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ESTIMATION OF A MIXTURE OF INPUT AND OUTPUT DISTANCE FUNCTIONS AND EFFICIENCY INDICES

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Abstract: In this paper we estimate parametric input and output distance functions as well as a mixture of input and output distance functions. The main advantage of the mixing model is that it lets the data determine the orientation of technical inefficiency, which is an open problem in efficiency analysis. Since estimates of technical inefficiency from the input and output distance functions are not directly comparable, we develop cross-indices that can be used to compute input (output) technical inefficiency from the estimates of output (input) distance function. These cross-indices are especially useful in the mixture model in which both the input and output distance functions are simultaneously estimated. The proposed technique is applied to a panel data on European Railways (1970-1994).

Keywords: Distance functions, technical inefficiency, railways.

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1. Introduction

Since the publication of papers by Aigner et al. (ALS, 1977) and Meeusen and van den Broeck (MB, 1977) stochastic frontier models are widely used in many different areas of economics, operations research, marketing, management science, and many other fields. The reason for the widespread use of it is that in estimating the technology producers are allowed to operate inefficiently, thereby generalizing the neoclassical approach that assumes every producer to be fully efficient.

Two measures of technical efficiency are primarily used in the efficiency literature. These are, (i) input-oriented (IO) technical efficiency, and (ii) output oriented (OO) technical efficiency.[1] ALS and MB used the OO approach. If output is endogenous (e.g., revenue maximization case) but inputs are exogenous, the proper measure would be the OO measure. On the other hand, if inputs are endogenous (e.g., cost minimization case) but output is exogenous the appropriate measure of technical efficiency is the IO measure. Since the choice of orientation can, in principle, have important consequences for efficiency measurement, estimation of both IO and OO models and formal model choice becomes necessary. Orea *et al*. (2003) addressed the choice between the IO and OO model from the viewpoint of cost minimizing behavior. They chose a single model for all firms whereas in the present approach we let the data determine which firm is using what orientation.

Econometric estimation of stochastic production models is problematic in the presence of multiple outputs. In such a case the distance function approach is very useful. Because of this, econometric estimation of distance functions is becoming popular in recent years (see, for instance, Lovell *et al*., 1994; and Coelli and Perelman, 2000). The technique proposed by ALS and MB can be used to estimate both the input and output distance functions. Furthermore, no price information is necessary to estimate the distance functions. Since price information is often hard to come by or is not considered reliable and/or sufficiently precise, applied researchers are inclining more and more to the distance function approach.

In the standard distance function approach the researcher chooses between the input and the output-oriented approaches and estimates the distance function of his/her

[1] See Fare and Lovell (1978) for an earlier discussion on these issues.

choice. Often both input and output distance functions are estimated and both sets of results are presented. This creates a big problem to the reader, who wants to know which results to take seriously. The results are expected to be different (unless the technology exhibits constant returns to scale) because they have different interpretations, and are based on two different technologies. (Note: In a distance function approach the technology associated with the input distance function is not the same (even theoretically) as the one associated with output distance function.)

In this paper we propose to estimate a mixture/latent class model (LCM) involving the output and input distance functions in the context of multi-input and multi-output production technology. First, we estimate the input and output distance functions separately. From these distance functions we estimate input- and output-oriented technical inefficiencies. Since estimates of technical inefficiency from the input and output distance functions are not directly comparable, we develop cross-indices that can be used to compute input (output) technical inefficiency from the estimates of output (input) distance function. Then we estimate the model in which the input and output distance functions are mixed. We justify this approach on the ground that for some producers the input distance function might be appropriate while for other the output distance function is appropriate. This is especially true when one associates input distance function with cost minimizing behavior and output distance function with revenue maximizing behavior. Thus the sample firms/producers may be classified into two groups depending on whether the input or the output distance function is appropriate for them (i.e., whether the producers minimize cost or maximize revenue). Since there is no a priori knowledge about which firms minimize cost (maximize revenue), we use the LCM that assumes that each firm has a non-zero probability to be in both groups. These prior probabilities can either be a constant or a function of covariates that are firm/time-specific. These covariates are assumed to explain the orientation. Finally, we compute (posterior) probability of group membership to determine whether the firm finally belongs to the IO or OO group. This is a natural approach, when one doesn't have precise information to determine whether the IO or OO approach is more appropriate for a specific firm. An advantage of this is that the input (output) distance function may not be appropriate for a firm throughout the entire sample period. Since the orientation might change over time and we do not have information about it, it becomes important to use data-based information to figure out the appropriate model.

Since the group membership is probabilistic, the estimates of technical inefficiencies are also probabilistic. To get a single measure one can use a weighted measure where weights are membership probabilities. This is possible if one uses mixing models with either input or output distance function. By doing so the composite measure will be either input or output technical inefficiency. However, if one mixes the technologies specified by the input and output distance functions (which are different unless the returns to scale is unity), the estimated inefficiencies are not in the same metric. We avoid this problem by first converting the input (output) inefficiencies into output (input) inefficiencies and then constructing an index for input (output) technical inefficiency.

This approach is different from the model-selection criterion used by Orea *et al*. (2003) primarily because in the present approach we do not *have* to classify firms into one or the other group. If so desired, we can accomplish this task by considering the posterior probabilities of group membership that can be computed after estimating the model. However, efficiency measures can be computed without knowing which firm is in what regime. From this point of view, we take model selection (orientation choice) uncertainty formally into account, and propose technical efficiency measures that are robust to this uncertainty.

The rest of the paper is organized as follows. The models are presented in section 2. The data are described in section 3. The empirical results are discussed in in section 4. The final section offers a summary of the paper and some concluding remarks.

2. Output, Input and Mixing Models

Let the technology be represented by a stochastic distance function that can be expressed in general terms as:

$$1 = f(y, x, \beta) \cdot \exp(v + u) \qquad (1)$$

or, in logs,

$$0 = \ln f(y, x, \beta) + v + u \qquad (2)$$

where y is a vector of outputs, x is a vector of inputs, v is a two-sided error term satisfying the classical assumptions, u is an one-sided random variable representing technical inefficiency; and β is a vector of technological parameters.

The use of the distance function requires the fulfillment of some restrictions. First, the inefficiency term *u* will be non-negative if the distance function is output oriented (i.e. $u=u^+$), whereas it will be non-positive if the distance function is input oriented (i.e. $u=-u^+$). Second, the orientation imposes homogeneity restrictions on outputs or inputs. In particular, if the distance function is output-oriented (hereafter, Output Model), the deterministic function in (1) and (2) must be homogeneous of degree one in outputs. Linearly homogeneity in outputs implies that $f(x,\mu\cdot y)=\mu\cdot f(x,y)$, $\mu>0$. Setting $\mu=1/y_M$, this property the output model can be expressed as:

$$-\ln y_M = \ln f(y/y_M, x, \beta) + v + u^+ \qquad (3)$$

Given appropriate distributional assumptions for v and u, the parameters of this stochastic output-oriented distance function can be estimated using the maximum likelihood procedure. Following ALS we assume that *v* are *iid* $N(0,\sigma_v^2)$, and distributed independently of u^+ which is assumed to be *iid* $|N(0,\sigma_u^2)|$. We denote the density function for a particular observation (y, x) drawn from the output-oriented model by $g_O(y,x,\theta_O)$, where θ_O is the vector of parameters associated with the output-oriented model.

