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# EMPIRICAL CONSEQUENCES OF DIRECTION CHOICE IN TECHNICAL EFFICIENCY ANALYSIS

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**Abstract:** This paper examines the effects of using input- and output-oriented measures of technical inefficiency (direction choice) on the estimated technology, its characteristics and efficiency rankings. If one assumes that the technology is known (as is done in theory) then this issue is trivial. Since the technology is rarely *known*, and *both* the technology and inefficiency are to be estimated *simultaneously*, the choice of direction is very important from an empirical point of view. Using a sample of dairy farms, we show that the estimated characteristics of the technology and efficiency rankings of farms depend on the choice of direction.

**Keywords:** Input- and output-oriented technical inefficiency, Returns to scale, Output elasticities, Efficiency rankings

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## 1. Introduction

In neoclassical production theory, producers are assumed to be fully efficient. In practice, however, inefficiency seems to be the rule. Farrell (1957) first introduced the notion of a frontier, which allowed inefficiency to be measured as the distance of each observation from the frontier. Since then the literature on the estimation of productive efficiency has been extended in many directions and used in many disciplines (operations research, management, marketing, economics, among others).

Among the different types of inefficiencies, technical inefficiency is perhaps the one most widely studied. A firm is said to be technically inefficient when it fails to produce the maximum possible output, given its current input use. Alternatively, a technically inefficient firm can produce the same level of output using less than its current level of inputs. Other definitions of efficiency that combine output increases and input reductions are also possible. In other words, given an inefficient production plan, it is possible to choose one of many possible directions to go to the frontier (e.g., output increase, proportional reduction in inputs, proportional increase in output and reduction in inputs). Associated with each direction there is a separate measure of technical efficiency (e.g., output-oriented, input-oriented, hyperbolic, etc.).

The presence of these competing measures of technical efficiency raises some interesting questions. Do these measures produce identical efficiency rankings of producers? Which measure should be used in practice? Do the features of the estimated production technology depend on the choice of technical efficiency measure? The key to understanding the issues embodied in the above questions lies on the fact that theoretical analysis of technical efficiency is always carried out under the assumption that the technology is known. However, in empirical work the problem is that the characteristics of the technology and the level of technical efficiency cannot be estimated independently. That is, it is necessary to choose an efficiency index (and therefore a direction) prior to estimating the production technology.

These important questions have not been sufficiently addressed in the efficiency literature. In fact, only Färe and Lovell (1968) and a couple of papers by Atkinson and Cornwell (1993, 1994) have attempted to answer the first question. Atkinson and

Cornwell estimated two cost functions, with an input and output-oriented index of technical efficiency, and found that the technical efficiency rankings associated with each cost function were different. A recent paper by Orea, Roibas and Wall (2002) has addressed the second question. They compared the estimation of various translog cost functions using different specifications of the efficiency index, and used the Vuong test to select the best model. The third question, although partially addressed in Orea et al., has not yet been systematically analyzed and is therefore the focus of the present paper.

The present paper analyzes the empirical consequences of index/direction choice in estimating production functions using parametric approach.[1] We focus on two measures of technical efficiency, viz., input-oriented (IO) and output-oriented (OO) measures. In addition to computing efficiency rankings of producers, we estimate other important features of the technology such as output elasticities, returns to scale, and elasticities of substitution. Therefore, the paper contributes to this tiny literature in two directions. First, it extends the results of previous papers using a primal approach. Second, it studies the effects of index/direction choice not only on efficiency rankings but also on some important characteristics of the estimated production technology.

The structure of the paper is as follows. Section 2 reviews the input- and output-oriented measures of technical efficiency. Section 3 discusses the data and the empirical models. Empirical results derived from the IO and OO measures of technical efficiency are discussed in Section 4. Section 5 points out the difficulties of comparing the OO and IO indices. The paper ends with some conclusions in Section 6.

## 2. Efficiency Index / Direction

Efficiency analysis compares an observed production plan (y, x) with a production point located on the efficient frontier. Since there are infinite "paths" that lead to the frontier, there are an infinite number of indexes that can be used to measure technical efficiency (i.e., the distance between these two points). The history of direction choice

[1] The recent book by Kumbhakar and Lovell (2000) presents a variety of parametric econometric models to measure technical efficiency.

in efficiency analysis goes back to Färe and Lovell (1978) who first distinguished between input-oriented and output-oriented indexes of technical efficiency and showed that they are equivalent under constant (unitary) returns to scale.

