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Decomposing Regional Productivity Growth using an Aggregate Production Frontier

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DECOMPOSING REGIONAL PRODUCTIVITY GROWTH USING AN AGGREGATE PRODUCTION FRONTIER

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Abstract: The main contribution of this paper is the implementation of a new model which combines the two parametric approaches most commonly used in the productivity literature: fixed effects and stochastic frontiers. This allows us to discuss whether it is better to use *average* or *frontier* functions to estimate regional productivity. The empirical section uses panel data of Spanish regions over the period 1980-1995. Additionally, we calculate and decompose total factor productivity growth for the Spanish regions.

Key words: Regional productivity, stochastic frontier, fixed effects.

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1. Introduction

The estimation of aggregate production functions is common in regional economics. Regional production functions have been used to study different topics including, among others, the existence of agglomeration economies, the evolution of productivity, the effect of knowledge spillovers and the existence of catching-up to the technological frontier.

One methodological issue that has not been widely discussed in this literature is whether it is best to estimate average production functions (where the random term has zero mean) or frontier production functions (where the random term follows a one-sided distribution). De la Fuente (1998) has questioned the use of stochastic frontiers. Specifically, he contends that by using the frontier method we are assuming that different regions use the same kind of technology in each time period. The most common alternative in the literature is to use the opposite assumption, namely that the efficiency differences are small and uncorrelated with the other explanatory variables (and can therefore be accommodated in the error term), as well as allowing for level differences between the regional production functions which are interpreted as indicators of the level of technological development of each economy. De la Fuente adds: "I think that everybody agrees that the ideal would be to quantify both factors and isolate their respective contribution to the productivity differentials which are observed across regions or countries. The problem is that it is almost impossible to separate these two things."

Therefore, the point is whether we can (separately) identify two unobservable phenomena for each region: "technical characteristics" and "productive efficiency". Under a given set of assumptions, both effects can be identified. In particular, assuming that the technical characteristics are time invariant and hence can be modelled as a fixed effect, efficiency can be modelled, following the stochastic frontier tradition, as a one-sided error component. This model, which was first suggested by Kumbhakar and Hjalmarsson (1993), has not been applied much in the empirical literature, most likely because the estimation by generalized least squares in its original formulation was very complicated. However, Greene (2002) has developed a maximum likelihood estimator which greatly simplifies its estimation. We will refer to this model as a "fixed-effects stochastic frontier" (FESF).

In this paper we estimate a FESF model using a panel data set of 17 Spanish regions over the period 1980-1995. As explained above, the distinguishing feature of this model is that, unlike the typical stochastic frontiers, it incorporates individual effects in the deterministic part. The efficiency term is allowed to vary over time. This specification allows us to test some interesting hypotheses. For example, we test if there are any significant differences across regions in the individual effects and in the efficiency levels. In the case that we cannot reject both hypotheses i.e., the existence of individual effects and efficiency differences, both formulations are in fact badly specified models and a more general model should be employed.

We use the estimation of this model to calculate and decompose total factor productivity (TFP) change for the Spanish regions. Although productivity is most commonly analysed at the firm-level, there is also great interest in analyzing productivity growth at the regional level.[1]

The paper is organized as follows. Section 2 presents an overview of total factor productivity and its decomposition. Section 3 briefly describes the data and in Section 4 we present the empirical model. Section 5 estimates and decomposes TFP for the Spanish regions. Finally, section 6 offers some concluding remarks.

2. TFP decomposition

This section develops the productivity measure as well as its decomposition.[2] Total factor productivity (TFP) can be defined as the ratio between an aggregate output index (Y) and an aggregate input index (X):

[1] The first studies of this kind took place in the early seventies (see Aarberg, 1973) and the literature in this area has been partially summarized by Gerking (1994).

[2] Diewert (1992) describes the theoretical foundations of the productivity measure. Nadiri (1970) summarizes the first advances in this field.

$$TFP = \frac{Y}{X} = \frac{Y(y_1, y_2, ..., y_m)}{X(x_1, x_2, ..., x_n)} \quad (1)$$

In the simple case in which there is only one output and one input, TFP becomes the average factor product:

$$TFP = \frac{y}{x} \quad (2)$$

Taking natural logs in (2) and deriving with respect to time, TFP growth can be written as follows:

$$T\dot{F}P = \dot{y} - \dot{x} \quad (3)$$

where a dot over a variable indicates a growth rate. Productivity growth is therefore measured as the difference between the growth rates of outputs and inputs.