The predicted value of the output distance function, $D_O=\exp(-u^+)$, is not directly observable because u is a part of the composed error term, $e=v+u^+$. Predictions may, however, be obtained using the Jondrow *et al.* (1982) formula. Notice that the output distance function is the inverse of the output-oriented Farrell (1957) measure of technical efficiency. This measure lies between 1 and $+\infty$, and the higher measure, lower is the efficiency. However, to be consistent with most of the parametric efficiency studies, we will use the value of the output distance function directly as a measure of efficiency that lies between zero and one. That is:

$$O\hat{T}E = \exp(-\hat{u}^+) \quad , \quad \hat{u}^+ = E\left[-u^+ \mid e\right] \qquad (4)$$

This index is a "natural" index since it has the same orientation (i.e. output) as the estimated output-oriented distance function (3). An input-oriented efficiency index can be also obtained from the estimated output distance function. We call this index a “cross-index” since it has the opposite orientation in comparison with the original distance function. The cross-indices are obtained in a way that is similar to Orea *et al*. (2003) who used it in a cost framework. In particular, the cross input efficiency index is obtained by solving the following problem for each observation:

$$\begin{aligned} &\min \quad a \\ &\text{s.t.} \quad \ln y_M^* + \ln f(\, y^* / y_M^*, x \cdot a\, ; \beta\,) = 0 \end{aligned} \tag{5}$$

where the output quantities with asterisk are adjusted by random noise using the estimated noise in the output model, that is

$$\ln y_j^* = \ln y_j + \hat{v} \quad , \quad j = 1,\ldots,M \tag{6}$$

The cross input efficiency index is then calculated as the minimum contraction of the input vector that makes the firm efficient, given its adjusted output vector. In other words, we try to evaluate the *input (cost) savings* that an efficient firm could achieve with the same output vector of an inefficient firm, given the technology estimated using the output model. Since estimated output distance function is, by construction, increasing in inputs, the resulting efficiency index (5) will take values equal or less than one.

The intuition behind (5) and (6) is illustrated in Figure 1 for the two output case. Assume we observe the output vector B. This observation is generated by the technology (point E), inefficiency (movement from E to A) and random noise (movement from A to B). In order to get any efficiency measure we have to eliminate the effect of random noise. Hence, in the first stage, output quantities are adjusted using (6). The movement from B to A shows this adjustment. In the second stage, we try to shift, by contracting the input vector; the production frontier up to it passes through A, making the firm to be efficient.

If, on the other hand, the distance function (1) is input-oriented (hereafter, Input Model), f(x,y) must be homogeneous of degree one in inputs, that is, $f(y,\mu \cdot x)=\mu \cdot f(x,y)$, $\mu>0$. Setting $\mu=1/x_N$, we get the following representation of the input distance function:

$$-\ln x_N = \ln f(y, x / x_N, \beta_I) + v - u^+ \tag{7}$$

The primary difference between (7) and (3) is that the non-negative error term will now be subtracted from the equation rather than added. Assuming again that the noise term is normal distributed and the inefficiency term follows a half-normal distribution[2], the density function of a particular observation (y, x) of the input-oriented model can be derived. We denote it as $g_I(y,x,\theta_I)$, where θ_I is the vector of parameters associated with the input-oriented model.

Once the parameters in equation (7) are estimated, the value of the input-oriented distance function would be predicted as $E[u^+|e]$, where $e=v-u^+$. Since this value will be greater than or equal to one, we propose using its inverse as a measure of technical efficiency, which is equivalent to the input-oriented technical efficiency measure introduced by Farrell (1957). That is:

$$I\hat{T}E = 1/\exp(\hat{u}^+) \quad , \quad \hat{u}^+ = E\left[u^+ \mid e\right] \tag{8}$$

Like the output efficiency index (4), the index in (8) is a "natural" index since it has the same orientation (i.e., input) as the estimated input-oriented distance function (7). The cross output-oriented efficiency index can also be obtained from (7). In this case, we first adjust the input vector for the random noise using the estimated value of v, i.e.,

$$\ln x_k^* = \ln x_k + \hat{v} \quad , \quad k = 1,...,K \tag{9}$$

As illustrated in Figure 2, the cross output efficiency index is then calculated as the minimum output contraction that is feasible, given the adjusted firm's input usage. In other words, we try to evaluate the *extra* output that an efficient firm could achieve with the *extra* inputs used by an inefficient firm, given the technology estimated from the input model. That is, we solve the following problem for each observation:

$$\begin{aligned} &\min \quad b \\ &\text{s.t.} \quad \ln x_N^* + \ln f(\, y/b \,, x^*/x_N^* \,; \hat{\beta}_I \,) = 0 \end{aligned} \tag{10}$$

Next, a mixture of both input and output oriented frontiers can be estimated using a stochastic frontier latent class model (hereafter, Mixing Model). This model is based on embedding the stochastic frontier approach into a latent class structure so that the technologies and the probability of using in the input (output) inefficiency model are estimated simultaneously.[3] This implies that all the observations in the sample are associated with a nonzero probability of being both output (revenue) maximizing firms and input (cost) minimizing firms.

[2] Other distributions for the one-sided error term can be introduced without changing the essentials of the present approach.

[3] See Greene (2002) for a survey of latent class models.

The determination of the efficiency orientation for each firm is addressed by adopting a latent class structure. In this formulation, the density function for a particular observation (y, x) is obtained as the weighted sum of both output-oriented and input-oriented density functions, where the weights are the probabilities of being output and input efficient. That is,

$$g(y,x,\theta,\delta) = g_O(y,x,\theta_O)\cdot P_O(\delta_O) + g_I(y,x,\theta_I)\cdot P_I(\delta_I) \qquad (11)$$

where $0 \leq P_j \leq 1$ (j=O,I), and $P_O+P_I=1$, $\theta=(\theta_O,\theta_I)$, $\delta=(\delta_O,\delta_I)$ and the probabilities of being output and input efficient are parameterized as a multinomial logit model,

$$P_j(\delta_j) = \frac{\exp(\delta_j' q)}{\sum_j \exp(\delta_j' q)} \quad , \quad j = O,I \qquad (12)$$

where *q* is a vector of variables. Under the mantained assumptions, maximum likelihood techniques will give asymptotically efficient estimates of all the parameters.[4]

Using Bayes' theorem the estimated parameters can be used to compute posterior probabilites as:

$$P(j\,|\,y,x) = \frac{g_j(y,x,\theta_j)\cdot P_j(\delta_j)}{\sum_j g_j(y,x,\theta_j)\cdot P_j(\delta_j)} \quad , \quad j = O,I \qquad (13)$$

These posterior probabilities can be used to classify firms. That is, if $P(j=O|y,x)>P(j=I|y,x)$ for a firm then we can include it in the Output-Model and vice versa. Once the classification is done we can estimate technical efficiency of the firms in the Output- Model from (4). Similarly, technical efficiency of firms belonging to the Input- Model can be obtained from (8).

