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THE EFFECTS OF RESOURCE DEPLETION ON COAL MINING PRODUCTIVITY

Carlos Arias[♣] and Xosé Antón Rodríguez[♦]

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Abstract: The Solow Residual has a direct interpretation as a measure of technical change under perfect competition and constant returns to scale. When these conditions do not hold, the residual has to be appropriately adjusted in order to be considered a correct measure of technical change. We argue that in extractive industries the Solow Residual is also affected by the continuous depletion of a non-renewable resource. Therefore, we provide a new decomposition of the Solow Residual for extractive industries in which the level of reserves is likely to affect extraction costs. Our empirical results illustrate the role played by the depletion of reserves in the measurement of productivity of coal mining in Spain.

Key words: Coal mining, Divisia Index, Non-renewable natural resources, Solow Residual

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1. Introduction

The steady supply of energy products plays an important role in economic activity and the price of energy is often the cause of growth spurs and economic slowdowns. Productivity growth in energy industries is important to guarantee both a steady supply and low prices. The bulk of energy requirements is satisfied by oil and natural gas. However, coal covers 15% of primary energy requirements in Spain, of which 30% is domestically produced (Ministerio de Industria, Turismo y Comercio, 2005). Moreover, coal mining in Spain has many peculiarities regarding geographic concentraction of the activity, mining system, coal quality and extraction costs (Colegio Oficial de Ingenieros de Minas del Noroeste de España, 1987). These features have stimulated interest in the evolution of coal mining productivity in Spain.

In a sense, coal mining is a special industry. Indeed, obtaining output (coal extracted) is not only a question of applying inputs efficiently but also depends on the characteristics of a non-renewable natural resource. For example, Harris (1993) shows that the characteristics of natural resources (reserves and geological characteristics) affect extraction costs and efficiency. Kulshreshtha and Pariskh (2002) show that coal mining productivity depends on the system of extraction (opencast or underground). A recent paper by Pickering (2007) analyzes the relationship between oil reserves and production. We add to the literature on productivity of coal extraction by analyzing the role of coal reserves in the measurement of productivity. We claim that the depletion of coal reserves can hide the extent of technical change in this industry. The idea is quite simple: decreasing reserves levels are likely to increase the cost of extraction while technical change contributes to decrease extraction costs. Since both effects occur simultaneously it might be difficult to determine the relative magnitude of each effect. In a more technical tone, the claim amounts to proposing an additional correction to the Solow Residual to take account of changes in the level of reserves. The Solow Residual is a correct measure of technical change under a concrete set of assumptions and has to be adjusted in an appropriate manner when these assumptions do not hold.

The structure of the paper is as follows. In section 2 we briefly discuss the measurement of technical change. We review some methodological issues related to the measurement of technical change using cost functions in section 3. In section 4, we illustrate the effects of including mineral reserves on productivity with an empirical application to coal mining in Spain. Some conclusions are provided in section 5.

2. The measurement of technical change

Divisia indexes, widely used in the analysis of productivity, measure the growth rate of an aggregate. For this purpose, the growth rates of the components of the aggregate are weighted by the share of each component in the aggregate and summed. For a multiple-output technology, a Divisia index of aggregate output ($\dot{Q}$) can be written as:

$$\dot{Q} = \sum_j \frac{P_j Q_j}{R} \dot{Q}_j \tag{1}$$

where Q_j is the quantity of the *j-th* output, P_j is the price of the *j-th* output, $R = \sum_j P_j Q_j$ is total revenue and $\dot{Q}_j = \frac{\frac{dQ_j}{dt}}{Q_j}$ the growth rate of the *j-th* output. Likewise, the Divisia index of aggregate input ($\dot{F}$) can be written as:

$$\dot{F} = \sum_i \frac{W_i X_i}{C} \dot{X}_i \tag{2}$$

where X_i is the quantity of the *i-th* input, W_i is the price of the *i-th* input, $C = \sum_i W_i X_i$ is total cost and $\dot{X}_i = \frac{\frac{dX_i}{dt}}{X_i}$ is the growth rate of the *i-th* input. Finally, the Divisia index of Total Factor Productivity growth can be obtained as:

$$T\dot{F}P = \dot{Q} - \dot{F} \tag{3}$$

This index, referred to in the literature as the Solow Residual (Hartley, 2000; Raa and Mohnen, 2002), measures the changes in the output aggregate not explained by changes in the input aggregate. Solow (1957) proves that, under constant returns to scale, in a long run competitive equilibrium the index in (3) can be interpreted as a measure of technical change. The starting point is a primal representation of technology such as:

$$Q = f\left(X_1, \ldots,\ X_n, t\right) \tag{4}$$

where the X_i*'s* are inputs, t denotes technology and Q output. It is straightforward to prove that:

$$T\dot{F}P = \frac{\partial \ln f}{\partial\ t} \tag{5}$$

The result in (5) shows that the Solow Residual can be interpreted as a shift of the production function not attributable to changes in inputs but to technical change. Alternatively, under cost minimization and some regularity conditions, the technology can be represented by the dual cost function (see Shephard (1953,1970), Uzawa (1964) and McFadden (1966, 1978)):

$$C = g\left(W_1, \ldots,\ W_n, Q, t\right) \tag{6}$$

where C is total cost and the W_i's are input prices. Under constant returns to scale and at a competitive equilibrium in the product and factor markets it can be shown that:

$$T\dot{F}P = -\frac{\partial \ln g}{\partial\ t} \tag{7}$$

Therefore, the Solow Residual can also be interpreted as a shift of the cost function not attributable to changes in input prices or output quantity but to technical change.

