ECONOMIC DISCUSSION PAPERS

Efficiency Series Paper 05/2005

Risk-adjusted Productivity Measurement with Application to Spanish Dairy Farms

Luis Orea y Alan Wall

Departamento de Economía

Universidad de Oviedo

Available online at: www.uniovi.es/economia/edp.htm

UNIVERSIDAD DE OVIEDO

DEPARTAMENTO DE ECONOMÍA

PERMANENT SEMINAR ON EFFICIENCY AND PRODUCTIVITY

RISK-ADJUSTED PRODUCTIVITY MEASUREMENT WITH APPLICATION TO SPANISH DAIRY FARMS

Luis Orea♣ and Alan Wall♦

Efficiency Series Paper 05/2005

Abstract: Production risk has generally not been taken into account when measuring productivity growth and traditional productivity indices thus provide an incomplete picture of producer performance and welfare when risk matters. Using standard concepts from the literature on uncertainty we introduce a measure of total factor productivity growth, defined in physical terms, which takes risk considerations into account. This measure is decomposed in order to identify the relative importance of the factors contributing to risk. The measure is illustrated using data from Spanish dairy farms.

Key words: Production risk, risk aversion, Total Factor Productivity (TFP)

♣ Departamento de Economía, Universidad de Oviedo, Spain.

♦ Corresponding author. Departamento de Economía, Universidad de Oviedo, Spain.
e-mail: awall@uniovi.es

1. Introduction

Production uncertainty and risk attitudes play a crucial role in producer decision making. When output is uncertain, risk-averse producers will take account not only of the expected output to be generated from a given input vector but also of the variability of this output. Thus, risk considerations will be a factor when choosing between production plans and hence will have influenced observed production results. It seems only natural therefore that they be taken into consideration when the production performance of producers is being evaluated. For example, it is well-known from banking and finance that choosing a more risky production plan which provides a higher expected profit may lead to a firm having a lower market value than if it chose a less risky plan with a lower expected profit (see, for instance, Hughes et al., 2000, and references therein). One of the most frequently used methods of measuring production performance is to construct a total factor productivity (TFP) index. However, if production performance is evaluated on the basis of a TFP measure based only on the level of expected output (which is one of the most widely-used methods), a higher measure would tend to be assigned to the risky plan, whereas it is clear that not all observers would agree that this represents better performance. To take an example from agricultural economics, consider the decision of a farmer to use pesticides. This will increase his input use and may not have much of an effect on his expected level of output, so a TFP index based on expected output will show a decrease. However, as the effect of the pesticide will probably be a significant decrease in output variability, the risk-averse producer (and many outside observers) will consider his production situation to be much better.

What these examples illustrate is that productivity measures based solely on the levels of output provide a somewhat incomplete method of evaluating producer performance when producers who are not risk neutral face production uncertainty. While much has been written on the measurement of productivity growth and its decomposition (see Morrison, 1993, Lovell, 1996, and Balk, 2003, for surveys), little attention has been paid in this literature to the role of production risk. Our aim in this paper is to construct a measure of production performance which incorporates production risk, where the impact of risk on performance depends on the producer's risk preferences. Using standard concepts from the literature on uncertainty, we introduce a measure of total factor productivity growth which takes these risk considerations into account and go on to show how the measure can be decomposed in order to identify the relative importance of its constituent factors.

To achieve this objective, we exploit the narrow relationship between productivity measures and welfare, highlighted recently by Hulten (2001) and Basu and Fernald (2002). While the former pointed out that welfare is restricted by the level of productivity (i.e. productivity represents a restriction on welfare maximization), the latter show that productivity itself can be viewed as a welfare measure. Here we use risk preferences to comprise expected output and the risk of output into an aggregated output. In this sense, our extended productivity index forms a welfare-type measure: a higher value of the index represents a higher utility level for the producer.

The paper is organised as follows. In section 2 we revise how total factor productivity growth is decomposed using parametric methods when output risk plays no role. In section 3 we discuss the role that output risk and risk preferences play when interpreting TFP indices as indicators of production performance. An index of TFP growth which takes these risk considerations into account is proposed and decomposed in section 4. Some issues surrounding the empirical implementation of the index are discussed in section 5. In section 6 an empirical application is presented, where the index is calculated and decomposed for a panel of Spanish dairy farms. Section 6 concludes and points to possible extensions of this work.

2. Measuring productivity growth ignoring uncertainty

While a variety of different approaches to productivity growth decomposition such as growth accounting and DEA exist, our focus is on parametric methods. The parametric literature on productivity growth decomposition usually assumes that output is generated by the following stochastic production function:

$$y = F(x,v) = f(x) + v \tag{1}$$

where F(x,v) is the stochastic version of the production function, f(x) is the deterministic part of production, y is the output, $x = (x_1 \ldots . x_K)$ is a vector of K inputs, and v is a random noise term. Under the classical assumptions of strict exogeneity and (conditional) homoskedasticity, we have that:

$$E(v \mid x) = 0 \tag{2}$$

$$Var(v \mid x) = E(v^2 \mid x) = \sigma^2 \tag{3}$$

Though production uncertainty exists, standard practice in productivity growth decomposition is to proceed as if the output generating process were deterministic. This is justified by the assumption in (2), as the effect of the random noise on production change disappears when averaging over time or firms. Thus, the error term and hence uncertainty are ignored. Assuming that there is only one input in order to keep the discussion simple, total factor productivity is measured by the ratio:

$$TFP = \frac{f(x)}{x} = \frac{E(y \mid x)}{x} \tag{4}$$

Therefore, ignoring uncertainty, total factor productivity can be interpreted as an expected average productivity ratio, given (i.e. conditional on) the input level.[1] Differentiating with respect to time and dividing by the productivity level we can express the rate of growth of total factor productivity as:

$$\frac{dTFP/dt}{TFP} = \frac{dE(y \mid x)/dt}{E(y \mid x)} - \frac{dx/dt}{x} \tag{5}$$

or in dot notation

$$\frac{\dot{TFP}}{TFP} = \frac{E(\dot{y} \mid x)}{E(y \mid x)} - \frac{\dot{x}}{x} \tag{6}$$

Total factor productivity growth is thus defined as the rate of growth of expected output minus the rate of growth in the input usage. Hence, changes in TFP capture changes in expected average productivity. Expected average productivity may change due to the effect of non-constant returns to scale when the input level expands over time. We show in Appendix A that (4) can be decomposed as:

[1] Total factor productivity can also be measured in terms of actual output rather than expected output, that is, as the ratio F(x,v)/x, where F(x,v) is the random production function (1). Since this index depends explicitly on the production disturbance, it can be interpreted as an ex post average productivity measure. This measure reflects the “appropriate” productivity when the production disturbance is known. However, in many production problems, firms’ decisions are made ex ante, i.e. before the realization of the production disturbance, so productivity must be defined in terms of conditional expected output. For this reason we prefer to present the classical regression assumptions (2) and (3) using conditional notation instead of the more usual E(v)=0 and Var(v)=σ^2, and we maintain this notation when defining TFP.

$$\frac{\dot{TFP}}{TFP} = (\varepsilon - 1)\frac{\dot{x}}{x} \tag{7}$$

where $\varepsilon = \partial \ln f(\cdot) / \partial \ln x$ is the scale elasticity. The term on the right-hand side measures the contribution of changes in scale efficiency when outputs expand over time (i.e. movements along the production function) and the technology exhibits increasing or decreasing returns to scale. This term depends on the degree of returns to scale, measured as the scale elasticity minus one. Increasing and decreasing scale economies are indicated by a positive value and negative value respectively. Hence, an expansion in input use leads to an increase (decrease) in productivity when increasing (decreasing) returns to scale exist. As illustrated in Figure 1, equation (7) can be easily extended to account the effect of other exogeous variables not treated as traditional inputs, such as, technical change, enviromental variables or regulatory/institutional variables, and multiple inputs can also be easily incorporated (see Appendix A).

3. Incorporating production risk and risk preferences

The value attributed to production performance as expressed by the measure of the total factor productivity growth in (6) is defined exclusively in terms of the growth of the level of expected output, with output risk playing no role. As a measure of performance, (6) effectively assumes that producers are risk neutral - only then can a positive productivity growth according to (6) be unambiguously associated with an improvement in production performance. If producers are not risk neutral, then they will be concerned not only about the effects on expected output but also about risk properties when they choose input levels and/or they consider the adoption of potentially risk-reducing or risk-increasing technologies.[2] A growth in TFP according to (6) may therefore not necessarily be perceived as positive from the perspective of the producer or an observer.[3]

[2] Formally, if production is stochastic then these agents will be concerned not only with the first moment of the output distribution but also with the second and possibly higher moments.

[3] For example, changes in input usage, or technical change, which increase expected output such that TFP rises according to (6) may lead to increases in output risk ("risk-increasing inputs") that makes a risk-averse producer perceive his situation to be worse than before. Equation (6) would therefore overstate the producer's performance (or understate it if the producer were a risk lover).

We illustrate some of the issues outlined in the above discussion using Figure 2, which represents the case of a producer who produces output with a single input, using a constant returns to scale technology where expected output is represented by f(x). From time *t* to time *t+1* the producer increases input use from x_t to x_{t+1}. Conditional on input usage, expected output increases from $E_t(y|x_t)$ to $E_{t+1}(y|x_{t+1})$, moving from point A to point B. According to the TFP index in (6), we see no change in the producer's productivity performance. Now introduce production risk, which will take the form of (conditional) output variability.The input *x* is therefore risk increasing as can be seen by the fact that $var_{t+1}(y|x_{t+1}) = cd > ab = var_t(y|x_t)$.[4] Hence, if the producer is not risk neutral, he will not regard his production performance as being the same in both periods. In particular, if he were risk-averse, he would consider that his performance in terms of productivity in time *t+1* has worsened with respect to time *t*. It could also be the case that the production risk does not change but that risk preferences do. Thus, take the case of a producer who operates at one of the points, say A, in both periods. If he is more risk-averse in time *t+1* then he will consider his production situation to have worsened, whereas a TFP index such as (6) which ignores preferences will not reflect this.

