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GRAPH EFFICIENCY AND PARAMETRIC DISTANCE FUNCTIONS WITH AN APPLICATION TO SPANISH SAVINGS BANKS

Rafael A. Cuesta[♣] **and José L. Zofío**[♠]

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Abstract: Distance functions were initially defined on the input or output production possibility sets by Shephard (1953, 1970) and extended to a graph representation of the technology by Färe, Grosskopf and Lovell (1985) through their graph hyperbolic distance function. Since then, different techniques, non parametric-DEA and parametric-SFA, have been used in order to calculate these distance functions, but in the latest case no study is known to have relaxed the restrictive input or output orientation. What we propose is to relax such partial dimensionality by defining and estimating a parametric hyperbolic distance function which simultaneously allows for the maximum equiproportionate expansion of outputs and reduction of inputs. Particularly, we introduce a translog hyperbolic distance function showing how this specification complies with the conventional properties that the hyperbolic distance function satisfies. Finally, to illustrate its applicability in efficiency analysis, we implement it using a data set of Spanish savings banks.

Keywords: Production frontiers, distance functions, parametric efficiency, banking efficiency.

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1. Introduction

Distance functions were initially defined in a producer context by Debreu (1951) and Shephard (1953). They had their first empirical application through Farrell's (1957) overall efficiency measure. The initial attempts to measure productive efficiency, as well as the huge number of theoretical models and empirical applications that followed afterwards, were usually based on input or output production possibility set representations of the technology. Färe, Grosskopf and Lovell (1985) explicitly defined a distance function on a graph representation of the production technology. Their graph distance function is the first attempt to relax these restrictive assumptions since it does not depend on a fixed levels of outputs or inputs in order to characterize the technology – as the input and output distance functions respectively do-, thus taking a step forward in applied production analysis. Actually, outputs and inputs are allowed to vary in the same proportion, but while outputs are increased, inputs are decreased.

Input or output distance functions are passive regarding the opposite orientation and this restriction on the analysis could be unacceptable by analysts if a measure of productive and economic performance that takes into accounts both inputs and outputs adjustability is desired. It is clear that researchers, when adopting a particular orientation, are focusing on what they consider is the most relevant orientation. By adopting an input orientation, one assumes that producers are fully capable of allocating resources when improving efficiency while outputs are exogenous. When an output orientation is selected, the production mix is the relevant variable while inputs are exogenous. This reasoning can be extended to economic efficiency through duality, since an input orientation identifies with cost minimizing behavior while the output oriented context focuses on revenue maximizing behavior.

The question is why not take advantage of the fact that all inputs and outputs could be freely adjusted subject only to the constraints imposed by the production technology. Thus, a graph hyperbolic distance function such as the parametric one that we propose here can enrich production performance analysis. Furthermore, if profit maximizing behavior is an adequate assumption for the market, then the hyperbolic distance function is a natural choice for profit efficiency analysis –as it is dual to the return to dollar function, see Färe and Grosskopf (2001)[1].

In empirical applications, it is worth noting that while the operations research field quickly grabbed the distance functions concept and expanded their applications through non parametric Data Envelopment Analysis techniques, *e.g.* Charnes, *et al.* (1994), the parametric field originated with Aigner, Lovell and Schmidt (1977) is quite new to distance functions estimation. Examples of recent parametric output distance functions are English *et al.* (1993), Grosskopf, Margaritis and Valdamis (1995), Coelli and Perelman (1996, 2000), Morrison *et al.* (2001) and Cuesta and Orea (2002). On the other hand, parametric input distance functions are found in papers by Grosskopf and Hayes (1993), Grosskopf, Hayes and Hirschberg (1995), Bosco (1996) and, once again, Coelli and Perelman (1996, 2000). Nevertheless, none of them are known to have developed the potential of the graph hyperbolic distance functions in the parametric field[2]. Regarding DEA some steps have been taken in this direction mainly in environmental efficiency analysis, *e.g.* Färe *et al.* (1989) and recent productivity analysis, Zofío and Lovell (2001).

The literature is then quite limited with regard to the definition and application of the graph hyperbolic function through DEA, and it has never been undertaken in the parametric field. The fact that such functions are not widely used is mainly due to the technical difficulties encountered in their calculation. In a non-parametric context, they require non-linear optimizing techniques, while in the parametric case they require special production function formulations as the one introduced here. We believe that this paper fulfills an important gap in the latter field. The paper is structured in the following sections. In the next section we introduce the desirable properties that output, input and hyperbolic distance functions should satisfy. Section three presents the translog hyperbolic distance function, THDF, and how it compares to its conventional input and output counterparts. The fourth section shows the empirical specification and the estimation procedure. An empirical application to the banking sector is undertaken in

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[1] Also, it would be possible to define a directional distance function, which is also defined on a graph representation of the technology, and is dual to the profit function, see Chambers, Chung and Färe (1996). However, its implementation in a parametric context would not be amenable because of its additive nature.

[2] Once again, directional distance functions have been applied in this field, see Chung, Färe and Grosskopf (1997). However, their implementation has been restricted to Data Envelopment Analysis, as this technique is better suited for additive model specifications.

section five. Finally, some conclusions are drawn in the last section.

2. Distance Functions and Technical Efficiency

Let us suppose a production technology transforming a series of input vectors $x_i = (x_{1i},..., x_{Ki}) \in \Re_+^K$ into the following output vectors $y_i = (y_{1i},..., y_{Mi}) \in \Re_+^M$ where the subscript $i = (1,2,...,N)$ refers to a set of observed processes –firms, DMU's, etc.-. Given this information, the technology can be represented by the graph set,

$$\mathrm{T}(x, y) = \{(x, y) : x \text{ can produce } y\} \tag{1}$$

This production structure can be expressed in equivalent terms through the output and input correspondences, $x \rightarrow \mathrm{P}(x) \subseteq \Re_+^M$ and $y \rightarrow \mathrm{L}(y) \subseteq \Re_+^K$, which respectively represent the set of all input vectors which yield y and the set of all output vectors obtainable from x. These output and input correspondences are inferred from the graph production possibility set (1): $\mathrm{P}(x)=\{y: (x, y) \in \mathrm{T}(x, y)\}$ and $\mathrm{L}(y) = \{x: (x, y) \in \mathrm{T}(x, y)\}$, while the graph can be also inferred from the input and output correspondences, $\mathrm{T}(x, y) = \{(x, y) \in \Re_+^{K+M} : x \in \mathrm{L}(y), y \in \Re_+^M\} = \{(x, y) \in \Re_+^{K+M} : y \in \mathrm{P}(x), x \in \Re_+^K\}$. Thus, it is verified that a given production process $(x, y) \in \mathrm{T}(x, y) \Leftrightarrow y \in \mathrm{P}(x) \Leftrightarrow x \in \mathrm{L}(y)$, Färe, Grosskopf and Lovell (1985:46). The following output and input distance functions defined in the technology set can be expressed in terms of the output and input correspondences while the latest graph hyperbolic distance function cannot be expressed in such terms. We assume that the graph, output and input production possibility sets satisfy a set of equivalent properties[3]:

T.1: $0 \in \mathrm{T}, (0, y) \in \mathrm{T} \Rightarrow y = 0$,

T.2 If $(x, y) \in \mathrm{T}$ then $(x, \lambda y) \in \mathrm{T}$ for $1 \geq \lambda \geq 0$.