The use of the Solow Residual as a measure of technical change relies on a number of simplifying assumptions. If these assumptions do not hold the residual has to be corrected accordingly. The effects of non-constant returns to scale and the violation of the various conditions necessary for long run competitive equilibrium have been analyzed empirically (Denny, Fuss and Waverman, 1981; Bauer, 1990; Morrison, 1992 and Boscá, Escribá and Murgui, 2002).

The Solow Residual is based on the idea that changes in the amount of output that can be produced with a given amount of inputs can be attributed only to technical change. However, we claim that this idea has to be qualified in the extractive industries where the extraction of natural resources is affected by the level of its reserves. In this case, the available technology defines which outputs are feasible given a set of inputs but the level of resource depletion also plays a role. The reason is that with increasing resource depletion less output is produced for given inputs or, alternatively, more inputs are necessary to extract a given quantity of mineral. The relationship between the level of reserves and extraction costs has been analyzed by Zimmerman (1981), Harris (1990) and Epple and Londregan (1993). However, to the best of our knowledge the analysis of the effects of the level of reserves of natural resources on the Solow Residual has not yet been analyzed. This is the objective of the present paper[1].

3. Dual estimation of the Solow Residual in extractive industries

In this section we outline a suitable correction of the Solow Residual in a cost function framework when the simplifying assumptions do not hold. In an empirical setting, it is reasonable to expect that some inputs can not be changed instantaneously when input prices change. In this case, the industry is in a short run instead of a long run competitive equilibrium. It is also frequent to observe decreasing or increasing returns to scale. Moreover, we make the case for the inclusion of a correction for the effects of the depletion of reserves of natural resources.

A measure of productivity in coal mining starts with the production function:

$$Q = f\left(X_L, X_E, X_M, X_K,\ t, R\right) \tag{8}$$

where Q is the output obtained with three variable inputs: Employment (X_L), Energy (X_E) and Materials (X_M); a quasi-fixed input, Capital (X_K); the technology (t); and a level of reserves of the natural resource denoted by R. Under certain regularity conditions of the production function (Lau, 1976) and under the assumption of cost-minimizing behaviour there is a dual variable cost function that contains all relevant information about the technology and which can be represented as:

$$VC = h\left(W_L, W_E, W_M, X_K, Q, t, R\right) \tag{9}$$

where VC denotes variable cost and W_L, W_E and W_M are the input prices of labour, energy and materials. The total cost function can be written as:

$$C = h\left(W_L, W_E, W_M, X_K, Q, t, R\right) + W_K X_K \tag{10}$$

where W_K is the user cost of capital.

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[1] Cuddington and Moss (2001) have analyzed the relationship between increasing extraction costs due to resource depletion and the adoption of cost-saving technologies. In this case, technical change can be disguised by the increasing difficulties of extracting mineral with depleting reserves (Livernois, 1988).

Next, we extend previous results by Denny, Fuss and Waverman (1981), Bauer (1990), Morrison (1992), De la Fuente (1999) and Boscá, Escribá and Murgui (2002) to take into account the role of reserves of non-renewable resources[2].

Differentiating with respect to time the log of total cost we have that:

$$\dot{C} = \sum_i S_i \dot{W}_i - (Z_K - W_K)\frac{X_K}{C}\dot{X}_K + \varepsilon_{CQ}\dot{Q} + \frac{1}{C}\frac{\partial h}{\partial t} + \frac{R}{C}\frac{\partial h}{\partial R}\dot{R} \quad i = L, E, M \tag{11}$$

where, $\dot{C}$ is the rate of change of total cost, S_i is the cost share of input *i*, $\dot{W}_i$ the rate of change of the price of variable inputs, $Z_k = -\frac{\partial h}{\partial X_K}$ is the shadow value of Capital, $\dot{X}_K$ is the rate of change of Capital, $\varepsilon_{CQ} = \frac{\partial \ln C}{\partial \ln Q}$ is the elasticity of cost with respect to output and $\dot{R}$ is the rate of change of reserves.

