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THE MEASUREMENT OF SPATIAL PRODUCTIVITY SPILLOVERS FROM PUBLIC CAPITAL

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Abstract: This paper analyzes the properties of a frequently-used test of spatial spillover effects of public capital when using regional panel data. This test compares the estimates of the coefficients of public capital in two different production functions. The benchmark model includes the public capital of each region as an explanatory variable. In the second model the public capital of each region is augmented with public capital in neighbouring regions. Using basic regression algebra, we show that the results of this "pseudo-test" are not necessarily related to the existence of spillover effects.

Keywords: productivity, public capital, spillover effects.

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1. Introduction

Since the appearance of certain seminal articles in the 1980s (Ratner, 1983; Aschauer, 1989), the productivity of public capital has been the subject of study in many papers.[1] Several surveys of this vast literature already exist, including Gramlich (1994), Draper and Herce (1994), and De la Fuente (1996, 2000). Due to the availability of high quality data sets, in Spain a large amount of empirical literature has appeared on this topic.[2]

Early papers found that the productivity of public capital was quite high. Aschauer, for example, estimated an output elasticity of public capital of 0.39, which was larger than the elasticity of private capital. However, these findings were soon criticized on several grounds. In particular, since these papers used aggregate national time-series data, some authors argued that the empirical results could be due to spurious correlation caused by common trends in the variables. Among the other main criticisms levelled against these studies were the lack of relevant variables (such as human capital) or the problem of reverse causality, i.e., the direction of causality may run from economic activity into public investment.

When researchers started using state-level data the estimated elasticities were much lower.[3] In fact, the empirical evidence shows that geographical disaggregation of data usually results in lower productivity of public capital. This finding has been attributed to spillover effects of public capital from one region onto neighbouring regions.[4] These spatial spillovers are due to the network effect of public capital. That is, since most elements of public capital have network characteristics (i.e., roads, telecommunications,

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[1] By public capital we mean the stock of infrastructure built by the public sector. As such, public capital is different from public expenditure. Moreover, a distinction is usually made between what is termed "productive" public capital (e.g., transport infrastructure) and "social" public capital (capital in education and health). The empirical literature cited below mainly uses the first concept, referring to it as "public infrastructure".

[2] Partial surveys of the Spanish literature can be found in Sanaú (1997), Fernández and Polo (2001) and Alvarez et al. (2003).

[3] See García-Milá et al. (1996) for a comprehensive review of the problems involved in estimating state-level production functions.

[4] The issue of spatial spillovers has also been addressed in other areas of economic research. In the economic growth literature, there is evidence that fast-growing countries cluster together, implying that location matters for growth (see, for example, Moreno and Trehan, 1997). In public economics, some researchers are interested in the degree to which state spending is influenced by the spending of neighbouring states (see, for example, Case et al., 1993). In development economics, some papers try to test the Myrdal-Hirschman core-periphery hypothesis of unbalanced growth which implies that the development of some regions may have a positive influence on nearby regions (see, for example, Ying, 2000). Finally, in regional analysis some

railways, etc.) it is expected that the stock of public capital in one region will affect production in other regions. However, it has also been argued that there may be negative spillovers from public infrastructure. The argument here is that “public infrastructure investments in one location can draw production away from other locations” since “it enhances the comparative advantage of that location relative to other places” (Boarnet, 1998).

The existence of spatial spillovers has been tested for in this literature. Holtz-Eakin and Schwartz (1995) parameterize the effect of neighbouring regions’ public capital and perform a statistical test in the framework of a model which is non-linear in parameters. However, this test has not been used in the papers that have looked for spatial spillovers using Spanish data.[5] In fact, most papers have used an alternative “test” developed by Mas et al. (1994) which is based on the comparison of two models. The procedure followed is that they first estimate a standard production model that includes the stock of public capital in each region as an explanatory variable. Then, they estimate another model where the public capital in each region has been augmented with some weighted sum of the public capital stock in neighbouring regions. Then, if the estimated elasticity of the model with the augmented stock is higher than the elasticity of the simpler model, this is interpreted as evidence of spillover effects. Since there is no way to statistically test the null hypothesis that the two coefficients are equal, we will refer to this two-step procedure as a “pseudo-test”. [6]

In this paper we show that the “pseudo-test” developed by Mas et al. (1994, 1996) is not an appropriate procedure to test for spillover effects, and that the higher elasticities found when using the augmented capital stock may not be related to the existence of a spillover effect.