Thus, the measure in (6) is somewhat incomplete when we recognise that producers may not be risk neutral. The question then arises as to how it can be extended to take these factors into account and thereby give a fuller picture of the productivity performance of producers. In general terms, a measure of productivity growth under uncertainty (TFPU) will be a function of a total factor productivity measure under certainty (TFP), an output risk measure (expressed in terms of some function of output variance) and a risk preference measure (based on the parameters of the producer's utility function). The next step therefore is to propose a specific, calculable index which incorporates these features.

4. Decomposition of productivity growth with production risk

[4] Note that input *x* is risk increasing if we define production risk in either absolute terms (as the conditional variance of output) or relative terms (using the coefficient of variation).

Given that risk-averse producers will be concerned not only with expected output but will take account of production risk, the question then is how to adjust expected output so that a more meaningful measure of production performance can be achieved. Assuming that producers maximize the expected utility of profits, this issue can be approached by asking what level of certain output would provide the producer with the same expected utility as that of a stochastic output which has mean E(y|x) but positive variance. The literature on uncertainty provides an answer to this question through the concepts of certainty equivalent and risk premium.

We begin by assuming that producers have access to an input x (which, for notational ease, is assumed again to be a scalar) that gives rise to a conditional distribution of output with expected value E(y|x) and variance σ^2(y|x). When output and input prices are exogenous, expected profit for any level x of the choice variable x is defined as $E(\pi \mid \mathrm{x}) = pE(y \mid \mathrm{x}) - wx$. Thus, an increase in x will contribute to expected profits through E(y|x). However, the choice of x also affects the variability of profits, as the variance of profits is $p^2\sigma^2$(y|x). The increases in the mean and variance of profits will both affect the utility of the producer. The Certainty Equivalent of *profits*, CE(π|x), is the level of riskless profit which would provide the producer with the same level of utility as the expected utility associated with the uncertain profit. Therefore, for any given choice of the input, x, there exists an output level y*(x) such that $\mathrm{CE}(\pi \mid \mathrm{x}) = \mathrm{py}^*(\mathrm{x}) - \mathrm{wx}$. Here, y*(x) represents the Certainty Equivalent *in output terms*, CE(y|x). That is, having chosen x, y*(x) is the amount of certain output that corresponds to a level of profits which generates the same utility as the expected utility of profits when production is stochastic. On the other hand, the Risk Premium in terms of profits, RP(π|x), defined as the difference between the expected profit under uncertainty and its certainty equivalent, represents the amount of profit that the producer would be willing to forego to avoid the risk, i.e. the cost of risk. By definition, having chosen x, $RP(\pi \mid \mathrm{x}) \equiv pE(y \mid \mathrm{x}) - wx - CE(\pi \mid \mathrm{x}) = p \cdot \left[E(y \mid \mathrm{x}) - y^*(x)\right]$. The term in brackets thus represents the amount in terms of output that the producer is willing to pay to avoid the risk arising from the stochastic nature of output. As such, it represents the Risk Premium *in terms of output*, RP(y|x).

Having defined the concepts of risk premium and certainty equivalent in terms of outputs, we now proceed to incorporate these into a measure of productivity performance.[5] In line with the above, the risk premium in output terms RP(y|x) is defined as:

$$RP(y \mid x) = E(y \mid x) - CE(y \mid x) \tag{8}$$

Following Pratt (1964), the risk premium can be approximated as:

$$RP(y \mid x) \approx \tfrac{1}{2}\sigma^2(y \mid x) \cdot r_A \tag{9}$$

where r_A is the Arrow-Pratt measure of absolute risk aversion in output terms.[6] In the analysis which follows, it will prove useful to consider the *proportional* risk premium, i.e. the fraction of output that the producer is willing to forego in order to avoid uncertainty. For a small variance, this is approximated by

$$\frac{RP(y \mid x)}{E(y \mid x)} \approx \tfrac{1}{2} m^2 r_R \tag{10}$$

where $m = \sigma(y \mid x) / E(y \mid x)$ stands for the coefficient of variation and $r_R = r_A \cdot E(y \mid x)$ is the coefficient of *relative* risk aversion.[7] Given that the risk premium measures the cost perceived by the producer of the risk associated with the conditional distribution of output, then expected output should be adjusted accordingly in order to provide a production performance measure under uncertainty. Clearly, the larger the risk premium, and hence the lower the certainty equivalent, the stronger is the negative impact of risk on producers and the worse is his perception of his performance.[8] If the risk premium is zero, on the other hand, then risk plays no role in the producer's valuation and he will be concerned only with the expected output, which in this case coincides with the certainty equivalent.

In line with the above, the appropriate index of TFPU can be expressed as:

$$TFPU = \frac{E(y \mid x) - RP(y \mid x)}{x} = \frac{CE(y \mid x)}{x} \tag{11}$$

[5] The idea of using the certainty equivalent to incorporate production risk into a productivity growth index was also discussed by Buccola (2002), but in a profit function framework

[6] We discuss in Appendix B how to get risk aversion coefficients in output terms from risk aversion coefficients in profit terms that corresponds to profit maximising producers.

[7] Thus, RP(y|x)/E(y|x) is proportional to the square of the coefficient of variation.

[8] From the definition of RP, this increased negative impact may be due to either an increase in risk, an increase in the producer's aversion to risk, or a combination of both factors.

where the substitution of expected output by the certainty equivalent brings with it the incorporation of the degree of risk and risk preferences to our measure. To express this index in detail, we begin with the identity:

$$\frac{CE(y\mid x)}{x} \equiv \frac{E(y\mid x)}{x}\frac{CE(y\mid x)}{E(y\mid x)} \tag{12}$$

Using (8) and substituting for the proportional risk premium from (10)

$$\frac{CE(y\mid x)}{E(y\mid x)} = 1-\tfrac{1}{2}m^2 r_R \tag{13}$$

Thus, substituting (13) into (12), yields the following index of TFP under uncertainty:

$$TFPU = \frac{E(y\mid x)}{x}\cdot\left[1-\tfrac{1}{2}m^2 r_R\right] \tag{14}$$

where the expression in brackets represents the certainty equivalent output expressed as a proportion of expected output. It can be seen that TFPU is expressed in terms of an adjustment to the certainty TFP index, with the direction and magnitude of this adjustment depending on the size of relative risk (as represented by the coefficient of variation m) and the nature of the producer's preferences towards risk. The latter is reflected in the sign and magnitude of the coefficient of relative risk aversion, r_R. As this coefficient can take values greater, less than, or equal to zero whenever the producer is risk-averse, risk-loving, or risk-neutral respectively, the CE expressed as a proportion of expected output can be greater, less than or equal to one, and hence TFPU can be greater, less than or equal to the certainty TFP index.

It is worth emphasizing that equation (14) can still be interpreted as a ratio between output and input levels, i.e. as a traditional average productivity measure. In particular, the numerator in (14) times the expression in brackets can be viewed as an "aggregate" output that incorporates the various facets of the productive process discussed earlier.[9] That is, this "aggregate" output reflects not only the expected output level (like a certainty TFP index) but also the existence of production risk. This risk, in turn, is evaluated taking into account the degree of risk aversion of the producer.[10]

[9] Recall that, in (14), the product of the term in brackets and the numerator is the certainty equivalent output, and hence denotes a level of output.

[10] In accordance with the discussion surrounding Figure 2, in the presence of production uncertainty a producer who is risk-averse and has an expected output $E_t(y|x_t)$ from the input vector x_t will receive a lower valuation under TFPU than he would receive under TFP, reflecting

Changes in (14) will thus accurately reflect changes in producer performance as changes in risk and/or producer preferences towards this risk will be accounted for. It should also be clear that, conditional on input levels, the TFPU index forms a welfare measure in the sense that a higher value of the index corresponds to a higher conditional certainty equivalent of output which in turn, for given input and output prices, corresponds to a higher certainty equivalent in profits. As utility is monotonically increasing in the certainty equivalent of profits, higher values of TFPU therefore correspond to a higher utility level for the producer.

Finally, some words are in order regarding the fact that the index is constructed *conditional on input levels*. The denominator in (14) is the observed input level, so the effect of unobservable inputs is accounted for in the numerator only. However, while inputs are taken as given, it is worth re-emphasizing that the same level of inputs can give rise to different values of the TFPU index through the role of preferences. Take, for example, the case of a risk-neutral producer and another producer who is non risk-neutral but does not know how to reduce (to increase) production risk by adjusting the input levels (we will label this producer “myopic"). Since neither producer takes the risk effect of inputs into account, the optimal input level and, hence, the TFP (growth) are the same for both risk-neutral and myopic producers. However, if the myopic producer is risk-averse (risk-loving), the risk level faced is negatively (positively) valued by the myopic producer, so his TFPU valuation will be lower (higher) than that of the neutral producer.

When producers are neither risk-neutral nor myopic, the input levels chosen by these producers are different from those selected by a neutral or a myopic producer. In this case, the observed production-risk level (measured by the coefficient of variation, m) faced by a risk-averse producer will be lower than that faced by a risk-neutral or a myopic producer. Thus, if the technology exhibits constant returns to scale (i.e. TFP is invariant), the index value of a risk-averse and non-myopic producer is higher than that for a myopic producer that has not managed his inputs in order to control output risk. In this sense, since our TFPU index is constructed conditional on input levels, and these levels have been adjusted, equation (14) give us a risk-adjusted measure of total factor productivity for those producers that have already partially adjusted their inputs levels.

