, $x \to 1$, and the technical efficiency remains fixed, then the second term on the right hand side of (12) determines whether the actual cost will increase or decrease. A proof of this result is provided in the appendix.

Intertemporal Cost Minimization and Efficiency

The challenge to the decision maker is to define the trajectory to move from x^0 to x^* in figure 2. The traditional approach involves following a ray from the origin to identify technical efficiency first which is measured as $0x^T/0x^0$. Upon identifying technical efficiency, allocative efficiency can be evaluated as the deviation of the technically efficient input bundle from the cost minimizing input bundle, $0x^E/0x^T$. This approach inevitably involves identifying the target location on the production frontier from which one can evaluate the technical inefficiency of the firm.

The distinction between the short and long run becomes a prime consideration in determining the appropriate time scale of economic decision making. These strategies focus on the choice of production factors assumed to be fixed when factor allocation decisions are to be made. All economic activity occurs in the short-run to the extent a factor (or factors) of production are taken as fixed. The long run refers to the firm planning ahead to select a future short-run production situation. The classical view of the long run (the case presented in intermediate microeconomic theory texts) is presented as the envelope of all possible short-run situations. When an infinite number of short-run situations are possible the long-run average cost curve is smooth (without kinks).

The problem with the classical description of the short- and long-run is that the story of the envelope curve is not entirely consistent with the story motivating the distinction between the short and long run. The long run consists of a range of possible short run situations available to the firm. As such, the firm always operates in the short run but plans for the long run. A more complete description of producer behavior in the long-run theory of cost concentrates on the planning problem involving the minimization of the discounted stream of costs. Such a characterization focuses on long-run cost as a stock rather than a flow concept.

The micro-level decision making character of input allocation changes can be addressed with an isoquant mapping. For a given period, t, the cost equation for the two input case is

$$\overline{C} = w_1 x_1^b(\cdot) + w_2 x_2^b(\cdot) + \Psi_1(x_{1,t-1}^b - x_{1,t}^b) + \Psi_2(x_{2,t-1}^b - x_{2,t}^b) \tag{13}$$

and the isocost expression is found by totally differentiating $\overline{C}$ and setting the result equal to zero (recognizing $dw_i = 0$ and $dx_{i,J-1} = 0$)

$$d\overline{C} = w_1 dx_{1,t}^b + w_2 dx_{2,t}^b - \Psi_1'(\cdot) dx_{1,t}^b - \Psi_2'(\cdot) dx_{2,t}^b = 0$$

or

$$\frac{dx_{2,t}^b}{dx_{1,t}^b} = -\frac{w_1 - \Psi_1'}{w_2 - \Psi_2'} \tag{14}$$

implying an isocost contour that is negatively sloped and becoming steeper in absolute value as $x_{2,t}$ increases since $\Psi'' > 0$. Figure 3 illustrates the current period decision compared to the long-run, fully efficient decision. For the firm at x^0, the cost minimizing decision rule in (14) presents isocost contours of the shape *B-B'* which direct the firm to move toward x^{1*} as a means to lower costs. However, the firm cannot instantaneously traverse this entire distance given the presence of transition costs. Say, the firm manages to reorganize its activities such that it ends the decision period at input bundle $x^{1'}$. The subsequent period commences with the firm conditioning its cost minimizing input decision (in the presence of transition costs) with a starting input bundle $x^{1'}$.

Figure 4 illustrates the next decision point and indicates that the shape of the isocost contours now adjust given these contours depend on the starting input bundle. The subsequent period's isocost contours are *C-C'* which direct the firm toward x^{2*} in the effort to lower cost. The firm traverses as far as $x^{2'}$, which becomes the starting point for the next period's input decision. Assuming input prices remain constant, as time goes on the components of $\Psi_1'(\cdot)$ and $\Psi_2'(\cdot)$ become smaller in magnitude leading to

$$\lim_{t \to T} \frac{dx_{2,t}^b}{dx_{1,t}^b} = \frac{dx_{2,t}^*}{dx_{1,t}^*} = -\frac{w_1}{w_2} \tag{15}$$

Figure 5 illustrates a transition trajectory that could be the result of such a situation. The isocost contours are steeper in absolute value as input allocation changes take place with a view toward improving efficiency.

The Dynamic Cost Minimization Problem

Consider the dynamic cost minimization problem in continuous time. The presence of allocative and technical inefficiency takes the form of

$$V(x_o, w, y, \boldsymbol{x}, \boldsymbol{h}) = \min_{\Delta} \int_0^T e^{-rs} \left[w' x^b (w^*, y, \boldsymbol{h}(x)) + \sum_i^n \Psi_i(\Delta_i) \right] ds \tag{16}$$

$$s.t. \quad \dot{x} = -\Delta, \quad x(0) = x_o.$$

The dynamic programming equation for this problem is

$$rV(x_o, w, y, \boldsymbol{x}, \boldsymbol{h}) = \min_{\Delta} \left\{ w' x^b (w^*, y, \boldsymbol{h}) + \sum_i^n \Psi_i(\Delta_i) - \Delta' V_x \right\} \tag{17}$$

The equation is interpreted as a flow version of long-run costs where the opportunity costs of this production plan, *rV*, equals the instantaneous cost in the presence of allocative and technical inefficiency, $w'x^b$, plus the transition cost, $\sum_i^n \Psi_i(\Delta_i)$, plus the cost reduction of changing the input allocation, *-Ä* V_x. The shadow value of changing the input allocation (or marginal factor cost), V_x, is the change in long-run costs associated with changing the input bundle. The optimality conditions for (23) are

$$\Psi_i'(\Delta_i) = V_{x_i} \tag{18}$$

which is the marginal transition cost equaling the shadow value of reducing the input bundle. For x_i less than the cost minimizing allocation, $V_x < 0$.

Differentiating both sides of the optimized dynamic programming equation with respect to x_o leads to

$$rV_x = w' \left[\frac{\partial x^b}{\partial x} \frac{\partial x}{\partial x_o} + \frac{\partial x^b}{\partial h} \frac{\partial h}{\partial x_o} \right] - V_{x_o x_o} \Delta \tag{19}$$

which implies that the opportunity cost of reducing the input allocation one unit equals the instantaneous decrease in cost arising from increasing efficiency (i.e., arising from the input reallocation) plus the instantaneous change in the shadow value of the input allocation, where $dV_{x_o}/dt = -V_{x_o x_o}\Delta$.

Efficiency Improving Trajectory

The dynamic optimization problem in (16) can be viewed also as a Calculus of Variations problem when it is rewritten as

$$V = \min_{x^b} \left\{ \int_{t=0}^{T} e^{-rs} \left[C^a \left(w^*, \bar{y}, \boldsymbol{h}\left(x^b\right) \right) + \Psi \left(-\dot{x}^b \right) \right] ds \right\} \tag{20}$$

Solving the Euler-Lagrange equation involves defining

$$H = e^{-rt} \left[C^a + \Psi\left(-\dot{x}^b\right)\right] \tag{21}$$

and finding

$$H_{x^b} = \frac{dH_{\dot{x}^b}}{dt} \tag{22}$$

where

$$H_{x^b} = e^{-rt} \cdot \frac{\partial C^a}{\partial x^b} \tag{23}$$

and

$$H_{\dot{x}^b} = e^{-rt} \Psi'\left(-\dot{x}^b\right). \tag{24}$$

Differentiating (24) with respect to time yields

$$re^{-rt}\Psi'\left(-\dot{x}^b\right) + e^{-rt}\Psi''\left(-\dot{x}^b\right) \cdot \ddot{x}^b \tag{25}$$

implying the Euler-Lagrange condition

$$w_i = r\Psi'\left(-\dot{x}^b\right) + \Psi''\left(-\dot{x}^b\right) \cdot \ddot{x}^b \tag{26}$$

With the terminal time free and the terminal stock variable x_T^b fixed (since x_T^b should be x^* at the terminal time), the transversality condition for (21) is

$$H_{\dot{x}^b} = 0. \tag{27}$$

From (24), this implies

$$-\Psi'\left(-\dot{x}^b\right) = 0. \tag{28}$$

which implies that the marginal transition cost at terminal time *T* should approach zero since input use has nearly attained fully efficient levels and thus input transitions at the associated time period will be negligible.

