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# Accounting for Unobservables in Production Models: Management and Inefficiency

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# UNIVERSIDAD DE OVIEDO

# DEPARTAMENTO DE ECONOMÍA

## PERMANENT SEMINAR ON EFFICIENCY AND PRODUCTIVITY

## ACCOUNTING FOR UNOBSERVABLES IN PRODUCTION MODELS: MANAGEMENT AND INEFFICIENCY

**Antonio Alvarez[♣], Carlos Arias[♠] and William Greene[♥]**

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**Abstract:** This paper explores the role of unobserved managerial ability in production and its relationship with technical efficiency. Previous analyses of managerial ability have been based on strong assumptions about its role in production or on the use of proxies. We avoid these limitations by introducing managerial ability as an unobserved random variable in a translog production function. The resulting empirical model can be estimated as a stochastic production frontier with random coefficients.

**Key words:** Managerial ability, technical efficiency, production frontier, random coefficients model, maximum simulated likelihood.

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## 1. Introduction

Management has always been considered an important factor of production. However, the modeling of management is problematic because it is unobservable and for this reason it has been omitted from many production models. This may be a source of important problems because the omission of relevant variables can lead to biased estimates of the remaining parameters of the production function (Griliches, 1957). Economists have coined the term 'management bias' to refer to this problem and two remedies have been proposed in the literature. Following Mundlak (1961), some authors have used covariance analysis [e.g. Massell (1987)] or similar tools such as the within transformation to control for the effect of time invariant management by effectively eliminating it from the equation to be estimated. Other studies have used 'proxies' for management [e.g. Dawson and Hubbard (1985); Mefford (1986)].

An alternative approach is to consider management as a random effect and model it as part of the stochastic element of the production function. This is the approach implicitly followed by the stochastic production frontier literature (Aigner et al., 1977) where the stochastic structure is composed of two terms: a symmetric term, which accounts for 'noise,' and an asymmetric term that accounts for technical inefficiency. In this literature, it has been common to assume that the inefficiency term picks up, among other things, differences in the level of managerial skills.

The production function and stochastic production frontier literatures have followed parallel but independent paths. On the one hand, the literature on 'average' production functions (i.e., production functions defined so that observations are stochastically arranged symmetrically around the function) recognizes the role of management but seldom mentions production inefficiency. On the other, the stochastic frontier literature focuses on estimating technical efficiency (TE) and recognizes that it is related to management ability but it has not provided an analytical linkage between the two concepts. For example, Farrell (1957) stated that "technical efficiency indicates the gain that can be achieved by simply 'gingering-up' the management," suggesting that technical inefficiency is the result of a lack of managerial ability. On the other hand, Leibenstein (1966) viewed technical *in*efficiency as the result of a lack of motivation or effort. In this case, the solution to inefficiency calls for better organization of the work process or greater motivation and supervision of employees, all of which are commonly considered to be management functions [Mefford (1986)].

In this paper we explore the analytical linkages between technical efficiency and management within the framework of a translog production model where management is treated as an unobservable fixed input in an otherwise fully specified production that accounts for other time invariant factors. Starting from an average production function, we construct a production frontier which allows us to derive an explicit relationship between technical efficiency and management. In particular, technical efficiency is shown to be a function of the difference between the firm's level of managerial input and the amount of managerial input that the firm would need to operate on the frontier.

In the empirical section of the paper we estimate the production function with the latent fixed input described above. It is important to note that in a translog production function the latent input enters the model both additively as well as in interaction with the remaining inputs. This feature is very important for modeling the role of management in production but greatly complicates the estimation of the model. The model that emerges is a type of random coefficients model that has to be estimated by maximum simulated likelihood.

The structure of the paper is as follows. In Section 2 we develop a model in which managerial ability is treated as an unobservable input in a flexible production function. Section 3 discusses estimation issues. Empirical results based on a study of dairy farmers are presented and discussed in Section 4. Finally, some conclusions are drawn in Section 5.