On the other hand, computing the rate of change of total cost using the cost identity we have that:

$$\dot{C} = \sum_i S_i \dot{W}_i + \dot{F} \quad i = L, E, M, K \tag{12}$$

Combining, (11) and (12) the rate of change of the input aggregator can be written as:

$$\dot{F} = -(Z_K - W_K)\frac{X_K}{C}\dot{X}_K + \varepsilon_{CQ}\dot{Q} + \frac{1}{C}\frac{\partial h}{\partial t} + \frac{R}{C}\frac{\partial h}{\partial R}\dot{R} \tag{13}$$

Finally, the Solow Residual is:

$$T\dot{F}P = \dot{Q} - \dot{F} = (Z_K - W_K)\frac{X_k}{C}\dot{X}_k + (1 - \varepsilon_{CQ})\dot{Q} - \frac{1}{C}\frac{\partial h}{\partial t} - \frac{R}{C}\frac{\partial h}{\partial R}\dot{R} \tag{14}$$

Following De la Fuente (1999) and Boscá, Escribá and Murgui (2002) the cost elastiticy (ε_{CQ}) can be written in terms of Capacity of Utilization (*CU*) and returns to scale (*RS*). Capacity of Utilization is defined as:

$$CU = \frac{W_L X_L + W_E X_E + W_M X_M + Z_K X_K}{W_L X_L + W_E X_E + W_M X_M + W_K X_K} = S_L + S_E + S_M + S_K^* \tag{15}$$

[2] We provide a more detailed derivation in Appendix 1.

where S_L, S_E, and S_M are the cost shares of Labor, Energy and Materials respectively, while S_K^* is the shadow cost share of Capital. The elasticity of scale (*RS*) can be computed as the sum of the output elasticities:

$$RS = \varepsilon_{QX_L} + \varepsilon_{QX_E} + \varepsilon_{QX_M} + \varepsilon_{QX_K} \tag{16}$$

where:

$$\varepsilon_{QXi} = \frac{X_i}{Q}\frac{\partial f}{\partial X_i} \quad i = L, E, M, K \tag{17}$$

Finally, it can be easily shown that:[3]

$$\varepsilon_{CQ} = \frac{CU}{RS} \tag{18}$$

Therefore, expression (14) can be written as:

$$T\dot{F}P = -\frac{1}{C}\frac{\partial h}{\partial t} + \left(Z_K - W_K\right)\frac{X_k}{C}\dot{X}_K + \left(1 - \frac{CU}{RS}\right)\dot{Q} - \frac{R}{C}\frac{\partial h}{\partial R}\dot{R} \tag{19}$$

Expression (19) splits the change in the Solow Residual ($T\dot{F}P$) into four components. The first component ($-\frac{1}{C}\frac{\partial h}{\partial t}$) is the corrected Solow Residual, a time shifter of the cost function commonly interpreted as a measure of technical change. The second component measures the effects on the Solow Residual of a non-optimal allocation of fixed factors. This term vanishes in a competitive equilibrium when the shadow value of the fixed factor (Z_K) is equal to its market price (W_K). The third component measures the effects on the Solow Residual of changes in the scale of production. Firms might find it difficult to change some inputs and, as a result, they do not necessarily operate at the optimum level of capacity utilization. This term vanishes when there are constant returns to scale (*RS=1*) and the capacity is fully utilized (*CU=1*). The last term measures the effects of resource depletion on the Solow Residual. This term vanishes only if the reserves of the resource do not change.

4. Estimation and decomposition of the Solow Residual

[3] A detailed derivation is provided in Appendix 2.

In this section we estimate the Solow Residual ($T\dot{F}P$) and its four components described in expression (19). This decomposition requires the estimation of a Variable Cost function in which the level of resource depletion is included as an explanatory variable. A central issue of this empirical analysis is the definition of a variable that measures the quantity of coal reserves available for extraction at each period of time.

The literature on non-renewable natural resources makes a distinction between *proven reserves* - a physical measure of the quantity of resources - and *reserves* defined as the portion of a known resource recoverable under current economic conditions (Harris, 1993). The idea is that at some point in time *t*, there are *n* known deposits of coal, CR_t = {c_{1t}, c_{2t}, ..., c_{nt}}, where *p* out of the *n* deposits, R_t = {c_{1t}, c_{2t}, …..,c_{pt}}, contain minerals recoverable under current economic conditions and *p-n* are deposits which are not recoverable, R_t'={c_{p+1t}, c_{2t}, …..,c_{nt}}. The set R_t corresponds to the definition of recoverable reserves while C_t is akin to the concept of proven reserves. Data on the level of reserves (R_t) are usually obtained through indirect methods. Harris (1993) mentions two different approaches. The first method consists of making an inference about the level of reserves based on geological information. The second method is to make an inference about the level of reserves based on the relationship between the level of production of the industry and the level of reserves. To the best of our knowledge, there is no annual estimation of R_t in Spain. However, the yearbook of the Association of Mining Engineers of Northwestern Spain reports 1765 millions tons of recoverable reserves of coal (anthracite, lignite and bituminous coal) in Spain in 1982 (Colegio Oficial de Ingenieros de Minas del Noroeste de España, 1987). Additionally, we have data on the quantities of coal mined from 1974 to 2001. As a result, an annual time series of reserves can be estimated using the following expression:

$$R_t = R_0 - \sum_{i=1}^{t} Q_{t-i} \qquad (20)$$

where R_t denotes coal reserves in year *t* and (Q_1, … , Q_{t-1}) the quantities of coal mined in previous periods of time. R_0 can be calculated using the reserves in any given year (e.g. 1982) and the amount of coal mined in previous years. The basic simplifying assumption contained in expression (20) is that coal extraction is the main force in the evolution of reserves while new discoveries are not very relevant. This assumption is not unrealistic in the case of coal mining in Spain[4].