Assume that changes in stock variables result in costs of transition and that these costs are strongly separable

$$\Psi_{\text{ijt}}\left(\Delta_{1\text{jt}}, \Delta_{2\text{jt}}, \cdots, \Delta_{\text{njt}}\right) \quad = \quad y_{1\text{jt}}(\Delta_{1\text{jt}}) + y_{2\text{jt}}(\Delta_{2\text{jt}}) \cdots + y_{\text{njt}}(\Delta_{\text{njt}}) \tag{29}$$

where

$$\Delta_{\text{ijt}} = -\left(\text{x}^{\text{b}}_{\text{ijt}} - \text{x}^{\text{b}}_{\text{ijt}-1}\right) \tag{30}$$

and

$$\frac{\partial \Psi_{kjt}}{\partial \Delta_{kjt}} > 0 \quad and \quad \frac{\partial^2 \Psi_{kjt}}{\partial \Delta^2_{kjt}} > 0 \tag{31}$$

where Ψ_{kjt} is increasing and convex in Δ_{kjt}), for all $k = 1 \cdots n,\ j = 1 \cdots m,\ t = 1 \cdots T$.

Let the transition cost associated with input *i* for the *j*[th] firm be

$$y_{\text{ijt}}(\Delta_{\text{ijt}}) = b_{\text{ijt}} \frac{\left(\Delta_{\text{ijt}}\right)^2}{2} \ \text{ for } \ \text{i} = 1, 2, \cdots, \text{n}. \tag{32}$$

With the transition cost function given in (29), the optimal condition given in (28) reduces to

$$w_i = -r b_i \dot{x}^b_i + b_i \ddot{x}^b_i \quad for\ i = 1, 2, 3. \tag{33}$$

The discrete time analogies to $\dot{x}^b$ and $\ddot{x}^b$ can be written as

$$\dot{x}^b_{i,t} = x^b_{i,t} - x^b_{i,t-1} \tag{34}$$

and

$$\ddot{x}_{i,t}^{b} = \left(x_{i,t+1}^{b} - x_{i,t}^{b}\right) - \left(x_{i,t}^{b} - x_{i,t-1}^{b}\right). \tag{35}$$

Substituting (33) and (34) into (35) results in

$$w_i = -r\boldsymbol{b}_i\left(x_{it}^{b} - x_{it-1}^{b}\right) + \boldsymbol{b}_i\left(\left(x_{it+1}^{b} - x_{it}^{b}\right) - \left(x_{it}^{b} - x_{it-1}^{b}\right)\right)$$

which can be simplified to

$$w_i = \boldsymbol{b}_i\left(x_{it+1}^{b} - (2+r)\,x_{it}^{b} + (1+r)\,x_{it-1}^{b}\right) \tag{36}$$

Holding the output level fixed at the initial output level and assuming input prices are fixed at their initial levels, equation (36) is a second-order difference equation. Dividing both sides of (36) by $\boldsymbol{b}_i$ results in

$$x_{it+2} - (2+r)x_{it+1} + (1+r)x_{it} = \frac{w_{i0}}{\boldsymbol{b}_i} \tag{37}$$

The particular solution of (37) is given as

$$x_{it}^{P} = \frac{-w_{i0}}{r\boldsymbol{b}_i}t \tag{38}$$

The characteristic equation to generate the complementary function is given as

$$b^2 - (2+r)b + (1+r) = 0 \tag{39}$$

Two characteristic roots for (39) are $b_1 = 1$ and $b_2 = 1 + r$, leading to the complementary function

$$x_{it}^{C} = A_1 + A_2(1+r)^t \tag{40}$$

Assuming that input uses at time 0 and time 1 are given as

$$x_{i0} = \bar{x}_{i0} \quad and \quad x_{i1} = \bar{x}_{i1} \tag{41}$$

A_1 and A_2 can be solved as functions of two initial input uses. Implementing this substitution and combining the complementary function with the particular solution results in

$$x_{it} = x_{it}^P + x_{it}^C = \frac{(1+r)\,\bar{x}_{i0} - \bar{x}_{i1}}{r} - \frac{\left(\bar{x}_{i0} - \bar{x}_{i1}\right)}{r} \cdot (1+r)^t - \frac{w_{i0}}{r\boldsymbol{b}_i} t \qquad (42)$$

which dictates the time paths of optimal input transition.

The time path of the input ratio, $K = x_2/x_1$, can be used to describe the efficiency improving trajectory. Assuming $\bar{x}_{i,0} = \bar{x}_{i,1}$,

$$\dot{K} = \frac{d\left(\frac{x_2^b}{x_1^b}\right)}{dt} = \left(\frac{\dot{x}_2^b}{x_2^b} - \frac{\dot{x}_1^b}{x_1^b}\right) \cdot \frac{x_2^b}{x_1^b} = \frac{1}{r}\left(\frac{w_1}{x_1^b \boldsymbol{b}_{12}} - \frac{w_2}{x_2^b \boldsymbol{b}_2}\right) \cdot \frac{x_2^b}{x_1^b} \qquad (43)$$

which suggests

$$\dot{K} \overset{>}{\underset{<}{=}} 0 \text{ leads to } \frac{w_1}{w_2} \overset{>}{\underset{<}{=}} \frac{\boldsymbol{b}_1}{\boldsymbol{b}_2} \cdot \frac{1}{x_2^b / x_1^b} \qquad (44)$$

Thus, the solution trajectory is involves relative prices, which should equal the relative marginal transition costs times the current input ratio.

3. Specification and Estimation

The estimation can proceed as a two-stage process. The first consideration is the estimation of the cost function in the presence of allocative and technical inefficiency. This is followed by the estimation of the transition cost function parameters.

Specification and Estimation of the Shadow Cost Function

Following Kumbhakar (1997) and Kumbhakar & Lovell (2000), the presence of inefficiencies is accounted for by using a shadow cost function. Assuming the cost function is homogeneous of degree 1 in factor prices, a shadow cost function can be written as

$$\mathrm{C}\left(\mathrm{y}_{jt}, h_j \mathrm{w}^*_{jt}; b\right) = \min_{h_j\, x^b_{jt}} \left\{ \left(h_j \mathrm{w}^*_{jt}\right)^{\mathrm{T}} \left(\frac{\mathrm{x}^b_{jt}}{h_j}\right) : \mathrm{f}\left(\frac{\mathrm{x}^b_{jt}}{h_j}\right) = \mathrm{y}_{jt} \right\} = h_j \mathrm{C}\left(\mathrm{y}_{jt}, \mathrm{w}^*_{jt}; b\right) \qquad (45)$$

where $j = 1, \cdots m$ indexes firms and $t = 1, \cdots, T$ indexes time periods. The parameters η_j's represent the inverse of firm specific but time invariant input-oriented technical inefficiency. The corresponding shadow input demand equations can be obtained using Shephard's Lemma and given as

$$x^b_{ijt}\left(y_{jt}, w^*_{jt}; \beta\right) = \eta_j \cdot \frac{\partial C\left(y_{jt}, w^*_{jt}; \beta\right)}{\partial w^*_{ijt}} \quad i = 1, \cdots, n \tag{46}$$

The shadow input cost share equations are given as

$$S_{ijt}\left(y_{jt}, w^*_{jt}; \beta\right) = \frac{w^*_{ijt} x^b_{ijt}\left(y_{jt}, w^*_{jt}; \beta\right)}{\eta_j C\left(y_{jt}, w^*_{jt}; \beta\right)} \quad i = 1, \cdots, n. \tag{47}$$

Because observed inputs minimize the shadow cost, observed input demand equations can be written as

$$x^b_{ijt}\left(y_{jt}, w^*_{jt}; \beta\right) = \eta_j \frac{\partial C\left(y_{jt}, w^*_{jt}; \beta\right)}{\partial w^*_{ijt}} = \eta_j \frac{C\left(y_{jt}, w^*_{jt}; \beta\right) \cdot S_{ijt}\left(y_{jt}, w^*_{jt}; \beta\right)}{w^*_{ijt}} \tag{48}$$