## 2. A production model with fixed managerial ability

Our starting point is a translog production function with one time-varying variable input, $x_{it}$, and managerial ability, $m_i$, which is considered a fixed input. The flexibility of the production function relaxes ex-ante constraints on the roles of $m_i$ and $x_{it}$ in the production process. The translog production function can be written as:

$$\ln y_{it} = \alpha + \beta_x \ln x_{it} + ½\beta_{xx}(\ln x_{it})^2 + \beta_m m_i + ½\beta_{mm} m_i^2 + \beta_{xm} \ln x_{it} m_i + v_{it} \qquad (1)$$

where subscripts $i$ and $t$ denote firms and time, respectively, and $y_{it}$ is the single output. The inputs in the translog model are conventionally expressed in logs, but since managerial ability and its units of measurement are unobservable we express it in generic form. We assume that $v_{it}$ is a symmetric random disturbance with zero mean. Therefore, the model in (1) corresponds to the typical 'average' production function. Its key feature for present purposes is the interaction of management with the variable input. Without this interaction, the two management terms collapse into an individual effect and the model does not differ in substance from the standard fixed or random effects production function model.

Greater managerial ability should allow the agent to produce more output from any given amount of input, so we expect production to be monotonically increasing in $m_i$. In the translog production function this assumption holds for a certain range of $m_i$; that is:

$$\frac{\partial \ln y_{it}}{\partial m_i} = \beta_m + \beta_{mm} m_i + \beta_{xm} \ln x_{it} > 0 \quad \text{iff} \quad m_i < \frac{\beta_m + \beta_{xm} \ln x_{it}}{-\beta_{mm}}. \tag{2}$$

Thus, for a given $x_{it}$, higher values of $m_i$ can imply higher levels of output. We assume that the level of managerial ability ($m_i$) is exogenously given and differs among producers. This is the stance taken in previous papers that deal with the issue of differences in managerial ability, such as Jovanovic (1982) or Alvarez and Arias (2003).

We define $m_i^*$ to be the amount of managerial ability that produces maximum output for given conventional inputs (i.e., the frontier output). Therefore, $m_i^*$ is exogenously given and no assumption of producer optimality is made about it. The stochastic production *frontier* may then be obtained by substituting $m_i^*$ for $m_i$:

$$\ln y_{it}^* = \alpha + \beta_x \ln x_{it} + ½\beta_{xx} (\ln x_{it})^2 + \beta_m m_i^* + ½\beta_{mm} m_i^{*\,2} + \beta_{xm} \ln x_{it} m_i^* + v_{it} \tag{3}$$

where $y_{it}$* denotes *efficient* output.

We can now establish a link between technical efficiency and management. This follows from the definition of an output-oriented index of technical efficiency as the ratio of observed to potential output. In log terms this is:

$$\ln TE_{it} = \ln y_{it} \text{-} \ln y_{it}^* = \left(\beta_m + \beta_{xm} \ln x_{it}\right)\left(m_i - m_{\mathrm{i}}^*\right) + ½\beta_{mm}\left(m_i^2 - m_i^{*2}\right) \le 0 \tag{4}$$

Note that when $m_i = m_i^*$, that is when the firm is using the amount of management that defines the frontier, $\ln TE_{it}$ =0 and the firm is therefore technically efficient. Equation (4) can be rewritten as:

$$\begin{aligned} \ln TE_{it} &= \theta_i + \theta_{xi} \ln x_{it}, \\ \theta_i &= \beta_m \left(m_i - m_i^*\right) + ½\beta_{mm}\left(m_i^2 - m_i^{*2}\right), \\ \theta_{xi} &= \beta_{xm} \left(m_i - m_i^*\right). \end{aligned} \tag{5}$$

Equation (5) shows that *TE* has two components. The first can be modeled as a time-invariant individual effect ($\theta_i$). The other term, reflecting the interaction of management with input use, is specified as a time-varying component in the production function ($\theta_{xi}\ln x_{it}$). Therefore, an interesting feature of expression (5) is that the implied technical efficiency will be time-varying because it depends on $x_{it}$ even when the observed level of management and the one at the frontier are constant over time. Given the specification in (1), this suggests that in contrast to earlier formulations of the panel data frontier model such as Schmidt and Sickles (1984), it does not seem appropriate to model technical efficiency as a fixed effect since it depends on $x_{it}$. A consequence of this specification is that the change in managerial input necessary to increase TE by a given amount differs according to input use. This idea appeared in an early paper. Hall and Winsten (1959) claimed that for similar firms (using the same amounts of inputs) more management would imply more output and therefore greater TE. In this case, there is a clear direct relationship between managerial ability and TE. However, things are less clear in the case of two firms using different inputs with the same level of TE. In this case, any increase in technical efficiency would require different increases in the levels of management for each firm.