We estimate a variable cost function using data on coal production, input prices and quantities reported in Mining Statistics (Estadística Minera), an annual publication of the Spanish Ministry of Economy[5]. The dataset contains aggregate data on coal mining operations in Spain from 1974 to 2001. The coal mining industry includes government and privately-owned firms carrying out both surface and underground mining. Some descriptive statistics of the variables used for the estimation are shown in Table 1.

Table 1. Descriptive statistics (1974-2001)

Variables	*Units*	*Maximum*	*Minimum*	*Mean*	*Std. Dev.*
Labor (X_L)	Hours (thousands)	86781	21867	63378	22445
Price of Labor (W_L)	Euro/hour	26.18	1.23	12.96	8.42
Capital (X_K)	Hours (thousands)	22176	12074	17546	2764
User cost of Capital (W_K)	Euro/hour	6.96	1.07	4.60	2.01
Materials (X_M)	Tons	15083	4984	10623	2878
Price of Materials (W_M)	Euro/Ton	15.94	3.24	9.14	3.62
Energy (X_E)	Tons of coal equivalent (TEC)	226261	92607	178973	44970
Price of Energy (W_E)	Euro/ TEC	0.49	0.06	0.31	0.14
Output (Q)	Tons of mineral (thousands)	42930	15197	30064	8412
Output price (P)	Euro/Ton	48.10	8.26	32.01	12.23
Variable cost (CV)	Euro (Millions)	1278.59	121.71	810.53	367.59
Total Cost (C)	Euro (Millions)	1390.70	140.68	889.56	398.56
Reserves (R)	Tons of mineral (thousands)	1942726	1123981	1547111	274191

Some clarifications on the construction of the variables are needed. First, the hours of Labor (X_L) are weighted by the cost share of each labor qualification. The tons of Materials (X_M) are weighted by the cost share of each type of material. Capital is measured as hours of machinery use. In this case, the hours are weighted by the power of each type of machinery. Finally, output is measured in Tons of mineral extracted.

The user cost of Capital (W_K) is defined as:

$$W_{Kt} = W_{K0} I_t \qquad (21)$$

[4] In fact, the number of active mines went from 150 in 1974 to 83 in 2001. This decrease is due to the closing of mines that reach the point in which reserves are not economically recoverable.

[5] These data were previously published by the Spanish Ministry of Industry and Energy.

where W_{K0} is the user cost of Capital in the base year and I_t is an index of price of machinery. In turn, the user cost of Capital for the base year is calculated as:

$$W_{K0} = \frac{S_0 \left(r + d \right)}{H_0} \tag{22}$$

where S_0 is an estimate of the value of the stock of Capital in the base year (Gómez Villegas, 1987), r is the interest rate, d a depretiation rate and H_0 is the stock of capital in the base year measured as hours of machinery use. Basically, we are calculating a user cost for the stock of capital measured in hours. The interest rate of the base year is 12%. This was the average interest rate charged then to mining firms for medium and long term loans. We have chosen the rate of depreciation provided for tax and accounting purposes in the mining industry by the Ministry of Economy of Spain. As is always the case in empirical applications, the choice of r and d is driven mainly by data availability.

Variable and Total Cost include only items directly related to coal extraction and do not include other costs (e.g. marketing costs). We believe that extraction costs are more likely to be affected by changes in productivity than non extractive costs.

The Translog functional form has been used in the estimation. This is a flexible function form used previously by Brown and Christensen (1981) and Berndt and Hesse (1986). The variable cost function can be written as:

$$\begin{aligned} \ln CV &= \alpha_0 + \alpha_Q \ln Q + \tfrac{1}{2} \alpha_{QQ} \left(\ln Q \right)^2 + \sum_i \alpha_i \ln W_i + \beta_k \ln X_k \\ &+ \tfrac{1}{2} \beta_{kk} \left(\ln X_k \right)^2 + \alpha_t t + \tfrac{1}{2} \alpha_{tt} t^2 + \alpha_R R + \tfrac{1}{2} \alpha_{RR} R^2 + \alpha_{QR} R \ln Q \\ &+ \alpha_{Qt} \ln Q \;\; t + \tfrac{1}{2} \sum_i \sum_j \alpha_{ij} \ln W_i \ln W_j + \sum_i \alpha_{iQ} \ln W_i \ln Q + \sum_i \delta_{ik} \ln W_i \ln X_k \\ &+ \sum_i \alpha_{it} \ln W_i t + \sum_i \alpha_{iR} R \ln W_i \end{aligned} \tag{23}$$

where i,j refer to the three variable inputs - Labor (L), Energy (E) and Materials (M) - while R is the measure of reserves in expression (20). All variables in the translog cost function are in natural logarithms with the exception of the time trend (t) and the measure of reserves (*R*) [6]. This practice is standard for the time trend since the passing of time (and not its "growth rate") is the interesting feature here. We have adopted the same approach for the level of reserves since we believe that we have a more precise measure of the absolute change in reserves (observed coal extraction) than of the growth rate (which depends on the estimated level of reserves).