Since $w^*_{ijt} = w_{ijt} \xi_{i1j}$, the observed total expenditure equation becomes

$$\begin{aligned} E_{jt} &= \sum_{i=1}^{n} w_{ijt} x^b_{ijt} = \sum_{i=1}^{n} \frac{w^*_{ijt}}{\xi_{i1j}} \cdot x^b_{ijt} = \sum_{i=1}^{n} \frac{w^*_{ijt}}{\xi_{i1j}} \cdot \eta_j \frac{C(\cdot) S_{ijt}(\cdot)}{w^*_{ijt}} \\ &= \eta_j C(\cdot) \sum_{i=1}^{n} \frac{S_{ijt}(\cdot)}{\xi_{i1j}} \end{aligned} \tag{49}$$

Using equations (48) and (49), observed input cost share equations can be rewritten as

$$S_{ijt} = \frac{w_{ijt} x^b_{ijt}}{E_{jt}} = \frac{S_{ijt}(\cdot)\left(\xi_{i1j}\right)^{-1}}{\sum_k \left[S_{kjt}(\cdot)\left(\xi_{k1j}\right)^{-1} \right]}, \quad i = 1, \cdots, n. \tag{50}$$

If the shadow cost function $C(y_{jt}, \eta_j w^*_{jt}; \beta)$ takes on a translog form, it can be written as

$$\ln E_{jt}^{shadow} = b_{0t} + b_y \ln y_{jt} + \sum_i b_i \ln w_{ijt}^* + \frac{1}{2} b_{yy} (\ln y_{jt})^2$$
$$+ \frac{1}{2} \sum_{k=1}^{n} \sum_{i=1}^{n} b_{ki} \left(\ln w_{kjt}^*\right)\left(\ln w_{ijt}^*\right) + \sum_{i=1}^{n} b_{iy} \left(\ln w_{ijt}^*\right)\left(\ln y_{jt}\right) + \ln h_j. \tag{51}$$

where symmetry and homogeneity of degree +1 in input prices are imposed through the parameter restrictions $b_{ik} = b_{ki}$ for $k \neq i$, $\sum_{i=1}^{n} b_i = 1, \sum_{k=1}^{n} b_{ik=0}$ for i = 1, …, n, and $\sum_{i=1}^{n} b_{iy} = 0$. The corresponding shadow input cost share equations can be written as

$$S_{ijt}\left(y_{jt}, w_{jt}^*; b\right) = b_i + \sum_{k=1}^{n} b_{kn} \ln w_{kjt}^* + b_{iy} \ln y_{jt}, \quad i = 1, \cdots, n. \tag{52}$$

Thus, the log of actual cost (49) becomes

$$\ln E_{jt} = b_{0t} + b_y \ln y_{jt} + \sum_i b_i \ln w_{ijt}^* + \frac{1}{2} b_{yy} (\ln y_{jt})^2$$
$$+ \frac{1}{2} \sum_{k=1}^{n} \sum_{i=1}^{n} b_{ki} \left(\ln w_{kjt}^*\right)\left(\ln w_{ijt}^*\right) + \sum_{i=1}^{n} b_{iy} \left(\ln w_{ijt}^*\right)\left(\ln y_{jt}\right) \tag{53}$$
$$+ \ln\left\{\sum_{i=1}^{n} \left(x_{i1j}\right)^{-1} \left[b_i + \sum_{k=1}^{n} b_{ki} \left(\ln w_{ijt}^*\right) + b_{iy} \ln y_{jt}\right]\right\} + \ln h_j$$

Observed input cost share equations (50) become

$$S_{ijt} = \frac{\left(x_{i1j}\right)^{-1}\left[b_i + \sum_{k=1}^{n} b_{ki} \ln w_{kjt}^* + b_{iy} \ln y_{jt}\right]}{\sum_{k=1}^{n} \left(x_{k1j}\right)^{-1}\left[{}_i\ b_l \sum_{k=1}^{n} b_l \ln w_{klt}^* + b_{ly} \ln y_{lt}\right]}, \quad i = 1, \cdots, n. \tag{54}$$

Estimation of the system consisting of equations (53) and (54) can be accomplished by following the fixed effects approach. With the loss of considerable degrees of freedom, we can make technical efficiency to be time varying.

The specification of equation (53) is altered as

$$\ln E_{jt} = b_{jt} + b_{y} \ln y_{jt} + \sum_{i} b_{i} \ln w^{*}_{ijt} + \frac{1}{2} b_{yy} (\ln y_{jt})^{2}$$
$$+ \frac{1}{2} \sum_{k=1}^{n} \sum_{i=1}^{n} b_{ki} \left(\ln w^{*}_{kjt}\right)\left(\ln w^{*}_{ijt}\right) + \sum_{i=1}^{n} b_{iy} \left(\ln w^{*}_{ijt}\right)\left(\ln y_{jt}\right) \qquad (55)$$
$$+ \ln \left\{ \sum_{i=1}^{n} \left(x_{i1j}\right)^{-1} \left[b_{i} + \sum_{k=1}^{n} b_{ki} \left(\ln w^{*}_{ijt}\right) + b_{iy} \ln y_{jt} \right] \right\} + v_{jt}$$

where

$$b_{jt} = b_{0t} + \ln h_{j} = b_{0t} + u_{jt} \qquad (56)$$

and $u_{jt} \geq 0$, v_{jt} is an additive error term. Note that b_{jt} is the intercept for producer j in time period t. Following Cornwell, Schmidt, and Sickles (1990), we can specify b_{jt} as

$$b_{jt} = \Omega_{j1} + \Omega_{2} t + \Omega_{3} t^{2} \qquad (57)$$

After adding classical error terms to equation (53) and deleting one input cost share equation, the remaining system of equations in (54) and (55) can be estimated using the seemingly unrelated regression (SUR) method. However, estimating the system (54) and (55) together can be computationally burdensome due to the inclusion of many dummy variables. Since the concern for a singular error covariance matrix with the system of share equations arises with the ITSUR, but not SUR, an alternative strategy involves estimating share equations by SUR and then estimating the cost function by MLE. In the second stage MLE estimation, coefficient estimate values that are obtained in the first stage regression (share equations estimation) are imposed into the model.

Upon obtaining estimates of b_{jt}'s, define $\hat{b}_{0t} = \min_{j} \left\{ \hat{b}_{jt} \right\}$ as the estimated intercept of the cost frontier in period t. Then, $\ln h_{jt}$ is obtained as

$$\ln h_{jt} = u_{jt} = \left(\hat{b}_{jt} - \hat{b}_{0t} \right) \qquad (58)$$

Thus, input oriented technical inefficiency h is obtained as

$$\mathrm{TE}_{\mathrm{jt}} = \mathrm{e}^{-\mathrm{u}_{\mathrm{jt}}} = \frac{1}{h_{\mathrm{jt}}} \tag{59}$$

Also, note that allocative inefficiency parameters, x_{i1}, are all estimated. The shadow input prices are identified and the estimated allocative inefficiency parameters are input pair-specific but not firm specific.

Estimating Input Transition Parameters

The theory governing the dynamics of efficiency-improving input transitions assumes fixed input prices and the output in obtaining the corresponding predicted cost and input demands. This is achieved by fixing the output and input prices at their initial levels.

Estimating the transition cost function is straightforward. Recognizing that estimated cost function is on log form, the Shephard's Lemma can be written as

$$\frac{\partial C_{jt}}{\partial w_{ijt}^{*}} = \frac{\partial \ln C_{jt}}{\partial \ln w_{ijt}^{*}} \cdot \frac{C_{jt}}{w_{ijt}^{*}} = S_{x_{ijt}^{b}} \cdot \frac{C_{jt}}{w_{ijt}^{*}} \tag{60}$$

where

$$x_{ijt}^{b} = \left(\frac{\partial C_{jt}^{b}}{\partial w_{ijt}^{*}} \right) = \left(\frac{\partial \ln C_{jt}^{b}}{\partial \ln w_{ijt}^{*}} \cdot \frac{C_{jt}^{b}}{w_{ijt}^{*}} \right) \tag{61}$$

holds, and (60) can be rewritten as

$$x_{ijt}^{b} = \frac{C_{jt}^{b}}{w_{ijt}^{*}} \cdot S_{ijt} \tag{62}$$

where S_{ijt} represents the share equation of the i[th] input at time *t*. C_{jt}^{b} represents the predicted cost obtained by taking exponential to the predicted cost. Similarly, we obtain x_{ijt-1}^{b} and x_{ijt+1}^{b} by simply changing the time subscripts of variables on the right hand side of (62). Substituting x_{ijt-1}^{b}, x_{ijt}^{b} and x_{ijt+1}^{b} into (37), we can estimate each optimality condition or the system of optimality conditions using MLE.