---

the negative influence of risk. In the absence of risk ($m = 0$) or if the producer is risk neutral ($r_R = 0$), the valuation of productive performance is the same.

To analyse TFPU growth, we first express (14) in logarithmic terms:

$$\ln \mathrm{TFPU} = \left[\ln E(y \mid x) - \ln x\right] + \ln\left[1 - \frac{1}{2} m^2 r_R\right] \tag{15}$$

Differentiating (15) with respect to time,

$$\frac{d\ln \mathrm{TFPU}}{dt} = \frac{d\left[\ln E(y \mid x) \quad \ln x\right]}{dt} + \frac{d\ln\left[1 - \frac{1}{2} m^2 r_R\right]}{dt} \tag{16}$$

and substituting from (6), we get:

$$\frac{\dot{TFPU}}{TFPU} = \frac{\dot{TFP}}{TFP} + \frac{d\ln\left[1 - \frac{1}{2} m^2 r_R\right]}{dt} \tag{17}$$

Carrying out the differentiation of the second term on the right-hand side, (17) can be expressed as:

$$\frac{\dot{TFPU}}{TFPU} = \frac{\dot{TFP}}{TFP} - W \cdot \left\{\frac{1}{2}\frac{\dot{r}_R}{r_R} + \frac{\dot{m}}{m}\right\} \tag{18}$$

where $W = r_R m^2 \left(1 - \frac{1}{2} m^2 r_R\right)^{-1}$. The growth of TFPU is thus expressed in terms of an adjustment to TFP growth, where the adjustment takes the form of a term involving the growth of risk and changes in preferences weighted by a factor, W, comprising the initial levels of risk aversion, magnitude of risk, and the expected output as a proportion of the CE.

From (18) it is clear that if the producer is risk neutral ($r_R = 0$), then the second term on the right-hand side disappears and productivity growth under uncertainty and certainty coincide. Moreover, if there is no production risk (m = 0) the certainty and uncertainty measures again coincide. Assume now that the producer is risk-averse so that $r_R > 0$.

Then, for a given production risk (m > 0), an increase in risk aversion ($\dot{r}_R / r_R > 0$) causes the second term on the right-hand side to be negative and $\dot{TFPU}/TFPU < \dot{TFP}/TFP$. An increase in production risk ($\dot{m}/m > 0$) will have the same effect as the second term will again be negative.

The decomposition above is developed in general terms, so in principle it can be applied for any functional form of the stochastic production technology, F(x,v). However, in order to get an estimate of the underlying technology, some restrictions should be imposed on the structural form of the production function, which in turn will allow us to extend the decomposition in (18). One of the more popular forms for F(x,v) is that proposed by Just and Pope (1978). These authors suggested a series of desirable properties that a stochastic production function should have and introduced a production function that accommodates both risk-increasing and risk-decreasing inputs. In particular, input usage can affect both the mean and variance of output, with no *a priori* restrictions placed on the risk effects of inputs (i.e $\partial \operatorname{var}(y)/\partial x_k$ can take any sign).

The Just-Pope production function has the general form:

$$y = f(x) + h(x)^{\frac{1}{2}} \cdot v \tag{19}$$

where again we assume strict exogeneity and conditional homoskedasticity on the random noise term (*v*). Thus, the conditional output standard error can be written as:

$$\sigma(y \mid x) = h(x)^{\frac{1}{2}} \cdot \sigma_v \quad where \quad \sigma_v = \operatorname{var}(v \mid x)^{1/2} \tag{20}$$

Taking logs in (20) and differentiating with respect to time, we get that the increase in (relative) production risk can be decomposed as:

$$\frac{\dot{m}}{m} = (\eta - \varepsilon)\frac{\dot{x}}{x} \tag{21}$$

where $\eta = 0.5 \cdot \partial \ln h(\cdot)/\partial \ln x$ is the elasticity of the conditional output standard deviation with respect to the input.[11] Introducing equations (7) and (21) into (18) we get the *overall* decomposition of total factor productivity growth under production risk:

---

[11] This decomposition can be easily extended to account for multiple inputs and other exogeous variables as:

$$\frac{\dot{m}}{m} = \sum_{k=1}^{K} (\eta_k - \varepsilon_k)\frac{\dot{x}_k}{x_k} + (\eta_z - \varepsilon_z)\dot{z}$$

where η_k and ε_k are respectively the elasticity of *h(·)* and *f(·)* with respect to the *k*th input.

$$\frac{\dot{TFPU}}{TFPU} = (\varepsilon - 1)\frac{\dot{x}}{x} - W \cdot \left\{ \frac{1}{2}\frac{\dot{r}_R}{r_R} + (\eta - \varepsilon)\frac{\dot{x}}{x} \right\} \tag{22}$$

where the contributions of scale effects to both expected output and output risk are made explicit.

Note that input *x* is risk increasing (reducing) in relative terms if η >(<) ε. It is thus made explicit that changes in input usage not only affect expected output (as captured in the traditional productivity growth under certainty) but may also affect the variance (and hence the risk) of output, and a risk-averse producer will take both considerations into account when planning input use.

5. Some estimation issues

As was mentioned before, the application of the proposed decomposition requires imposing some structural restrictions on F(x,v). For instance, since Just and Pope, a separability restriction such as that in (19) is often imposed on F(x,v) in order to distinguish the input's marginal effects on expected output from its impact on production risk (see Love and Buccola, 1991).[12]

Just and Pope suggested estimating the production function (19) in two stages, where the mean function and the variance (risk) function are estimated separately.[13] The procedure involves estimating the mean function, and then fitting the variance function through a regression on residuals. That is, rewrite the production function (19) as:

$$y = f(x) + u \tag{23}$$

[12] Other authors (e.g. Newbery and Stiglitz, 1981) have argued for a production function of the form Y=f(x)·g(z,v), where z is a vector of stochastic inputs, such as rainfall. The main disadvantage of this multiplicative model over the additive model in (19) is that input's marginal effects on variance is a function of its marginal effect on mean, which makes difficult the estimation of both technology and preference parameters.

[13] Both the mean function and the variance function can also be estimated by FGLS and ML techniques (see, for instance, Harvey, 1976 and Saha et al., 1997). The ML estimator provides asymptotically more efficient estimates of the variance function than FGLS. However, a problem with the ML estimator is that parameters often do not converge.

where $u = h(x)^{\frac{1}{2}} \cdot v$. The presence of production risk makes the stochastic term u heteroscedastic. However, the parameters of the mean function can be consistently estimated using just the production function, even without correcting for heteroscedasticity, under the assumption of exogeous regressors.[14] If panel data is available, the mean function equation can be also extended by including a firm-specific effect in order to account for possible endogeneity problems due to the fact that there may be non-controllable and unobservable inputs (e.g. rainfall) that might be correlated with some of the observed inputs (foodstuffs, irrigation,..). In the context of recent developments in production theory where uncertainty is modelled in a state-contingent framework, these firm-specific effects could be interpreted as variables which capture the average state of nature faced by the firm over the period under review.[15]

Given the estimate of f(x) one can obtain the estimated residuals $\hat{u} = y - \hat{f}(x)$. An estimate of the variance can then be obtained from a regression of the logarithm of the square of û on the input set in a second step, that is:

$$\ln(\hat{u}^2) = \ln h(x) + \psi \tag{24}$$

where Ψ is an error term. As in the mean function equation, this specification can be also extended, if panel data is available, by including a firm-specific effect in order to account unobservable variables that might be correlated with the observed inputs.

On the other hand, the application of the proposed decomposition requires not only an estimate of the underlying technology, but also the coefficient of relative risk aversion. The introduction of this coefficient will depend on the perspective from which productivity performance is being evaluated. If the evaluation is being carried out by some interested observer, such as for example a sector's regulator or the managing director of a firm who is evaluating the performance of subsidiaries or branches, then

[14] Such an assumption can be made when firms' decisions are made ex ante, i.e. before the realization of the production disturbance (see Blair and Lusky, 1975). See also Shankar and Nelson (1999) and Love and Buccola (1999) regarding the controversy on consistency of the estimators obtained from (23) alone.
[15] This is similar to the one of the basic models in a recent paper on state-contingent analysis of production risk by O'Donnell and Griffiths (2005) where the effects of changes in the states of nature are captured by (changes in) the intercept term. In a more general model in the same paper, these authors extend their analysis to allow the parameters of the inputs to vary with states of nature.

(an estimate of) only the observer's risk aversion can be used.[16] In this case we can calibrate the theoretical decomposition under different assumptions/conjectures on the degree of risk aversion of producers. Thus, when calculating (22) we impose a common degree of risk aversion on each producer which corresponds to that of the observer.

If we are interested in estimating how the individual producers evaluate their performance, we need estimates of each producers's risk preferences. These preferences can be aproximated using information from other sources, such as interviews, questionnaires, etc. Otherwise, they must be estimated. However, the estimation of the individual's risk preferences requires not only imposing more assumptions (for instance, on producer's behaviour or the risk preference function, quasi-fixed inputs, etc.), but also using more information relevant to firm or household optimal choices (such as input and output prices). This kind of information might be not available in many applications. If data on input and output prices is available, the estimation of firm preferences is often based on a system of input demand equations.