Using Shephard's lemma in equation (23) we have that:

$$\mathrm{S_i} = \frac{\partial \ln CV}{\partial \ln W_i} = \alpha_i + \sum_j \alpha_{ij} \ln W_j + \alpha_{iQ} \ln Q + \delta_{ik} \ln X_k + \alpha_{it} t + \alpha_{iR} R \qquad i = L,\ E,\ M \tag{24}$$

where S_i denotes the cost share of input *i*. Moreover, we consider the following equilibrium condition:

$$P = \frac{\partial CV}{\partial Q} = \frac{CV}{Q}\left(\alpha_Q + \alpha_{QQ} \ln Q + \sum_i \alpha_{iQ} \ln W_i + \alpha_{QR} R + \alpha_{Qt} t\right) \tag{25}$$

This equation imposes optimizing behaviour in the output market (Morrison and Schwartz, 1996). The system of equations (23), (24) and (25) is estimated after imposing parametric restrictions of symmetry and homogeneity of degree one on input prices. The share equation of materials is dropped to avoid singularity of the system. Prices and Variable Cost have been divided by the price of Materials to impose homogeneity of degree one on input prices of the cost function. As a result, the coefficients associated with the price of Materials are not estimated directly although they can be estimated easily using the parametric restriction implied by the linear homogeneity restriction. The resulting system of equations is estimated by Iterative Seemingly Unrelated Regression (*ISURE*). This method provides consistent estimates of the parameters of the system. Kmenta and Gilber (1968) show the computational equivalence of ISURE and maximum likelihood. Barten (1969) shows how the parameter estimates of a system of equations estimated by maximum likelihood do not depend on which equation is omitted.

[6] It is well known that when a variable is introduced into the model in levels, the associated coefficient measures the effect of a unitary change in this variable on the dependent variable. However, if the variable is introduced in logarithmic terms, the coefficient measures the effect on the dependent variable of a unitary increase *in the growth rate* of the variable in question.

The estimation produced a number of coefficients not significantly different from zero. Moreover, we could not reject the null hypothesis that the non significant coefficients of the quadratic terms were jointly different from zero. Therefore, in order to deal with what looks like a multicollinearity problem we decided to re-estimate the cost function without the quadratic terms whose coefficients were jointly non significant. The final results of the estimation are shown in Table 2.

Table 2: Restricted Translog Variable Cost Function

Parameters	*Estimate*	*Std. Error*	*t-Statistic*	*Prob.*
α_0	-0.0529	0.0298	-1.7753	0.0791
α_Q	1.0381	0.0320	32.4234	0.0000
α_L	0.8080	0.0389	20.7327	0.0000
α_E	0.0697	0.0009	75.9374	0.0000
β_k	-0.0320	0.0126	-2.5347	0.0129
α_t	-0.0529	0.0041	-12.7733	0.0000
α_R	-0.0249	0.0113	-2.2013	0.0302
α_{QR}	-0.2650	0.0441	-5.9989	0.0000
α_{LL}	0.0869	0.0173	5.0015	0.0000
α_{EE}	0.0266	0.0055	4.7677	0.0000
α_{LE}	-0.0330	0.0047	-6.9060	0.0000
α_{LQ}	-0.0337	0.0216	-1.5603	0.1221
α_{EQ}	0.0265	0.0056	4.6606	0.0000
δ_{LK}	0.0936	0.0435	2.1503	0.0342
δ_{EK}	-0.0177	0.0110	-1.6034	0.1123
$\alpha_{L\,t}$	-0.0028	0.0049	-0.5792	0.5639
$\alpha_{E\,t}$	-0.0058	0.0012	-4.6791	0.0000
α_{LR}	0.0095	0.0126	0.7517	0.4541
α_{ER}	-0.0096	0.0032	-2.9725	0.0038
α_{Qt}	-0.0083	0.0039	-2.0956	0.0389

Estimation Method: Iterative Seemingly Unrelated Regression using Eviews 4.1

The R-squared of the estimated equations are 0.90 for the Variable Cost function, 0.56 for the Labour Cost share equation, 0.94 for the Energy Cost share equation and 0.82 for market equilibrium equation. We have checked for the presence of autocorrelation using the Durbin-Watson test in each equation of the system (Durbin, 1957; Malinvaud, 1970). The values of the Durbin-Watson test are 1.47 for the Variable Cost function, 1.61 for the Labour cost share equation, 1.74 for the Energy cost share equation and 1.63 for the market equilibrium equation. These results suggest that autocorrelation is not a severe problem since the values of the test for the first two equations are in the indeterminacy zone of the test while for the last two equations the values indicate an absence of autocorrelation.