4. The Case of U.S. Agricultural Banks

The agricultural banking data are obtained from Call Reports for the first quarter of 1996 to the second quarter of 2000. Quarterly Call Reports contain comprehensive operating cost data for all banks in U.S. The Call Report (CR) database, provided by the Chicago Federal Reserve Bank, contains the information required to evaluate efficiencies in financial institutions. Information includes various loans, deposits and operating costs. The database covers the time period between the first quarter of 1976 and the second quarter of 2000. Information for approximately 10,000 financial institutions is available via the database. The exact number of financial institutions in a specific quarter varies.

The agricultural banks are defined using the FDIC criterion to identify agricultural banks as those financial institutions whose agricultural loan ratio is no less than 25% which can present a focused set of banks supporting agricultural activities.[2]

The time period to be covered in the sample is chosen to be short enough to preclude the impact of technological change on input allocation decisions while being long enough to guarantee time-variant and bank-specific technical inefficiencies. For these reasons, we chose the time period between the first quarter of 1996 and the second quarter of 2000 to construct our panel data.

Definitions of Inputs, Output and Input Prices

Following the value added approach the single output is defined as the sum of total loans and total deposits. The three inputs are labor, expenses for the premises and fixed assets, and the sum of interest and other expenses. Labor input is defined as the total number of

[2] Various agencies such as Federal Reserve System (FRS), Federal Deposit Insurance Corporation (FDIC) and American Bankers Association (ABA) use their own criteria to identify agricultural financial institutions. FRS defines an agricultural bank as a bank that has more than an average agricultural loan ratio. The agricultural loan ratio is defined as the dollar amount of agricultural loans divided into the dollar amount of total loans where agricultural loans are loans to finance agricultural production and other loans to farmers. With the average agricultural loan ratio at approximately 10% and varying quarter by quarter, the FRS's criterion defines a heterogeneous set of agricultural banks. The FDIC regards any financial institution whose agricultural loan ratio is no less than 25% as an agricultural bank. The ABA uses two criteria to identify agricultural banks. The first is the absolute dollar volume of agricultural loans and the other is the agricultural loan ratio. ABA classifies any bank that has an agricultural loan ratio greater than 50% or provides more than $ 2.5 million to the agricultural sector as an agricultural bank. The first criterion for ABA may be too strict while the second criterion may too loose. Applying the ABA's criteria, the financial institutions included in the sample will present very heterogeneous agricultural loan ratios.

employees. Expenses for the premises and fixed assets include all non-interest expenses related to the use of premises, equipment, furniture, and fixtures. Other expenses involve interest paid on deposits and other non-interest operating expenses. Except for the labor input, no explicit input price information is available. Following Ferrier and Lovell (1990), input prices for the second and the third inputs are obtained by dividing each input into total deposits where total cost is defined as $TC = w_1 \times x_1 + x_2 + x_3$.[3] Definitions for output, inputs and input prices for the 906 sample banks are provided in Table 1.

Cost Function Estimation Results

Obtaining time-variant technical inefficiency parameters involves estimating 2,718 parameters in the cost function. In the first stage, three share equations are estimated using seemingly unrelated regression (SUR), which incorporate restrictions to guarantee the homogeneity of the cost function with respect to shadow input prices. The first stage system regression results in estimates for most of slope parameters for the cost function and time-invariant allocative inefficiency parameters.

In the second stage, the system of equations comprising three share equations is re-estimated for each period to obtain time-variant allocative inefficiency parameters. The slope parameters are restricted such that they are equal to those obtained from the first stage regression and the restrictions to guarantee the homogeneity of the cost function are retained.

In the third stage, the translog cost function is estimated using Full Information Maximum likelihood estimation after imposing slope parameter estimates and time-variant allocative efficiency parameter estimates that were obtained in the first and the second stage estimations. In this regression, the intercept term is omitted and variables to capture technical efficiencies of banks (bank specific dummy variables, bank specific dummy variables multiplied by time and bank specific dummy variables multiplied by time-squared)

[3] In the process of creating inputs and input prices, it was found that twelve sample banks have zero inputs for at least one of their inputs and these banks were eliminated from the sample. Additionally, it was found that two sample banks among the 908 remaining sample banks have negative inputs for at least one of their inputs and these banks were removed from the sample. Thus, we have 906 sample banks and the total number of observations for the balanced panel is 16,308 (906 banks $\times$ 18 quarters).

are included. The third stage regression yields the parameter estimates of Ω_{1i}, Ω_{2i} and Ω_{3i} for each bank in addition to the slope parameters that were not estimated in the first and the second stage estimation process. Using these estimates, the predicted values for $\Omega_{1i} + \Omega_{2i} \cdot t + \Omega_{3i} \cdot t^2$ are generated. Defining the minimum of the predicted as the minintercept, time variant and bank specific technical efficiencies are obtained by taking the exponential as the predicted minintercept. Technical inefficiency parameters are the inverse of these technical efficiency measures.

Table 2 presents the decomposition of overall cost efficiency into allocative and technical efficiency component. Overall efficiency measures (OE) are obtained by dividing the cost frontier into the predicted cost and presenting the mean value of 0.64. Technical efficiency (TE) measures and Pseudo Allocative Efficiency (SAE) measures are obtained by mimicking the DEA efficiency decomposition; i.e., OE = TE x SAE or SAE = OE/TE. The results suggest that agricultural banks are technically inefficient (TE measures are less than one) but allocatively efficient. This implies that technical barriers rather than allocatively inefficient decision-making are the main causes of agricultural banks' inefficiencies. Stefanou, Choi and Stokes (2002) present a more detailed analysis addressing efficiency differences and examining the imacts of firm-specific characteristics on the types of efficiency changes for this entire sample. The remainder of this study focuses on a subset of agricultural banks exhibiting increasing cost efficiency over time and uses the model of efficiency transition to explain their adjustment paths.

Estimation

The subset of agricultural banks demonstrating increasing cost efficiencies over the estimation period are described in Table 3 with descriptive statistics for the entire sample are presented in parentheses for comparison purposes. The average size of the selected banks is smaller than that of the entire sample suggesting that smaller banks have demonstrated an ability to increase efficiency. This result coincides with Rangan et al (1988) who attributed the positive relationship between technical efficiency and bank to the fact that larger banks are more likely to be located in larger, urban areas and thus are faced with greater competition. This implies that smaller banks may have more room for efficiency improvement especially since agricultural banks tend to be smaller and located in more rural areas. .

Transition Cost Estimation for the Selected Agricultural Banks

Table 4 presents the regression results for the transition costs equations. None of transition cost parameter estimates from the system estimation is statistically significant at 5% significance level. All estimates from the single estimation are statistically significant at 5% significance level.

Using the cost function and transition cost parameter estimates, cost frontier, predicted cost with TE & AE and transition costs for agricultural banks presenting increasing cost efficiency trends are obtained. The transition cost parameter estimates obtained by single equation estimation are used to obtain transition costs which are presented in Table 5. Agricultural banks presenting increasing cost efficiency trends are all allocatively efficient but are not technically efficient. The average cost efficiency for these selected agricultural banks (0.64) is the same as the average cost efficiency for the entire sample (0.64). Being allocatively efficient implies that the input uses of agricultural banks are on a plane from the origin. Table 6 decomposes the total transition cost into components for each input. On average, the contribution of transition cost for x_3 is the greatest followed by that for x_1, with x_2 exhibiting instantaneous transition.

The Characteristics of Transition Trajectories

A standard time path analysis indicates that the time path of each optimal input use will be explosive since the dominant root (the root with the largest absolute value) for the complementary function is greater than unity. However, two factors prevent the time path from being explosive: 1) the particular solution also influences the time path, and 2) the input use x_{it} (inefficient input use) cannot be less than x_i^* (fully efficient input use) by definition.