T.3 If $(x, y) \in \mathrm{T}$, then $(\lambda x, y) \in \mathrm{T}$ for $\lambda \geq 1$,

T.4: $(\mathrm{T} \cap \{(x, y): x \leq \bar{x}\})$ is bounded for each $\bar{x} \in \Re^K_+$,

T.5 T is a closed set,

[3] $\mathrm{T}(x, y)$ satisfies strong disposability of inputs and outputs if given $(x, y) \in \mathrm{T}(x, y)$, $\forall (x' \geq x, y' \leq y) \Rightarrow (x', y') \in \mathrm{T}(x, y)$ or, alternatively, if $x \in \mathrm{L}(y), x' \in \mathrm{L}(y)$, and $y \in \mathrm{P}(x), y' \in \mathrm{P}(x)$. The weak disposability axioms imply that if $(x, y) \in \mathrm{T}(x, y)$, $\forall (\mu x, \lambda y) \Rightarrow (\mu x, \lambda y) \in \mathrm{T}(x, y)$, $\mu \geq 1, 0 < \lambda \leq 1$.

T.6 T is a convex set

2.1 Output and Input Distance Functions

Given this technology characterization, Shephard (1953) introduced the output distance function as the maximum feasible expansion of the output vector necessary to reach the boundary of the technology $T(x, y)$.

Definition 1: The output distance function D_O: $\Re^K_+ \times \Re^M_+ \rightarrow \Re_+ \cup \{+\infty\}$ is defined by

$$D_O(x, y) = \inf\{\varphi > 0 : (x, y/\varphi) \in T(x, y)\} \tag{2}$$

The output distance function range is $0 < D_O(x, y) \leq 1$ and completely characterizes the technology assuming weak disposability of outputs (Färe and Primont, 1995). Some relevant properties of $D_O(x,y)$ deriving from the axioms of the technology set can be stated:

D_O.1 $D_O(x,\mu y) = \mu D_O(x, y)$, $\mu > 0$. $D_O(x, y)$ is homogeneous of degree one in outputs,
D_O.2 $D_O(x, \lambda y) \leq D_O(x, y)$, $\lambda \in [0,1]$. $D_O(x, y)$ is non-decreasing in outputs,
D_O.3 $D_O(\lambda x, y) \leq D_O(x, y)$, $\lambda \geq 1$. $D_O(x, y)$ is non-increasing in inputs.

With regard to the homogeneity condition, D_O.1 states that for a scalar $\mu > 0$ and any (x,y):

$$\begin{aligned} D_O(x,\mu y) &= \inf\{\varphi > 0 : (x, \mu y/\varphi) \in T(x,y)\} = \inf\left\{\left(\frac{\mu\varphi}{\mu}\right) > 0 : \left(x, \frac{y}{\varphi/\mu}\right) \in T(x,y)\right\} = \\ &= \mu \inf\left\{(\varphi/\mu) > 0 : \left(x, \frac{y}{\varphi/\mu}\right) \in T(x,y)\right\} = \mu\, D_O(x, y) \end{aligned} \tag{3}$$

An alternative representation of the technology is provided by the input distance function as the maximum feasible reduction of the input vector necessary to place (x, y) on the boundary of technology $T(x, y)$.

Definition 2: The input distance function D_I: $\Re^K_+ \times \Re^M_+ \rightarrow \Re_+ \cup \{+\infty\}$ is given by

$$D_I(x, y) = \sup\{\phi > 0 : (x/\phi, y) \in T(x, y)\} \tag{4}$$

The input distance range is $1 \leq D_I(x, y) \leq +\infty$ and completely characterizes the technology assuming weak disposability of inputs. It is linearly homogeneous of degree one and non-decreasing in inputs, while it is non-increasing in outputs. The homogeneity condition can be shown in a similar way to (3).

2.2. Hyperbolic Distance Function

It is now possible to introduce the hyperbolic distance function as the maximum equiproportionate expansion of the output vector and reduction of the input vector that places a given observation on the boundary of the technology $\mathrm{T}(x, y)$.

Definition 3: The hyperbolic distance function $\mathrm{D_H}$: $\Re_+^K \times \Re_+^M \rightarrow \Re_+ \cup \{+\infty\}$ defines as:

$$\mathrm{D_H}(x,y) = \inf\{\theta > 0 : (x\theta, y/\theta) \in \mathrm{T}(x, y)\}. \tag{5}$$

This distance function inherits its name from the hyperbolic path that it yields toward the production frontier. Its range is $0 < \mathrm{D_H}(x, y) \leq 1$ and, once again, completely characterizes the technology assuming weak disposability of outputs and inputs. If the technology satisfies T.1-T.5, then the graph distance function verifies (Färe, Grosskopf and Lovell, 1985:111):

$\mathrm{D_H}$.1 $\mathrm{D_H}(\mu^{-1} x, \mu y) = \mu \mathrm{D_H}(x, y)$, $\mu > 0$,
$\mathrm{D_H}$.2 $\mathrm{D_H}(x, \lambda y) \leq \mathrm{D_H}(x, y)$, $\lambda \in [0,1]$,
$\mathrm{D_H}$.3 $\mathrm{D_H}(\lambda x, y) \leq \mathrm{D_H}(x, y)$, $\lambda \geq 1$.