The output-oriented technical efficiency (TE) index can be defined as the proportion in which a firm can increase output from a given quantity of inputs. In a nonparametric setting this index can be represented as:

$$a = \left\{ \min \ \theta \,|\, \frac{y}{\theta} \in P(x) \right\} \tag{1}$$

where x is an input vector, y represents output, P(x) is the production possibilities set, and a denotes the output-oriented index of technical efficiency ($a \leq 1$).

On the other hand, the input-oriented measure of technical efficiency can be defined as the maximum equi-proportional reduction in all inputs that still permits production of a given quantity of output. That is:

$$b = \{ \min \theta \,|\, \theta x \in L(y) \} \tag{2}$$

where L(y) is the input requirement set,[2] and b is the input-oriented index of technical efficiency ($b \leq 1$).

The efficiency measures can also be defined using a parametric approach. The parametric analysis requires the specification of both a functional form and a direction for the efficiency index. Here, two alternatives exist. One alternative is to model technical efficiency as a parameter, an approach pioneered by Lau and Yotopoulos (1971). The production function with an output-oriented technical efficiency index is:

$$y = a\, f(x) \exp(v) \tag{3}$$

where *v* represents random noise, while the production function with an input oriented

[2] Input and output indices of TE do not exhaust the range of index direction choices. For example, the hyperbolic measure has been used in this literature (Färe et al., 1985). This index can be defined as:

$c = \left\{ \min \ \theta \,|\, \left( \theta x, \frac{y}{\theta} \right) \in T \right\}$ where T represents the technology set.

technical efficiency index is:

$$y = f(bx)\exp(v) \tag{4}$$

The other alternative, following Aigner and Chu (1968), is to model technical efficiency as a random variable that becomes a part of the error term. After the path-breaking article by Aigner, Lovell and Schmidt (1977) the literature has adopted the error component structure that gave birth to stochastic frontier models. A frequent specification of a stochastic production frontier with an output-oriented TE is:

$$y = f(x)\exp(v-u) \tag{5}$$

and exp(-u) $\leq$ 1 is the output-oriented TE index. The specification of a production function with an input-oriented TE is:

$$y = f(xe^{-u})\exp(v) \tag{6}$$

where exp(-u) $\leq$ 1 is the input-oriented TE index[3].

The empirical literature in this field has used the output-oriented model when estimating production functions. That is, the models in (3) and (5) are the ones most commonly estimated. The main reason for this is that the input-oriented models in (4) and (6) are far more difficult to estimate and this probably explains why the concern about getting different empirical results with different index specifications in primal models is not voiced in the efficiency literature.[4]

There should be concerns from theoretical points as well. For example, the output-oriented index implies a neutral shift of the production function, which in turn implies that the key production characteristics (e.g. output elasticities, returns to scale, etc.) are independent of technical inefficiency. Perhaps these results led Schmidt (1985-86; p. 320) to conclude "… the only compelling reason to estimate a production frontier is to measure efficiency". However, the opposite is true for the input-oriented index, since in

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[3] It is important to note that expressions (5) and (6) (or alternatively, (3) and (4)) represent exactly the same model if the production function f(·) is homogenous.

[4] The difficulties of estimating the model in (6) as well as concerns about exogeneity of inputs have triggered the search for alternative procedures. For example, Gathon and Perelman (1992) estimate an input specific (labor) factor requirement frontier.

this case TE affects output elasticities, returns to scale and input substitution elasticities.[5]

Therefore, it is very likely that the OO and IO models provide different empirical representations of the technology. This is probably the reason why Atkinson and Cornwell found that the two models result in different efficiency rankings. In the next section we estimate a production frontier with OO and IO technical efficiency indexes and compare the characteristics of the technology estimated in both models.

### 3. Data and the econometric model

The empirical analysis is based on a balanced panel data set of 80 Spanish dairy farms for the years 1993 to 1998. These are all small family farms. We consider one output (liters of milk) and four variable inputs (viz., number of cows, kilograms of concentrates, hectares of land and labor (measured in man-equivalent units)). Table 1 presents sample mean of these variables for each year.