One agricultural bank presenting increasing cost efficiencies is selected to examine the time path of input use. Based on transition cost parameter estimates, solutions for optimal input uses and the assumed interest rate (5%), optimal input uses are simulated given two initial input use (time zero and time one input uses). When the input use at time one is less than or equal to than that at time zero, input transitions are almost instantaneous. The simulations depends on solving a two-point boundary problem for a second order linear difference equation. With the initial position fixed, the subsequent decision that moves further away from the frontier (i.e., greater input use than the initial level) and still permits the firm to achieve the full efficient input allocation is defined as the threshold value. The first period input use exceeding the threshold value leads to a trajectory that never achieves full efficiency. As such, these threshold values reflect the magnitude of flexibility in making mistakes and still having the prospect of being efficient eventually. These threshold values are generated for the selected agricultural bank. Based on these threshold values, table 7 categorizes the time paths of input uses into two classes: 1) converging to the full efficiency (CTFE) and 2) diverging from the full efficiency (DFFE).

Figures 6 through 8 present each simulated input use. The simulation results show that transitions for all three input uses are gradual. Reflecting the magnitudes of transition cost parameter estimates for each input (transition cost parameter for the first input is the greatest and that for the third input is the smallest), the pace of the first input transition is the slowest while the second input is the fastest to adjust.

Based on the simulation, figure 9 presents the efficiency improving input transition trajectory on the (x_1, x_3) plane since x_2 presents no transition costs. With the allocative efficiency parameter for x_3 near unity, the initial input pair (x_{10}, x_{30}) is nearly on the ray line from the origin and passes through the fully efficient input pair (x_1^*, x_2^*). The bank achieves full efficiency after 10 periods as the speed of transition slows over time. The transition trajectory shows that the bank is allocatively inefficient until it achieves full efficiency. This may imply that in achieving full efficiency, improving the technical efficiency is the priority.

5. Concluding Comments

A dynamic production theory-based model describing how inefficient producers move toward efficient input allocations was developed. Taking an exogenous perspective to adjustment is akin to estimating the time path of measured cost efficiency and projecting the time to when full efficiency is realized. For the selected banks in this study, regressing cost efficiency against time yields the predicted equation $CE(t) = 0.6204 + 0.0028 \quad t$ (which is the single equation form that performs best) which suggests that it will take 136 time periods for an agricultural bank (whose time 1 cost efficiency is 0.6231) to obtain full cost efficiency. Using the same behavioral estimation of economic behavior under inefficiency, a model of transition finds that these banks can converge to full cost efficiency by 10 time periods. The firm transitions toward an efficient allocation by becoming simultaneously improving on technical inefficiency instantaneously and allocative efficiency intertemporally, which suggests a nonlinear path toward the most efficient allocation.

Definitions of efficiency in an intertemporal context are needed. Input allocation X^o in figure 5 and all points along the trajectory (X^o, X^*) are allocatively and technically inefficient when using the static measures of allocative and technical inefficiency. However, in a dynamic sense, all points along this trajectory are intertemporally efficient in that the optimality conditions are satisfied for the objective of intertemporal cost minimization, which motivates efficiency improving decisions. Silva and Stefanou (2001) develop measures of temporal efficiency as a flow notion of dynamic efficiency in that the firm's decisions are assumed to be made in the short run with a view to the long run.

The temporal notion is conditioned on past decisions but reflects dynamic linkages of past decisions to future prospects. As a result, static measures will not provide an accurate characterization of producer efficiency behavior, which can span more than one time period.

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Appendix

Proposition: *As* $x \rightarrow 1$, $\overset{\bullet}{C}^{a} < 0$, *where* $x' = \left(1x_1 \cdots x_n\right)$ *and* $1' = \left(1 \cdots 1\right)$ *for* $i = 1, \cdots N$ *.*

That is, the actual cost decreases over time if allocative efficiencies improve over time.

Proof: Full allocative efficiency ($x = 1$) implies that the first order condition holds for each input pairs. That is,

$$\frac{w_i}{w_j} = \frac{f_{x_i^b}}{f_{x_j^b}} \; for \; i, \; j = 1, \cdots N \, , \tag{A.1}$$

where $f_{x_i^b}$ indicates the first derivative of production function with respect to i^{th} input use and the production function is given as

$$y = f\left(\frac{x^b}{\boldsymbol{h}}\right) \tag{A.2}$$

Define the actual cost when the first order condition (A.1) holds for all input pairs as

$$C^{a*} = w' x^b \left(\boldsymbol{h}, w, \bar{y}\right) \tag{A.3}$$

Suppose the allocative efficiency parameter for some input i is not 1 while allocative efficiency parameters for all the other inputs are 1. This implies that

$$\frac{w_i}{w_j} \neq \frac{f_{x_i^b}}{f_{x_j^b}} \; for\, all \; j, \; j \neq 1 \, and \;\; j = 1 \cdots N \tag{A.4}$$

$$\frac{w_i x_i}{w_j} = \frac{f_{x_i^b}}{f_{x_j^b}} \; for \; all \;\; j, \; j \neq 1 \, and \;\; j = 1 \cdots N \, . \tag{A.5}$$

The observed cost for this case is given as

$$C^{a} = w' x^b \left(\boldsymbol{h}, w^*, \bar{y}\right) \tag{A.6}$$

Comparing (A3) with (A.6), the inequality

$$C^{a*} < C^{a} \tag{A.7}$$

follows since input prices are given as w', given the output and the technical inefficiency level, C^{a*} is the minimum cost that can be achieved.