However, in the Mixing Model, there are other ways of measuring efficiency of a firm. The Mixing Model proposes that a firm may be, with some probability, an revenue-maximizing firm *or* a cost-minimizing firm. Since for each firm, both objectives (orientations) apply with some probability, we can calculate *mixtures* of efficiency indices that merge (natural and cross) efficiency indices from both output and input distance functions.

In particular, we can use two *mixtures* of technical efficiency indices. The first one combines two output-oriented efficiency indices, viz. the natural output index obtained from the output distance function part of the Mixing Model, and the cross output efficiency index obtained from input distance function part. That is:

$$M\hat{OT}E = O\hat{T}E^{P(j=O|y,x)} \cdot CO\hat{T}E^{P(j=I|y,x)} \tag{14}$$

where COTE denotes a cross output-oriented efficiency index. The second mixture index combines two input-oriented efficiency indices, viz. the natural input index obtained from the input distance function part of the Mixing Model, and the cross input efficiency index obtained from output distance function part. That is:

$$MI\hat{T}E = CI\hat{T}E^{P(j=O|y,x)} \cdot I\hat{T}E^{P(j=I|y,x)} \tag{15}$$

where CITE denotes a cross output-oriented efficiency index.[5] The results obtained from using (14) and (15) would be, in general, different from those based on output or input distance functions alone. This is illustrated in Figure 3 for the simple one-output-one-input case. In this case, the output oriented mixture index can be written as:[6]

$$MO\hat{T}E = \left(\frac{y}{y^*}\right)^{P_O} \cdot \left(\frac{y}{y'}\right)^{P_I} \tag{16}$$

where y* and y' are the maximum output that can be achieved using the input vector x and taking the production frontier associated with the output and input oriented distance functions respectively as the reference technology. This index can be rewritten as:

$$MO\hat{T}E = \frac{y}{F_O(x)^{P_O} F_I(x)^{1-P_O}} = \frac{y}{F(x,P_O)} \tag{17}$$

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[4] A *necessary* condition for identifying δ, the parameters of the latent class probabilities, is that the sample must be generated from a non-constant returns to scale technology. In this particular case, the frontier parameters in (3) and (7) are the same (i.e. $\theta_O = \theta_I$) and the first derivative of the likelihood (11) with respect to δ is always equal to zero.

[5] A third mixture index take into account both directions of firm's performance and combines the two natural efficiency indices associated to both output and input distance function parts of the Mixing Model. That is:

$$M\hat{T}E = O\hat{T}E^{P(j=O|y,x)} \cdot I\hat{T}E^{P(j=I|y,x)}$$

This index, that can be viewed as a directional-type or hyperbolic-type index, has some interpretation problems since it implies both output expansions and input reductions.

[6] Similar comments deserve the input oriented mixture index, MITE.

The denominator in (17) can be interpreted as the expected technology of a firm with output class probability P_O. This means that, given the estimated posterior class probabilities, the reference technology used for calculating a mixture of efficiency indices is specific for each firm, unlike in OTE and ITE where the production frontier is the same for all the firms.

3. Sample and Data

The Output, Input and Mixing models introduced in Section 2 are estimated using data from the European Railways during the period 1970 to 1994.[7] This industry has historically been thought, in most European countries, as a natural monopoly requiring unitary ownership at the network level and either public control or ownership. Public control over rail industry is justified by the idea that this industry is considered as an integrative mechanism able to overcome geographical barriers in certain areas, aid in the economic development of undeveloped zones, and even as a guarantee of minimum transport services for a particular segment of the population. Since government (and regulators) encouraged spreading railways services, the European rail industry had rapid growth during the mid 2oth century.

However, in the 80s a substantial fall in the European rail activity (specially in freight transportation) started that apparently stabilized during mid 1990s and improved since 1995. This reduction can be attributed to both exogenous causes (e.g. the rapid development of alternative modes of transport, especially by road) and endogenous causes (e.g., regulation) that restrict the industry's adaptation to changing conditions of its economic environment (Campos and Cantos, 1999). Because of a declining market share and worsening financial performance of railways, in 1984 the European Commission proposed using decentralised methods of management by sectors or business units, separating accounts and clearing targets to increase profitability or

[7] This application can be viewed as an extension of the "classical" distance function paper written by Coelli and Perelman (2000) using data from 1988-1993.

reduce the scale of losses.[8] These measures were mainly input oriented. For instance, management contract in 1984-1986 in Spain had an agreement to close 882 km of lines and to reduce the workforce by 15,000 persons in four years.

In 1991, the Directive 91/440 presented by the European Commission advocated a system of competitive access to the infrastructure, based on the principle of vertical disintegration between infrastructures and operations. This approach is explained by the fact that infrastructure costs are largely sunk and infrastructure provision exhibits natural monopoly characteristics.[9] Next, various Directives were presented by the European Commission (1994, 1995, 1996, 1998). Finally, Directive 12/2001 required to separate accounting for passenger and freight transport services and the White Paper, 2001 (Commission of the European Communities) established that full competition across Europe will take place by 2008.[10]

The Mixing Model proposed in Section 2 is suitable for this data set since firms (or regulators) have changed their strategies from maximizing market share in the 70s and early 80s to reducing costs at the end of 80s and in the 90s.This suggests that both orientations have played an important role in the European railroad industry and, therefore, a model should take both orientations into account.

The data used in this paper are taken from the reports published by the *Union Internationale des Chemins de Fer* (UIC) and covers the period 1970-1994 for the railways of 17 countries of the European Union: BR (U.K); CFF (Switzerland); CFL (Luxembourg); CH (Greece); CIE (Ireland); CP (Portugal); DB (Germany); DSB (Denmark); FS (Italy); NS (Holland); NSB (Norway); OBB (Austria); RENFE (Spain); SJ (Sweden); SNCB/NMBS (Belgium); SNCF (France) and VR (Finland).

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[8] This approach was adopted in Britain form the early 1980s to 1994, where the five business sector were given their own directors, separated accounts and clear targets to increase profitability/reduce the scale of losses. Costs were allocated to the business sector on the basis that each sector was responsible for the costs of assets (including infrastructure) and staff. Following the British experience, many European Railways, such as Spain, The Netherlands and Germany have reorganized on a business sector basis (OECD, 1998).