**Table 1. Sample mean of the variables used**

| | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 |
|---|---|---|---|---|---|---|
| **Milk production** | 112030 | 121410 | 126658 | 138403 | 134179 | 139232 |
| **Cows** | 20.59 | 21.68 | 21.67 | 23.38 | 22.63 | 23.14 |
| **Land** | 13.35 | 13.56 | 13.52 | 13.37 | 13.57 | 13.54 |
| **Labor** | 1.48 | 1.49 | 1.49 | 1.49 | 1.49 | 1.49 |
| **Concentrates** | 47362 | 53385 | 54137 | 62919 | 57906 | 61507 |

It can be seen from the above table that there are very little year-to-year variations in the use of land and labor. Most variations are in the production milk and the use of concentrates.

[5] These results are reversed when technical efficiency is analyzed in a cost function framework. In fact, the returns to scale and output elasticities do not depend on technical efficiency with an input-oriented index while they do under the output-oriented specification (Arias and Alvarez, 1998).

We estimate the technology of farms in the sample using a translog production function. The translog production function with an output-oriented index of technical efficiency is the empirical counterpart of expression (3). It can be written as:

$$\ln y_{it} = \ln a_i + \alpha_o + \sum_{j=1}^{4} \alpha_j \ln x_{jit} + \frac{1}{2} \sum_{j=1}^{4} \sum_{k=1}^{4} \alpha_{jk} \ln x_{jit} \ln x_{kit} + v_{it} \qquad (7)$$

where $a_i$ is the output-oriented index of technical efficiency, the α's are the parameters of the translog production function and $v_{it}$ is a random term with mean zero and variance $\sigma_v^2$. Schmidt and Sickles (1984) suggest estimating model (7) as:

$$\ln y_{it} = \alpha_{0i} + \sum_{j=1}^{4} \alpha_j \ln x_{jit} + \frac{1}{2} \sum_{j=1}^{4} \sum_{k=1}^{4} \alpha_{jk} \ln x_{jit} \ln x_{kit} + v_{it} \qquad (8)$$

where $\alpha_{0i} = \alpha_0 + \ln a_i$ are the farm-specific fixed-effects.

Following Schmidt and Sickles (1984) the output-oriented index of technical efficiency can be computed using the individual effects, $\alpha_{0i}$, as:

$$TE_{0i} = \exp(\alpha_{0i} - \max_i \alpha_{0i}) \qquad (9)$$

On the other hand, the translog production function with an input-oriented index of technical efficiency can be written as:

$$\ln y_{it} = \beta_0 + \sum_{j=1}^{4} \beta_j (\ln x_{jit} + \ln b_i) + \frac{1}{2} \sum_{j=1}^{4} \sum_{k=1}^{4} \beta_{jk} (\ln x_{jit} + \ln b_i)(\ln x_{kit} + \ln b_i) + w_{it} \qquad (10)$$

where $b_i$ represents the input-oriented index of technical efficiency, the β's are the parameters of the translog function and $w_{it}$ is a random term with mean zero and variance $\sigma_w^2$.

In order to calculate the index of TE in the input-oriented model it is necessary to normalize the estimates of $\ln b_i$ as well. The normalization procedure is more cumbersome than in the output-oriented model. First, it is necessary to fix the value of the individual effect for one farm in order to get an estimate of the intercept and the individual effects for the remaining farms in the sample. In a first run, we set an arbitrary farm to have $\ln b_i = 0$. In practical terms, this implies settting the efficiency parameter of that farm to 1 and estimate the efficiency parameters of the rest of the farms in the sample. The value of the individual effect for other farms is then measured relatively to this arbitrary farm. If the estimates of $\ln b_i$ are positive for one or more farms then in the second run we set the farm with the largest value of $\ln b_i$ equal to zero. This makes the values of $\ln b_i$ for the other farms less than zero.

## 4. Estimation and results

Prior to estimating (8) and (10), all the variables were divided by their respective geometric means. The output-oriented model was estimated using the least squares dummy variable approach.[6] The input-oriented model was estimated by nonlinear least squares since treating the index of technical efficiency as an individual parameter makes equation (10) non linear in parameters. The estimated parameters are reported in Table 2.[7]

In general, the coefficients of the translog function with both specifications are roughly of the same magnitude, although some differences show up. In particular, the coefficients associated with land are different. Another exception is the coefficient of concentrate squared. In this case, the estimates have different signs, although the estimates are very small and not significantly different from zero. We now compare some other features of the production functions estimated from both models.