TFP growth can be obtained using several methods including growth accounting, index numbers, the estimation of production functions, or cost functions (see Diewert, 1992 for a survey). In this paper we follow a parametric approach in a primal framework. To do so, we start from a production function that incorporates technical change in a general way:

$$y = f(x_1, ..., x_n, t) \quad (4)$$

Taking logs, differentiating (4) with respect to time and operating we obtain the following well-known expression for output growth:

$$\dot{y} = \sum_{j=1}^{n} e_j \dot{x}_j + TC \quad (5)$$

where e_j is the output-elasticity of input j and TC=∂lnf/∂t measures the contribution of technical change to output growth. In accordance with this, if the elasticities were observable, technical change could be obtained as a residual:

$$TC = \dot{y} - \sum_{j=1}^{n} e_j \dot{x}_j \quad (6)$$

Under the assumption that the first order conditions for profit maximization hold, the marginal product of each factor is equal to its real price. Then, if the factors are paid their marginal products, the production elasticities are equal to the factor revenue shares[3]. That is:

$$e_j = \frac{\partial y}{\partial x_j}\frac{x_j}{y} = \frac{p_j}{p_y}\frac{x_j}{y} \tag{7}$$

Hence, under these conditions the production elasticities in (6) can be replaced by the observed factor shares. By doing so, we can obtain an estimation of the contribution of technical change to output growth.[4]

In expression (6), the output growth not explained by the growth in inputs cannot be interpreted strictly as a change in TFP because a TFP index captures both the technological change effect and the influence of the changes in size when there are no constant returns to scale. However, the right hand side of equation (6) obviously accounts only for technological change. This index could be interpreted as a TFP index if the input weights sum to one. For this reason we modify equation (6) by dividing the weight received by each input by the scale elasticity (e). That is:

$$e = \sum_{j=1}^{n} e_j \quad \Rightarrow \quad \sum_{j=1}^{n} \frac{e_j}{e} = 1 \tag{8}$$

After some algebraic manipulation the following decomposition of productivity growth is obtained:

$$T\dot{F}P = \dot{y} - \sum_{j=1}^{n} \frac{e_j}{e}\dot{x}_j = TC + (e-1)\sum_{j=1}^{n} \frac{e_j}{e}\dot{x}_j \tag{9}$$

The second term on the right-hand side of this equation measures the scale change. If there are constant returns to scale, the scale elasticity will be equal to one (e=1), so the term vanishes. The productivity decomposition into technical and scale changes was first carried out by Denny, Fuss and Waverman (1981).

[3] This also requires competitive markets and efficient allocation of both output and inputs.

[4] Using aggregate data from the US economy for the years 1909-49, Solow (1957) obtained a 1.5% annual rate of technical change.

The following step is to consider the possibility that changes in efficiency could exist. Nishimizu and Page (1982) was the first paper that explicitly incorporated efficiency changes into the productivity measure, though they ignored the scale change[5]. We start from the usual Farrell (output oriented) technical efficiency index, which is defined as the ratio of observed output (y) to potential output (y*):

$$E = \frac{y}{y^*} \quad \Rightarrow \quad \ln E = \ln y - \ln y^* \tag{10}$$

Taking derivatives with respect to time, the efficiency change is obtained as the difference between the observed output change and the potential output change:

$$\dot{E} = \dot{y} - \dot{y}^* \quad \Rightarrow \quad \dot{y} = \dot{y}^* + \dot{E} \tag{11}$$

Therefore, in order to include the efficiency change, we augment TFP change as follows:

$$T\dot{F}P = TC + (e-1)\sum_{j=1}^{n} \frac{e_j}{e} \dot{x}_j + \dot{E} \tag{12}$$

Once a production frontier is estimated and the three productivity components are calculated, the TFP change is obtained as the sum of these components.[6]

3. Data

The empirical model is estimated using panel data for 17 Spanish regions over the period 1980-1995. The dependent variable is Value Added (VA) at market prices, measured in 1986 millions of pesetas. The data have been taken from Cordero and Gayoso (1996). In our analysis there are two inputs: capital and labor. Private capital has been taken from *Instituto Valenciano de Investigaciones Economicas* (IVIE) and is also measured in 1986 millions of pesetas. To consider only productive private capital, residential capital is

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[5] Bauer (1990) extended the Nishimizu and Page (1982) results by adding a scale effect using both primal and dual approaches.