All variables appearing in natural logarithms were divided by their geometric mean prior to estimation. The variable that measures reserve levels was rescaled by subtracting the average of reserves and the time trend was set at zero in 1986. As a result, the coefficients of the first order terms of the variables in natural logarithms can be interpreted as cost elasticities in that year evaluated at the geometric mean of the explanatory variables and at the arithmetic mean of the level of reserves. These cost elasticities have the expected signs and are significantly different from zero at conventional levels of significance.

The coefficient of the first order term of the time trend (α_t) is negative and significantly different from zero. The negative value (-0.05) can be interpreted as the average annual growth rate of total variable cost. In other words, total variable cost decreases by five percent annually, keeping all other explanatory variables constant. A particularly interesting result is the value of the coefficient of the first order term of the variable that represents the level of coal reserves (α_R). The negative value (-0.02) is significantly different from zero and can be interpreted as the growth rate of total variable cost associated with a unitary change in the level of reserves evaluated at the geometric mean of the sample. These two results together show the conflicting effects of technical change and the decrease in coal reserves on extraction cost. Since both effects occur simultaneously, the effect attributed to technical change (the Solow Residual) might be downward biased when the variable that measure coal reserves is not included in the cost function.

The decomposition of the Solow Residual following equation (19) is shown in Table 3. The four components of $T\dot{F}P$ in equation (19) are shown in the first four columns of Table 3 while the estimated value of $T\dot{F}P$ appears in the fifth column. The results show that, in each year, non-optimal allocation of fixed factors, non-constant returns to scale and resource depletion have sizeable effects on the Solow Residual. Therefore, they should be taken into account for a correct measure of technical change.

Table 3: Total Factor Productivity growth and its components (Equation (19)**)**

Year	*Corrected Solow Residual*	*Fixed factors*	*Scale*	*Level of Reserves*	*Solow Residual* $\dot{TFP}$
1975	0.0407	0.0010	0.0137	-0.0074	0.0481
1976	0.0416	-0.0002	0.0067	-0.0077	0.0404
1977	0.0441	-0.0007	0.0356	-0.0077	0.0713
1978	0.0452	0.0001	0.0310	-0.0085	0.0678
1979	0.0469	-0.0032	0.0405	-0.0092	0.0749
1980	0.0515	-0.0010	0.0530	-0.0108	0.0927
1981	0.0523	-0.0058	0.0419	-0.0140	0.0744
1982	0.0542	-0.0045	0.0331	-0.0167	0.0661
1983	0.0532	-0.0016	0.0164	-0.0187	0.0492
1984	0.0537	0.0050	-0.0055	-0.0198	0.0334
1985	0.0545	-0.0009	-0.0002	-0.0191	0.0342
1986	0.0561	-0.0039	-0.0037	-0.0181	0.0304
1987	0.0548	0.0072	-0.0093	-0.0177	0.0349
1988	0.0523	-0.0025	-0.0074	-0.0165	0.0260
1989	0.0515	-0.0012	0.0105	-0.0153	0.0456
1990	0.0538	0.0050	-0.0019	-0.0166	0.0403
1991	0.0554	0.0064	-0.0059	-0.0154	0.0404
1992	0.0540	-0.0033	-0.0004	-0.0147	0.0355
1993	0.0548	0.0090	-0.0053	-0.0142	0.0444
1994	0.0556	-0.0005	-0.0074	-0.0130	0.0347
1995	0.0540	0.0142	-0.0041	-0.0118	0.0522
1996	0.0551	-0.0052	-0.0047	-0.0109	0.0342
1997	0.0538	0.0046	-0.0041	-0.0104	0.0438
1998	0.0545	0.0157	-0.0033	-0.0093	0.0576
1999	0.0535	0.0128	-0.0078	-0.0090	0.0496
2000	0.0537	0.0072	-0.0032	-0.0085	0.0492
2001	0.0514	-0.0271	-0.0030	-0.0080	0.0133
Average	**0.0519**	**0.0010**	**0.0076**	**-0.0129**	**0.0476**

The last row of Table 3 shows the average values of the decomposition. The average value of the effect of non-optimal allocation of fixed factors (Capital) is positive but quite small (0.1%), the reason being that although the shadow value of Capital is smaller than the user cost ($Z_K<W_K$) fixed Capital decreases in the period analyzed. The average value of the scale effect is positive but also quite small (0.76%). The small average value is driven by the existence of positive values in the period 1975-1983 followed by negative values thereafter. For all years in the sample the elasticity of scale (*RS*) is larger than the index of Capacity Utilization (*CU*), with average values of 1.12 and 0.91 respectively. As a result, the term $\left(1-\frac{CU}{RS}\right)$ is always positive. Therefore,

the negative values of the scale effect are due exclusively to the negative rate of output growth in the period 1984-2001.