In order to estimate producer risk preferences, we follow Kumbhakar (2002) and Love and Buccola (1991) by assuming that firms maximise the expected utility of anticipated profit, $E[U(\pi)]$ - which is equivalent to maximising the utility of the certainty equivalent of profits - where anticipated profit is normalised by the output price so that $\pi = y - w \cdot x = f(x) + h(x)^{\frac{1}{2}} \cdot v - w \cdot x$, with w representing the input prices relative to output price. The first order conditions of this maximization problem can be written as:

$$\frac{\partial f(x)}{\partial x_k} = \frac{w_k}{P} - \theta(\cdot)\frac{\partial h(x)^{\frac{1}{2}}}{\partial x_k} \qquad \forall\, k = 1,....,K \tag{25}$$

where w_k is the k^{th} input price relative to the output price and $\theta = E[U'(\pi) \cdot v] / E[U'(\pi)]$ is a risk preference function. This function takes values less than, equal to or higher than zero when producers are risk averse, risk neutral or risk loving respectively (see Chambers, 1983). Love and Buccola (1991) show that under negative exponential utility, the risk preference function can be expressed as $\theta = r_A(\mu_\pi) \cdot \sigma_\pi$, where σ_π is

[16] Note that this is equivalent to analysing how the producers themselves would evaluate their performance under the assumption that they all have the same risk preferences.

the standard deviation of profit, and $r_A(\mu_\pi) = U''(\mu_\pi)/U'(\mu_\pi)$ is the Arrow-Pratt coefficient of absolute risk aversion where $\mu_\pi = f(x) - w \cdot x$. Thus, the system of input demands (25) to be estimated is:

$$\frac{\partial f(x)}{\partial x_k} = \frac{w_k}{P} - r_A(\mu_\pi) \cdot h(x)^{\frac{1}{2}} \cdot \frac{\partial h(x)^{\frac{1}{2}}}{\partial x_k} \qquad \forall\, k = 1,....,K \tag{26}$$

This system can be estimated jointly with the Just and Pope stochastic production function (see, for instance, Love and Buccola, 1991; Saha, Shumway, and Talpaz, 1994) or alone, replacing the mean and variance marginal products by their predicted values from the previous estimation of the mean and variance function, (23) and (24). Both approaches have their own relative advantages and disadvantages. While the joint estimation approach yields more efficient (and, if some inputs are endogenous, consistent) estimators, the technological parameter estimates, i.e. the mean and variance function, might be biased if the additional assumptions required by (26) are not satisfied or input and output price information is not believable.

Finally, note that the Arrow-Pratt coefficient of relative risk aversion (RRA) is $r_R(\mu_\pi) = r_A(\mu_\pi) \cdot \mu_\pi$. However, this coefficient is defined in terms of profits whereas in (22) the RRA coefficient (r_R) is defined in terms of output. In Appendix B we show that the RRA coefficient in output terms is the RRA coefficient in profit terms mutliplied by the ratio of expected revenue (μ_{rev}) to expected profits (μ_π). This provides the last component needed to construct the TFPU index.

In the next section we present an empirical application where we compare the growth of TFPU with that of TFP ignoring risk. Clearly, the application of the proposed decomposition requires estimates of the coefficient of relative risk aversion. While the joint approach, though more involved, we have chosen the latter perspective to avoid contaminating the technological parameter estimates by introducing extra assumptions.

6. Empirical illustration: Evidence from Spanish dairy farms

In this section we calculate and compare TFPU growth with TFP growth using a panel data set of 71 dairy farms from Asturias, a northern region of Spain and a principal producer of milk, covering the years 1993 to 1998. This dataset has several advantages for our purposes. Firstly, we can exploit the panel structure of the data in order to account for unobservable variables that might be correlated with the observed inputs. Secondly, we can compare our estimated technology with that obtained in previous papers dealing with the same sector in the same region. Thirdly, the output price and some input prices are available, so not only can we estimate the technology but also producers' risk preferences. Fourthly, even though farms' milk production is restricted by production quotas it shows a high variability, as suggested by the highly changeable annual milk per cow growth rates depicted in Figure 3, and the imprecise relationship between the partial productivities, as illustrated in Figure 4 for the case of feedstuffs. Finally, this industry is highly endebted, so revenue variability as a consequence of output risk might endanger farms' bank repayments.

To calculate the TFPU and TFP indices, a Just-Pope production function is estimated in two stages. The procedure involves estimating the mean function (23), and then fitting the variance function (24) through a regression on residuals. Functional forms for the mean function, f(·), and the variance function, h(·), must be chosen and we choose a linear quadratic form for the mean function and a restricted linear quadratic form for the (logaritmth of the) variance function,[17] where the variance function is given an exponential specification following Harvey (1976) in order to ensure positive output variances. In particular, the mean function to be estimated takes the form:

$$f(x_{it}, z_t, \beta) = \beta_{0i} + \sum_{k=1}^{K} \beta_k x_{kit} + \frac{1}{2} \sum_{k=1}^{K} \sum_{h=1}^{K} \beta_{kh} x_{kit} x_{hit} + \sum_{\tau=2}^{T} \beta_\tau z_{\tau t} \tag{26}$$

while the variance function can be expressed in log form as:

$$\ln h(x_{it}, z_t, \alpha) = \alpha_{0i} + \sum_{k=1}^{K} \alpha_k x_{kit} + \frac{1}{2} \sum_{k=1}^{K} \sum_{h=1}^{K} \alpha_{kh} x_{kit} x_{hit} + \sum_{\tau=2}^{T} \alpha_\tau z_{\tau t} \tag{27}$$

where the z_t vector is a set of (T-1) time-dummy variables (i.e. $z_{\tau t}=1$ if $\tau=t$ and $z_{\tau t}=0$ otherwise), which are included to capture time effects not accounted for by other regressors, and the firm-specific effects β_{0i} and α_{0i} are included respectively in the mean function and variance function in order to account for possible non-controlable and unobservable inputs, which we assume invariant over time. The following inputs

[17] Tveteras (2000) estimates Just-Pope production functions with linear quadratic, generalized Leontief and translog mean functions and shows that the estimates are not dramatically affected by the choice of functional form.

are considered: cattle (COWS), measured as the number of cows; chemical feedstuffs (FEED), measured in thousand of kilos; veterinary and medical expenses (VET); and artificial fertilizer (FERT), where the latter two variables are measured in monetary terms (in thousands of 1998 Euros).[18]

The results of the estimation are shown in Table 1. Although our main interest in the parameter values is to use them to calculate the input elasticities, some comments on the estimates are in order. We first carried out an *F* test for the joint significance of the firm effects and there was strong support for the existence of a firm-specific effect in the data. Given the evidence in favour of a firm-specific effect, the question arises as to how to estimate these firm-specific effects. We carried out a Hausman test to check the hypothesis that the individual effects are uncorrelated with the observed inputs and this hypothesis was rejected. On the basis of this, the estimation of the mean function was carried out using a WITHIN estimator. Regarding the variance function, an *F* test could not reject the existence of a common constant in the data so a pooled OLS estimator was used. As the R^2 statistic could be considered to be relatively low, we carried out an *F* test for the joint significance of the inputs. The hypothesis that the inputs are jointly insignificant was strongly rejected.

The value of the inputs is expressed as deviations with respect to the arithmetic sample mean, so the first order coefficients can be interpreted as marginal productivities evaluated at that point. The estimates corresponding to the mean function show that it is well-behaved at the sample mean in that concavity in all inputs is complied with: all the first-order coefficients are positive and significant, and all the second-order coefficients are negative and significant, with the exception of fertilizer and cows for which a zero value cannot be rejected. The estimates show that cattle increase output variability. This result is to be expected as our data show that labour is invariant, so an increase in the number of cows would mean that managerial time is distributed over more animals, leading to a greater variability in milk production. Concentrate feed also increases variability. Veterinary expenses has a negative coefficient and thus reduce variability.

[18] Note that land and labour do not appear as inputs. While data were available for these inputs for each production unit, they were almost invariant over time (and, for the labour input, also across firms), so their effect on the $f(\cdot)$ and $h(\cdot)$ functions is captured through the firm-specific effects. Moreover, since both inputs were virtually invariant over time, they do not contribute to productivity change.

The average elasticities of the inputs in the mean and variance functions over the period, calculated on the basis of these estimates, are shown in Table 2. The sum of the mean function elasticities yields a scale elasticity of 1.005, implying the existence on average of constant returns to scale. The magnitude of these returns is slightly lower than that found in other empirical studies of dairy farms.[19] Taking into account that the contributions of land and labour to RTS are excluded due to the fact that they are captured by the individual effects, our RTS are likely underestimated. However, our estimate of returns to scale is still comparable to those obtained in previous work on this regional sector. For example, Cuesta (2000) cannot reject constant returns to scale, while Álvarez and González (1999) and Orea, Roibás and Wall (2004) find evidence of moderate but significant increasing returns to scale.

Regarding the variance function, it can be seen that fertilizer is the unique risk-reducing input while feedstuff and cows are the most risk-increasing in that they increase the variance of output. However, given that we define risk in *relative* terms (through the coefficient of variation), the effect of increases in the inputs on the TFPU index will depend on the relative magnitudes of the mean and standard deviation elasticities as well as on the sign of the standard deviation elasticity. The fact that veterinary expenditure has almost no effect on expected output but it has a negative effect on the variance function implies that increases in this input will tend to increase the TFPU indices for risk-averse producers. Cows also increases the variance function but this effect is smaller than that on the mean function, so increases in cows reduce the coefficient of variation and thus would increase the TFPU indices for risk-averse producers. Increases in the other two inputs, feed and fertilizer, on the other hand would have little effect on TFPU as their effect on both the mean and standard deviation function increase are quite similar.