[6] It is important to emphasize that these productivity components are not intended to explain the factors that cause productivity growth. Behind these components there are several variables (R+D expenditure, public capital resources, human capital investment …) which are the true driving forces of productivity change.

subtracted from the aggregate value. In addition, to control for possible cyclical effects we multiply the capital stock by a capacity utilization index, which is common to all regions and reported by the Spanish Statistical Institute. Lastly, labor is measured as the employment level. This data proceeds from the IVIE study "*Capital Humano, series históricas 1964-2001*".

In order to account for differences in the regions' productive structures, we have included a production specialization index as a control variable. This index is defined as follows:

$$SI_i = \sum_{j=1}^{5} \left(\frac{VA_{ji}}{VA_i} - \frac{VA_{jN}}{VA_N} \right)^2 \tag{13}$$

where subscript *j* denotes sector (agriculture, industry, energy, construction and services); *i* represents region; and N indicates that the value refers to Spain. This index is zero when the regional productive structure is equal to the national average and increases with the level of specialization.

4. Empirical model

The empirical model implemented in this study is based on a stochastic frontier function (Aigner, Lovell and Schmidt, 1977) with a Cobb-Douglas specification which assumes Hicks neutral technical change:

$$\ln y_{it} = \alpha + \sum_j \beta_j \ln x_{jit} + \delta_t t + \frac{1}{2}\delta_{tt} t^2 + v_{it} - u_{it} \tag{14}$$

where y_{it} is the production of the region i in year t; x_j, j = 1,2 are the inputs (capital and labor); and t is a time trend. The error is composed of two terms, v and u. v is a symmetric random disturbance which captures the effect of statistical noise and which is assumed to be distributed as a $N(0,\sigma_v^2)$, whereas u is a non-negative random disturbance that captures technical inefficiency and which is assumed to follow a half-normal distribution, that is, $u \sim |N(0,\sigma_u^2)|$. Production frontiers have been estimated using regional data in several papers (e.g., Beeson and Husted, 1989; Brock, 1999; Puig-Junoy, 2001).

In this paper we distinguish between the inefficiency term and other components of the 'unobserved heterogeneity'. We do this by including a set of regional dummy variables which will capture certain characteristics of each region (localization, natural resources or the climate) that affect regional production. These regional characteristics can be assumed 'time invariant' since they have very little or no temporal variability. On the other hand, we have included the specialization index defined in the previous section as an explanatory variable in order to account for the different productive structures of the regions. In this way we hope to control for an important part of the time-varying heterogeneity. Therefore, the model to be estimated can be written as:

$$\ln y_{it} = \alpha_i + \sum_j \beta_j \ln x_{jit} + \gamma \ln z_{it} + \delta_t t + \frac{1}{2}\delta_{tt} t^2 + v_{it} - u_{it} \tag{15}$$

where the α_i are the regional fixed effects and z is the specialization index.

This model, which was first suggested by Kumbhakar and Hjalmarsson (1993), differs from the usual stochastic frontiers because it combines the individual effects with a composed error specification. Since we have included an additional explanatory variable (the specialization index), productivity change is obtained adding the effect of this variable into expression (12):

$$T\dot{F}P = \left[TC + (e-1)\sum_{j=1}^{n} \frac{e_j}{e} \dot{x}_j + \dot{E} \right] + e_z \dot{z} \tag{16}$$

Therefore, TFP change can be decomposed into technical change (TC), scale change (SC), technical efficiency change (EC) and the change in regional productive specialization (ZC). These components of TFP growth are calculated from the production function (15) using discrete-time versions of the continuous-time model (16).I In particular, the following expressions are used:

$$
\begin{aligned}
&TC: \quad TC = \hat{\delta}_t + \hat{\delta}_{tt} t \\
&SC: \quad (e-1)\cdot\sum_{j=1}^{n}\frac{e_j}{e}\dot{x}_j \cong (\hat{\beta}-1)\cdot\sum_{j=1}^{n}\frac{\hat{\beta}_j}{\hat{\beta}}(\ln x_{jit} - \ln x_{jit-1}); \quad \hat{\beta} = \sum_j \hat{\beta}_j \\
&EC: \quad \dot{E} \cong \ln E_{it} - \ln E_{it-1} = -(\hat{u}_{it} - \hat{u}_{it-1}) \\
&ZC: \quad e_z \dot{z}_{it} \cong \hat{\gamma}(\ln z_{it} - \ln z_{it-1})
\end{aligned}
\tag{17}
$$

Technical change is calculated evaluating the production function derivative at period t, while the other three components are approximated through discrete changes, i.e., by taking differences between two consecutive periods. Productivity change can then be calculated by aggregating the different components which appear in (17).