The structure of this paper is as follows. In section 2, we review the modeling of spillover effects. In section 3, we show that the “pseudo-test” to check the existence of public capital spillovers can lead, in many instances, to erroneous results. Section 4 concludes.

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studies of regional convergence make convergence conditional upon location (see, for example, Chua, 1993).

[5] One exception is the paper by Avilés et al. (2003) where a linear version of the model is estimated.

[6] Some papers check for across-region spillovers using different models or approaches. Moreno et al. (1997) uses spatial econometric techniques. Martínez (2000) tests for spillovers in the framework of a convergence equation. Pereira and Roca (2003).

2. Spatial spillover effects of public capital

The early claims by Munnell (1990) and others that the use of state-level data misses part of the spillover benefits of public capital did not result in rigorous statistical testing of this hypothesis. In fact, the papers that tried to address this issue used an indirect "test": they estimated the same model at different levels of geographical aggregation. This approach was used by Holtz-Eakin (1994), who failed to find greater elasticities at higher regional aggregation.

The first statistical test of this hypothesis was carried out by Holtz-Eakin and Schwartz (1995). They estimate a Cobb-Douglas production function, such as the following:

$$\ln Y_{it} = \alpha_i + \alpha \ln K_{it} + \beta \ln L_{it} + \gamma \ln G_{it}^e \tag{1}$$

where subscript *i* indexes regions, subscript *t* indexes time, Y is aggregate production, K is private capital, L is labor and G^e is the effective stock of public capital.

The effective stock of public capital in region *i* (G_i^e) differs from the observed stock (G_i) due to the contribution of the stock of other regions. This variable can be defined in general terms as,

$$G_i^e = G_i \prod_j^N G_j^{\delta w_j} \tag{2}$$

where j indexes nearby regions (j≠i), w_j is the weight of other region's capital, and the parameter δ measures the effect of the public capital of other regions on the effective capital stock.[7] When δ=0 the effective actual measures are equal. In the other hand if δ is significantly greater than zero it can be viewed as a test of the spillover effect.

However, some papers have used the "pseudo-test" to check for the existence of spillovers by comparing two regressions based on different specifications of the public capital variable[8]. First, the Cobb-Douglas production function in equation (1) is estimated

[7] In practice, some assumptions usually take place: the number of nearby regions (N) is limited to the bordering regions, the weights (w_j) are assumed to be equal to one for these regions (i.e. w_j is zero for non-bordering regions). Other alternatives are to define the weights in terms of the inverse of the distance of other regions or to build the weights so that they reflect the commercial relationship among regions.

[8] Some examples of papers that have followed that approach are Mas et al. (1996), Gil et al. (1997,1998), Cantos et al. (2002), Alvarez et al. (2003).

with the effective capital stock defined as $G_i^e = G_i + \sum_j^N G_j$ and then estimated using the region's own public capital stock (G) as follows:

$$\ln Y_{it} = \alpha_i^* + \alpha^* \ln K_{it} + \beta^* \ln L_{it} + \gamma^* \ln G_{it} \qquad (3)$$

If the output elasticity of public capital using G^e (γ) is higher than using G (γ*), this is interpreted as evidence of the existence of spillover benefits of public capital. In section 3 we will show that the positive value of the "pseudo-test is related to the existence of spatial spillovers only under strong and unrealistic assumptions about the correlation among stocks of public capital in a region and neighboring regions.

3. An analysis of the "pseudo-test" for spillover effects

The properties of the "pseudo-test" described above can be analyzed using a basic result of regression algebra on omitted variables. Our analysis uses a benchmark model which includes as separate regressors the public capital in the region (lnG) and the public capital in neighboring (adjacent) regions ($\ln G^a$).