[9] The most radical experience in the process of vertical separation is The British railways which after strong improvements in the later 1980s began to deteriorate in the early 1990s. The infrastructure was place in the hands of a new company, which was privatised in 1996. Less extreme experiences are Germany and Netherlands (for details see Cantos *et al.*, 2002 and OECD, 1998).

We have considered two outputs: freight-tonnage per kilometer (y_1) and passengers per kilometer (y_2). Three inputs have been considered: Energy (x_1), Labour (x_2), and Capital (x_3). For the energy variable, motor equipment energy consumption has been used, in thousand million kilocalories. We use observations until 1994 because data after 1994 are not directly comparable due to changes in accounting procedures. Labour is measured using the number of employees. For capital measurement we follow Coelli and Perelman (2000) and consider rolling stock, which is measured by the sum of available freight wagons and coach transport capacities in tonnes and seats, respectively. The sample means of all variables are presented in Table 1 for each of the 17 companies over the period 1970-1994.

4. Empirical results

The parameter estimates for the Output, Input and Mixing Model are presented in Table 2. All variables have been mean-corrected prior to estimation. That is, each output and input variable has been divided by its geometric mean. In this way, the first order coefficients can be interpreted as distance elasticities evaluated at the sample means. While in the output distance function the linear homogeneity is imposed using the output y_2 as a numeraire, in the input distance function it is imposed using x_3 as a numeraire. Since all the elasticities possess the expected signs at the geometric mean, the estimated distance functions satisfy the property of monotonicity, i.e. the output distance functions are non-decreasing in outputs and decreasing in inputs, and the input distance functions are non-decreasing in inputs and decreasing in outputs.

The mean log-likelihood value in the Mixing Model is twice that obtained in other models. This suggests that the Mixing Model is a significant improvement over the traditional Output and Input models. Indeed, unlike the Mixing Model, the traditional models impose a common orientation for all firms and over time. In our application this restriction seems to be quite restrictive, as shown in Table 3. This table provides information to allocate each observation into two classes: i) Output-Model, which includes those observations where $P_O > P_I$; and ii) Input-Model, which includes those

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[10] As Claes *et al*. (2003) point out, all of these directives are designed to liberalize Europe's rail systems, but they do not mandate specific means to achieve liberalization. Each member

observations where $P_O<P_I$. The information contained in Table 2 shows that half of the sample consists of output maximizing firms and, the other half consists of input minimizing firms. On the other hand, the efficiency orientation has changed over time, especially in the cases of U.K, Greece, Ireland and Belgium.

Following Färe and Primont (1995), the scale elasticity in an output distance function can be calculated as the negative of the sum of the input elasticities. The sum of first order input coefficients in the Output Model is equal to 1.045, indicating slight increasing returns to scale, as found in many other empirical analyses of railways. The same results hold for the Input Model. In this model, the scale elasticity can be calculated as the inverse of the negative sum of the output elasticities. The sum of the first-order output coefficients in the Input model (i.e. –0.913) is less than one in absolute value, indicating again the presence of increasing returns to scale. Similar comments apply to the scale elasticity estimated from the input distance function part of the Mixing Model. However, at the sample mean, we cannot reject constant returns to scale using the output distance function part.

Technical change can be calculated using the derivative of the distance function (in logs) with respect to time trend. Table 4 reports the annual rates of technical change calculated from the Output, Input and Mixing Models. The results in this table show, in general, positive technical change throughout the period. All the models indicate, however, the existence of technical "regress" during the 1970s. This result seems to corroborate the apparently worsening performance of railways up to 1984 when the European Commission's policy changed. The positive rates of technical change in the latter 1980s and early 1990s suggest that the new course of the European policy produced an overall improvement in the railway industry.

The estimated parameters can be used to obtain output-based, input-based and mixtures of technical efficiency indices. A summary of the technical efficiency indices is presented in Table 5 and the results are also presented in Figure 4. In general, we obtain average technical efficiency over 80 percent, as in previous railways studies. Moreover, the output-oriented indices are higher than the input oriented indices; except for the mixture indices that yield similar average values. As expected, the efficiency levels rise as the efficiency orientation is allowed to vary over firms and over time, like in the

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country must decide how it will comply with the directives.

Mixing Model. This suggests that when orientation is restricted a priori to be input- or output- based, the estimated efficiency levels are likely to be biased downward (due to model misspecification).

We now examine the behavior of technical efficiency over time. All measures, graphed in Figure 4, show an increase in technical efficiency levels of European railways during the 1970s and 1980s, and a decrease in the early 1990s (except for the Output-Model). Note that, while the increase in efficiency matches in time with negative or moderate rates of technical change, the efficiency deterioration takes place during the period characterized by strong technical progress.

Productivity growth can be measured from the estimated distance functions by combining the effect of technical change (i.e., shifts in the distance funtion over time) and changes in technical efficiency into a Malmquist productivity index. Figure 5 provides the temporal path of the Malmquist index of productivity. The plots for the Output and Input models are quite similar, but are quite different from the one obtained using the Mixing Model. Figure 5 shows that *productivity growth estimates seem to be quite sensitive to restricting the efficiency orientation among firms and/or over time.*

Finally, we examine the temporal behavior of the class probabilities and the coefficients of the logit probability functions. The yearly average class probabilities are depicted in Figure 6. Both the prior and posterior probabilities for Input Model are, in general, higher than those in the Output-Model, especially during the 1970s. The relative importance of both orientations has changed drastically over time, as indicated by the significance of the coefficients of the probability function. In fact, the output orientation during the 1980s is almost as important as the input orientation. This Figure also suggests that the policy advocated by the European Commission since 1984 allowed rail companies to have more control on their inputs (compared with the control on their outputs), increasing the input orientation approach in the latter 1980s and early 1990s.

5. Conclusions

Traditional Output and Input models impose a common orientation for all firms and over time. We show using European railway data that this restriction seems to be quite restrictive. In particular, productivity growth estimates seem to be quite sensitive to imposing a common orientation and the estimated efficiency levels are likely to be downward biased. Collectivelly, these results highlight the importance of estimating less restrictive models, such as a Mixing Model that combines both output and input distance functions.