The special implications for economic analysis of our measure of TE can be better understood by looking at the effects on TE of changes in managerial ability and input use. These effects are given by the following derivatives:

$$\begin{aligned} \frac{\partial \ln TE_{it}}{\partial m_i} &= \beta_m + \beta_{mm} m_i + \beta_{xm} \ln x_{it} \\ \frac{\partial \ln TE_{it}}{\partial \ln x_{it}} &= \beta_{xm}\left(m_i - m_i^*\right). \end{aligned} \tag{6}$$

The derivative of TE with respect to managerial input corresponds exactly to the condition for monotonicity of production with respect to managerial ability shown in expression (2). Therefore, an increase in managerial ability increases TE given conventional inputs if the production function is monotonic in managerial ability. The derivative of TE with respect to the level of input use is negative if $\beta_{xm}$ is positive because $m_i$ is smaller than $m_i^*$ by definition. Therefore, when $\beta_{xm}$ is positive the increase in the use of conventional inputs decreases TE for a given amount of managerial ability.

In this section we have shown that the model in (1) raises interesting issues about the relationship between fixed management and technical efficiency. In particular, it shows that TE is not necessarily a fixed effect but instead can vary over time, and that the relationship between TE and managerial ability depends on the amount of managerial ability and usage of the conventional inputs. In the next section we discuss the estimation of this model in more general terms, with more than one observed input.

## 3. Estimation issues

The model in (1) cannot be directly estimated because the individual level of management is unobservable. Previous authors have dealt with this problem by introducing a proxy for management in a cost function. [See, e.g., Dawson and Hubbard (1985) and Alvarez and Arias (2003)]. Since the use of a proxy introduces new complexities (such as measurement error) into the model, we will employ a more direct approach to the problem which takes advantage of the panel nature of the data set we will analyze. We will now translate the model in the preceding section into an empirically estimable form.

### 3.1. Stochastic Frontier Model with Fixed Management

We consider production with $K$ inputs, $x_1 \ldots, x_K$. As before, let $m_i^*$ denote the level of management that defines the frontier and $m_i$ the actual management input for firm *i.* We continue to employ the translog form. The production model will then be

$$\begin{aligned}\ln y_{it} &= \ln y^*_{it} - u_{it} \\ &= \alpha + \sum_{k=1}^{K}\beta_k \ln x_{itk} + \tfrac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K}\beta_{kl}\ln x_{itk}\ln x_{itl} + \\ &\qquad \beta_m m^*_i + \tfrac{1}{2}\beta_{mm} m^{*2}_i + \sum_{k=1}^{K}\beta_{km}\ln x_{itk} m^*_i + v_{it} - u_{it}\end{aligned} \tag{7}$$

where $u_{it}$ corresponds to the standard definition of technical inefficiency, so that $TE_{it} = \exp(-u_{it})$.

For estimation, a critical assumption is the absence of correlation between $u_{it}$ and the input levels in (7). Therefore, it is important to show explicitly the definition of $u_{it}$ in the model:

$$\begin{aligned}u_{it} &= \ln y^*_{it} - \ln y_{it} \\ &= \left(\beta_m + \sum_{k=1}^{K}\beta_{km}\ln x_{kit}\right)\left(m^*_i - m_i\right) + \tfrac{1}{2}\beta_{mm}\left(m^{*\,2}_i - m^2_i\right) \geq 0.\end{aligned} \tag{8}$$

While $\ln x_{it}$ does appear in $u_{it}$ we assume that it does not influence ($m^*_i - m_i$). Thus, $u_{it}$ is of the form $a_i + \sum_{k}\left(m^*_i - m_i\right)\mathrm{g}\left(x_{kit}\right)$, and each term $\left(m^*_i - m_i\right)g\left(x_{kit}\right)$ will, by virtue of the presence of the freely varying ($m^*_i - m_i$) be uncorrelated with $x_{kit}$. We note, this is essentially the argument of Zellner et al. (1966), who argued that in a production function, the input levels would be uncorrelated with the deviation of output from the optimal output, even though they would obviously be correlated with the actual output, itself.