As a result of the data transformation, the first order coefficients (i.e., the $\alpha_j$ and $\beta_j$ parameters) can be interpreted as output elasticities evaluated at the geometric mean of the inputs. Both models yield similar elasticitites with the exception of the output elasticity of land, which is more than double in the IO model.

[6] Alternatively, the parameters of the production function can be estimated by applying least squares to the model in which all the variables are expressed in mean deviation form. This is called 'within estimator' in the panel data literature (Hsiao, 1986; p. 31).

[7] We used Gauss v.3.1 to estimate the models.

**Table 2. Parameters of the production function models**

| | Output-oriented TE model | | Input-oriented TE model | |
|---|---|---|---|---|
| | Parameter | s.e. | Parameter | s.e. |
| CONSTANT | --- | --- | 0.2751 | 0.0552 |
| COWS | 0.7256 | 0.0484 | 0.7949 | 0.0671 |
| LAND | 0.1149 | 0.0515 | 0.2443 | 0.0934 |
| LABOR | -0.0763 | 0.1912 | -0.0619 | 0.2293 |
| CONCENTRATE | 0.3101 | 0.0211 | 0.2851 | 0.0329 |
| COWS x COWS | -0.0946 | 0.2371 | -0.1377 | 0.2358 |
| COWS x LAND | 0.1154 | 0.1198 | 0.1588 | 0.1204 |
| COWS x LABOR | 0.2294 | 0.2028 | 0.2651 | 0.1867 |
| COWS x CONCENTRATE | -0.0512 | 0.1027 | -0.0375 | 0.1025 |
| LAND x LAND | -0.1300 | 0.1705 | -0.0917 | 0.1789 |
| LAND x LABOR | 0.3010 | 0.2634 | 0.4343 | 0.2489 |
| LAND x CONCENTRATE | -0.0019 | 0.0586 | -0.0244 | 0.0614 |
| LABOR x LABOR | -0.6967 | 1.1057 | -0.7440 | 0.8487 |
| LABOR x CONCENTRATE | 0.0192 | 0.1071 | -0.0137 | 0.1029 |
| CONCENTRATE x CONCENTRATE | 0.0008 | 0.0580 | -0.0017 | 0.0582 |

Another important production characteristic is returns to scale. The value of returns to scale from both models are shown in Table 3. The descriptive statistics of returns to scale in these models are strikingly different. Although the minimum values are very similar (around 0.8), the mean and the maximum values are very different. The input-oriented model seems to produce higher values of returns to scale. At the mean point of the data difference in the estimates of returns to scale is caused by the big differences in the estimates of the output elasticity of land.

**Table 3. Returns to scale**

| | MIN | MEAN | MAX |
|---|---|---|---|
| Output-oriented TE model | 0.814 | 1.074 | 1.560 |
| Input-oriented TE model | 0.890 | 1.262 | 1.973 |

In order to further analize the differences in the technologies estimated under both approaches, we check the Allen elasticities of substitution. The Allen elasticity of substitution between two inputs (j,k) can be computed from:

$$\sigma_{jk} = \frac{\sum_j f_j x_j}{x_j x_k} \frac{|F_{jk}|}{|F|}$$

where $f_j$ is the first derivatives of the production function with respect to input j (the marginal product of input $x_j$), F is the bordered hessian of first and second derivatives of the production function, |F| is the determinant of the bordered hessian and $|F_{jk}|$ is the $(j,k)^{th}$ minor of F.