5. Estimation and results

Table 1 presents the maximum likelihood estimates of the stochastic frontier production function in (15). Due to space limitations we omit the individual effects (shown later). All estimated coefficients are significant at the 1% level and they display the expected sign. The hypothesis of constant returns to scale in capital and labor cannot be rejected.

Table 1. Stochastic frontier function estimation

	Parameter	*Coefficient*	*t-ratio*
Capital	β_1	0.1241	17.93
Labor	β_2	0.8793	371.0
Specialization index	γ	0.0740	92.08
Trend	δ_t	0.0309	24.62
Trend x Trend	δ_{tt}	-0.0014	-10.82
σ_u / σ_v	λ	12.31	14.73
	σ_v	25.20	139.5

A positive value is found for the coefficient of the specialization index. This result suggests that higher specialization contributes positively to regional production.[7] Neutral technical change is significant, although this effect is decreasing over time. The significant value obtained for λ, which is equal to the ratio between the standard deviations of inefficiency and statistical noise, indicates that inefficiency explains part of the difference in production across regions which is not accounted for by the explanatory variables. This result allows us to conclude that for this data set the frontier model is better than the standard fixed effects model (which, if inefficiency is the same for all regions, is nested within the frontier model).

Table 2. TFP change by region (%)

Region	*1980 - 1995*	*1981 - 1985*	*1986 - 1990*	*1991 - 1995*
Andalusia	1.17	2.05	0.32	1.14
Aragon	2.19	4.01	1.01	1.55
Asturias	1.95	2.26	0.65	2.95
Balearic Islands	1.22	3.69	-0.40	0.39
Basque Country	1.86	2.97	0.46	2.13
Canary Islands	2.18	5.64	1.03	-0.11
Cantabria	2.61	2.47	3.20	2.15
Castille-La Mancha	1.73	1.21	1.80	2.19
Castille-Leon	2.14	2.91	-0.01	3.52
Catalonia	1.63	2.55	0.41	1.91
Extremadura	2.61	5.13	0.31	2.40
Galicia	1.84	0.20	2.38	2.95
La Rioja	2.67	6.21	-1.80	3.60
Madrid	1.56	2.78	0.87	1.03
Murcia	0.88	0.80	1.20	0.64
Navarre	1.56	2.49	1.49	0.70

[7] Nadiri (1970) points out that if productivity change is not equal in all sectors then this may induce changes in the productive structure. In this case, the specialization index would be endogenous, requiring a different econometric treatment.

Valencia	1.36	3.49	-0.34	0.96
National average	1.83	2.99	0.74	1.77

Table 2 presents the average TFP change over the sample period as well as for three sub-periods. The first column in Table 2 shows that all regions have positive TFP growth during the period analyzed (1980-95). The five regions with the highest growth are (in order of importance) La Rioja, Extremadura, Cantabria, Aragon and the Canary Islands. The remaining columns in this table present TFP growth by sub-periods. It is noteworthy that no region appears among the top five with the highest growth for all three sub-periods. In general, there has been positive growth, although in the period 1986-1990 we found four regions with decreasing productivity. In addition, in the 1991-1995 period the Canary Islands also show a slight decrease. In the last sub-period we can highlight the high growth rates (over 3.5%) in La Rioja and Castille-Leon.

Table 3. TFP decomposition by region (1980-1995 average, in %)

Region	*TFP Change*	*Technical Change*	*Specialization Change*	*Efficiency Change*
Andalusia	1.17	1.76	-0.071	-0.51
Aragon	2.19	1.76	-0.012	0.44
Asturias	1.95	1.76	-1.262	1.45
Balearic Islands	1.22	1.76	-0.079	-0.45
Basque Country	1.86	1.76	-0.211	0.30
Canary Islands	2.18	1.76	-0.019	0.44
Cantabria	2.61	1.76	-0.442	1.28
Castille-La Mancha	1.73	1.76	-0.241	0.21
Castille-Leon	2.14	1.76	0.370	0.003
Catalonia	1.63	1.76	-0.108	-0.02
Extremadura	2.61	1.76	0.036	0.81
Galicia	1.84	1.76	-0.206	0.28
La Rioja	2.67	1.76	0.251	0.65
Madrid	1.56	1.76	-0.289	0.08
Murcia	0.88	1.76	-0.451	-0.43