This model can be derived from the original model in expression (1), where for simplicity, we drop the subscripts, the intercept[9], the private capital and the labor terms, so we can focus on the estimation of the parameter on effective public capital. We also simplify the definition of effective capital assuming that the weights (w_j) equal one and that there is only one adjacent region. Thus,

$$G_i^e = G_i \left(G_i^a\right)^{\delta} \qquad (4)$$

where G_i^a is a Cobb-Douglas aggregator of public capital in adjacent regions ($G_i^a = \prod_j^N G_j$).

Substituting, expression (4) in the simplified version of (1), we have the basic formulation of our benchmark model:

[9] The intercept can be assumed away by rescaling all the variables so that they have zero mean.

$$\ln Y = \gamma \ln G^e = \gamma \ln G + \gamma\delta \ln G^a \tag{5}$$

This model can be estimated by least squares. Making θ=γδ, the estimated equation is:

$$\ln Y = \hat{\gamma} \ln G + \hat{\theta} \ln G^a + e \tag{6}$$

where $\hat{\gamma}$ and $\hat{\theta}$ are the parameter estimates and e denotes the least squares residual.[10] Since $\hat{\theta}$ is a direct estimate of δγ, a positive sign of $\hat{\theta}$ can be interpreted as evidence of spillover effects (δ>0).

There are two restricted versions of the benchmark model that are of interest for the analysis of the "pseudo-test". In the first model, public capital of neighboring regions is added to that of the region analyzed (in logs). This is equivalent to estimating model (6) imposing the restriction that $\gamma=\theta$. The result of estimating this model is:

$$\ln Y = \hat{\gamma}^r \left(\ln G + \ln G^a \right) + e^r \tag{7}$$

where $\hat{\gamma}^r$ is the least squares estimate and e^r denotes the least squares residual. Since this is a *restricted* version of model (6), we denote its estimates with superscript *r*.[11]

On the other hand, the model which includes exclusively the public capital of the region analyzed is equivalent to estimating model (6) imposing that θ is equal to zero. The result of estimating this model is:

$$\ln Y = \hat{\gamma}^s \ln G + e^s \tag{8}$$

where $\hat{\gamma}^s$ is the least squares estimate and e^s denotes the least squares residual. We use superscript s to denote the estimates of this *short* model[12].

[10] We assume throughout the discussion that the parameter estimates $\hat{\gamma}$ and $\hat{\theta}$ are non-negative.

[11] At this point, we have to recognize that, instead of ($\ln G+\ln G^a$), the aggregation used by Mas et al. (1994, 1996) is $\ln(G+G^a)$. The use of $\ln(G+G^a)$ makes the analysis of the spillover effect far more difficult. In the Appendix we prove that the results obtained for the aggregation ($\ln G+\ln G^a$) hold approximately for the aggregation $\ln(G+G^a)$.

[12] Goldberger (1991. p. 183) uses the concepts of "short regression" and "long regression" in the discussion of the effects of omitted variables in regression models.

Now, in order to study the properties of the pseudo-test, we analyze the differences between models (7) and (8) using some well known results on omitted variables in least squares estimation (Goldberger, 1991, p. 184). We first rewrite the benchmark model in expression (6) using as explanatory variables the public capital in the neighboring region ($\ln G^a$) and the effective public capital ($\ln G+\ln G^a$). That is:

$$\ln Y = \hat{\gamma}\left(\ln G + \ln G^a\right) + \left(\hat{\theta} - \hat{\gamma}\right)\ln G^a + e \tag{9}$$

Comparing equations (7) and (9), the parameter estimates in (7) can be seen as the result of omitting the variable $\ln G^a$ from the model in (9). Therefore, the least squares coefficient in (7) can be written as:

$$\hat{\gamma}^r = \hat{\gamma} + \left(\hat{\theta} - \hat{\gamma}\right)\hat{b}_1 \tag{10}$$

where $\hat{b}_1$ is the least squares slope of the regression of the omitted variable ($\ln G^a$) on the included variable ($\ln G+\ln G^a$). That is:

$$\hat{b}_1 = \frac{\operatorname{cov}\left(\ln G^a, \ln G + \ln G^a\right)}{\operatorname{var}\left(\ln G + \ln G^a\right)} \tag{11}$$

where, var(·) and cov(·) refer respectively to the sample variance and covariance of data. Alternatively, expression (11) can be written as:

$$\hat{b}_1 = \frac{\operatorname{cov}\left(\ln G^a, \ln G\right) + \operatorname{var}\left(\ln G^a\right)}{\operatorname{var}\left(\ln G\right) + \operatorname{var}\left(\ln G^a\right) + 2\operatorname{cov}\left(\ln G^a, \ln G\right)} \tag{12}$$