The graph hyperbolic distance function is non-decreasing in outputs, $\mathrm{D_H}$.2, non-increasing inputs, $\mathrm{D_H}$.3, while it presents the special homogeneity degree corresponding to $\mathrm{D_H}$.1. Since this specific homogeneity condition is a key issue when dealing with concrete functional specifications, it requires further attention. To be precise, it can be related to the concept of almost homogeneity introduced by Aczel (1966, Ch.7)[4]:

Definition 4: A function $F(x, y)$ is almost homogeneous of degrees k_1, k_2 and k_3 if

$$F(\mu^{k_1} x, \mu^{k_2} y) = \mu^{k_3} F(x, y), \quad \forall \mu > 0. \tag{6}$$

Therefore, if the set of outputs y is increased by a power of a given proportion while the set of inputs x is increased by other power of that proportion, then the distance function will be increased by yet another power of that proportion. According to this generic definition, the particular case represented by $\mathrm{D_H}$.1 implies that the hyperbolic distance

[4] Lau (1972) defines almost homogeneity in a slightly different way.

function is almost homogeneous of degrees -1, 1, 1. In other words, if the set of outputs is increased by a given proportion and the set of inputs is reduced by the same proportion, the function increases by that same proportion.

In this case, the almost homogeneity condition corresponding to D_H.1 states that for a scalar $\mu > 0$ and any (x, y)

$$D_H\left(\frac{x}{\mu}, \mu y\right) = \inf\left\{\theta > 0 : \left(\frac{\theta x}{\mu}, \frac{\mu y}{\theta}\right) \in T(x,y)\right\} = \inf\left\{\left(\frac{\mu\theta}{\mu}\right) > 0 : \left(\frac{\theta}{\mu}x, \frac{y}{\theta/\mu}\right) \in T(x,y)\right\} =$$

$$= \mu \inf\left\{\left(\frac{\theta}{\mu}\right) > 0 : \left(\frac{\theta}{\mu}x, \frac{y}{\theta/\mu}\right) \in T(x,y)\right\} = \mu\, D_H(x, y) \tag{7}$$

When the production technology exhibits constant returns to scale, Färe, Grosskopf and Lovell (1994) remark as an additional property of the graph hyperbolic distance its output and input zero degree of homogeneity:

D_H.4. $D_H(\mu x, \mu y; CRS) = D_H(x, y; CRS)$, $\mu > 0$.

When coming to the interpretation of the output, input and graph distance functions, these representations of the technology can be regarded as measures of technical efficiency[5]. It is then necessary to define certain subsets of $T(x, y)$ which can be regarded as production frontiers. With regard to a graph representation of the technology, any feasible production process (x, y) is efficient if it belongs to the graph efficient subset of $T(x, y)$:

$$\text{Eff } T(x,y) = \{(x, y) : (x, y) \in T(x, y), (x', -y') \leq (x, -y) \Rightarrow (x, -y) \notin T(x, y)\} \tag{8}$$

Hence, if (x, y) belongs to $\text{Eff } T(x, y)$, any increase and/or reduction in the (x, y) vectors will render infeasible production vectors and $D_O(x, y) = D_I(x, y) = D_H(x, y) = 1$. However, when strong disposability characterizes the technology, even if the production vector (x,y) presents a unitary distance function, it may not be efficient according to (8). However, It does present a weaker notion of efficiency by belonging to the isoquant subset,

$$\text{Isoq } T(x, y) = \{(x, y): (x, y) \in T(x, y), (\theta x, y/\theta) \notin T(x, y), 0 < \theta < 1\} \tag{9}$$

[5] Färe and Primont (1995) show how the output and input distance functions are inversely related to Farrell's (1957) output and input efficiency measures.

Thus, if a vector $(x,\ y)$ is efficient, it is isoquant efficient but not conversely. The measurement of technical efficiency through distance functions do not characterize efficiency in the Koopmans (1951) sense but in the weaker sense provided by (9); *i.e.* whether the evaluated unit belongs to the isoquant subset or not.

Empirical implementation of technical efficiency measures requires calculation of distance functions either through mathematical programming DEA techniques or parametric frontier regressions. In any case, the true technology is unknown but it can be approximated through ex-post or ex–ante representations. The technique known as Data Envelopment Analysis approximates the technology from the observed units through piecewise linear combinations thus defining an ex-post representation of $\mathrm{T}(x,y)$ –for an updated discussion of DEA see Cooper, Seiford and Tone (2000). Econometric analysis requires an ex-ante functional definition of the technology, which should satisfy several properties -regularity conditions- to adequately characterize it, *e.g.* Chambers (1988). In the next section we present a parametric hyperbolic distance function based on the flexible translog production function, which has been extensively used in the literature since it was introduced by Christensen, Jorgenson and Lau (1971, 1973).

3. Translog Distance Functions

Before introducing the translog hyperbolic distance function we briefly survey its conventional output and input counterparts.

3.1. Translog Output and Input Distance Functions

As previously noted, in production analysis based on parametric techniques it is mandatory to choose an ex-ante functional form for the distance function. The form for the distance function would ideally be flexible, amenable to homogeneity imposition, and easy to calculate. Many authors have selected the translog form because it satisfies these requirements, see Lovell *et al.* (1994) and Grosskopf *et al.* (1997). For the particular *i*-th process in the sample, the translog output distance function in the case of K inputs and M outputs is specified as:

$$\ln \mathrm{D}_{\mathrm{O}i} = \alpha_0 + \sum_{m=1}^{\mathrm{M}} \alpha_m \ln y_{mi} + \frac{1}{2} \sum_{m=1}^{\mathrm{M}} \sum_{n=1}^{\mathrm{M}} \alpha_{mn} \ln y_{mi} \ln y_{ni} + \sum_{k=1}^{\mathrm{K}} \beta_k \ln x_{ki} + \\ + \frac{1}{2} \sum_{k=1}^{\mathrm{K}} \sum_{l=1}^{\mathrm{K}} \beta_{kl} \ln x_{ki} \ln x_{li} + \sum_{k=1}^{\mathrm{K}} \sum_{m=1}^{\mathrm{M}} \delta_{km} \ln x_{ki} \ln y_{mi}, \quad i = 1,2,\ldots,\mathrm{N}, \tag{10}$$

complying with the usual regularity conditions –including $\mathrm{D_O}$.1-$\mathrm{D_O}$.3. Note that in order to obtain the production frontier (*i.e.*, the transformation function) one would set $\mathrm{D_O}(x,y) = 1$, which implies that the left-hand side of equation (10) is equal to zero.