Navarre	1.56	1.76	0.246	-0.44
Valencia	1.36	1.76	0.116	-0.51
National average	1.83	1.76	-0.13	0.21

In Table 3, TFP change is decomposed according to the definitions given in equation (17). Given that constant returns to scale cannot be rejected, scale change is not computed. Therefore, we only present the estimates for technical change, specialization change and efficiency change. Average technical change during the 1980-95 period is 1.76% and this value is common to all regions due to the empirical specification employed. The productivity change caused by changes in the regional productive structure takes positive values when a region increases its specialization. Given that only five regions increase their specialization, the mean effect is negative (-0.13%) with the reduction reaching to 1.2% in the case of Asturias. The growth in productivity due to technical efficiency change is relatively unimportant (0.21% at the national level), although this component is rather large for some regions such as Asturias (1.4%) and Cantabria (1.2%). It should also be noted that there are six regions with negative technical efficiency change.

An interesting issue is whether there is any relationship between productivity growth and the initial level of TFP. The TFP level with respect to the national average can be calculated according to the following formula:

$$TFP_{it} = (\ln y_{it} - \ln y_{Nt}) - \sum_j \frac{1}{2}(S_{ij} + S_{Nj})(\ln x_{ijt} - \ln x_{Njt}) \qquad (18)$$

where S_{ij} is the cost share of input j in region i. Subscript N (standing for ‘national’) indicates that the arithmetic mean of all regions has been taken, so the value is interpreted as the national average. Given that the costs shares are not known and constant returns to scale cannot be rejected, the S_{ij} are approximated by the output elasticities (e_j), which are common across regions.

Table 4. Initial and final TFP, and individual effects

Region	*TFP Change 80 - 95*	*Initial TFP (1980)*	*Final TFP (1995)*	*Individual Effects*

Andalusia	1.17	105	95	6.80
Aragon	2.19	96	101	6.91
Asturias	1.95	92	96	6.85
Balearic Islands	1.22	113	103	6.82
Basque Country	1.86	118	118	6.90
Canary Islands	2.18	91	97	6.76
Cantabria	2.61	92	103	6.87
Castille-La Mancha	1.73	94	93	6.64
Castille-Leon	2.14	91	95	6.70
Catalonia	1.63	112	108	6.88
Extremadura	2.61	65	77	6.45
Galicia	1.84	71	72	6.49
La Rioja	2.67	117	129	6.95
Madrid	1.56	120	116	6.93
Murcia	0.88	110	95	6.87
Navarre	1.56	113	108	6.86
Valencia	1.36	101	94	6.91

Table 4 shows the TFP levels for 1980 and 1995. These values are expressed as indexes with respect to the national average, which has been set equal to 100. At first sight there is no clear relation between TFP growth and its initial value. Likewise, the third column contains the TFP values for the last year in the sample. It can be observed that the regions with higher initial TFP values are those with higher TFP values in the last period.

The estimated fixed effects are shown in the last column of Table 4. These effects can be interpreted as indicators of persistent technical efficiency.[8] In other words, they capture time-invariant characteristics which allow some regions to systematically produce more than others (regardless of the period considered or input quantity employed). The highest individual effects belong to traditionally wealthier regions, except for Aragon.

[8] Schmidt and Sickles (1984) propose measuring technical efficiency as: $TE_i=\exp(\alpha_i-\max\alpha_j)$.

6. Conclusions

In this paper we have estimated total factor productivity change for 17 Spanish regions between 1980 and 1995. The results show that TFP has increased in all regions during the sample period. The decomposition of TFP growth suggests that technical change is the most important component of productivity change.

The model implemented in this paper incorporates time-invariant individual effects jointly with a composed error specification. For this reason, we refer to it as a "fixed-effects stochastic frontier". The model allows us to split unobserved heterogeneity into two components: "technical characteristics" and "productive efficiency". In our empirical application we have found that both are important elements in explaining the economic performance of Spanish regions. In conclusion, the higher flexibility of this model over the classical fixed-effects or the standard stochastic frontier models makes it a good candidate for empirical applications in regional economics given the considerable amount of unobserved heterogeneity that generally exists across regions.

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