Although (7) involves an unobservable variable $m^*_i$, we can translate it into an empirically estimable form. The result follows from the fact that the unobservable can be seen as a 'random effect' in a panel data model. For that purpose, we rewrite (7) as follows:

$$\begin{aligned}\ln y_{it} &= \left(\alpha + \beta_m m^*_i\right) + \tfrac{1}{2}\beta_{mm} m^{*2}_i \\ &\quad + \sum_{k=1}^{K}\left(\beta_k + \beta_{km} m^*_i\right)\ln x_{itk} + \tfrac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K}\beta_{kl}\ln x_{itk}\ln x_{itl} + v_{it} - u_{it}.\end{aligned} \tag{9}$$

If we write the model in (9) in the form

$$\ln y_{it} = \alpha_i + \sum_{k=1}^{K} \beta_{km,i} \ln x_{itk} + \tfrac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K} \beta_{kl} \ln x_{itk} \ln x_{itl} + v_{it} - u_{it}$$
$$\alpha_i = \alpha + \beta_m m_i^* + \tfrac{1}{2}\beta_{mm} m_i^{*2}$$
$$\beta_{km,i} = \beta_k + \beta_{km} m_i^*$$

we find that it bears a superficial resemblance to the random parameters (RP) stochastic frontier model proposed in Greene (2005) and Tsionas (2002). It differs from a conventional RP model in several crucial respects. Most importantly, from a logical standpoint, the 'randomness' of the parameters in the specification is not a reflection of cross firm unobserved and unexplained heterogeneity as it is in an RP model; it is a specifically modeled unobserved factor of the production model. Thus, the random component is not an element of the parameter vector, it is a component of the 'regressor' vector. In practical terms, note that the random component in the structural model enters quadratically in the constant term and linearly in the 'slopes,' (and is absent from the second order terms) and, moreover, *the same* random element, $m_i$* enters all of the first order coefficients. Thus, as noted, the resemblance to a more familiar RP model is only coincidental; indeed, the model in (9) cannot be obtained as a special case of either Greene's or Tsionas's RP model.

The estimation of the model has two important requirements that remain to be considered. First, the random coefficients model requires as an identification condition that the random components of the coefficients be uncorrelated with the explanatory variables. The random component of the coefficients in our model is the level of management that defines the frontier ($m_i^*$), which likely *is* correlated with at least some of the inputs. (Note this is a different issue from the deviation of $m_i$ from $m_i^*$ which we argued above *is* uncorrelated with the inputs.) In order to avoid this problem, we take the approach suggested for random effect models by Chamberlain (1984) (and borrow from early work by Mundlak) and specify $m_i$* as a function of inputs in the following way:

$$m_i^* = \boldsymbol{\tau}'\overline{\mathbf{lnx}}_i + w_i \qquad (10)$$

where $\overline{\mathbf{lnx}}_i$ is the vector of the means of the logs of inputs, $\boldsymbol{\tau}$ is a vector of parameters to be estimated (a constant term will not be identified) and $w_i$ is a random term that follows a standard normal distribution and which we assume is uncorrelated with the inputs.

The second issue concerns the stochastic specification of $u_{it}$. Maximum likelihood estimation of the model in (9) requires a distributional assumption about $u_{it}$. In this paper, we model the distribution of $u_{it}$ as half normal, which produces a model that is logically a type of random coefficients stochastic frontier model in the spirit of Aigner et al. (1977).

### 3.2. Estimation of the Random Coefficients Stochastic Frontier Model

This section will describe the method used to estimate the parameters of the stochastic frontier model. From (7), we define

$$\varepsilon_{it} = v_{it} - u_{it} \tag{11}$$

In what follows, it is useful to note explicitly that $\varepsilon_{it}$ will be conditioned on $m_i$*. The conditional density for a single observation in the half normal stochastic frontier model is

$$f(\varepsilon_{it} \mid m_i*) = \frac{2}{\sigma}\phi\left(\frac{\varepsilon_{it} \mid m_i*}{\sigma}\right)\Phi\left(\frac{-\lambda\varepsilon_{it} \mid m_i*}{\sigma}\right) \tag{12}$$

where $\phi(z)$ and $\Phi(z)$ denote the density and CDF of the standard normal variable, respectively. The parameter $\lambda$ is the ratio $\sigma_u/\sigma_v$, while $\sigma^2=\sigma_v^2+\sigma_u^2$. [See Aigner et al., (1977)]. The joint density for $T$ observations on firm $i$ is

$$f(\varepsilon_{i1},\ldots,\varepsilon_{iT} \mid m_i*) = \prod_{t=1}^{T} f(\varepsilon_{it} \mid m_i*). \tag{13}$$