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Table 1. Descriptive Statistics

Variable	Mean	Std. Dev.	Minimum	Maximum
y_1	13778	18642	310	76424
y_2	14356	16762	207	64256
x_1	133708126	166327003	27000	755328000
x_2	82573	97281	3289	405713
x_3	2381012	3420845	70564	12493852

Table 2. Parameter estimates

	Mixing Model		Output Model		Input Model	
Parameters	Estimates	Est./s.e.	Estimates	Est./s.e.	Estimates	Est./s.e.
$\ln(y_1/y_2)$	0.561	55.738	0.460	22.056	-	-
$\ln x_1$	-0.321	-31.341	-0.161	-10.904	-	-
$\ln x_2$	-0.478	-25.591	-0.449	-12.674	-	-
$\ln x_3$	-0.199	-10.557	-0.436	-10.828	-	-
$0.5\cdot\ln(y_1/y_2)^2$	-0.032	-0.474	0.113	1.646	-	-
$0.5\cdot\ln(x_1)^2$	-0.141	-26.137	-0.060	-7.821	-	-
$0.5\cdot\ln(x_2)^2$	-0.327	-2.183	0.817	4.110	-	-
$0.5\cdot\ln(x_3)^2$	-0.635	-4.678	0.294	1.972	-	-
$\ln(y_1/y_2)\cdot\ln x_1$	0.073	6.247	0.021	2.047	-	-
$\ln(y_1/y_2)\cdot\ln x_2$	-0.028	-0.468	0.451	5.157	-	-
$\ln(y_1/y_2)\cdot\ln x_2$	-0.062	-1.079	-0.406	-5.156	-	-
$\ln x_1\cdot\ln x_2$	0.018	0.976	0.034	2.195	-	-
$\ln x_1\cdot\ln x_3$	0.164	12.466	0.053	3.428	-	-
$\ln x_2\cdot\ln x_3$	0.365	2.708	-0.508	-2.993	-	-
t	0.013	3.400	0.015	2.407	-	-
$0.5\cdot t^2$	-0.001	-4.415	-0.002	-4.520	-	-
Intercept	0.308	9.744	-0.213	-3.624	-	-
Sigma	0.065	6.964	0.289	14.429	-	-
Lambda	2.497	1.606	2.844	3.656	-	-
$\ln y_1$	-0.443	-18.629	-	-	-0.439	-24.334
$\ln y_2$	-0.464	-21.120	-	-	-0.475	-25.304
$\ln(x_1/x_3)$	0.100	8.499	-	-	0.131	12.221
$\ln(x_2/x_3)$	0.379	9.221	-	-	0.442	14.062
$0.5\cdot\ln(y_1)^2$	-0.266	-3.422	-	-	-0.183	-2.475
$0.5\cdot\ln(y_2)^2$	-0.106	-1.479	-	-	0.044	0.661
$0.5\cdot\ln(x_1/x_3)^2$	0.037	5.389	-	-	0.055	7.938
$0.5\cdot\ln(x_2/x_3)^2$	-0.082	-0.477	-	-	-0.271	-1.835
$\ln y_1\cdot\ln y_2$	0.085	1.284	-	-	-0.020	-0.314
$\ln y_1\cdot\ln(x_1/x_3)$	-0.003	-0.253	-	-	-0.003	-0.236
$\ln y_1\cdot\ln(x_2/x_3)$	-0.348	-3.626	-	-	-0.462	-5.236
$\ln y_2\cdot\ln(x_1/x_3)$	-0.001	-0.102	-	-	-0.008	-0.707
$\ln y_2\cdot\ln(x_2/x_3)$	0.256	3.959	-	-	0.284	4.408
$\ln(x_1/x_3)\cdot\ln(x_2/x_3)$	0.004	0.311	-	-	0.009	0.695
t	-0.009	-1.472	-	-	-0.017	-3.043
$0.5\cdot t^2$	0.002	4.000	-	-	0.002	5.126
Intercept	0.273	6.438	-	-	0.239	5.321
Sigma	0.231	13.412	-	-	0.303	20.527
Lambda	5.510	2.514	-	-	5.491	4.063
Probabilities						
Intercept	-1.462	-2.745	-	-	-	-
t	0.164	1.895	-	-	-	-
$0.5\cdot t^2$	-0.010	-1.629	-	-	-	-
Mean log-likelihood:		0.644544		0.279167		0.338808
Number of cases:		405		405		405

Table 3. Most probable orientation

	BR	CFF	CFL	CH	CIE	CP	DB	DSB	FS	NS	NSB	OBB	RENFE	SJ	SNCB	SNCF	VR
1970	-	I	I	-	I	I	I	-	O	I	I	O	I	I	I	I	I
1971	-	I	O	-	I	I	O	-	O	I	I	O	I	O	I	I	I
1972	-	O	I	I	I	I	O	-	O	I	I	O	I	I	I	I	I
1973	I	O	I	I	I	I	O	-	O	O	I	O	I	I	I	I	I
1974	O	O	I	I	I	I	O	-	O	O	O	O	I	I	I	I	I
1975	O	I	I	I	I	I	O	-	O	I	O	O	I	I	O	I	I
1976	O	I	O	I	I	I	O	-	O	I	O	O	I	I	O	I	I
1977	O	O	I	I	I	I	O	I	O	I	I	O	I	I	O	I	I
1978	O	O	O	O	I	I	O	I	O	I	O	O	I	I	O	I	I
1979	O	O	O	O	O	I	O	O	O	O	O	I	I	I	O	I	I
1980	O	I	O	O	I	I	O	I	O	O	O	O	I	I	O	I	I
1981	O	I	O	O	I	I	O	I	O	O	O	O	I	I	O	I	I
1982	O	O	O	I	I	I	O	I	O	I	I	O	I	I	O	I	I
1983	I	O	O	I	I	I	O	I	O	I	I	O	I	I	O	I	I
1984	I	O	O	O	O	I	O	I	O	I	O	O	I	I	O	I	I
1985	I	I	O	O	O	I	O	I	O	I	O	O	I	I	O	I	I
1986	I	O	O	O	I	O	O	I	O	I	O	O	I	I	O	I	I
1987	I	O	I	O	I	O	O	I	O	I	O	O	I	I	O	I	I
1988	I	O	O	O	I	I	O	I	O	I	O	O	I	I	O	I	I
1989	I	O	O	O	O	O	O	I	O	I	O	O	I	I	O	I	I
1990	I	O	O	O	O	O	I	I	O	I	O	O	I	I	O	I	I
1991	-	O	O	-	O	O	I	I	O	I	-	O	I	I	O	I	I
1992	O	O	O	I	O	I	O	I	O	I	-	O	I	I	I	I	I
1993	O	O	O	I	O	I	O	I	O	I	-	O	O	I	O	I	I
1994	O	I	O	I	I	I	I	-	O	-	-	O	I	I	O	I	I

Note: I=Input orientation; O=Output orientation

Table 4. Rates of Technical Change (%)

Year	Mixing Model		Output Model	Input Model
	Output Part	Input Part		
1971	-1.07	-0.53	-1.13	-1.32
1972	-0.95	-0.36	-0.93	-1.12
1973	-0.83	-0.19	-0.73	-0.92
1974	-0.71	-0.02	-0.53	-0.72
1975	-0.59	0.15	-0.33	-0.52
1976	-0.47	0.32	-0.13	-0.32
1977	-0.35	0.49	0.07	-0.12
1978	-0.23	0.66	0.27	0.08
1979	-0.11	0.83	0.47	0.28
1980	0.01	1.00	0.67	0.48
1981	0.13	1.17	0.87	0.68
1982	0.25	1.34	1.07	0.88
1983	0.37	1.51	1.27	1.08
1984	0.49	1.68	1.47	1.28
1985	0.61	1.85	1.67	1.48
1986	0.73	2.02	1.87	1.68
1987	0.85	2.19	2.07	1.88
1988	0.97	2.36	2.27	2.08
1989	1.09	2.53	2.47	2.28
1990	1.21	2.70	2.67	2.48
1991	1.33	2.87	2.87	2.68
1992	1.45	3.04	3.07	2.88
1993	1.57	3.21	3.27	3.08
1994	1.69	3.38	3.47	3.28
Average	0.31	1.425	1.17	0.98