The restrictions required for homogeneity of degree one in outputs are obtained from the application of the Euler Theorem:

$$\sum_{m=1}^{\mathrm{M}} \alpha_m = 1, \tag{11}$$

$$\sum_{n=1}^{\mathrm{M}} \alpha_{mn} = 0, \qquad m = 1,2,\ldots,\mathrm{M}, \tag{12}$$

$$\sum_{m=1}^{\mathrm{M}} \delta_{km} = 0, \qquad k = 1,2,\ldots,\mathrm{K}, \tag{13}$$

completed with the usual symmetry conditions. A convenient method of imposing the homogeneity constraints upon (10) is to follow Lovell *et al.* (1994). From $\mathrm{D_O}$.1, homogeneity implies that $\mathrm{D_O}(x,\mu y) = \mu\, \mathrm{D_O}(x, y)$, $\mu > 0$ and by arbitrarily choosing one of the outputs –such as the M-th output–, we can set $\mu = 1/y_{\mathrm{M}}$:

$$\mathrm{D_O}\left(x, \frac{y}{y_{\mathrm{M}}}\right) = \frac{\mathrm{D_O}(x,y)}{y_{\mathrm{M}}} \tag{14}$$

For the translog form this provides:

$$\ln\left(\mathrm{D}_{\mathrm{O}i} / y_{\mathrm{M}i}\right) = \alpha_0 + \sum_{m=1}^{\mathrm{M}-1} \alpha_m \ln y^*_{mi} + \frac{1}{2} \sum_{m=1}^{\mathrm{M}-1} \sum_{n=1}^{\mathrm{M}-1} \alpha_{mn} \ln y^*_{mi} \ln y^*_{ni} + \sum_{k=1}^{\mathrm{K}} \beta_k \ln x_{ki} + \\ + \frac{1}{2} \sum_{k=1}^{\mathrm{K}} \sum_{l=1}^{\mathrm{K}} \beta_{kl} \ln x_{ki} \ln x_{li} + \sum_{k=1}^{\mathrm{K}} \sum_{m=1}^{\mathrm{M}-1} \delta_{km} \ln x_{ki} \ln y^*_{mi}, \quad i = 1,2,\ldots,\mathrm{N} \tag{15}$$

where $y^*_{mi} = y_{mi}/y_{\mathrm{M}i}$. Note that when $y_{mi} = y_{\mathrm{M}i}$, the ratio y^*_{mi} is equal to one and its log is zero. Thus, all terms involving the M-th output also become zero, *i.e.* summations involving y^*_{mi} in the above expression are over the M-1 outputs not used for normalization, and not over M.

In the input case, a translog distance function is obtained by imposing homogeneity of degree one in inputs. Setting $\mu = 1/x_K$ one obtains $D_I\ (x/x_K, y) = D_I\ (x,y)\ /x_K$, and the translog input distance function becomes:

$$\ln\left(D_{Ii}/x_{Ki}\right) = \alpha_0 + \sum_{m=1}^{M}\alpha_m \ln y_{mi} + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M}\alpha_{mn}\ln y_{mi}\ln y_{ni} + \sum_{k=1}^{K-1}\beta_k \ln x^*_{ki} + \\ + \frac{1}{2}\sum_{k=1}^{K-1}\sum_{l=1}^{M-1}\beta_{kl}\ln x^*_{ki}\ln x^*_{li} + \sum_{k=1}^{K-1}\sum_{M=1}^{M}\delta_{km}\ln x^*_{ki}\ln y_{mi}\ , \quad i = 1,2,...,N \tag{16}$$

where $x^*_{ki} = x_{ki}/x_{Ki}$.

3.2. Translog Hyperbolic Distance Function, THDF.

As was earlier remarked, we intend to define a parametric distance function that allows estimation of hyperbolic technical efficiency using a translog form. For this purpose, to impose the necessary conditions of almost homogeneity on the translog distance function we follow Cuesta, Kumbhakar and Zofío (2001). These authors rely on the modified Euler Theorem introduced by Lau (1972: 283) to obtain the restrictions required for almost homogeneity. Departing from (6), and assuming that $F(x,y)$ is a continuously differentiable function, to be almost homogenous it should satisfy**:**

$$k_1\sum_{k=1}^{K}\frac{\partial F}{\partial x_k}x_k + k_2\sum_{m=1}^{M}\frac{\partial F}{\partial y_m}y_m = k_3 F\ , \tag{17}$$

which taking logs, and for the particular hyperbolic case that requires almost homogeneity of degrees -1, 1, 1, yields:

$$-\sum_{k=1}^{K}\frac{\partial \ln F}{\partial \ln x_k} + \sum_{m=1}^{M}\frac{\partial \ln F}{\partial \ln y_m} = 1\ . \tag{18}$$

Assuming now a translog specification for $F(x,y)$:

$$\ln F = \alpha_0 + \sum_{m=1}^{M} \alpha_m \ln y_{mi} + \frac{1}{2} \sum_{m=1}^{M} \sum_{n=1}^{M} \alpha_{mn} \ln y_{mi} \ln y_{ni} + \sum_{k=1}^{K} \beta_k \ln x_{ki} + \\ + \frac{1}{2} \sum_{k=1}^{K} \sum_{l=1}^{K} \beta_{kl} \ln x_{ki} \ln x_{li} + \sum_{k=1}^{K} \sum_{m=1}^{M} \delta_{km} \ln x_{ki} \ln y_{mi}, \quad i = 1,2,...,N, \tag{19}$$

and taking derivatives with respect to inputs and outputs:

$$\frac{\partial \ln F}{\partial \ln x_k} = \beta_k + \sum_{l=1}^{K} \beta_{kl} \ln x_l + \sum_{m=1}^{M} \delta_{km} \ln y_m, \quad k = 1,2,...,K, \tag{20}$$

$$\frac{\partial \ln F}{\partial \ln y_m} = \alpha_m + \sum_{n=1}^{M} \alpha_{mn} \ln y_n + \sum_{k=1}^{K} \delta_{km} \ln x_k, \quad m = 1,2,...,M, \tag{21}$$

it is possible to obtain the following relation using (20) and (21) in (18):

$$-\sum_{k=1}^{K} (\beta_k + \sum_{l=1}^{K} \beta_{kl} \ln x_l + \sum_{m=1}^{M} \delta_{km} \ln y_m) + \sum_{m=1}^{M} (\alpha_m + \sum_{n=1}^{M} \alpha_{mn} \ln y_n + \sum_{k=1}^{K} \delta_{km} \ln x_k) = 1. \tag{22}$$