The final column shows the overall average effects of a scale increase. An increase in scale will lead to a greater proportional increase in the mean function compared with the variance function, so the riskiness of output will decrease. From this we can draw some policy implications. For example, in the EU there are voluntary abandonment schemes and similar type measures to encourage increases in farm size. These

[19] For example, Ahmad and Bravo-Ureta (1996) estimate a variety of different specifications for production functions for US dairy farms and find scale elasticities ranging from 1.04 to 1.15.

measures have been promoted with the aim of increasing average productivity. To evaluate such policies, not only the effect on average productivity (which is expected to be moderate) should be taken into account, but also their effect on risk. In the case the sector we study, increases in farm size will lead to additional gains in the form of a reduction in output risk, thus providing further justification for promoting policies to increase scale.

The magnitude of the overall impact on TFPU from changes in input use will also depend on the degree of risk aversion, which, along with the magnitudes of the changes in the inputs, is the final component we need in order to calculate the TFPU indices. To incorporate risk preferences to our productivity measure, we first choose the simplest possible empirical specification of (26) to estimate risk preferences and then we aggregate these into two groups, "low risk aversion" and "moderate risk aversion". In particular, we have estimated the system of equations (26),assuming that $r_A(\mu_\pi)$ is common to all firms and invariant over time. Thus, we assume constant absolute risk aversion, which in turn implies increasing relative risk aversion.

We have obtained each farm's coefficient of absolute risk aversion, $r_A(\mu_\pi)$ by estimating the system of equations (26),[20] where the mean and variance marginal products are replaced by their predicted values from the prior estimation of the mean and variance functions. This replacement induces, however, a measurement error bias that tends to reduce the value of the estimated $r_A(\mu_\pi)$ if this parameter is positive (i.e. if farms are risk averse). Hence, here we will get a lower bound for the risk aversion coefficient. The system (26) was estimated by three-stage least squares (3SLS) and the estimated $r_A(\mu_\pi)$ was 0.551 with a standard error of 0.068 (yielding a t-statistic of 8.079).

The relative risk aversion coefficients $r_R(\mu_\pi)$ associated with the risk preference function $\theta(\cdot)$ are depicted in Figure 5. The coefficient of relative risk aversion varies from 0.20 to 8.94, with a mean estimate of 2.88. These values are similar to that found in the literature on risk aversion estimation in agriculture, where the relative risk aversion

[20] In order to estimate (26) we have considered three inputs as variable inputs: feedstuffs, veterinary and fertilizer expenses. Since the latter two variables are measured in monetary terms, their prices are normalized to be equal to one. All prices are expressed in real terms.

varies from 0 to over 7.5, but with a median estimate of close to 1 (see Chavas and Holt, 1996).[21]

The average relative risk aversion coefficient of the first group ($r_R(\mu_\pi)$ = 1.70) and second group ($r_R(\mu_\pi)$ = 4.03) represent low and moderate risk aversion respectively. To convert these into relative risk aversion coefficients in terms of ouput, we have to multiply these figures by the ratio of expected revenue to expected profits as mentioned above and outlined in Appendix B. Using overall sectoral data we find that the average value of this ratio is 1.64[22]. Thus, low and moderate risk aversion in output terms will be represented by r_R = 2.79, and r_R = 6.61 respectively.

The average TFPU rates of growth for the five annual periods corresponding to these two degrees of risk aversion are shown in Table 3, which also presents the TFP rates of growth as well as the changes in inputs and the contribution of these inputs and temporal effects on productivity growth. In order to implement the decomposition of TFPU, a discrete approximation to (22) has been made.

While the absolute values of productivity change are relative small for the sector as a whole over the period studied, the results presented in Table 3 clearly show how the incorporation of risk preferences can affect the evaluation of a sector's productivity. The first block of Table 3 shows how inputs have changed over the period, and it can be seen that farms have expanded over time in that, with the exception of 1996/97 and the use of fertilizer in 1994/95, all inputs have risen for all years. The next block shows the evolution of average TFP growth. Two aspects here are worth highlighting. Firstly, the scale effect, which captures the effect of input growth on TFP growth, is relatively small. This is to be expected given that we have constant constant returns to scale. The exception is the final year, where the scale effect is much larger. This latter result is influenced by large changes in input use by a few farms which had significntly strong increasing returns to scale. Secondly, the temporal effect (loosely speaking, “technical change”), is generally much stronger than the scale efect. Finally, we note that average

[21] See also Table 2 in Saha et al (1994) for estimates of relative risk aversion coefficients from a variety of applied risk studies.

[22] Since many inputs in the sector under study can be considered quasi-fixed (see, for example, Maietta, 2000 and Reinhard and Thijssen, 2000), this ratio is obtained using variable profits rather than total profits as a denominator.

expected TFP growth was positive in all years except 1995/96, where there was a strong negative temporal effect.

Given our assumption that risk preferences are invariant over time, the difference between the growths of TFP and TFPU basically depend on the the magnitude of the changes in output risk. The changes in output risk, measured by changes in the coefficient of variation, are shown in the table and have also been decomposed into a scale effect and a temporal effect. Except for the the year 1997/98, these two effects go in the same direction and thus reinforce each other. Again, the temporal effect is generally much higher. As is to be expected, in years when inputs expand, output risk as measured by the coefficient of variation falls. Furthermore, we observe large increases and descreases in output risk over the period analysed, ranging from decreases of over 20% in the first two years to an increase of over 46% in the year 1996/97.

Given the large changes in output risk experienced by producers, their perception of productivity performance will vary substantially from that implied by changes in TFP when they are risk averse. This is particulary well illustrated in the values of productivity change from 1996 to 1997. In this period, the TFP index, which ignores risk preferences, shows that productivity increased by 1.52%. However, the TFPU index shows that a moderately risk-averse observer would consider that productivity performance has actually worsened, with a change in the TFPU index of –0.06. Similarly, from 1997 to 1998, there was a small improvement in productivity performance according to the TFP index (an increase of 0.47%), whereas risk-averse observers would consider that productivity performance has worsened as illustrated bu the negative values of the TFPU index. The indices thus give very different pictures of productivity performance.

Closer inspection of Table 3 provides explanations as to why the risk situation of producers changes from one period to the next. For example, in the period from 1996 to 1997, there was a positive shock to production captured by the time dummy for 1997 in the production function which produces an increase in TFP. However, at the same time the large value of the dummy variable for 1997 in the variance function reflects a shock to output variability which drives a large increase in the coefficient of variation of output, thus considerably worsening the risk situation of producers.

Overall, these results show that production risk can be significantly affected by scale (input) effects and exogenous (temporal) effects. This in turn will affect the evaluation of production performance, and therefore highlights the importance of accounting for risk and risk preferences in estimating and decomposing total factor productivity growth.

7. Conclusions

Focusing on physical production, in this paper we extend previous work on productivity measurement in order to incorporate the impact of risk on production performance. In this context, we outline certain desirable characteristics that we believe an index of total factor productivity under uncertainty should have if it is to capture the impact of risk on the production performance perceived by the producer. Drawing on familiar concepts from the literature on the measurement of risk aversion, a total factor productivity index under uncertainty is proposed. This index depends on input and output levels and risk preferences, and is decomposed in order to isolate the contributions of changes in scale (inputs) and in non-controllable variables (e.g. technical change) on both the expected and risky facets of production process. We conclude with an empirical application to dairy farms in Spain which illustrates how the incorporation of production risk, and risk preferences, provides a different picture of the sector's average production performance compared with that provided by a productivity index which uses only levels of output.

The fact that the picture of a sector's production performance can substantially change when uncertainty is incorporated into the analysis has important policy implications. Care must be taken when considering or evaluating the impact of policy measures which affect production risk. If such measures are in place, use of a traditional productivity index based solely on levels of output may well show gains in productivity, whereas the picture may be very different if the policy measure has affected the variability of output as well. These measures may include promoting the introduction of new technologies; policy measures which affect scale by promoting increases in the average size of firms, such as voluntary abandonment schemes in agriculture; or measures which promote the substitution of certain inputs by others, such as replacing pesticides with other less polluting inputs. Incorporation of the risk effects of a policy measure and the attitudes towards risk of the agents to be affected is crucial for a complete evaluation of the policy.

There are several directions in which the research in this paper can be extended. The first is to allow for multiple outputs. This could be done by exploiting some of the recent primal representations introduced in the literature such as output-oriented distance functions (see Färe and Primont, 1995; Coelli and Perelman, 2000; Orea, 2002), and would allow us to study the risk implications of joint production. Second, our focus is on measuring physical production performance. Although this has the advantage that our productivity measure can be applied to any producer regardless of its final objectives (profit maximization, market share, social welfare maximization, etc), it is limiting in that the role of prices and price uncertainty are ignored (see Chambers, 1983, for a discussion of productivity and scale elasticities under output price uncertainty). The third related extension is to explicitly accommodate our model to the literature on the state-contingent treatment of production risk (see Chambers and Quiggin, 2000). The empirical side to this literature is still at a relatively incipient stage but is proving to be an exciting line of research in this field.

References

Ahmad, M. and Bravo-Ureta, B.E., 1996. 'Dairy farm technical efficiency measures using panel data and alternative model specifications', *Journal of Productivity Analysis*, 7, 399-415.

Álvarez, A. and González, E., 1999. 'Using cross-section data to adjust technical efficiency indexes estimates with panel data', *American Journal of Agricultural Economics*, 81, 894-901.

Anderson, J. R. and Dillon, J.L., 1992. 'Risk analysis in dryland farming systems', in *Farming Systems Management Series*, No. 2, Rome: FAO,.