Table 5. Average efficiency indices

Year	Output Model	Input Model	Mixing Model			
			Output Class	Input Class	Mixture of indices	
	OTE	ITE	OTE	ITE	MOTE	MITE
1970	78.8	74.2	96.5	82.5	81.5	83.8
1971	76.1	70.4	95.5	78.8	80.5	82.9
1972	78.5	73.6	94.6	81.0	80.9	83.0
1973	80.8	76.0	96.7	80.8	82.8	84.5
1974	82.2	77.8	97.5	83.9	85.6	87.7
1975	80.5	76.4	95.1	85.0	85.2	86.9
1976	79.9	75.2	95.2	82.9	85.2	87.2
1977	80.4	76.5	93.7	85.5	85.2	87.2
1978	81.2	77.0	95.0	86.2	87.4	88.9
1979	82.5	78.5	97.0	83.6	89.5	90.6
1980	84.2	80.8	97.0	87.5	89.8	90.9
1981	83.5	80.6	95.6	86.0	89.6	90.6
1982	81.2	78.1	94.0	86.6	87.3	88.9
1983	81.4	78.8	92.2	85.4	86.2	87.4
1984	83.0	81.1	95.2	88.0	90.6	91.0
1985	83.7	82.2	96.3	87.9	90.9	91.2
1986	81.7	79.4	95.0	86.6	89.0	89.7
1987	81.5	79.6	94.5	87.7	89.1	89.9
1988	82.5	81.1	95.3	90.7	91.0	91.8
1989	83.1	81.3	96.9	89.3	91.6	92.1
1990	82.0	79.5	95.2	85.2	88.8	89.7
1991	81.9	80.3	97.2	84.8	89.6	89.8
1992	82.5	79.2	95.5	81.0	84.5	86.3
1993	80.0	76.5	93.8	84.7	85.6	87.7
1994	81.0	78.5	96.3	83.3	84.8	87.2
Sample	81.4	78.2	95.5	84.9	87.0	88.4