**Table 4. Allen elasticities of substitution**

| | | | | |
|---|---|---|---|---|
| | | OUTPUT-ORIENTED MODEL | | |
| | COWS | LAND | LABOR | CONCENTRATE |
| COWS | -0.521 | -0.212 | -0.398 | 1.201 |
| LAND | --- | -8.923 | -4.676 | 2.652 |
| LABOR | --- | --- | -4.316 | 1.602 |
| CONCENTRATE | --- | --- | --- | -3.399 |
| | | INPUT-ORIENTED MODEL | | |
| COWS | -0.489 | 0.300 | -0.115 | 1.081 |
| LAND | --- | -8.234 | -5.598 | 3.857 |
| LABOR | --- | --- | -5.820 | 3.857 |
| CONCENTRATE | --- | --- | --- | -6.468 |

The Allen elasticities of substitution (evaluated at the geometric mean of the sample) for both models are shown in Table 4. Both models provide elasticities of substitution of the same sign except for the elasticity between cows and land. Most of the estimates are roughly of the same magnitude, with the exceptions of the elasticity between cows and labor (larger in the output oriented model by a factor of three), and three out of four elasticities of substitution of concentrate. In this case, the elasticities of substitution in the input oriented model are larger by a factor of two. Again, these results provide evidence that specification of the technical efficiency index affects the estimated technology and its characteristics.

## 5. Comparing the two indexes

In our empirical exercise the specification of different index directions gives different estimates of the technology. These different estimates of the frontier are likely to produce different efficiency rankings. In this section, we analyze the relationship between efficiency indexes computed from the two models estimated above.

Previous research on input-output orientation is based on the theoretical relationship between indices of technical efficiency when the technology f(.) is known. By equalizing outputs in expressions (3) and (4), we get

$$\ln a = \ln f(bx) - \ln f(x) \tag{11}$$

which shows the link between the IO and OO measures of technical efficiency. Expression (11) can be written as

$$\ln a = \ln f(e^{\ln b} x) - \ln f(x) \tag{12}$$

which upon differentiation with respect to lnb yields

$$\frac{\partial \ln a}{\partial \ln b} = \frac{1}{f(e^{\ln b} x)} \sum_j f_j(e^{\ln b} x) e^{\ln b} x_j = \frac{\sum_j f_j(bx) b x_j}{f(bx)} = RTS(bx) \tag{13}$$

where RTS(bx) denotes the elasticity of scale evaluated at bx. This result shows that lna is a (positive) monotonic function of lnb (since RTS > 0). Thus, efficiency rankings of farms under the IO and OO models are the same (for a given x), assuming that the technology is *known*.

If the technology is *unknown* and it has to be estimated, things are far more complicated. In this case, we have a different technology for each index, viz.,

$$\begin{aligned} &\text{i) } \ln y = \ln \hat{a} + \ln \hat{f}_a(x) + e_a \\ &\text{ii) } \ln y = \ln \hat{f}_b(\hat{b}x) + e_b \end{aligned} \tag{14}$$

where $\hat{a}$ and $\hat{b}$ are the estimated indices, $\hat{f}_a$ and $\hat{f}_b$ are estimates of the technology and $e_a$ and $e_b$ are the residuals. As a result, the relationship between the estimated indexes can be written as:

$$\ln \hat{a} = \ln \hat{f}_b(\hat{b}x) - \ln \hat{f}_a(x) + e_b - e_a \tag{15}$$

In this case, the relationship between the indices is affected by the differences between estimated technologies ($\hat{f}_a$ and $\hat{f}_b$) and the residuals ($e_a$ and $e_b$). For example, when $\hat{b}$ is unity, there are two reasons why $\hat{a}$ can be less than unity ($\ln \hat{a} < 0$): (i) $\hat{f}_b(x) \neq \hat{f}_a(x)$ and (ii) $e_b \neq e_a$. Thus, it does not look feasible to analyze the relationship between the indexes without looking at a particular data. Given this difficulty, it seems that the relationship between the estimated indexes has to be explored empirically.

The estimation of the OO model (equation 8) and the IO model (equation 10) provide an estimated OO and IO index of technical efficiency, respectively. These indices are not strictly comparable since they are defined in output and input spaces, respectively. However, in each model it is possible to map the index defined in the output space to the input space (or vice versa). This mapping can be done using the following expression (derived from (11) using the translog function in (10))

$$\ln a_i = RTS(x) \ln b_i + (\ln b_i)^2 \quad \frac{1}{2} \sum_j \sum_k \gamma_{jk} \tag{16}$$

where RTS is defined at the frontier (when $b_i = 1$).[8]

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[8] For a homogeneous production function $\sum_j \sum_k \gamma_{jk} = \sum_j \sum_k \beta_{jk} = 0$. Therefore, $\ln a_i = RTS \ln b_i$ where *RTS* is a constant.