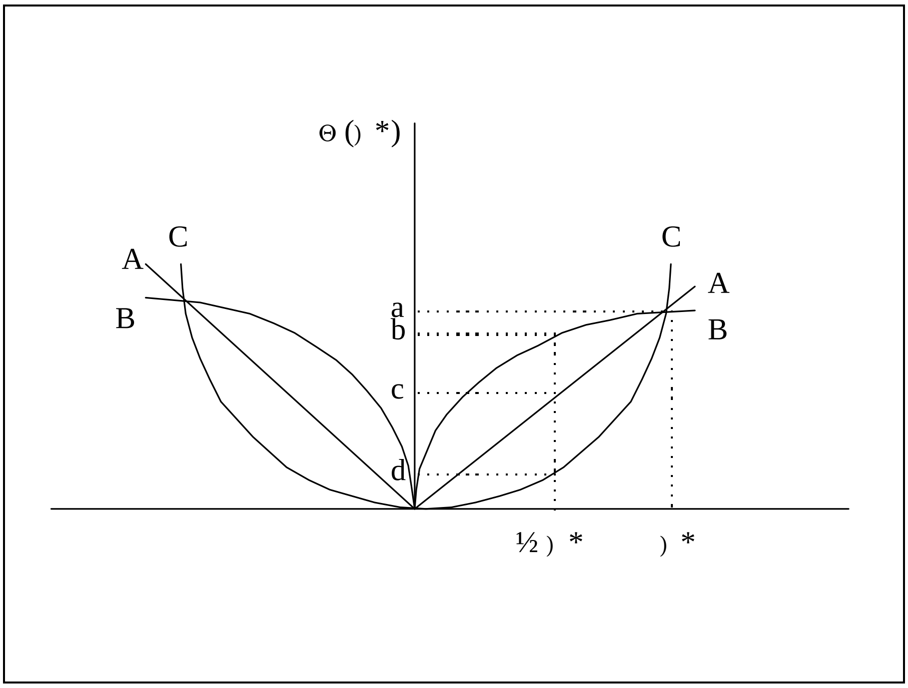

Figure 1: Possible forms of transition costs

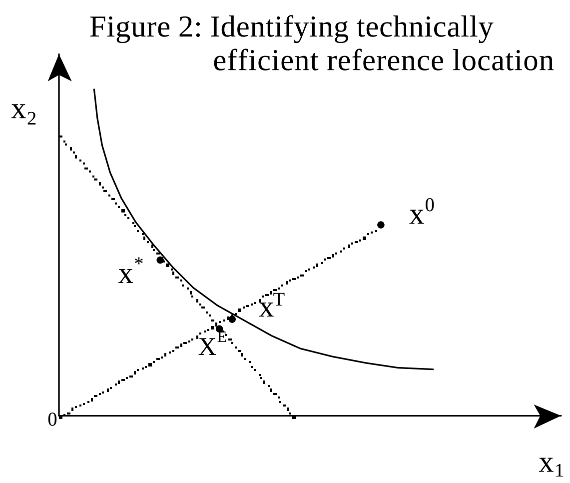

Figure 2: Identifying technically efficient refe rence location

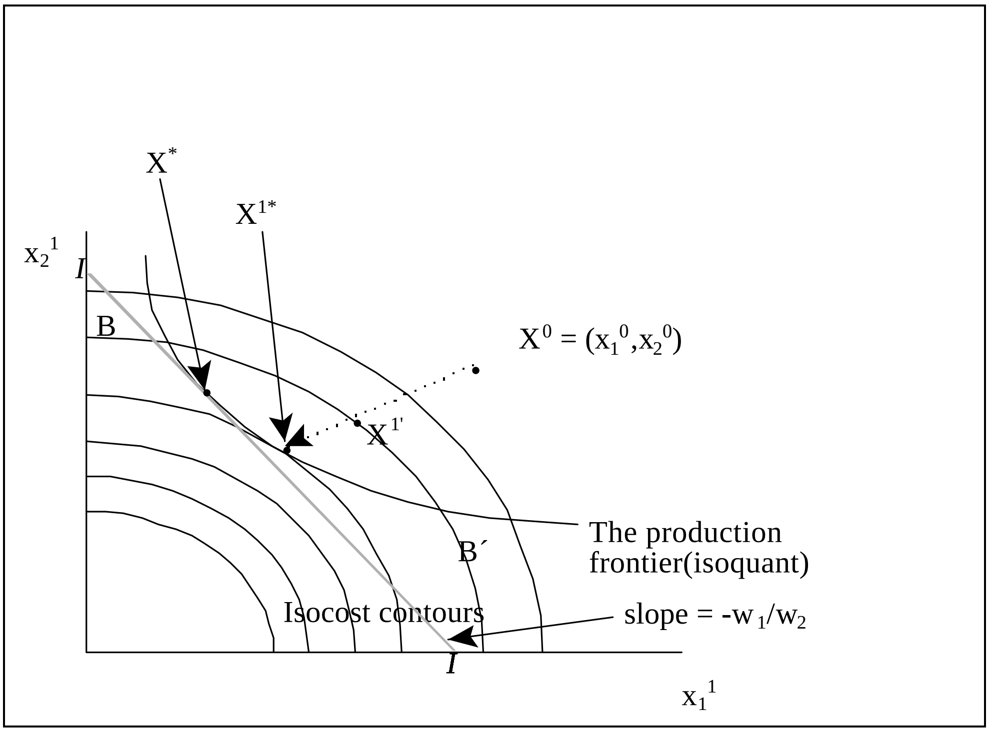

Figure 3: Two period cost minimization problem at time 1

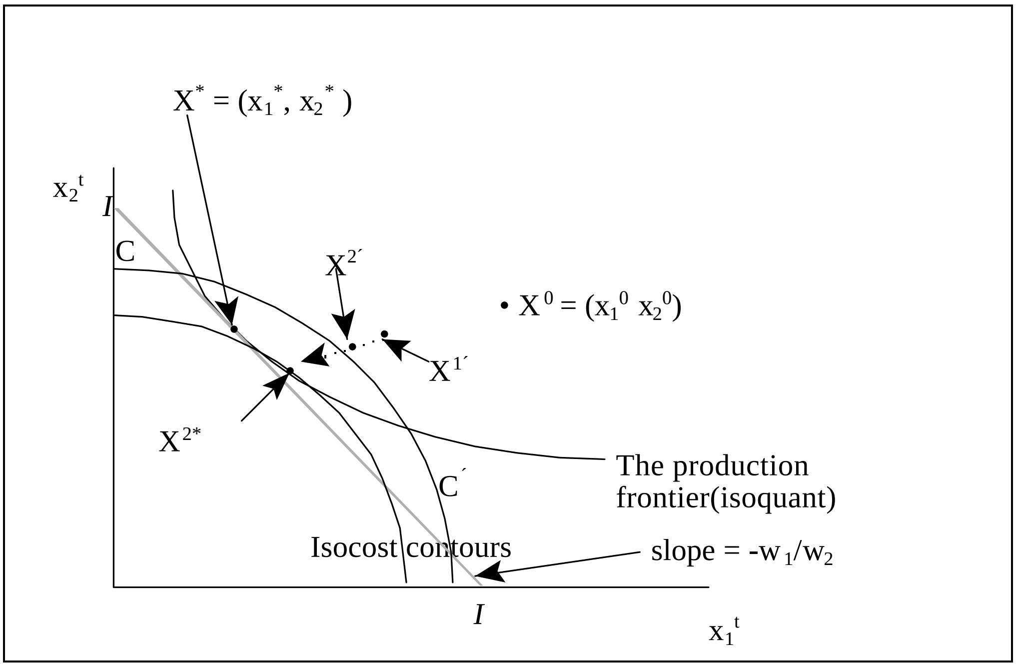

Figure 4: Multi period cost minimization problem at time t (1<t<T)

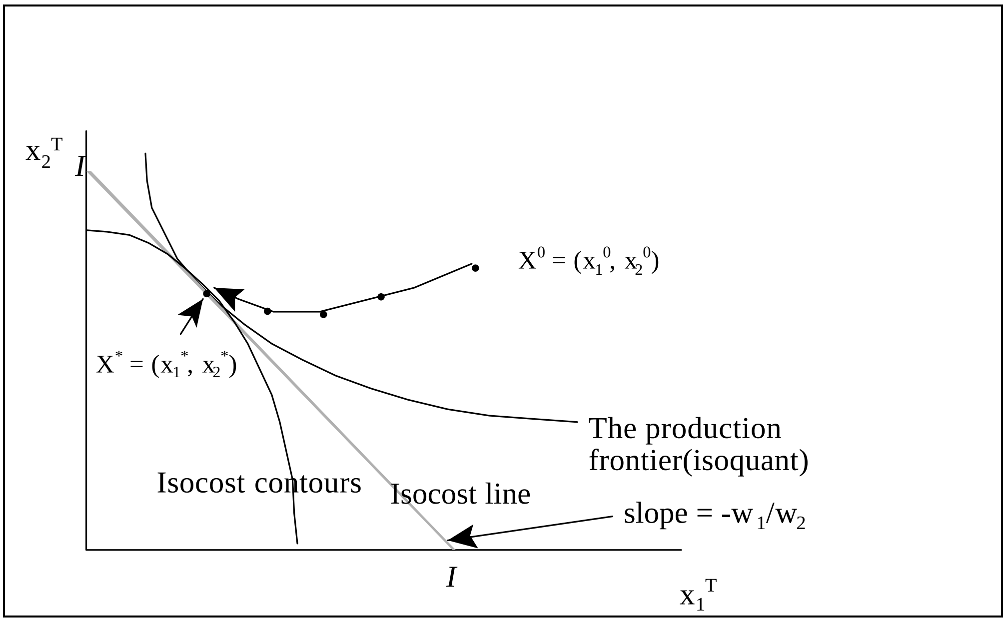

Figure 5: Multi period cost minimization problem at time *T*

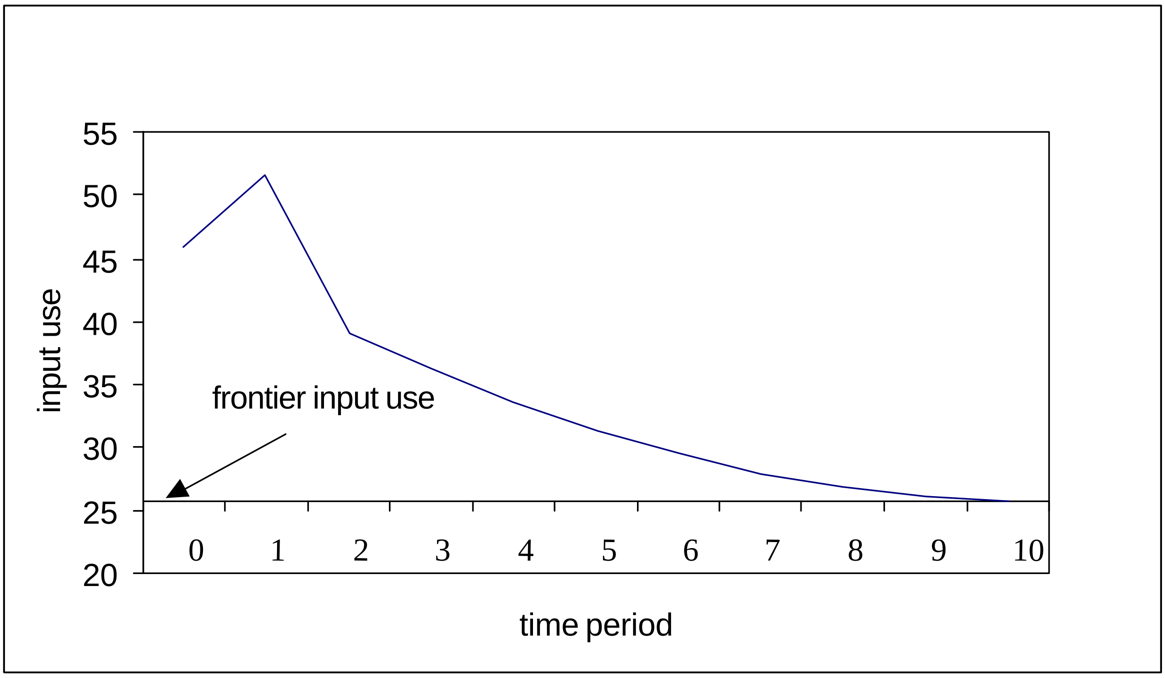

Figure 6: The trend of the first input use when the input transition at time one equals the threshold value