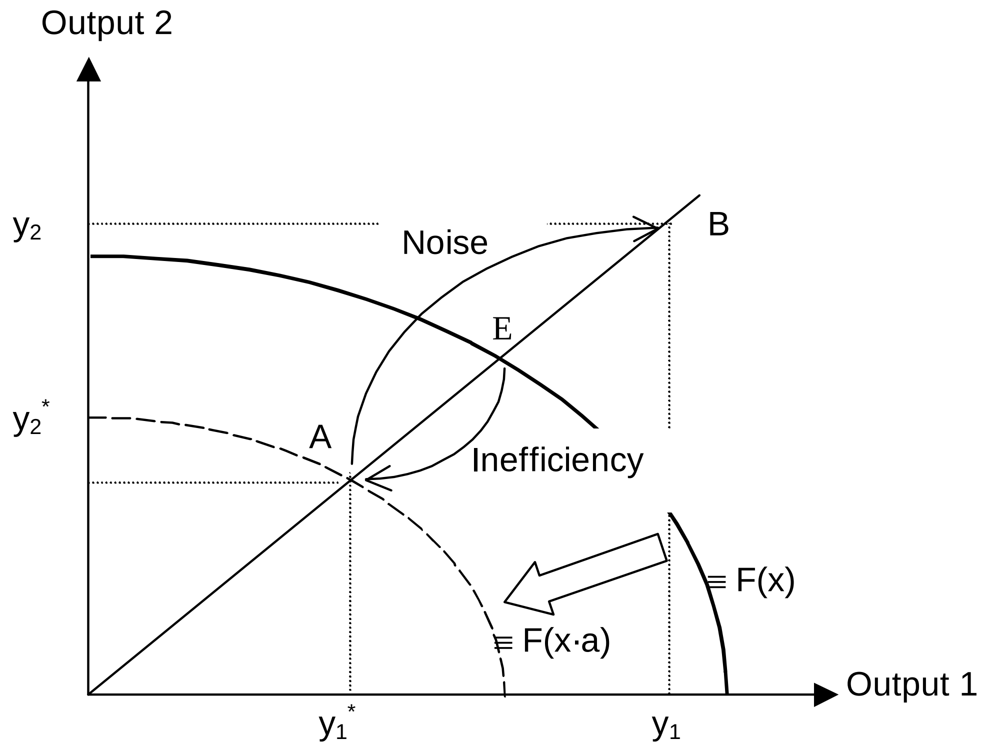

Figure 1. Cross input efficiency index

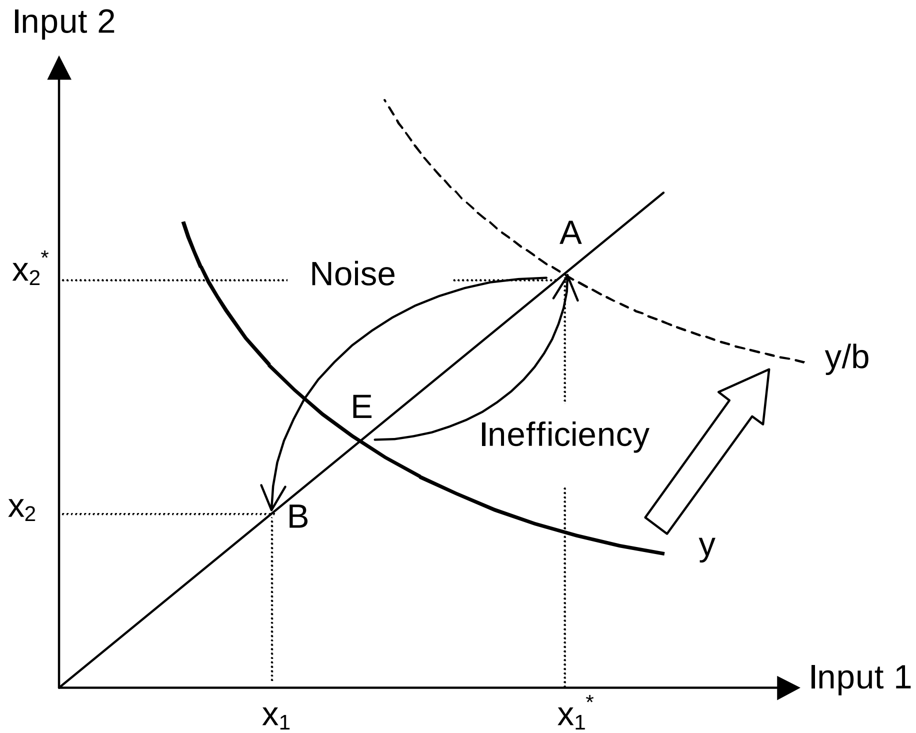

Figure 2. Cross Output efficiency index

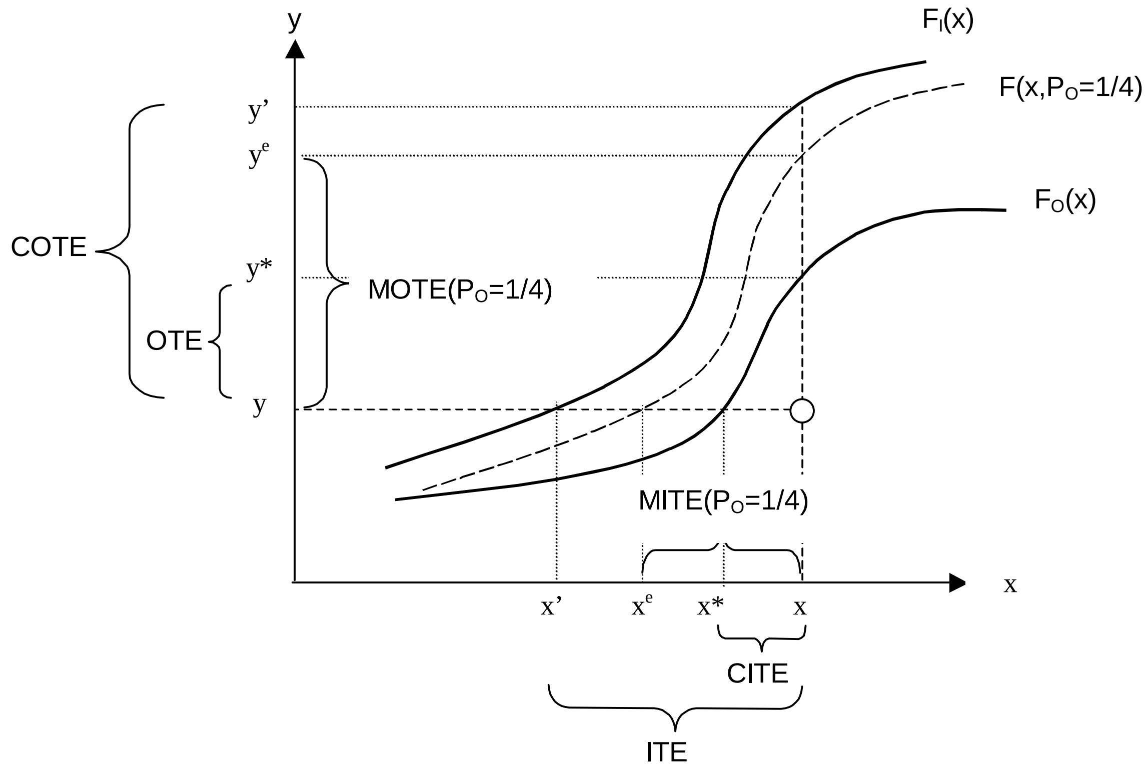

Figure 3. Output, input and mixtures of efficiency indices