On the other hand, the parameter estimates in (8) can be seen as the result of omitting the variable $\ln G^a$ from model (6). Therefore, the least squares coefficient in (8) can be written as:

$$\hat{\gamma}^s = \hat{\gamma} + \hat{\theta}\hat{b}_2 \tag{13}$$

where $\hat{b}_2$ is the least squares slope of the regression of the omitted variable ($\ln G^a$) on the included variable ($\ln G$). That is:

$$\hat{b}_2 = \frac{\operatorname{cov}\left(\ln G^a, \ln G\right)}{\operatorname{var}\left(\ln G\right)} \tag{14}$$

At this point, we are ready to study the pseudo-test for the existence of spillover effects. Since the "pseudo-test" compares the estimates in models (7) and (8), it is given by:

$$\hat{\gamma}^r - \hat{\gamma}^s = \left(\hat{\theta} - \hat{\gamma}\right)\hat{b}_1 - \hat{\theta}\hat{b}_2 \tag{15}$$

where a positive sign is interpreted as the existence of spillovers.

This test of the existence of spillovers has a number of important shortcomings. First, the test can produce a positive sign in the absence of any spillover, that is, when $\hat{\theta} = 0$. Under this restriction, expression (15) can be written as:

$$\hat{\gamma}^r - \hat{\gamma}^s = -\hat{\gamma}\hat{b}_1 \tag{16}$$

Therefore, in absence of spillovers, the "pseudo-test" can be greater than zero if $\hat{b}_1$<0. Using expression (12), we have that:

$$b_1 \leq 0 \Rightarrow \operatorname{cov}\left(\ln G^a, \ln G\right) \leq -\operatorname{var}\left(\ln G^a\right) \Rightarrow \frac{\operatorname{cov}\left(\ln G^a, \ln G\right)}{\operatorname{var}\left(\ln G^a\right)} \leq -1 \tag{17}$$

Therefore, there are totally plausible correlations between public capital in a region (lnG) and public capital in neighboring regions ($\ln G^a$) under which the "pseudo-test" does not work at all. In fact, in the absence of spillovers ($\hat{\theta} = 0$) the "pseudo-test" provides an accurate measure of spillovers only if $\hat{b}_1$=0. Using expression (17), this condition holds only if the slope of the least squares line between public capital in a region and public capital in a neighboring region is -1. This is a very restrictive condition that requires that the sum of public capital in a region and in the neighboring regions be equal for all the regions in the sample.

Now, we turn to analyze the behavior of the "pseudo-test" when there is a large spillover. For this purpose, we analyze the case in which the spillover is as large as the elasticity of public capital in the region analyzed ($\hat{\theta} = \hat{\gamma}$). In this case, expression (15) becomes:

$$\hat{\gamma}^r - \hat{\gamma}^s = -\hat{\theta}\hat{b}_2 \tag{18}$$

In this case, the "pseudo-test" is positive only if $\hat{b}_2$ is negative. From (14), $\hat{b}_2$ is negative if there is a negative correlation between public capital in a region and public capital in neighboring regions.

Finally, there is a situation where the “pseudo-test” correctly identifies the existence of spillovers (i.e., $\hat{\theta}$ >0). Making $\hat{b}_1$=0 in (15) yields expression (18). Under this restriction and when $\hat{b}_2$<0, the “pseudo-test” is proportional to $\hat{\theta}$, therefore yielding the same result as the true test. These odd conditions assure a reasonable performance of the “pseudo-test”. We believe that a reasonable test for the existence of spillovers should work under more general correlations among the key variables.

In summary, we have shown that the “pseudo-test” is not directly related to the existence of an spillover effect. In fact, for a given level of spillover, the results of the test depend heavily on the correlation between public capital in different regions.