Finally, since (22) involves all $\ln x$ and $\ln y$, to impose the almost homogeneity condition of degrees $-1, 1, 1$, the following (1+K+M) constraints apply:

$$\sum_{m=1}^{M} \alpha_m - \sum_{k=1}^{K} \beta_k = 1, \tag{23}$$

$$\sum_{m=1}^{M} \delta_{km} - \sum_{l=1}^{K} \beta_{kl} = 0, \quad k = 1,2,...,K, \tag{24}$$

$$\sum_{n=1}^{M} \alpha_{mn} - \sum_{k=1}^{K} \delta_{km} = 0, \quad m = 1,2,...,M. \tag{25}$$

One can impose (23), (24) and (25) on the translog specification modifying the approach initiated by Lovell *et al.* (1994). Using the almost homogeneity condition (7), and choosing once again the M-th output for normalizing purposes, $\mu = 1/y_M$, we obtain:

$$D_H\left(x y_M, \frac{y}{y_M} \right) = \frac{D_H(x,y)}{y_M}. \tag{26}$$

For the translog hyperbolic distance function THDF this provides:

$$\ln\left(D_{Hi}/y_{Mi}\right)=\alpha_0+\sum_{m=1}^{M-1}\alpha_m\ln y_{mi}^{*}+\frac{1}{2}\sum_{m=1}^{M-1}\sum_{n=1}^{M-1}\alpha_{mn}\ln y_{mi}^{*}\ln y_{ni}^{*}+\sum_{k=1}^{K}\beta_k\ln x_{ki}^{**}+$$
$$+\frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K}\beta_{kl}\ln x_{ki}^{**}\ln x_{li}^{**}+\sum_{k=1}^{K}\sum_{m=1}^{M-1}\delta_{km}\ln x_{ki}^{**}\ln y_{mi}^{*}\ ,\quad i=1,2,...,N \tag{27}$$

where $y_{mi}^{*}=y_{mi}/y_{Mi}$ and $x_{ki}^{**}=x_{ki}y_{Mi}$. Once again when $y_{mi}=y_{Mi}$ the ratio y_{mi}^{*} is equal to one. Thus all log terms involving the normalizing M-th output are null; however, this is not observed on the input side. That is why summations involving y_{mi}^{*} in equation (27) are over M-1 while summations involving x_{ki}^{**} are over K.

Therefore, using (27) we can compute the maximum equiproportionate input reduction and output expansion required to place (x,y) on the production frontier while fulfilling D_H.1-D_H.3. With respect to the relevant homogeneity condition D_H.1, it is easy to demonstrate how the translog hyperbolic distance function (27) satisfies it by dividing inputs and multiplying outputs by a scalar $\lambda > 0$. Concerning the monotonicity conditions, the hyperbolic distance function must be non-decreasing in outputs and non-increasing in inputs, D_H.2 and D_H.3. In the THDF case, one needs to evaluate the derivatives for each data point. The output derivatives should be non-negative and the input derivatives non-positive.

With regard to the THDF when taking into account constant returns to scale, D_H. 4, it must be homogeneous of degree 0 in inputs and outputs. To proof that our THDF satisfies this property, it is mandatory to derive the constant returns to scale restrictions. The scale elasticity is given by

$$E(x,y)=\frac{\partial\ \ln\theta}{\partial\ \ln\lambda}, \tag{29}$$

where

$$D_H\ (\lambda x,\theta y)=1, \tag{30}$$

and applying the implicit function theorem to (27) we obtain:

$$E(x,y)=-\frac{\Delta_x D_H\ (x,y)}{\Delta_y D_H\ (x,y)}\cdot\frac{x}{y}, \tag{31}$$

and the scale elasticity is defined as the negative of the sum of the input elasticities over the sum of the output elasticities. Simple manipulations of (27) to obtain the sale elasticity of the translog hyperbolic distance functions yield:

$$\mathrm{E}(x,y)=-\frac{\sum_{k=1}^{\mathrm{K}}\beta_k+\sum_{l=1}^{\mathrm{K}}\beta_{kl}\ln x_{ki}^{**}+\sum_{k=1}^{\mathrm{K}}\delta_{km}\ln y_{mi}^{*}}{1+\sum_{k=1}^{\mathrm{K}}\beta_k+\sum_{l=1}^{\mathrm{K}}\beta_{kl}\ln x_{ki}^{**}+\sum_{k=1}^{\mathrm{K}}\delta_{km}\ln y_{mi}^{*}}. \tag{32}$$

From (32), the constant returns to scale restrictions are:

$$\sum_{k=1}^{\mathrm{K}}\beta_k=-\frac{1}{2}, \quad k=1,2,...,\mathrm{K}, \tag{33}$$

$$\sum_{l=1}^{\mathrm{K}}\beta_{kl}=0, \quad l=1,2,...,\mathrm{K}, \tag{34}$$

$$\sum_{k=1}^{\mathrm{K}}\delta_{km}=0, \quad m=1,2,...,\mathrm{M}. \tag{35}$$

Hence, when the translog hyperbolic distance function is homogeneous of degree –1/2 in inputs, it exhibits constant returns to scale. Imposing these restrictions on the THDF (27) we obtain the following specification:

$$\begin{aligned}-\ln y_{\mathrm{M}i}+\frac{1}{2}\ln(y_{\mathrm{M}i}x_{\mathrm{K}i})&=\alpha_0+\sum_{m=1}^{\mathrm{M}-1}\alpha_m\ln y_{mi}^{*}+\frac{1}{2}\sum_{m=1}^{\mathrm{M}-1}\sum_{n=1}^{\mathrm{M}-1}\alpha_{mn}\ln y_{mi}^{*}\ln y_{ni}^{*}+\\&+\sum_{k=1}^{\mathrm{K}-1}\beta_k\ln x_{ki}^{*}+\frac{1}{2}\sum_{k=1}^{\mathrm{K}-1}\sum_{l=1}^{\mathrm{K}-1}\beta_{kl}\ln x_{ki}^{*}\ln x_{li}^{*}+\sum_{k=1}^{\mathrm{K}-1}\sum_{m=1}^{\mathrm{M}-1}\delta_{km}\ln x_{ki}^{*}\ln y_{mi}^{*}, \quad i=1,2,...,\mathrm{N},\end{aligned} \tag{36}$$

where $y_{mi}^{*} = y_{mi}/y_{\mathrm{M}i}$ and $x_{ki}^{*} = x_{ki}/x_{\mathrm{M}i}$. Observe that when $y_{mi} = y_{\mathrm{M}i}$ and $x_{ki} = x_{\mathrm{K}i}$ the ratios y_{mi}^{*} and x_{ki}^{*} are equal to one and hence their log is zero. Thus, all terms involving the M–th output and K–th input also become zero, *i.e.* all summations in the above expression are over M-1 and K-1. For (36), if we multiply both inputs and outputs by the same positive scalar, $\lambda > 0$, we obtain the same function:

$$
\begin{aligned}
&-\ln(\lambda\, y_{Mi}) + \frac{1}{2}\ln(\lambda y_{Mi}\lambda\, x_{Ki}) = \alpha_0 + \sum_{m=1}^{M-1}\alpha_m \ln\left(\frac{\lambda\, y_{mi}}{\lambda\, y_{Mi}}\right) \\
&+\frac{1}{2}\sum_{m=1}^{M-1}\sum_{n=1}^{M-1}\alpha_{mn}\ln\left(\frac{\lambda\, y_{mi}}{\lambda\, y_{Mi}}\right)\ln\left(\frac{\lambda\, y_{ni}}{\lambda\, y_{Mi}}\right) + \sum_{k=1}^{K-1}\beta_k \ln\left(\frac{\lambda\, x_{ki}}{\lambda\, x_{Ki}}\right) \\
&+\frac{1}{2}\sum_{k=1}^{K-1}\sum_{l=1}^{K-1}\beta_{kl}\ln\left(\frac{\lambda\, x_{ki}}{\lambda\, x_{Ki}}\right)\ln\left(\frac{\lambda\, x_{li}}{\lambda\, x_{Ki}}\right) \\
&+\sum_{k=1}^{K-1}\sum_{m=1}^{M-1}\delta_{km}\ln\left(\frac{\lambda\, x_{ki}}{\lambda\, x_{Ki}}\right)\ln\left(\frac{\lambda\, y_{mi}}{\lambda\, y_{Mi}}\right).
\end{aligned}
\tag{37}
$$

Once we have shown all the properties and relationships that allow a complete characterization of the translog hyperbolic distance function, it is important to remark its potential uses in production analysis. The usefulness of this proposal when imposing the almost homogeneity condition on the THDF can be easily verified in all applications where a flexible functional form such as (27) is required. There are two immediate examples: (i) environmental efficiency and (ii) Malmquist productivity indexes. Looking at the relevant literature in the first case, hyperbolic distance functions have been implemented to measure environmental performance. Relevant articles are Färe *et al.* (1989) -paper mills-, Ball *et al.* (1994) -US agriculture- and Zofio and Prieto (2001) –OECD manufacturing industry–. In the second case, Zofio and Lovell (2001) extended efficiency analysis to productivity analysis, calculating and decomposing productivity change by means of hyperbolic distance functions. All these applications have been developed using non-parametric DEA techniques, and the translog hyperbolic distance function can be used to calculate environmental efficiencies and productivity indexes taking advantage of the possibilities offered by parametric regressions[6].

4. Empirical Implementation of the Translog Hyperbolic Distance Function.

To implement the translog hyperbolic distance function, one may choose from all the available programming and regression analysis techniques when estimating deterministic

[6] With regard to environmental efficiency as well as Malmquist productivity indexes incorporating undesirable outputs, one would define three sets of variables: desirable outputs, undesirable outputs and inputs. Imposing the almost homogeneity condition on the output sets, the environmental efficiency index that one obtains would yield the maximum equiproportionate undesirable output reduction and desirable output expansion for a given input set. This could be extended to productivity change allowing for a contemporary reduction in input usage.

or stochastic specifications. Here, we rely on a panel data model with a stochastic specification. Considering the $i = 1,2,...,N$ observed firms in $t = 1,2,...,T$ time periods, the stochastic hyperbolic distance function model is:

$$1 = D_{Hi}(y_{it}, x_{it}; \alpha, \beta, \delta)\, h(\varepsilon_{it}), \tag{38}$$

where deviations from one are accommodated in the composed error $h(\varepsilon_{it})$. As in Aigner Lovell and Schmidt (1977), an empirical formulation of the error term that includes the stochastic specification is the following one:

$$h(\varepsilon_{it}) = \exp(u_i + v_{it}) \tag{39}$$

In this case, the additive error includes a one-sided component u_i, and a standard noise term symmetrically distributed around zero, $v_{it} \rightarrow N(0, \sigma_v^2)$. The one side component u_i, captures the distance between the observed output vector and the production possibility set, *i.e*. it is assumed to have a one tail –half normal– distribution, $u_i \rightarrow |N(0, \sigma_u^2)|$. Taking logs from (38) and substituting (39), it is possible to obtain the actual hyperbolic distance function to be estimated

$$0 = \ln D_{Hi}(y_{it}, x_{it}; \alpha, \beta, \delta) + u_i + v_{it}, \tag{40}$$

where the proposed specification for $D_{Hi}(y_{it}, x_{it}; \alpha, \beta, \delta)$ corresponds to (27).

Finally, handling this panel data characterization through the standard maximum-likelihood methodology introduced by Pitt and Lee (1981) and extended by Battese and Coelli (1988), it is possible to obtain the individual conditional distribution of the one side error, $E(u_i|\varepsilon_{it})$. These values are then substituted into

$$TE_i = \exp[\ln D_H(y_{it}, x_{it}, \alpha, \beta, \gamma)] = \exp(-u_i) \tag{41}$$

to obtain time invariant hyperbolic technical efficiency estimates for each producer.

5. Empirical illustration to Spanish savings banks

5.1. Data

In order to illustrate our model, the proposed THDF distance function is estimated using a sample of Spanish savings banks, Confederación Española de Cajas de Ahorros (CECA). The observed data corresponds to the 1985-1994 period when an important number of mergers and acquisitions occurred, thus raising questions about the best way to treat this issue. The approach followed in this paper leads to an unbalanced panel. The observations to merge disappear from the sample in a given period to show up in the subsequent year as different ones result from the mergers or acquisitions[7]. In the present panel, the number of firms declines from 77 in 1985 to 34 in 1998.

To select the relevant variables, we follow the commonly accepted intermediation approach proposed by Sealey and Lindley (1977), which treats deposits as inputs and loans as outputs. Four inputs are employed to produce three outputs. Inputs are time and saving deposits (x_1), deposits from banks and other funds (x_2), personnel expenses (x_3), and capital (x_4), measured by the value of fixed assets in the balance. On the output side, production is represented by loans to non-banks (y_1), bonds, cash and other assets (y_2), and non-interest income (y_3). This last output is included to account for off-balance-sheets activities such as securitization, brokerage services and management of financial assets for customers –a growing activity in the Spanish banking sector. These monetary variables are expressed in millions of real 1985 Spanish pesetas by means of the GDP deflator index. A summary of their descriptive statistics appears in Table 1.