Balk, B.M., 2003. 'The residual: on monitoring and benchmarking firms, industries, and economies with respect to productivity', *Journal of Productivity Analysis*, 20, 5-47.

Basu, S. and Fernald, J.G., 2002. 'Aggregate productivity and aggregate technology', *European Economic Review*, 46, 963-991.

Blair, R. D., and Lusky, R., 1975. 'A note on the influence of uncertainty on estimation of production function models', *Journal of Econometrics*, 3, 391-394.

Buccola, S. T., 2002. 'Productivity Growth when output is risky', Paper presented at the North American Productivity Workshop II, Schenectady, NY, June 20 - 22, 2002.

Chambers, R. C., 1983. 'Scale and productivity measurement under risk', *American Economic Review*, 73, 802-805

Chambers, R.C. and Quiggin, J., 2000. *Uncertainty, production, choice, and agency : the state-contingent approach*, New York: Cambridge University Press.

Chavas, J.P. and Holt, M.T., 1976. 'Economic behavior under uncertainty: a joint analysis of risk preferences and technology', *Review of Economics and Statistics*, 76, 329-335.

Cuesta, R.A., 2000. 'A production model with firm-specific temporal variation in technical efficiency: with application to Spanish dairy farms', *Journal of Productivity Analysis*, 13, 139-158.

Coelli, T.J. and Perelman, S., 2000. 'Technical efficiency of european railways: a distance function approach', *Applied Economics*, 32, 1967-1976.

Färe, R. and Primont, D., 1995. *Multi-output production and duality: theory and applications*, Boston/London/Dordrecht: Kluwer Academic Publishers.

Harvey, A. C., 1976. 'Estimating regression models with multiplicative heteroscedasticity', *Econometrica*, 44 (3), 461-465.

Hulten, C.R., 2001. 'Total factor productivity: a short biography', in C.R. Hulten, E.R. Dean and M.J. Harper (eds.), *New developments in productivity analysis, studies in income and wealth, volume 63*. Chicago and London: the University of Chicago Press.

Just, R. E. and Pope, R.D., 1978. 'Stochastic specification of production functions and economic implications', *Journal of Econometrics*, 7, 67-86.

Kumbhakar, S. C., 2002. 'Specification and estimation of production risk, risk preferences and technical efficiency', *American Journal of Agricultural Economics*, 84 (1), 8-22.

Love, H.A. and Buccola, S.T., 1991. 'Joint risk preference-technology estimation with a primal system', *American Journal of Agricultural Economics*, 73, 765-774

Love, H.A. and Buccola, S.T., 1999. 'Joint risk preference-technology estimation with a primal system: reply', *American Journal of Agricultural Economics*, 73, 765-74.

Lovell, C.A.K., 1996. 'Applying efficiency measurement techniques to measurement of productivity change', *Journal of Productivity Analysis*, 7, 329-340.

Maietta, O.W., 2000. 'The decomposition of cost efficiency into technical and allocative components with panel data of Italian farms', *European Review of Agricultural Economics*, 27 (4), 473-495.

Morrison, C.J., 1993. *A Microeconomic Approach to the Measurement of Economic Performance: Productivity Growth, Capacity Utilization and Related Performance Indicators*, New York: Springer-Verlag,).

Newbery, D.M.G. and Stiglitz J.E., 1981. *The Theory of Commodity Price Stabilization: A Study in the Economics of Risk*, New York: Oxford University Press.

O'Donnell, C.J. and Griffiths W., 2005. 'Estimating State-Contingent Production Frontiers', *American Journal of Agricultural Economics* (forthcoming).

Orea, L., 2002. 'Parametric decomposition of a generalized malmquist productivity index', *Journal of Productivity Analysis*, 18, 5-22.

Orea, L., Roibás, D. and Wall, A., 2004. 'Choosing the technical efficiency orientation to analyze firms' technology: a model selection approach', *Journal of Productivity Analysis*, 22, 51-71.

Pratt, J. W., 1964. 'Risk aversion in the small and in the large', *Econometrica*, 32, 122-136.

Reinhard, S. and Thijssen, G., 2000. 'Nitrogen efficiency of Dutch dairy farms: a shadow cost system approach, *European Review of Agricultural Economics*, 27 (2), 167-186.

Saha, A., Shumway, C.R. and Talpaz, H., 1994. 'Joint estimation of risk preference structure and technology using expo-power utility', *American Journal of Agricultural Economics*, 76, 173-184.

Shankar, B., and Nelson, C. H., 1999. 'Joint risk preference-technology estimation with a primal system: comment', *American Journal of Agricultural Economics*, 81, 241-244.

Tveterås, R., 2000. 'Flexible panel data models for risky production technologies with an application to salmon aquaculture', *Econometric Reviews*, 19 (3), 367-389.

Appendix A. Decomposing TFP growth under certainty

As noted in Section 1, the standard framework for estimating (decomposing) productivity change under certainty is derived from the deterministic production function:

$$y = f(x, z) \tag{A.1}$$

where x is the input vector and z is a vector of exogenous variables not treated as a traditional input variable. The vector *z* often includes, for instance, a simple time trend as an indicators of the technological level, a set of firm-specific dummy variables, enviromental variables or regulatory/institutional variables.

As customary, a total factor productivity index can be obtained by logarithmically differentiating (A.1) to obtain

$$\frac{\dot{y}}{y} = \sum_{k=1}^{K} \varepsilon_k \frac{\dot{x}_k}{x_k} + \varepsilon_z \dot{z} \tag{A.2}$$

where $\varepsilon_k = \partial \ln f(\cdot) / \partial \ln x_k$ is the elasticity of output with respect to input *k* and $\varepsilon_z = \partial \ln f(\cdot) / \partial z$ is the rate of growth of output associated with changes in z. Taking into account that

$$\frac{\dot{y}}{y} = \frac{f(\dot{x}, z)}{f(x, z)} = \frac{E(\dot{y} \mid x, z)}{E(y \mid x, z)}, \tag{A.3}$$

we can rewrite expression (A.2) as

$$\frac{E(\dot{y} \mid x, z)}{E(y \mid x, z)} - \sum_{k=1}^{K} \varepsilon_k \frac{\dot{x}_k}{x_k} = \varepsilon_z \dot{z} \tag{A.4}$$

The left-hand side can be viewed as an index of total factor productivity, defined as the difference between the growth of (expected) output and the weighted average rates of growth of inputs. Using input elasticities as weights for aggregating the rate of growth of inputs, (A.4) measures the effect of changes in the environmental variable z on (expected) output. The expression above can be extended to allow for the effect of non-constant returns to scale. This can be accomplished by aggregating the growth of inputs using input elasticities shares rather than input elasticities. Defining the elasticitiy share of the *k*th input, e_k, as

$$e_k = \frac{\varepsilon_k}{\varepsilon} = \frac{\varepsilon_k}{\sum_{k=1}^{K} \varepsilon_k} \tag{A.5}$$

total factor productivity growth in (6) can be rewritten in the case of multiple inputs as:

$$\frac{\dot{TFP}}{TFP} = \frac{\dot{E}(y \mid x, z)}{E(y \mid x, z)} - \sum_{k=1}^{K} e_k \frac{\dot{x}_k}{x_k} \tag{A.6}$$

Using expression (A.2), and after some manipulation, equation (A.6) can then be decomposed into two terms:

$$\frac{\dot{TFP}}{TFP} = (\varepsilon - 1)\sum_{k=1}^{K} e_k \frac{\dot{x}_k}{x_k} + \varepsilon_z \dot{z} = (\varepsilon - 1)\frac{\dot{x}}{x} + \varepsilon_z \dot{z} \tag{A.7}$$

Note, finally, that the TFP growth rate in (A.7) collapses to

$$\frac{\dot{TFP}}{TFP} = (\varepsilon - 1)\frac{\dot{x}}{x} \tag{A.8}$$

when output is produced using a unique input and the environmental variables do not exist or they are time-invariant.

Appendix B. Relationship between coefficients of risk aversion in terms of output and profits

Let us assume that firms select inputs in order to maximize expected utility of profit. The appropriate profit depends on the existence of quasi-fixed inputs. If all inputs in the x vector are variable, profit is defined in the long-run as $\pi = Py - w \cdot x$. In the presence of quasi-fixed inputs, total cost ($w \cdot x$) is replaced by variable cost and profit (π) will represent short-run variable profits. Hereafter, we will assume that all inputs are variable, so the firm's objective can be written as

$$\max_{x} \ \mathrm{E}[\mathrm{U}(\pi)] \tag{B.1}$$

where $\pi = Py - w \cdot x$, $y = F(x,z,v)$ is the stochastic output, P is the output price, *z* is a vector of exogeous variables not treated as traditional inputs, x and w are vectors of inputs quantities and input prices respectively, and *v* is a random noise term.

The problem (B.1) is the same as maximizing U[CE(π)], i.e. the utility of the certainty equivalent of profits, which can be defined as:

$$\mathrm{CE}(ð) = \mathrm{E}(ð) \ \ \mathrm{RP}(ð) \tag{B.2}$$

where RP(π) is the risk premium, which in turn can be expressed as

$$RP(\pi) = \frac{\sigma_\pi^2 r_A(\mu_\pi)}{2} \tag{B.3}$$

where $r_A(\mu_\pi)$ is the Arrow-Pratt coefficient of absolute risk aversion in terms of profits.

Now we assume that output and input prices are non-stochastic, and the input quantities selected by firms are given. In this case, the variance of profits can be written as

$$\sigma_\pi^2 = Var(\pi \mid P, w, x, z) = P^2 \sigma^2 (y \mid x, z) \tag{B.4}$$

and the expected level of profits as

$$\mu_\pi = E(\pi \mid P, w, x, z) = \mu_{rev} - w \cdot x \tag{B.5}$$

where $\mu_{rev} = P \cdot E(y \mid x, z)$ represents expected revenue.