The input-oriented index of technical efficiency (IO) estimated in (10) is mapped into the output space using expression (16). This gives us a computed value of $\ln a_i$ (labeled as COO). Following the same procedure, the output-oriented index of technical efficiency (OO) is estimated in (9) and is mapped into the input space using (16), to get a computed value of $\ln b_i$ (labeled as CIO). In this case, the parameters in (8) are used to compute RTS.

Table 5 contains some descriptive statistics and the relative frequencies for four intervals of the empirical distribution of the OO index estimated in equation (9), the IO index estimated in equation (10) and the CIO and COO indexes. The IO and CIO indexes provide similar information while the OO and COO are different, as shown by the mean and relative frequencies.

**Table 5. Descriptive statistics of estimated and computed indexes of TE**

| | MIN | MEAN | MAX | 0.6-0.7 | 0.7-0.8 | 0.8-0.9 | 0.9-1 |
|---|---|---|---|---|---|---|---|
| OO | 0.59 | 0.81 | 1 | 0.16 | 0.25 | 0.33 | 0.22 |
| COO | 0.45 | 0.75 | 1 | 0.11 | 0.26 | 0.25 | 0.17 |
| IO | 0.52 | 0.81 | 1 | 0.17 | 0.26 | 0.32 | 0.21 |
| CIO | 0.54 | 0.82 | 1 | 0.12 | 0.26 | 0.32 | 0.25 |

In Table 6 we show the Spearman rank coefficient of correlation between the four TE indexes. The correlations are very high. Therefore, the first conclusion is that, in this particular empirical example, the indexes are different but provide similar information about efficiency rankings. A surprising result is that the highest correlated indexes are the actual input-oriented and output-oriented indexes. These indexes are defined in different spaces (input and output) but seem to contain very similar information about efficiency rankings.

**Table 6. Rank correlation between the estimated and computed TE indexes**

| | OO | COO | IO | CIO |
|---|---|---|---|---|
| OO | 1 | 0.9458 | 0.9793 | 0.9675 |
| COO | --- | 1 | 0.9641 | 0.8543 |
| IO | --- | ---- | 1 | 0.9524 |
| CIO | --- | --- | --- | 1 |

To examine the agreement of the rankings further, we compute Kendall's coefficient of concordance for a set of ranks from *j* types (denoted by $rank_{i,j}$),

$$W = \frac{\sum_{i=1}^{n}\left[\left(\sum_{i=1}^{k} rank_{i,j}\right) - \frac{1}{2}k(n+1)\right]^2}{k^2 n(n^2-1)/12}$$

where *k* is 2 (that is, ranks based on the IO and OO measures) and *n* equals 80. The value of zero for *W* indicates complete disagreement while a value of unity indicates complete agreement. The null hypothesis of similar ranking between the IO and OO models can be tested by referring $k(n\text{-}1)W$ to a $\chi^2$ table with $n$-1 (79) degrees of freedom. Table 7 reports the W statistics and the p values for all pairs of IO and OO ranks. These results suggest that TE indexes from the IO and OO models give similar efficiency rankings, although the level of TE differs among different measures.

**Table 7. Kendall's concordance statistic between the estimated and computed TE**

| | COO | IO | CIO |
|---|---|---|---|
| OO | 0.9657<br>(0.0000) | 0.9912<br>(0.0000) | 0.8979<br>(0.0000) |
| COO | -- | 0.9797<br>(0.0000) | 0.9247<br>(0.0000) |
| IO | ---- | -- | 0.9748<br>(0.0000) |

## 6. Conclusions

This paper deals with the empirical consequences of estimating technical efficiency using different directions under a primal approach. We find that input- and output-oriented technical efficiency models yield different estimated frontiers, which in turn implies that the estimated production characteristics (such as output elasticities, returns to scale and elasticities of substitution) are different. We also find that the two indexes of technical efficiency provide different efficiency values but very similar efficiency rankings. Since the focus in an empirical study on production is to estimate the underlying technology as well as technical efficiency, we argue that one should take extreme care in deciding the orientation problem before estimating the model routinely using an orientation that is easy to estimate.

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