This is the contribution to the conditional likelihood for firm $i$, $L_i|m_i$*. The unconditional contribution to the likelihood function is

$$L_i = \int_{m_i^*}\prod_{t=1}^{T} f(\varepsilon_{it} \mid m_i*)g(m_i^*)dm_i^* \tag{14}$$

where $g(m_i*)$ is the marginal density of $m_i$*. Consistent with the preceding discussion, there are no new parameters in this density. The log likelihood is

$$\log L(\boldsymbol{\delta}) = \sum_{i=1}^{N}\log L_i(\boldsymbol{\delta}) \tag{15}$$

where we use $\delta$ to denote the full vector of parameters in the model. The maximum likelihood estimates of the parameters are obtained by maximizing (15) with respect to $\delta$. Since the integral in (14) will not have a closed form, it is not possible to maximize (15) directly. We will use the method of maximum simulated likelihood, instead. [See Train (2003), Greene (2003), Gourieroux and Monfort (1996) and the Appendix for discussion].

## 4. Empirical application

Our empirical application uses a balanced panel of 247 dairy farms located in Northern Spain. We have data on these farms for a period of six years (1993-1998). Since the farms are specialized in milk production we consider only one output. The variables used in the estimation of the production frontier are described in Table 1.

**Table 1. Variables Used in Production Analysis**

| | | Mean | Std. Dev. | Minimum | Maximum |
|---|---|---|---|---|---|
| Milk | Liters of milk production | 131,107 | 92538 | 14110 | 727,281 |
| Cows | Number of milking cows | 22.12 | 11.27 | 4.5 | 82.3 |
| Labor | Number of man-equivalent units | 1.67 | 0.55 | 1 | 4 |
| Land | Hectares of land | 12.99 | 6.17 | 2 | 45.1 |
| Feed | Kilograms of feedstuffs | 57941 | 47981 | 3924 | 376,731 |

We wish to explore the empirical consequences of restrictions on the role of management in the production function. For that purpose, we first estimate a conventional (pooled) stochastic production frontier with four inputs and including time-effects (this is equivalent to estimating equation (7) without considering the level of management at the frontier). The results of the estimation of the production frontier are given in the first column of estimates in Table 2. Since the explanatory variables in the original data were divided by their geometric mean, the first order coefficients can be interpreted as output elasticities evaluated at the geometric mean of the sample. They are positive and significantly different from zero at conventional levels of significance.

**Table 2. Estimated frontier models**

| *Variable* | *Param.* | *Stochastic Frontier* | *Random Coefficients Model* | |
|---|---|---|---|---|
| | | | *Means of Random Parameters,* $\beta_k$ | *Management* $\times$ *Inputs,* $\beta_{km}$ |
| *Cows* | $\beta_1$ | *0.6106* *(0.0223)*** | *0.6597* (0.0113)** | *-0.0130* *(0.0067)*** |
| *Land* | $\beta_2$ | *0.0254* *(0.0128)*** | *0.0406* (0.0081)** | *0.0167* (0.0046)** |
| *Labor* | $\beta_3$ | 0.0239 (0.0146)** | *0.0251* (0.0013)* | *-0.0173* (0.0053)** |
| *Feed* | $\beta_4$ | 0.4393 (0.0121)** | *0.3090* (0.0060)** | *0.0103* (0.0040)** |
| *Constant* | $\alpha$ | *11.719* *(0.0128)*** | *11.6761* (0.0041)** | |
| *Management* | $\beta_m$ | | | *0.1083* *(0.0014)*** |
| *Management x Management* | $\beta_{mm}$ | | | *-0.0143* *(0.0018)*** |
| *Cows* $\times$ *Cows* | $\beta_{11}$ | *0.5556* *(0.1158)*** | *0.1512* (0.0479)** | |
| *Land* $\times$ *Land* | $\beta_{22}$ | *-0.1042* *(0.0457)*** | -0.0425 (0.0173)** | |
| *Labor* $\times$ *Labor* | $\beta_{33}$ | *0.2116* *(0.0371)*** | -0.1275 (0.0334)** | |
| *Feed* $\times$ *Feed* | $\beta_{44}$ | *-0.01150* *(0.0964)* | 0.0962 (0.0151)** | |
| *Cows* $\times$ *Land* | $\beta_{12}$ | *-0.1078* *(0.0507)*** | -0.0186 (0.0209) | |
| *Cows* $\times$ *Labor* | $\beta_{13}$ | *-0.3390* *(0.0638)*** | 0.0936 (0.0241)** | |
| *Cows* $\times$ *Feed* | $\beta_{14}$ | *0.2230* *(0.0753)*** | -0.0780 (0.0258)** | |
| *Land* $\times$ *Labor* | $\beta_{23}$ | *0.0694* *(0.0263)*** | *0.0003* (0.0148) | |
| *Land* $\times$ *Feed* | $\beta_{24}$ | *-0.0027* *(0.0500)* | *-0.0186* (0.0108) | |
| *Labor* $\times$ *Feed* | $\beta_{34}$ | *-0.0695* *(0.0411)* | *-0.0074* (0.0143) | |
| *Year 1994* | $\delta_{94}$ | *-0.0363* *(0.0120)*** | *0.0315* (0.0044)** | |
| *Year 1995* | $\delta_{95}$ | *-0.0198* *(0.0126)* | *0.0578* (0.0044)** | |
| *Year 1996* | $\delta_{96}$ | *-0.0013* *(0.0117)* | *0.0675* (0.0044)** | |
| *Year 1997* | $\delta_{97}$ | *-0.0067* *(0.0121)* | *0.0715* (0.0043)** | |
| *Year 1998* | $\delta_{98}$ | *-0.0110* *(0.0125)* | *0.0902* (0.0044)** | |
| $\lambda$ | | *1.8657* *(0.1521)*** | *1.1994* *(0.0490)*** | |
| $\sigma$ | | *0.1932* *(0.0061)*** | *0.0961* *(0.0014)*** | |
| *Log L* | | *860.649* | *1401.562* | |