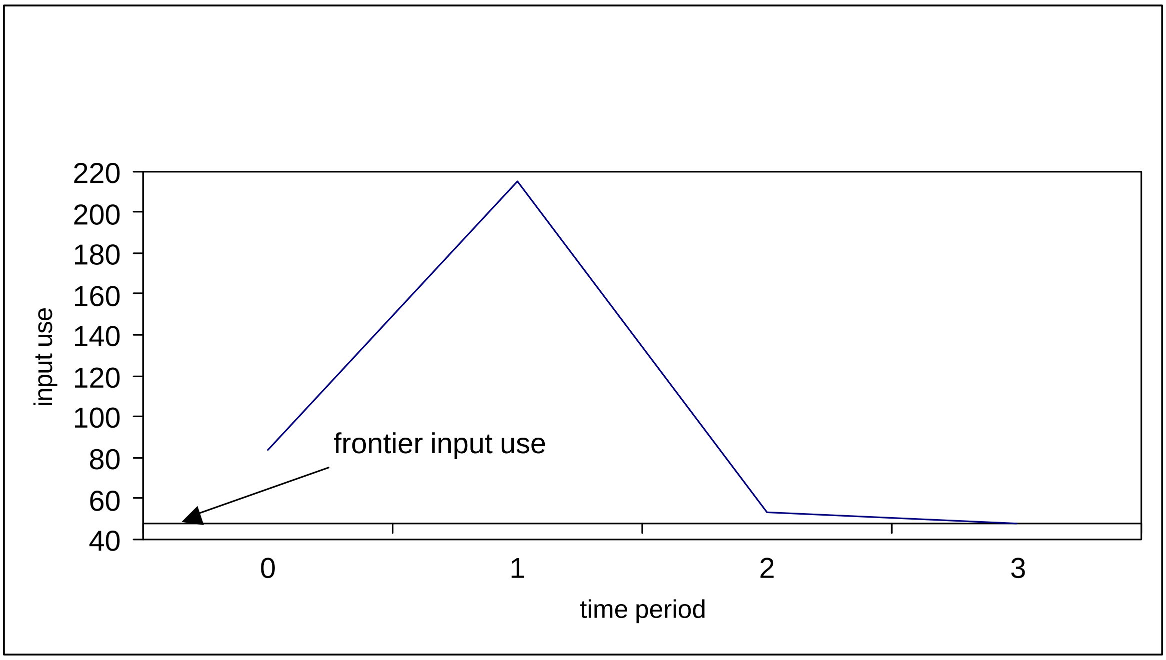

Figure 7: The trend of the second input use when the input transition at time one equals the threshold value

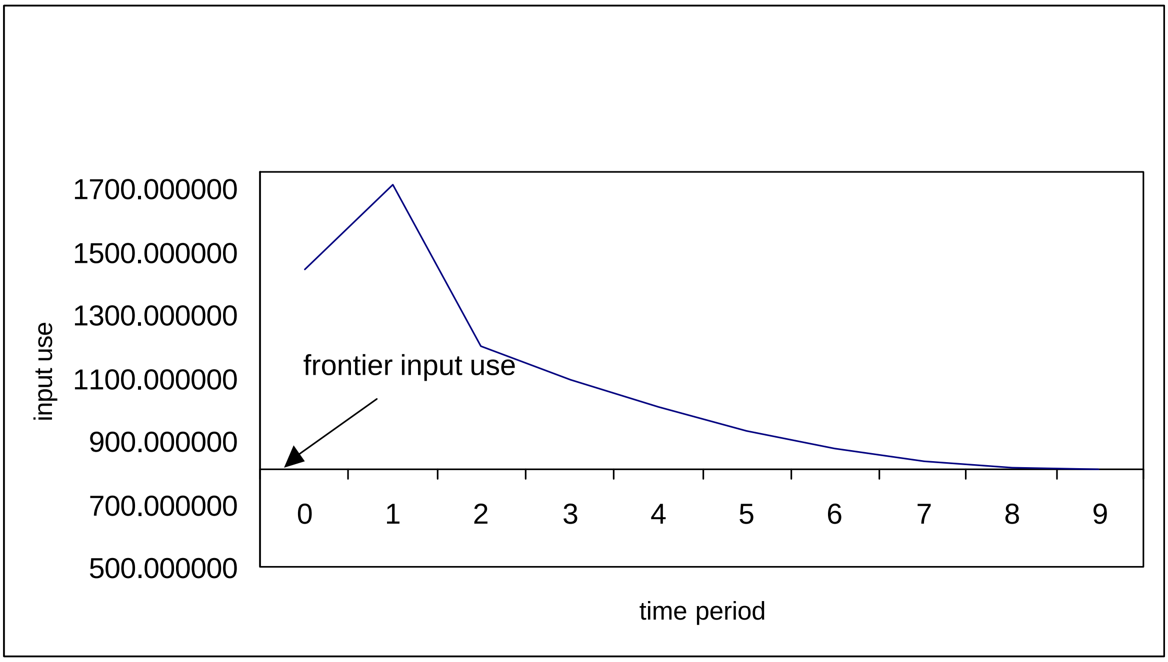

Figure 8: The trend of the third input use when the input transition at time one equals the threshold value

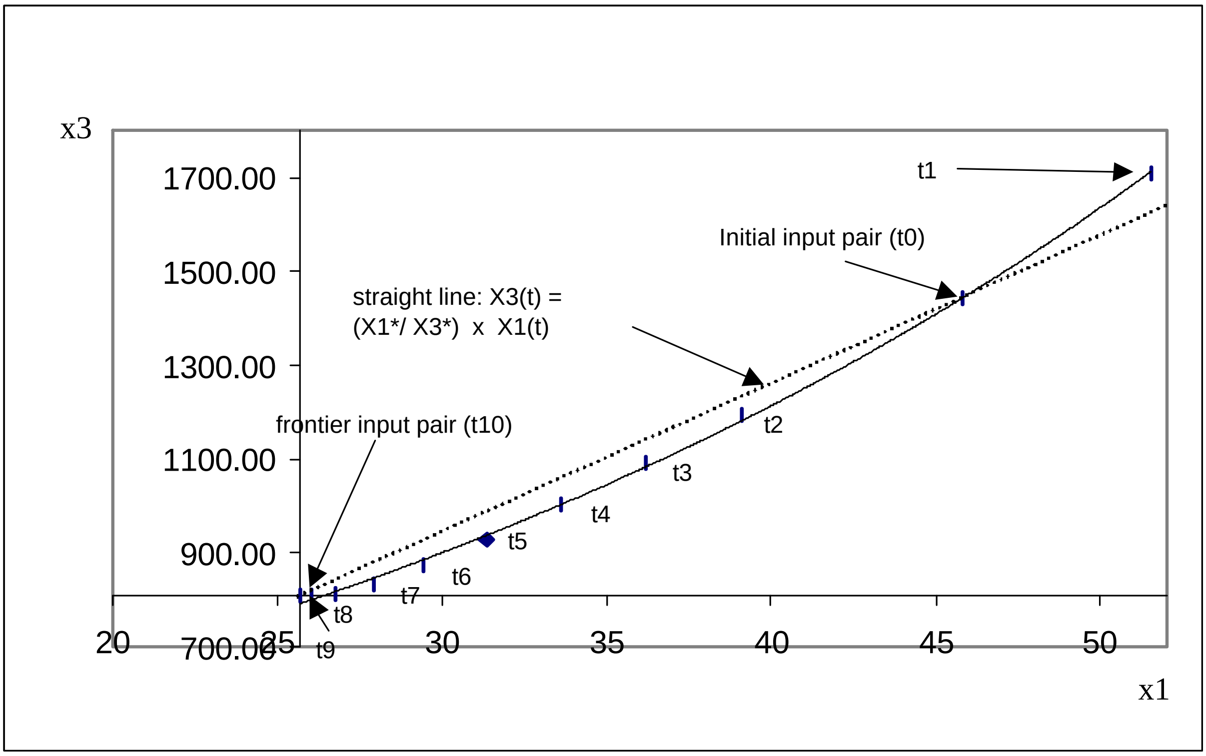

Figure 9: The efficiency improving input transition trajectory based on the simulation