Using (B.2)-(B.5), the certainty equivalent of profit for a given level of input can be expressed as:

$$CE(\pi \mid P, w, x, z) = \mu_{rev} - w \cdot x - RP(\pi \mid P, w, x, z) \tag{B.6}$$

where

$$RP(\pi \mid P, w, x, z) = \frac{1}{2} P^2 \sigma^2 (y \mid x, z) \cdot r_A(\mu_\pi) \tag{B.7}$$

Thus, the certainty equivalent of profit for a given level of input will depend on the expected level of output and the variance of output.

In order to get the certainty equivalent in terms of output, which is what enters into our productivity growth decomposition, we concentrate on the risk premium. The risk premium associated with profit is the amount of profit that the producer is willing to forego in order to avoid the profit risk arising from production risk. For a given input level and given input and output prices, the risk premium in profits can be written as

$$RP(\pi \mid P, w, x, z) = P \cdot RP(y \mid x, z) \tag{B.8a}$$

or equivalently

$$RP(y \mid x, z) = \frac{RP(\pi \mid P, w, x, z)}{P} \tag{B.8b}$$

where RP(y|x,z) is the output-counterpart of the risk premium in profits. This risk premium measured in output terms can be interpreted as the output reduction corresponding to the profit that a risk-averse producer is willling to forego in order to avoid the uncertainty associated with output, given an input level.

Next we can follow Pratt (1964) and express the output risk premium (B.8b) as

$$RP(y \mid x, z) = \frac{1}{2} \sigma^2 (y \mid x, z) \cdot r_A \tag{B.9}$$

where r_A is the equivalent coefficient in output terms that has to be inferred from the coefficient of absolute risk aversion in profits. Substituting (B.7) and (B.9) in (B.8b) we get

$$r_A = P \cdot r_A(\mu_\pi) \quad \text{(B.10)}$$

Given mean profit μ_π defined in (B.5), the relative risk aversion coefficient in profits can be written as

$$r_R(\mu_\pi) = r_A(\mu_\pi) \cdot \mu_\pi \quad \text{(B.11)}$$

Multiplying both sides of (B.10) by expected output and rearranging using (B.11), we get the equivalent coefficient in output terms, r_R, of the coefficient of relative risk aversion in profits, $r_R(\mu_\pi)$, that is:

$$r_R = r_A \cdot E(y \mid x, z) = r_R(\mu_\pi) \cdot \left[\frac{\mu_{rev}}{\mu_\pi} \right] \quad \text{(B.12)}$$

Thus, the relative risk aversion coefficient in terms of output is the relative risk aversion coefficient in terms of profit multiplied by the ratio of expected revenue to expected profits. In particular, (B.12) shows that $r_R > r_R(\mu_\pi)$, with the magnitude of this difference depending on the ratio of total revenue to profit. Alternatively, the lower the ratio of total costs to total revenue, the greater the size of r_R relative to $r_R(\mu_\pi)$.

To summarize, if data on output prices are available then the relationship in (B.10) can be used to construct the absolute risk aversion coefficient regarding *output* from plausible estimates for $r_A(\mu_\pi)$. Estimates for $r_R(\mu_\pi)$ can be used to construct r_R if the ratio of revenue to profits is known, as can be seen from (B.12).

Table 1. Parameter estimates: mean and variance functions

	Mean Function (Dependent variable=milk)		Variance Function ***(Dep. variable= ln(error)^2)***	
Parameter	***WITHIN Estimates***	***t-statistic [(a)]***	***OLS Estimates***	***t-statistic***
CONSTANT	-	-	-1.8402	-6.1454
VET	0.6859	4.1177	-0.4760	-2.0094
FEED	0.0558	10.0668	0.0171	2.4018
FERT	0.3693	2.1553	0.1757	0.5893
COWS	0.3958	13.8056	0.0471	1.7448
VET2	-0.3477	-1.6776	-0.0397	-0.2190
FEED2	-0.0006	-2.0464	0.0003	1.0023
FERT2	-0.0332	-0.1595	0.0969	0.4761
COWS2	0.0052	0.9920	0.0063	1.6667
VET*FEED	0.0014	0.2372	0.0033	0.3163
VET*FERT	-0.4902	-2.2842	-0.1215	-0.2951
VET*COWS	0.0812	3.2936	0.0226	0.5883
FEED*FERT	0.0006	0.1125	0.0042	0.4203
FEED*COWS	0.0009	0.7713	-0.0032	-1.8889
FERT*COWS	0.0319	1.3454	-0.0446	-1.0251
Z_{94}	0.0326	0.1948	-0.3675	-0.9913
Z_{95}	0.2998	1.8022	-0.5526	-1.4879
Z_{96}	0.0492	0.2625	-0.6257	-1.6487
Z_{97}	0.2019	1.1404	0.2715	0.7252
Z_{98}	0.1618	0.8848	0.7346	1.9538
R^2 Statistic	***0.9861***		***0.1516***	
Adjusted R^2 Statistic	***0.9824***		***0.1119***	
Hausman Test: RE vs FE [(b)]	***49.355***		***-***	
Firm-specific effects [(c)]	***10.927***		***1.1580***	
Overall inputs [(d)]	***-***		***3.7394***	

Notes:

(a) Test robust to heteroskedasticity.

(b) The Hausman test for fixed or random effects was carried out using a chi-squared statistic with 14 degrees of freedom.

(c) The firm-specific effect test is carried out using an *F*-statistic with 70 restrictions and 336 degrees of freedom.

(d) The overall input test is carried out using an *F*-statistic with 14 restrictions 420 degrees of freedom

Table 2. Average elasticities for the period 1993/98

	Vet	Feed	Fertilizer	Cows	Scale effect
Mean effect (ε_k)	0.060	0.225	0.020	0.700	1.005
Standard Dev. effect (η_k)	-0.240	0.265	0.012	0.273	0.309
Risk effect (η_k - ε_k)	-0.300	0.040	-0.009	-0.427	-0.696

Note: The scale effect is the sum of the input elasticities for each function.

Table 3. Decomposition of Total Factor Productivity Growth (%)

Period	Changes in inputs				Expected productivity growth [1]			Changes in risk [2]			Productivity growth under risk [3]					
						Decomposition			Decomposition		Low risk aversion ($r_R(\pi) = 1.70$)			Moderate risk aversion ($r_R(\pi) = 4.03$)		
												Decomposition			Decomposition	
	Vet	Feed	Fert.	Cows	$\frac{\dot{TFP}}{TFP}$	*Scale Effect*	*Temporal Effects*	$\frac{\dot{m}}{m}$	*Input effect*	*Time effect*	$\frac{\dot{TFPU}}{TFPU}$	$\frac{\dot{TFP}}{TFP}$	Risk Effect	$\frac{\dot{TFPU}}{TFPU}$	$\frac{\dot{TFP}}{TFP}$	Risk Effect
1993/94	16.48	12.90	28.45	4.20	0.24	-0.12	0.36	-21.55	-2.81	-18.74	0.59	0.24	0.34	1.07	0.24	0.83
1994/95	0.75	0.30	-9.98	0.10	2.87	0.10	2.77	-20.69	-8.66	-12.02	3.04	2.87	0.17	3.28	2.87	0.41
1995/96	16.06	17.05	48.32	8.52	-2.37	0.06	-2.43	-3.45	-2.23	-1.22	-2.34	-2.37	0.02	-2.31	-2.37	0.06
1996/97	-0.81	-7.76	-38.56	-4.59	1.52	0.07	1.45	46.53	3.13	43.41	0.86	1.52	-0.66	-0.06	1.52	-1.58
1997/98	3.39	5.24	16.59	1.70	0.47	0.86	-0.38	9.03	-14.51	23.54	-0.08	0.47	-0.55	-0.90	0.47	-1.38