[7] Cuesta and Orea (2002) further discuss these issues. All firms involved in mergers could have been removed from the data set, however this procedure would have reduced the sample dramatically due to the high number of entities in such situation. An alternative way to handle the problem, which is broadly used in the literature, considers the merged firms as unique entities in the periods previous to the merger, thus aggregating their data. This procedure results in a balanced panel but requires the use of fictitious firms, which may have great influence on the efficient frontier. Therefore, as none of these methods seem to be fully appropriate, we settle for an unbalanced panel which, nevertheless, also poses some limitations since data loss might not just be related to mergers and acquisitions, but also to attrition problems. In this scheme, if the probability of observation disappearance from the sample is correlated with the experimental answer, then traditional statistical methods result in biased and inconsistent estimates, Hsiao (1986). We will not pursue this issue here, since the correction of the possible bias would require a new investigation in itself using a limited dependent variables model, Heckman (1976). All these issues underlie this application and, therefore, caution must be exercised when interpreting the results beyond the model illustrating goal.

Table 1. Descriptive statistics and selected variables.

Variable	Mean	Standard Dev.	Minimum	Maximum
y_1	134,814	248,596	1,066	2,641,366
y_2	74,122	167,109	583	1,737,633
y_3	1,473	3,866	5	51,670
x_1	225,926	409,389	2,802	3,539,309
x_2	32,002	95,096	38	1,184,926
x_3	4,516	7,427	69	68,081
x_4	2,924	5,106	33	46,767

Source: Confederación Española de Cajas de Ahorro, CECA

For convenience purposes all variables have been mean-corrected prior to estimation, *i.e.* each output and input variable is divided by its geometric mean. Proceeding this way, first order coefficients can be interpreted as distance elasticities evaluated at the sample means. In addition, the almost homogeneity condition is imposed using loans, y_1, as numeraire.

The particular translog hyperbolic distance function specification corresponding to (40) is the following one –enhanced with time dummies intended to capture the presence of neutral technical change as well as other temporal effects:

$$-\ln y_{1it} = \left[\alpha_0 + \sum_{m=1}^{2}\alpha_m \ln y^*_{mit} + \frac{1}{2}\sum_{m=1}^{2}\sum_{n=1}^{2}\alpha_{mn}\ln y^*_{mit}\ln y^*_{nit} + \sum_{k=1}^{4}\beta_k \ln x^{**}_{kit} + \right.$$
$$\left. + \frac{1}{2}\sum_{k=1}^{4}\sum_{l=1}^{4}\beta_{kl}\ln x^{**}_{kit}\ln x^{**}_{lit} + \sum_{k=1}^{4}\sum_{m=1}^{2}\delta_{km}\ln x^{**}_{kit}\ln y^*_{mit} + \sum_{\tau=1}^{T}\psi_\tau D_{\tau,t}\right] + u_i + v_{it} \ , \qquad (42)$$

where $y^*_{mit} = y_{mit}/y_{1it}$, $x^{**}_{kit} = x_{kit}y_{1it}$ and $D_{\tau,t} = 1$ for $\tau = t$, $D_{\tau,t} = 0$ for $\tau \neq t$.

5.2. Results and discussion

The obtained results for the estimated model are presented in Table 2. All elasticities estimates for inputs and outputs present the expected sign and their associated *t*-ratios indicate that they are significantly different from zero. This outcome is consistent with the above stated monotonicity conditions $D_H.2$ and $D_H.3$, reflecting how the translog hyperbolic distance function is non-decreasing in outputs and non-increasing in inputs. On the input side, time and savings deposits x_1, present the highest elasticity. Its value doubles that of personnel expenses x_3, and it is way ahead of the remaining and less important productive factors. On the output side, elasticities show how bonds, cash and other assets y_2, are still far more relevant that the increasingly important non-interest income, y_3. It is noteworthy to remark that alternative normalizations using y_1 or y_2 do not alter the estimates.

With respect to the time dummies intended to capture neutral technical change, all coefficients present the expected negative sign, reflecting the existence of technical progress by an aggregated value of 19,04% in the fourteen years period –which corresponds to a mean annual growth rate of 1,35%. The existence of technical progress is a well established result in international banking studies and quite specifically in the Spanish saving banks literature; e*.g.* using the same database, Grifell-Tatjé and Lovell (1997) report a 1,90% average annual rate of technical progress from 1986 to 1993.

One may use the obtained parameters values for the translog hyperbolic distance function to estimate firm specific efficiency, (41). With regard to technical efficiency, the significant parameters σ^2 and λ indicate that the one side error is a relevant source when explaining a producer's deviation from the transformation function. Average hyperbolic technical efficiency is 0.95, showing how the Spanish savings banks sector can improve its productive performance by increasing its outputs by 5,6% while simultaneously reducing its inputs by 5%.

As discussed in what follows, this average value is among the higher ones reported in banking studies, showing the consistency of our model with distance function theory. Färe, Grosskopf and Lovell (1985) demonstrate how under a constant returns to scale assumption, the hyperbolic distance function is related to its output and input oriented counterparts[8]: $D_H(x,y;CRS) = D_O(x,y;CRS)^{1/2} = D_I(x,y;CRS)^{-1/2}$, *i.e.* the hyperbolic distance function is numerically greater its output counterpart. In the present application, where variable returns to scale are allowed, the difference between hyperbolic and output efficiency scores depends on the magnitude of returns to scale. However, since our estimated scale elasticity (–0.51) is not far from the constant returns to scale value of

[8] Cuesta and Zofio (1999) define and provide an example of this equivalence between distance functions for the translog hyperbolic and output distance functions.

–0.5 introduced in (33), it would be expected that our hyperbolic efficiency scores were greater than their output oriented counterparts. To confirm the case we have solved an output oriented model equivalent to (40) using (15) as the parametric specification for the distance function. As in the hyperbolic specification, the estimated scale elasticity of 1.02 is not far from the unitary value corresponding to constant returns to scale, being the average output efficiency value 0.90 –quite smaller than its hyperbolic counterpart.

Also, the magnitude of our average hyperbolic efficiency value is consistent with recent results obtained from the same database by Cuesta and Orea (2002). Also deciding for an output oriented translog specification –but allowing for a time varying formulation of the one side error, these authors report similar scale elasticity and average efficiency values to those obtained by us for our output oriented model[9]. Therefore, as scale normally plays a limited role in banking applications, one would generally expect hyperbolic technical efficiency estimates to be higher than their output oriented counterparts[10].