4. Conclusions

This paper shows that the practice of comparing estimates of coefficients of public capital of two models with different specifications of public capital is not a good test of the spatial spillovers of public capital. The result of this test depends on the correlation of public capital in a region and neighboring regions, not on the effects of public capital on output. Therefore, in order to check for the existence of spillover effects it is better to use the test derived by Holtz-Eakin and Schwarz (1995).

References

Álvarez, A., Orea, L. and Fernández, J. (2003), "La productividad de las infraestructuras en España", *Papeles de Economía Española,* 95: 125-136.

Aschauer, D.A. (1989), "Is Public Expenditure Productive?", *Journal of Monetary Economics*, 23(2): 177-200.

Avilés, C. A., Gómez García, R. and Sánchez Maldonado, J. (2003), "Capital público, actividad económica privada y efectos desbordamiento: Un análisis por Comunidades Autónomas de los sectores Industria y Construcción en España", *Hacienda Pública Española*, 165: 25-51.

Boarnet, M.G., (1988), "Spillovers and the locational effects of public infrastructure", *J. of Regional Science*, 38(3): 381-400.

Cantos, P., Gumbau-Albert, M. and Maudos, J. (2002), "Transport Infrastructure and Regional Growth: Evidence of the Spanish Case", WP-EC 2002-27, IVIE.

Case, A.C., Hines, J.R. Jr, and Rosen, H. (1993), "Budget Spillovers and Fiscal Policy Interdependence", *Journal of Public Economics* , 52: 285-307.

Chua, H.B. (1993), *On Spillovers and Convergence*, PhD Thesis, Harvard.

De la Fuente, A. (1996), "Infraestructuras y productividad: Un panorama de la evidencia empírica", *Información Comercial Española*, 757: 25-41.

De la Fuente, A. (2000), "Infrastructure and Productivity: A Survey", Instituto de Análisis Económico, Barcelona (mimeo).

Draper, M. and Herce, J.A. (1994), "Infraestructuras y crecimiento: un panorama", *Revista de Economía Aplicada*, 6: 129-168.

Fernández Fernández, M. and Polo Andrés, C. (2001), "Capital público y productividad privada en España: una panorámica", *Revista Galega de Economía*, 10 (1): 1-28.

Garcia-Mila, T., McGuire, T. and Porter, R.H. (1996), "The Effect of Public Capital in State-level Production Functions", *Review of Economics and Statistics*, 78(1): 177-180.

Gil, C., Pascual, P. and Rapún, M. (1997), "Evaluación del impacto de las infraestructuras en los costes de las regiones españolas", *Cuadernos Aragoneses de Economía*, 7(2): 361-381.

Gil, C., Pascual, P. and Rapún, M. (1998), "Public Capital , Regional Productivity and Spatial Spillovers", DT 98/11, Dpto. de Economía, Univ. Pública de Navarra.

Goldberger, A., (1991), *A course in Econometrics*, Cambridge, Mass:Harvard University Press.

Gramlich, E.M. (1994), "Infrastructure Investment: A Review Essay", *Journal of Economic Literature*, 32: 1176-1196.

Holtz-Eakin, D. (1994)," Public sector capital and the productivity puzzle", *Review of Economics and Statistics*, 76(1): 12-21.

Holtz-Eakin, D and A. Schwartz, (1995), "Spatial Productivity Spillovers from Public Infrastructure: Evidence from State Highways", *National Bureau of Economic Research*, Working Paper 5004.

Martínez, D. (2000), "Is there any relationship between public investment and economic growth in the Spanish regions?", WP 0002/13, Dpto. de Economía Aplicada, Universidad de Jaén.

Mas, M., Maudos, J., Pérez F. and Uriel E. (1994), "Capital público y productividad en las regiones españolas", *Moneda y Crédito,* 198: 163-206.

Mas, M., Maudos, J., Pérez F. and Uriel E. (1996), "Infrastructures and Productivity in the Spanish Regions", *Regional Studies*, 30 (7): 641-649.

Moreno, R. and Trehan, B. (1997), "Location and the Growth of Nations", *Journal of Economic Growth*, 2: 399-418.