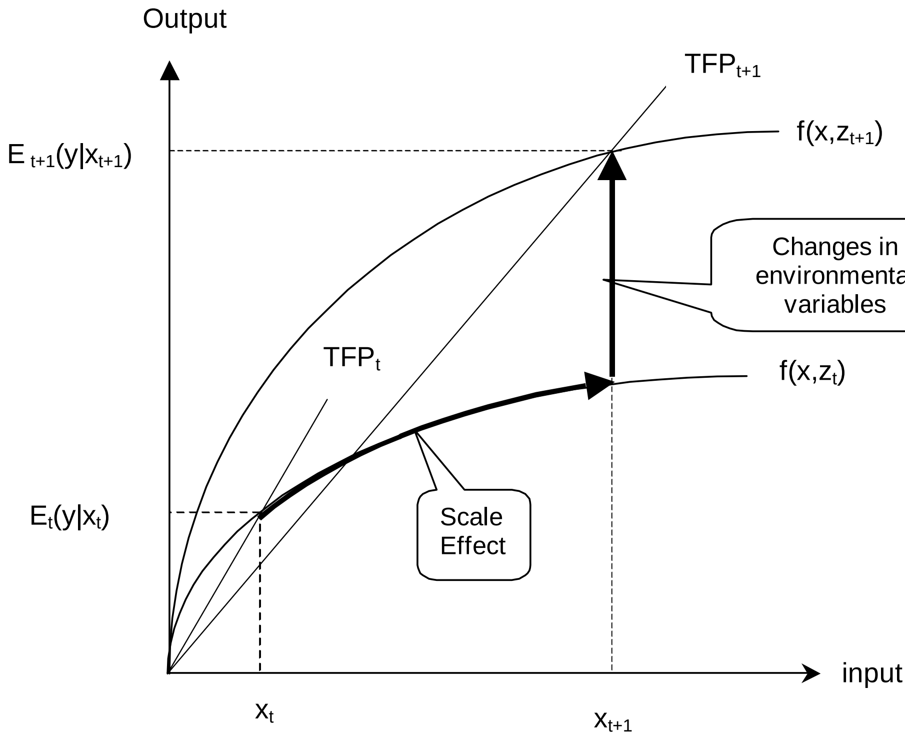

Figure 1. Productivity decomposition under certainty

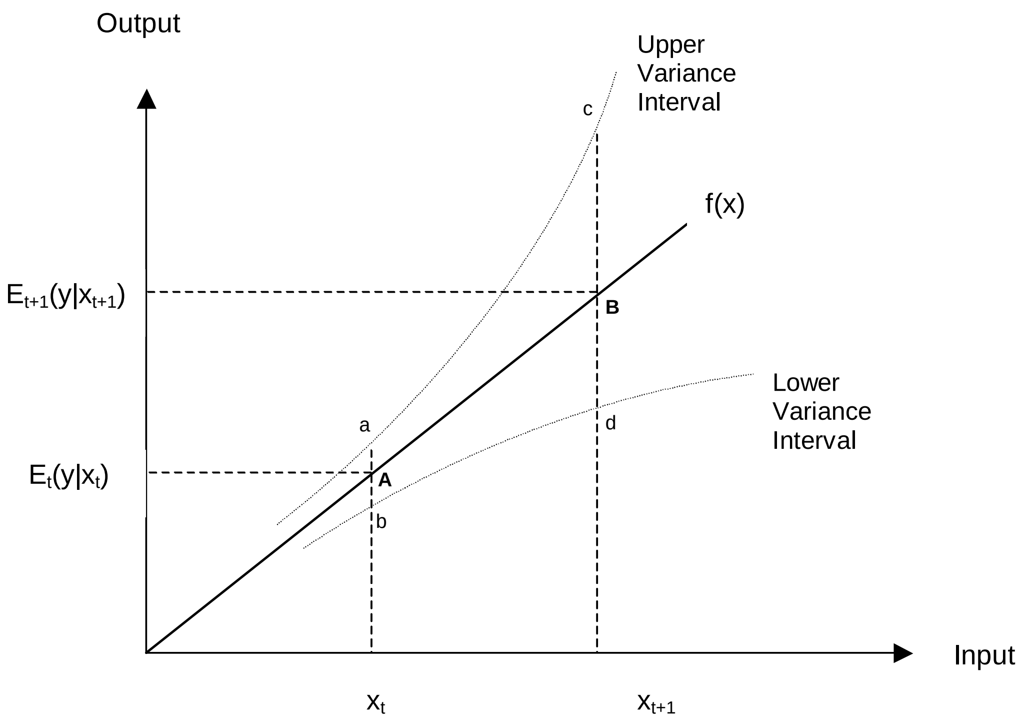

Figure 2. Productivity and risk

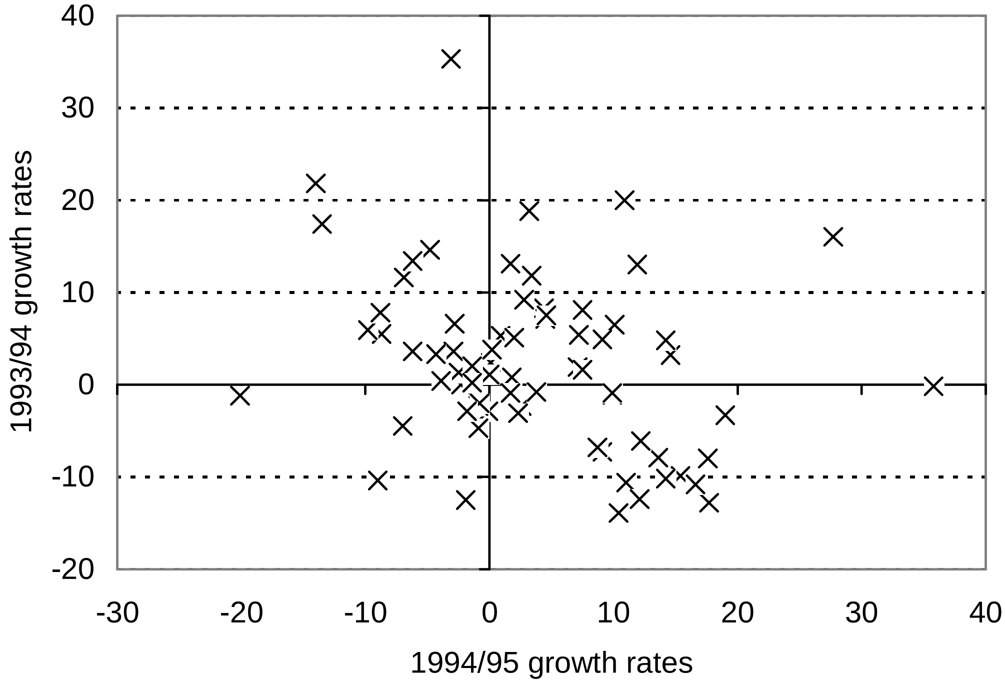

Figure 3. Milk per cow growth rates (%)

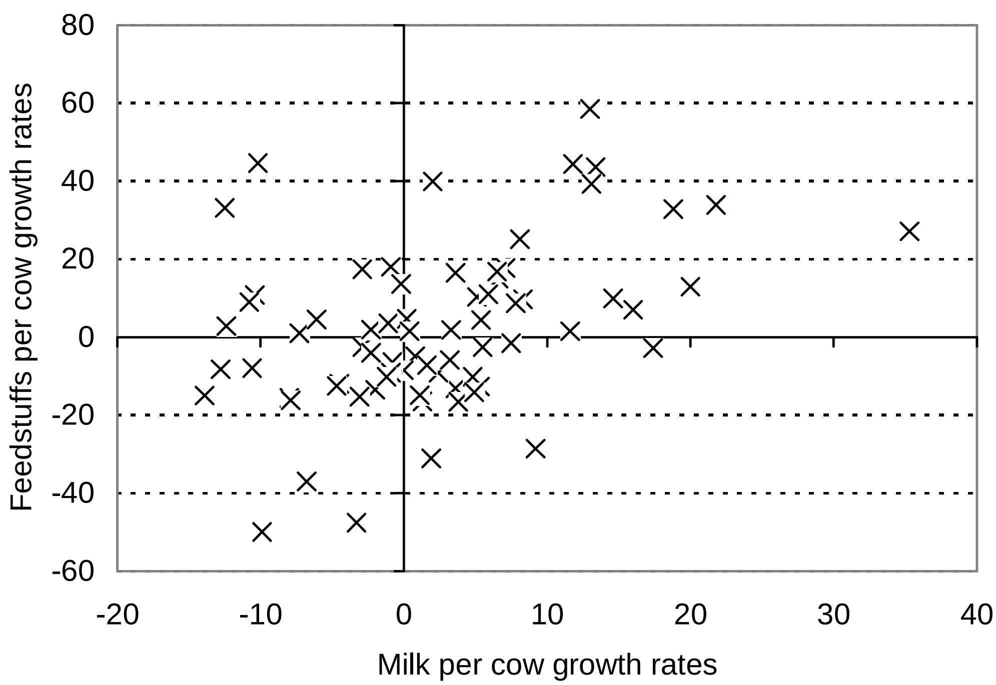

Figure 4. Milk per cow vs. Feedstuffs per cow growth rates (%)

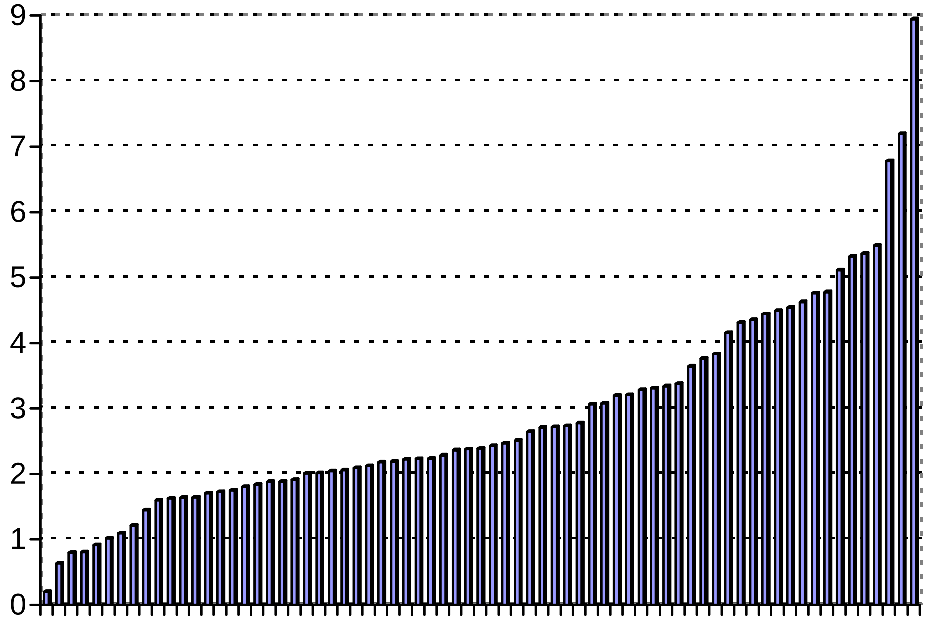

Figure 5. Farms' relative risk aversion coefficient (in profits)