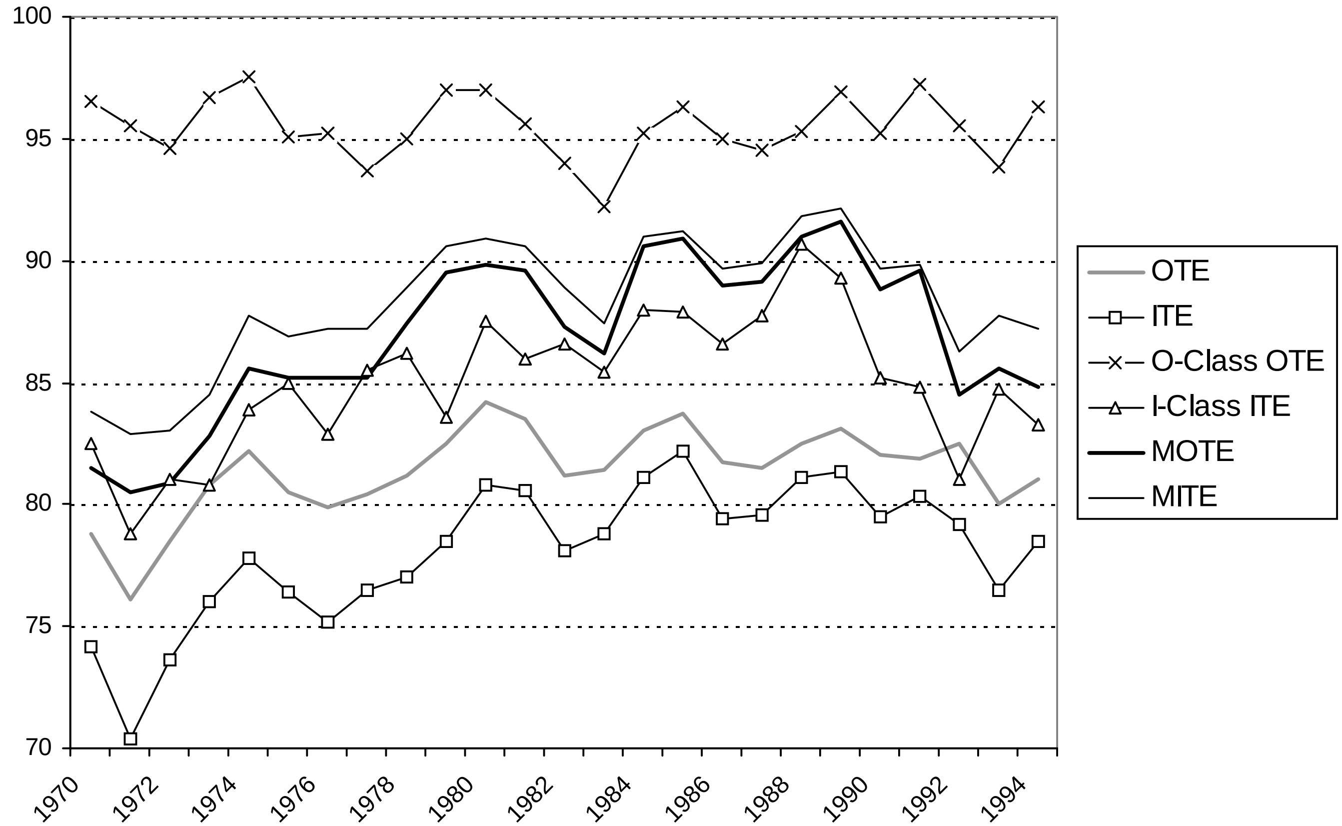

Figure 4. Technical Efficiency Indices

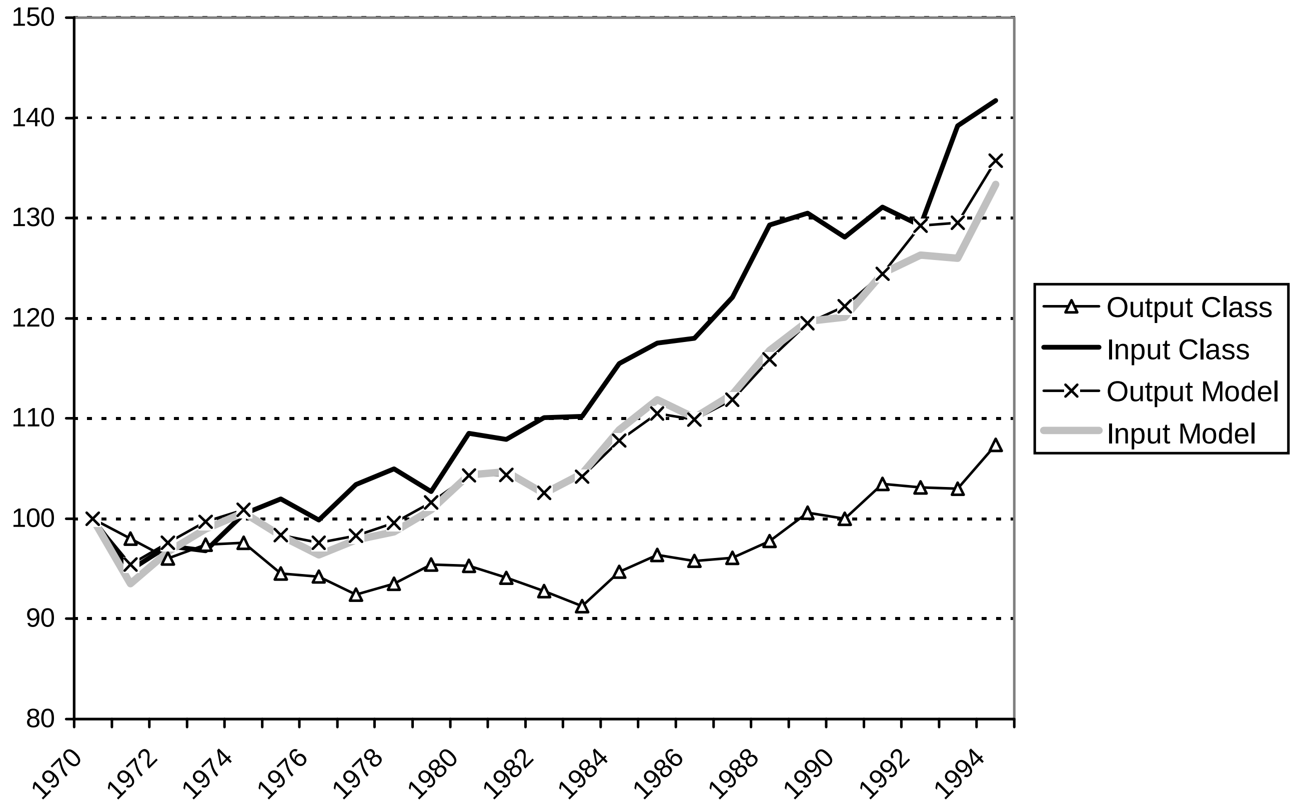

Figure 5. Productivity Growth

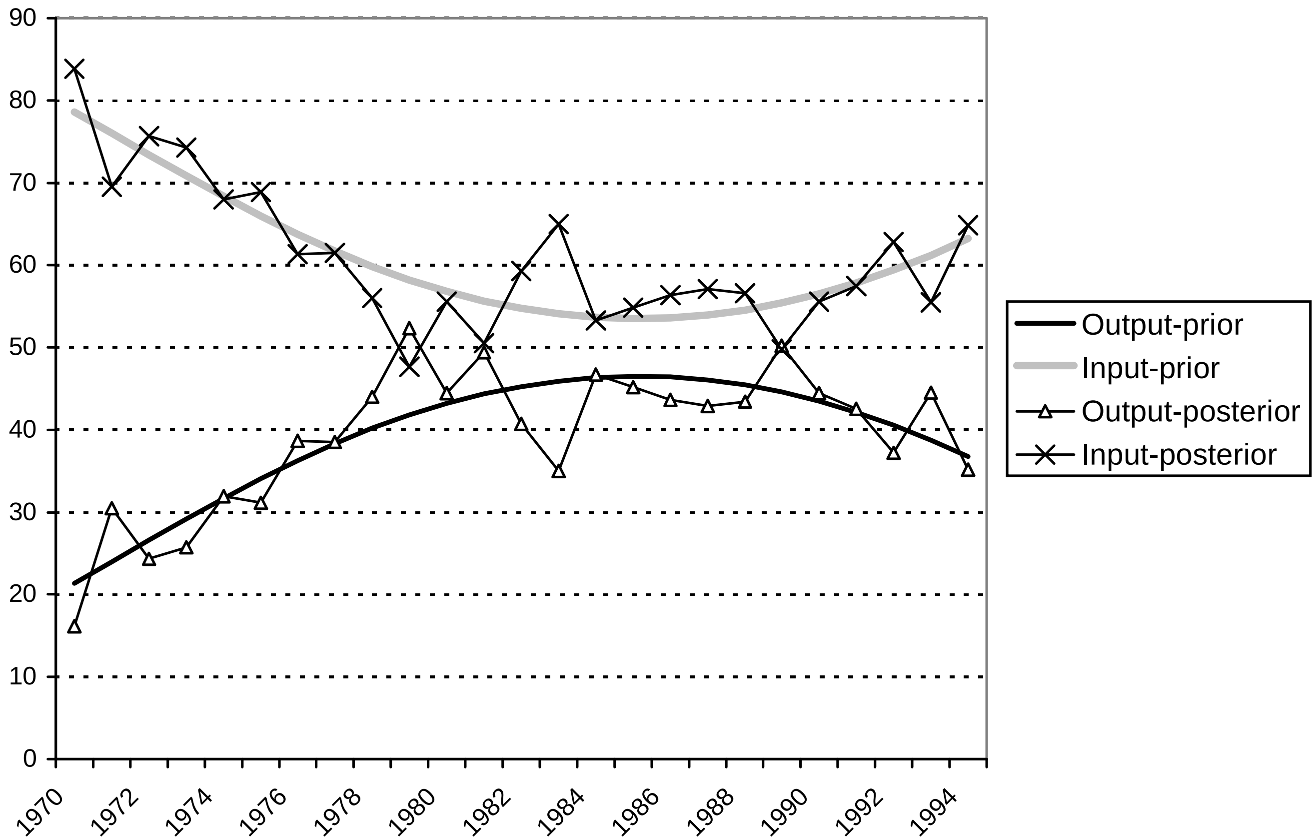

Figure 6. Output and Input Class Probabilities