Table 1: Defining output, inputs and input prices

Variable	Definition
Y	The sum of total loans and total deposits
x_1	Labor input, the total number of employees
x_2	Capital input, expenses of premises and fixed assets
x_3	The sum of interest expenses on deposits and other non-interest operating expenses
w_1	Total salaries and employment benefits divided into the total number of employees
w_2	Expenses of premises and fixed assets divided into total deposits
w_3	The sum of interest expenses on deposits and other non-interest operating expenses divided into total deposits
TC	The sum of total salaries & employment benefits, occupancy expenses and interest and other non-interest operating expenses

Table 2: Average values of cost frontier, predicted cost with technical & allocative inefficiencies and transition cost

Time	Overall efficiency	Technical efficiency	Pseudo allocative efficiency
1	0.65235	0.65235	1.00000
2	0.66606	0.66606	1.00000
3	0.66612	0.66612	1.00000
4	0.66209	0.66209	1.00000
5	0.65825	0.65825	1.00000
6	0.65461	0.65461	1.00000
7	0.65114	0.65114	1.00000
8	0.64784	0.64784	1.00000
9	0.64471	0.64471	1.00000
10	0.64173	0.64173	1.00000
11	0.63892	0.63892	1.00000
12	0.63627	0.63627	0.99999
13	0.63378	0.63378	1.00000
14	0.63145	0.63145	1.00000
15	0.62929	0.62929	1.00000
16	0.62732	0.62732	1.00000
17	0.62553	0.62553	1.00000
18	0.60604	0.60604	1.00000
1-18	0.64297	0.64297	1.00000

1. Overall efficiency is obtained by dividing the cost corresponding to the frontier for the predicted cost.
2. Pseudo Allocative Inefficiency measure is the ratio of overall to technical efficiency.

Table 3: Descriptive statistics for selected agricultural banks

Variable	Mean	Standard deviation
Y	58123.67 (63230.18)	52334.30 (64944.70)
x_1	16.3258 (17.2111)	15.7020 (18.0229)
x_2	85.308 (84.592)	133.468 (121.093)
x_3	1045.24 (1117.84)	1324.51 (1387.03)
w_1	24.9675 (23.6745)	14.5042 (13.1752)
w_2	0.00222 (0.00223)	0.00136 (0.0015)
w_3	0.02872 (0.02919)	0.01356 (0.01400)
TC	1517.90 (1595.88)	1854.05 (1939.26)

In the parentheses are statistics for the entire sample.

Table 4: Estimation of transition cost parameters using the first step cost function for chosen agricultural banks

Estimation method	Parameter	Estimate	S.E.	Pr > |t|
Single estimation	$\exists_1$	14.06328	4.1878	0.0009
	$\exists_2$	0.133372	0.0658	0.0434
	$\exists_3$	0.05046	0.0163	0.0022
System estimation	$\exists_1$	-0.14169	0.6188	0.8190
	$\exists_2$	0.011685	0.0120	0.3301
	$\exists_3$	2.121928	481.80	0.9965

Table 5: Average values of cost frontier, predicted cost with inefficiencies and transition cost for the selected agricultural banks

Time	Cost Frontier	Cost With TE&AE	Transition Cost	OE	TE	SAE
1	399.433	667.334	NA	0.61470	0.61470	1.00000
2	399.433	651.066	7.20045	0.62975	0.62975	1.00000
3	399.433	648.130	0.32457	0.63234	0.63234	1.00000
4	399.433	648.843	0.12433	0.63141	0.63142	1.00000
5	399.433	649.006	0.08998	0.63106	0.63106	1.00000
6	399.433	648.619	0.10121	0.63128	0.63128	1.00000
7	399.433	647.683	0.09976	0.63206	0.63206	0.99999
8	399.433	646.197	0.12118	0.63341	0.63341	1.00000
9	399.433	644.167	0.16531	0.63533	0.63534	1.00000
10	399.433	641.596	0.23306	0.63784	0.63784	1.00000
11	399.433	638.494	0.24923	0.64092	0.64092	1.00000
12	399.433	634.868	0.30867	0.64459	0.64460	0.99998
13	399.433	630.718	0.89410	0.64888	0.64889	1.00000
14	399.433	626.066	0.68255	0.65378	0.65378	1.00000
15	399.433	620.922	0.72677	0.65931	0.65931	1.00000
16	399.433	615.300	0.71672	0.66548	0.66548	1.00000
17	399.433	609.212	1.02050	0.67232	0.67232	1.00000
18	399.433	620.468	3.40360	0.66035	0.66035	1.00000
1(2)-18	399.433	638.261	0.96835	0.64193	0.64194	1.00000

1. In the case of transition cost, values for the first period is not reported because to obtain the transition cost the differences of input demands are required.
2. The estimated parameter and predicted input uses are used to obtain transition costs
3. TE stands for Technical Efficiency
4. OE represents Overall Efficiency and is obtained by dividing the cost corresponding to the frontier for the predicted cost.
5. SAE stands for Psuedo Allocative Efficiency measure and obtained by OE/TE.
6. Cost with inefficiencies = Cost frontier x TE effect x AE effect or
ln(Cost with inefficiencies) = ln(Cost frontier)+ln(TE effect)+ln(AE effect).
7. To calculate transition cost, predicted input uses were used.

Table 6. Decomposition of transition costs for the selected agricultural banks

	Total	X1	X2	X3
2	7.20045	1.78697	0.07387	5.33962
3	0.32457	0.07947	0.01378	0.23133
4	0.12433	0.02889	0.00095	0.09450
5	0.08998	0.02168	0.00095	0.06736
6	0.10121	0.01998	0.01327	0.06796
7	0.09976	0.01929	0.02873	0.05174
8	0.12118	0.03057	0.00380	0.08681
9	0.16531	0.03660	0.00176	0.12695
10	0.23306	0.05749	0.00210	0.17347
11	0.24923	0.05855	0.06151	0.12917
12	0.30867	0.07352	0.12518	0.10996
13	0.89410	0.15453	0.07980	0.65977
14	0.68255	0.16184	0.00288	0.51783
15	0.72677	0.18548	0.00372	0.53757
16	0.71672	0.20271	0.05258	0.46142
17	1.02050	0.24385	0.01995	0.75670
18	3.40360	0.86244	0.14076	2.40040
2-18	0.96835	0.23670	0.03680	0.69486

In each parenthesis, the percent ratio of each component to total transition cost is reported.

Table 7. The threshold values for input transitions at time 1

x_1		x_2		x_3	
$\Delta_{11}\left(\frac{\Delta_{11}}{x_{11}}\times 100\right)$	dynamics	$\Delta_{21}\left(\frac{\Delta_{21}}{x_{21}}\times 100\right)$	dynamics	$\Delta_{31}\left(\frac{\Delta_{31}}{x_{31}}\times 100\right)$	dynamics
≥ -5.7507 (11.7%)	CTFE	≥ -131.4086(61.2%)	CTFE	≥ -266.0472 (15.6%)	CTFE
< -5.7507 (11.7%)	DFFE	< -131.4086(61.2%)	DFFE	< -266.0472(15.6%)	DFFE

1. Δ_{i1} and x_{i1} stand for the i^{th} input transition and the i^{th} input use at time one.
2. In the parentheses are percent ratios of input transitions to input uses at time one.
3. CTFE indicates Converging To the Full Efficiency.
4. DFFE indicates Diverging from the Full Efficiency.