Standard errors in parentheses.

We now estimate the random coefficients model specified in equation (9) following the estimation procedure described previously. The results are given at the right in Table 2.

The means for the random parameters remain positive and significant at the geometric mean of the sample. These means differ slightly from the coefficients of the conventional stochastic frontier. The coefficients of management ($\beta_m$, $\beta_{mm}$ and $\beta_{km}$) are significantly different from zero at conventional levels of significance. This can be interpreted as evidence in favor of the random coefficients model with respect to the conventional stochastic frontier approach since the coefficients of the production function change with the level of management of the farm. The log likelihood function for the management model is far larger than that for the restricted model; the likelihood ratio statistic of roughly 280 is far larger than the critical value for 6 degrees of freedom. Thus, the restricted model is rejected on this basis.

The first order parameters of the translog production function ($\beta_k$'s) can be interpreted as output elasticities evaluated at the geometric sample mean provided that the original data were divided by the geometric sample mean. This is the case in our empirical application. In turn, a positive $\beta_m$ can be seen as evidence of a positive effect of managerial ability in production.

The second order parameters of the translog production function ($\beta_{kl}$'s, $\beta_{km}$'s and $\beta_{mm}$) do not have a direct economic interpretation. However, they affect the value of relevant economic effects, such as the following:

a) The marginal product of management:

$$\frac{\partial y_{it}^*}{\partial m_i^*} = e_{mit} y_{it}^*$$
$$\text{where } e_{mit} = \frac{\partial \ln y_{it}^*}{\partial m_i^*} = \beta_m + \beta_{mm} m_i^* + \sum_{l=1}^{K} \beta_{lm} \ln x_{itl} \tag{16}$$

b) The effect of managerial ability on the marginal product of management:

$$\frac{\partial \left( \dfrac{\partial y_{it}^*}{\partial m_i^*} \right)}{\partial m_i^*} = \left( \beta_{mm} + e_{mit}^2 \right) y_{it}^* \tag{17}$$

c) Output elasticities at the frontier

$$e_{kit} = \frac{\partial \ln y_{it}^{*}}{\partial \ln x_{itk}} = \beta_k + \sum_{l=1}^{K} \beta_{kl} \ln x_{itl} + \beta_{km} m_i^{*} \tag{18}$$

d) The effect of managerial ability on the marginal product of inputs

$$\frac{\partial \left( \frac{\partial y_{it}^{*}}{\partial x_{itk}} \right)}{\partial m_i^{*}} = \left( \beta_{km} + e_{mit} e_{kit} \right) \frac{y_{it}^{*}}{x_{itk}} \tag{19}$$

The computation of these relevant economic effects requires an estimate of $m_i^{*}$. We develop such estimate below.