The average value of the component that measures the effect of the decrease in the level of reserves has a negative contribution to the average Solow Residual (-1.29%)[7]. This component increases in absolute value until 1984 and decreases thereafter. This result is driven mainly by the effect of decreasing output in our measure of reserves. A reduction in production implies a reduction, in absolute value, of the negative growth rate of reserves.

Finally, the average value of the growth rate of the Corrected Solow Residual (5.19%) is larger than the average value of the traditional Solow Residual (4.46%). However, the null hypothesis of both means being equal can not be rejected using a paired *t* test at conventional levels of significance[8]. In the present empirical exercise, the difference between the Corrected and traditional Solow Residual caused by decreasing reserves is obscured by the small but positive effects of fixed factors and scale of operation.

The average growth rate of the corrected Solow Residual (5.19%) indicates that technical change has been quite intense in the period analyzed. In fact, production (coal extraction) grew at an average annual rate of 1.56% while employment and fixed Capital decreased substantially (-5% and -0.86% respectively).

5. Conclusions

The analysis of productivity in mining requires special care due to the likely effects of depletion of reserves on extraction costs. As a result, it is reasonable to expect that the measure of productivity change might be affected by the evolution of reserves.

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[7] Using a *t* test, the null hypothesis of the population mean being equal to zero can be rejected at the 1% significance level. However, the observations are the result of quite involved computations using yearly data. This feature of the series cast some doubts on the independence of the observations. The development of a specific statistical test is beyond the scope of the present paper.

[8] The results of the paired *t* test are likely to be affected by the statistical problems described in the previous footnote.

The analysis in the present paper unveils an interesting issue: measured technical progress is affected by the depletion of reserves that increasingly impedes coal extraction. We find that the depletion of natural resources requires an annual increase of input use of 1.29%.

The results in the present paper show the importance of correcting the Solow Residual for the effects of coal reserves on extraction costs. It is reasonable to expect that such a correction is necessary in any extractive industry in which the level of reserves is likely to affect extraction costs. The results of the estimation show that other corrections of the Solow Residual due to non-constant returns to scale and non-optimal allocation of fixed inputs are also necessary. However, the magnitude of both corrections is quite small.

Finally, we conclude with some policy consequences of the results in the present paper. First, resource management is important not only on environmental grounds but also in terms of its consequences for the productivity of the industry. Second, the relevance of reserves for productivity suggests the convenience of gaining a better knowledge of reserves. Third, the results in the paper indicate quite intense technical change in coal mining in Spain. The magnitude of technical change in this sector is usually obscured by the role of decreasing reserve levels. We believe that this insight has been largely overlooked in the design of coal mining policy in Spain.

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Appendix 1

The rate of change of Total Factor Productivity can be written as:

$$T\dot{F}P = \dot{Q} - \dot{F} \qquad \text{(A1.1)}$$

where the dot over a variable denotes rate of change of that variable. The input aggregator can be written as:

$$\dot{F} = \sum_i S_i \dot{X}_i \quad i = L, E, M, K \qquad \text{(A1.2)}$$

where $S_i = \dfrac{W_i X_i}{C}$ is the cost share of input *i*. The total cost identity can be written as:

$$C = \sum_i X_i W_i \quad i = L, E, M, K \qquad \text{(A1.3)}$$

Differentiating the cost identity with respect to time we have that:

$$\frac{\partial C}{\partial t} = \sum_i X_i \frac{\partial W_i}{\partial t} + \sum_i W_i \frac{\partial X_i}{\partial t} \quad i = L, E, M, K \qquad \text{(A1.4)}$$

The rate of change of total cost can be written as:

$$\dot{C} = \frac{\frac{\partial C}{\partial t}}{C} = \sum_i S_i \dot{W}_i + \sum_i S_i \dot{X}_i = \sum_i S_i \dot{W}_i + \dot{F} \quad i = L, E, M, K \qquad \text{(A1.5)}$$

The total cost function can be written as:

$$C = h\left(W_L,\ W_E, W_M, X_K, Q, t, R\right) + W_K X_K \qquad \text{(A1.6)}$$

Differentiating the total cost function with respect to time we have that:

$$\frac{\partial C}{\partial t} = \sum_i \frac{\partial h}{\partial W_i}\frac{\partial W_i}{\partial t} + \frac{\partial h}{\partial X_K}\frac{\partial X_K}{\partial t} + \frac{\partial h}{\partial Q}\frac{\partial Q}{\partial t} + \frac{\partial h}{\partial t} + \frac{\partial h}{\partial R}\frac{\partial R}{\partial t} + X_K \frac{\partial W_K}{\partial t} + W_K \frac{\partial X_K}{\partial t}$$
$$i = L, E, M$$

(A1.7)