In fact, our hyperbolic results yield higher average efficiency values than most of studies focused on depository financial institutions, thus reinforcing our conclusion. In an extensive review of the bank efficiency literature, Berger and Humphrey (1997:181-183) survey all relevant studies in the international field. For the Spanish case, average efficiency values are in the 70%-80% range. Nine out of the ten surveyed studies –all of them, as ours, using the same database– report much lower average technical efficiency scores –except for Lovell and Pastor (1992), who dealing with branch performance, report a 0.91 value. We can remark here the studies that culminate in Grifell–Tatjé and Lovell (1997), who employed Data Envelopment Analysis deterministic techniques to obtain an average value of 0.82 under a constant returns to scale assumption; Maudos (1996), who use Stochastic Frontier Analysis, 0.82, and Lozano-Vivas (1997), who employ Thick Frontier Analysis, 0.72. Even if these studies present a

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[9] Cuesta and Orea (2002) introduce a –quadratic– time varying specification of technical efficiency in order to capture if mergers had an effect on the efficiency paths of non-merged banks compared to the newly created firms.

[10] The existence of limited scales elasticities (economies) in the banking sector is well documented in the literature. However recent studies employing innovative production and cost functions specifications (Fourier, Kernel and Spline), as well as alternative explanatory variables (capital structure and risk-taking), suggest that substantial scales economies may be available, see Hughes, Mester and Moon (2001).

wide range of model specifications –although authors tend to settle for a value added approach when defining the production process, see Berger and Humphrey (1992)– and estimation techniques, their calculated average efficiency scores are quite lower than the one presented in Table 1.

Table 2. Estimated parameters for the translog hyperbolic distance function.

Parameter	Estimated Value	*t*-statistic	Parameter	Estimated Value	*t*-statistic
α_0	0.0553	8.0200	δ_{13}	0.0347	1.9480
α_2	0.1118	19.9790	δ_{22}	-0.0018	-0.5210
α_3	0.0360	5.5330	δ_{23}	0.0061	1.4230
α_{22}	0.0847	9.1110	δ_{32}	0.0565	3.3220
α_{33}	-0.0139	-1.4360	δ_{33}	-0.0193	-1.2170
α_{23}	-0.0283	-4.2590	δ_{42}	0.0299	1.9640
β_1	-0.3012	-21.8190	δ_{43}	-0.0205	-1.2030
β_2	-0.0332	-9.3930	ψ_{86}	-0.0098	-2.2170
β_3	-0.1190	-7.6200	ψ_{87}	-0.0335	-6.8210
β_4	-0.0556	-4.6150	ψ_{88}	-0.0639	-11.8590
β_{11}	0.1382	1.7850	ψ_{89}	-0.0549	-9.7660
β_{22}	-0.0223	-6.7230	ψ_{90}	-0.0953	-16.9330
β_{33}	0.0925	1.0640	ψ_{91}	-0.0964	-15.9060
β_{44}	0.0036	0.0960	ψ_{92}	-0.0987	-15.470
β_{12}	0.0230	2.1160	ψ_{93}	-0.0809	-12.0850
β_{13}	-0.1409	-1.6990	ψ_{94}	-0.1093	-16.3240
β_{14}	-0.0222	-0.7550	ψ_{95}	-0.1187	-17.3470
β_{23}	0.0035	0.3590	ψ_{96}	-0.1348	-18.9440
β_{24}	-0.0023	-0.2280	ψ_{97}	-0.1597	-21.1510
β_{34}	0.0325	0.7500	ψ_{98}	-0.1904	-22.9940
δ_{12}	-0.0856	-4.7300			
σ^2	0.0057	6,323	Mean L.L.F.	2.0365	
λ	0.1475	5,186	Mean T.E.	0.9466	

Source: Own elaboration.
Note: The following parameterization applies: $\sigma^2 = \sigma^2_u + \sigma^2_v$, $\lambda = \sigma^2_v / \sigma^2_u$

This same pattern can be found when considering results for additional countries reported in the cited survey. For the extensively studied American case in the target period corresponding to the late eighties and early nineties, and focusing mainly on panel data SFA applications as the one employed here, Bauer, Berger and Humphrey (1993) report values around 0.85, Elyasiany and Mehdian (1990), 0.88, Ferrier and Lovell (1993), 0.79, Hasan and Hunter (1996), 0.81, Kaparakis, Miller and Noulas (1994), 0.90, Kwan and Esenbeis (1994), 0.86, Mester (1997), 0.88, Pi and Timme

(1993), 0.87, and Zhu, Ellinger and Shumway (1995), 0.85. Thus, our proposal to assess hyperbolic technical efficiency yield results which situate on the upper bound of these studies. As researches normally employ partially oriented approaches to assess technical efficiency such as the output or input functions, the resulting efficiency patterns tend to be substantially lower that those attained when allowing for a more comprehensive path toward the frontier, *i.e.* the hyperbolic distance function projects the input-output vector onto the boundary of the technology by equiproportionally expanding outputs and reducing inputs.

6. Conclusions

In this paper we have introduced a new definition and estimation procedure of distance functions through parametric techniques. Distance functions are usually defined in the partially oriented input or output production possibility sets, which restricts technical and economic efficiency analysis to the output (revenue) or input (cost) side of the production process. It can be anticipated that such a restrictive analytical framework may cause unrest among researches who do not welcome these limitations, so more flexible specifications which simultaneously take into account outputs and inputs are required. Therefore, we relax the output or input partial orientations by defining a new tool: the translog hyperbolic distance function THDF, which is based on the graph representation of the technology and allows for simultaneous output increase and input reduction.

We show how this new distance function satisfies customary theoretical properties and regularity conditions and how it relates to its output and input counterparts. Also we illustrate how the THDF function can be empirically implemented by means of standard econometric techniques. Actually, practical implementation of this function proves to be rather simple even in a panel data context. To illustrate efficiency analysis based on our translog hyperbolic distance function, we rely on an extensively researched panel data set of Spanish savings banks and depart from the well-known and widely applied model of Battese and Coelli (1988).

The potential of the translog hyperbolic distance function is not exhausted with productive efficiency analysis. On the contrary, it can be extended to many research fields where hyperbolic distance functions are being increasingly applied. Particularly, environmental efficiency and productivity change analysis where just DEA applications have been undertaken. We hope that this new analytical tool will be welcomed by practitioners in the efficiency and productivity field, as it provides yet another possibility to study firms performance in a parametric context.

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