Moreno, R., Artis, M. López-Bazo E. and Suriñach J. (1997), "Evidence on the Complex Link Between Infrastructures and Regional Development", *International Journal of Development Planning Literature,* 20: 81-108.

Munnell, A. (1990), "How Does Public Infrastructure affect regional economic performance?", *New England Economic Review* (September/October), 11-32.

Pereira, A. and Roca-Sagalés, O. (2003), "Spillover effects of public capital formation: evidence from the Spanish Regions", *Journal of Urban Economics*, 53: 238-256.

Ratner, J. (1983), "Government capital and the production function for US private output", *Economics Letters*, 13: 213-217.

Sanaú, J. (1997), "La contrastación de la hipótesis del capital público en España: principales aportaciones", *Cuadernos Aragoneses de Economía*, 7 (2): 281-314.

Ying, L.G. (2000), "Measuring the spillover effects: Some Chinese evidence", *Papers in Regional Science*, 79: 75-89.

APPENDIX

The natural logarithm of the sum of public capital in a region and in neighboring regions can be written as:

$$\ln\left(G+G^{a}\right)=\ln\left(e^{\ln G}+e^{\ln G^{a}}\right) \tag{A1}$$

The partial derivatives of this function are:

$$\begin{aligned}\frac{\partial \ln\left(G+G^{a}\right)}{\partial \ln G}&=\frac{e^{\ln G}}{e^{\ln G}+e^{\ln G^{a}}}=\frac{G}{G+G^{a}}\\ \frac{\partial \ln\left(G+G^{a}\right)}{\partial \ln G^{a}}&=\frac{e^{\ln G^{a}}}{e^{\ln G}+e^{\ln G^{a}}}=\frac{G^{a}}{G+G^{a}}\end{aligned} \tag{A2}$$

Now, we can write a first-order Taylor approximation of the function at the point $(G_0,G^a{}_0)$ as:

$$\begin{aligned}\ln\left(G+G^{a}\right)\approx{}&\ln\left(G_0+G_0^{a}\right)-\frac{G_0}{G_0+G_0^{a}}\ln G_0-\frac{G_0^{a}}{G_0+G_0^{a}}\ln G_0^{a}\\ &+\frac{G_0}{G_0+G_0^{a}}\ln G+\frac{G_0^{a}}{G_0+G_0^{a}}\ln G^{a}\end{aligned} \tag{A3}$$

This expression indicates that the log of the sum can be approximated by a weighted sum of logarithms. Evaluating, expression (A3) at the point $(G=1,G^a=1)$, we have that:

$$\ln\left(G+G^{a}\right)\approx\ln 2+0.5\left(\ln G+\ln G^{a}\right) \tag{A4}$$

The approximation at the point $(G=1,G^a=1)$ is totally reasonable. In fact, the elasticities of the model do not change with changes in units. Therefore, it is possible to make the variables close to one by dividing by its geometric mean without affecting the estimation of the elasticities.

The counterpart of expression (7) with the new aggregation of public capital is:

$$\ln Y=\hat{\gamma}^{*}\ln\left(G+G^{a}\right)+e^{*} \tag{A5}$$

In this case, the "pseudo-test" can be written as:

$$\hat{\gamma}^{*} - \hat{\gamma}^{s} \tag{A6}$$

Substituting the approximation in (A4) in (A5), we have that:

$$\ln Y \approx \hat{\gamma}^{*} \ln 2 + \frac{\hat{\gamma}^{*}}{2}\left(\ln G + \ln G^{a}\right) + e^{*} \tag{A7}$$

Comparing, (A7) with (7), we have that:

$$\frac{\hat{\gamma}^{*}}{2} \approx \hat{\gamma}^{r} \tag{A8}$$

The "pseudo-test" can be written as:

$$2\hat{\gamma}^{r} - \hat{\gamma}^{s} = \hat{\gamma}\left(1 - \hat{b}_{1}\right) + \hat{\theta}\left(2\hat{b}_{1} - \hat{b}_{2}\right) \tag{A9}$$

Therefore, with this aggregation of public capital, the test is related not only to the magnitude of the spillover ($\hat{\theta}$) but to the correlation between public capital in a region and its neighbors ($\hat{b}_1$ and $\hat{b}_2$).