Applying Shephard lemma's in expression (A1.7) and rearranging terms we have that:

$$\frac{\partial C}{\partial t} = \sum_i X_i \frac{\partial W_i}{\partial t} - \left(Z_K - W_K\right)\frac{\partial X_K}{\partial t} + \frac{\partial h}{\partial Q}\frac{\partial Q}{\partial t} + \frac{\partial h}{\partial t} + \frac{\partial h}{\partial R}\frac{\partial R}{\partial t} \qquad \text{(A1.8)}$$
$$i = L, E, M, K$$

where $Z_K = -\dfrac{\partial h}{\partial X_K}$ is the shadow price of capital. The rate of change of total cost can be written as:

$$\dot{C} = \frac{\frac{\partial C}{\partial t}}{C} = \sum_i S_i \dot{W}_i - \left(Z_K - W_K\right)\frac{X_K}{C}\dot{X}_K + \varepsilon_{CQ}\dot{Q} + \frac{1}{C}\frac{\partial h}{\partial t} + \frac{R}{C}\frac{\partial h}{\partial R}\dot{R} \qquad \text{(A1.9)}$$
$$i = L, E, M, K$$

where $\varepsilon_{CQ} = \frac{Q}{C}\frac{\partial h}{\partial Q}$. Using (A1.5) and (A1.9), we have that:

$$\dot{F} = -\left(Z_K - W_K\right)\frac{X_K}{C}\dot{X}_K + \varepsilon_{CQ}\dot{Q} + \frac{1}{C}\frac{\partial h}{\partial t} + \frac{R}{C}\frac{\partial h}{\partial R}\dot{R} \tag{A1.10}$$

Finally, the rate of change of total factor productivity can be written as:

$$T\dot{F}P = \dot{Q} - \dot{F} = \left(Z_K - W_K\right)\frac{X_K}{C}\dot{X}_K + \left(1 - \varepsilon_{CQ}\right)\dot{Q} - \frac{1}{C}\frac{\partial h}{\partial t} - \frac{R}{C}\frac{\partial h}{\partial R}\dot{R} \tag{A1.11}$$

Appendix 2

The variable cost function can be defined as:

$$h\left(W_L,\ W_E, W_M, X_K, Q, t, R\right) = \min_{X_L, X_E, X_M} W_L X_L + W_E X_E + W_M X_M$$
$$st\ Q = f\left(X_L, X_E, X_M, X_K,\ t, R\right) \tag{A2.1}$$

The associated lagrangian is:

$$L = W_L X_L + W_E X_E + W_M X_M + \lambda\left[Q - f\left(X_L, X_E, X_M, X_K,\ t, R\right)\right] \tag{A2.2}$$

The F.O.C. are:

$$\begin{aligned}
\frac{\partial L}{\partial X_L} &= W_L - \frac{\partial f}{\partial X_L} = 0\\
\frac{\partial L}{\partial X_E} &= W_E - \frac{\partial f}{\partial X_E} = 0\\
\frac{\partial L}{\partial X_M} &= W_M - \frac{\partial f}{\partial X_M} = 0\\
\frac{\partial L}{\partial \lambda} &= Q - f\left(X_L, X_E, X_M, X_K,\ t, R\right) = 0
\end{aligned} \tag{A2.3}$$

A straightforward application of the envelope theorem gives the following results:

$$\begin{aligned}
\frac{\partial h}{\partial Q} &= \lambda\\
\frac{\partial h}{\partial X_K} &= -\lambda \frac{\partial f}{\partial X_K} = -\frac{\partial h}{\partial Q}\frac{\partial f}{\partial X_K}
\end{aligned} \tag{A2.4}$$

The shadow price of capital Z_K can be defined as:

$$Z_K = -\frac{\partial h}{\partial X_K} = \frac{\partial h}{\partial Q}\frac{\partial f}{\partial X_K} \tag{A2.5}$$

The index of capacity utilization (*CU*) is defined as:

$$CU = \frac{W_L X_L + W_E X_E + W_M X_M + Z_K X_K}{C} \tag{A2.6}$$

where:

$$C = W_L X_L + W_E X_E + W_M X_M + W_K X_K \tag{A2.7}$$

Using in (A2.6) the F.O.C in (A2.3) and the envelope theorem results in (A2.4), we obtain:

$$CU = \frac{\frac{\partial h}{\partial Q}\frac{\partial f}{\partial X_L} X_L + \frac{\partial h}{\partial Q}\frac{\partial f}{\partial X_E} X_E + \frac{\partial h}{\partial Q}\frac{\partial f}{\partial X_M} X_M + \frac{\partial h}{\partial Q}\frac{\partial f}{\partial X_K} X_K}{C} \tag{A2.8}$$

Rearranging terms we have that:

$$CU = \frac{Q}{C}\frac{\partial h}{\partial Q}\left(\frac{X_L}{Q}\frac{\partial f}{\partial X_L} + \frac{X_E}{Q}\frac{\partial f}{\partial X_E} + \frac{X_M}{Q}\frac{\partial f}{\partial X_M} + \frac{X_K}{Q}\frac{\partial f}{\partial X_K}\right) = \varepsilon_{CQ} RS \qquad \text{(A2.9)}$$

where ε_{CQ} is the output cost elasticity and RS is the elasticity of scale, a measure of returns to scale obtained by adding-up the output elasticities of the four inputs in the production function.