The efficiency levels can be computed according to Jondrow et al.'s (1982) prescription conditioned on $m_i$*.

$$E\left[ u_{it} \mid \varepsilon_{it}, m_i * \right] = \frac{\sigma\lambda}{(1+\lambda^2)} \left[ \frac{\phi(-(\varepsilon_{it} \mid m_i *)\lambda / \sigma)}{\Phi(-(\varepsilon_{it} \mid m_i *)\lambda / \sigma)} - \frac{(\varepsilon_{it} \mid m_i *)\lambda}{\sigma} \right] \tag{20}$$

Like other quantities that involve $m_i$*, this can be estimated by simulation. We have, instead, computed estimates of $m_i$* as described below, then computed the values in (20) conditioned on the estimates of $m_i$* and the other data for farm *i*. The value of $m_i$* can be computed from the conditional distribution of $m_i$* given the data on farm *i* using Bayes theorem as follows: Let $\mathbf{y}_i$ denote the $T\times1$ vector of logs of the outputs for farm *i* for the six years. Let the $T\times K$ matrix $\mathbf{X}_i$ denote the other data (inputs and year dummy variables, linear and quadratic terms in logs) for farm *i*. The conditional distribution of $m_i$* given $\mathbf{y}_i$ is

$$\begin{aligned} f(m_i^{*} \mid \mathbf{y}_i, \mathbf{X}_i) &= \frac{f(\mathbf{y}_i \mid m_i^{*}, \mathbf{X}_i) g(m_i^{*})}{f(\mathbf{y}_i \mid \mathbf{X}_i)} \\ &= \frac{f(\mathbf{y}_i \mid m_i^{*}, \mathbf{X}_i) g(m_i^{*})}{\int_{m_i^{*}} f(\mathbf{y}_i \mid m_i^{*}, \mathbf{X}_i) g(m_i^{*}) dm_i^{*}} \end{aligned} \tag{21}$$

The denominator is the contribution of farm *i* to the likelihood function for the sample (not the log likelihood) in equation (14). Thus, we can estimate $m_i$* for farm *i* as the conditional mean from this distribution. This would be

$$\hat{m}_i{*} = \hat{E}(m_i^* \mid \mathbf{y}_i, \mathbf{X}_i) = \frac{\int_{m_i^*} m_i^* f(\mathbf{y}_i \mid m_i^*, \mathbf{X}_i) g(m_i^*) dm_i^*}{\int_{m_i^*} f(\mathbf{y}_i \mid m_i^*, \mathbf{X}_i) g(m_i^*) dm_i^*} \tag{22}$$

Like the likelihood, itself, this quantity cannot be computed directly, as the integrals will not have a closed form. But, they can be simulated, in the same fashion. Thus, the simulation based estimator of $m_i$* is

$$\ldots\ldots\ldots\ldots\ldots\ldots.. \hat{E}(m_i^* \mid \mathbf{y}_i, \mathbf{X}_i) = \frac{(1/R)\sum_{r=1}^{R} m_{i,r}^* \hat{f}(\mathbf{y}_i \mid m_{i,r}^*, \mathbf{X}_i)}{(1/R)\sum_{r=1}^{R} \hat{f}(\mathbf{y}_i \mid m_{i,r}^*, \mathbf{X}_i)}. \tag{23}$$

where $m_{i,r}$* is a draw from the population of $m_i$*. Note, $\hat{f}$ denotes the contribution to the likelihood function for farm *i*, evaluated at the parameter estimates and the current draw of $m_{i,r}$*. Draws on $m_i$* are obtained by drawing $w_i$ in (10) from the standard normal distribution using Halton sequences. With these estimates in hand, estimated inefficiencies for the farms are produced using (20).

### 4.1 Discussion

Our model allows for a complex role of management in the production function. For example, it is possible to gauge the effects of management in the technological characteristics of the frontier. In this section we will calculate the main characteristics of interest developed in the preceding sections, which correspond to the marginal effects contained in equations (16) – (19). Since $m_i$* enters all of these effects, we estimate it using equation (23). The results are shown in Table 3 below.

**Table 3. Descriptive Statistics for Estimated Management ($m_i$*)**

| | Mean | Standard Deviation | Minimum | Maximum |
|---|---|---|---|---|
| Frontier Management, $m_i$* | 0.0634 | 1.0799 | -2.8240 | 3.4772 |

Since $m_i$* is unit free, these values do not have a direct interpretation relative to any observed quantities. However, as mentioned above, they are necessary to compute the marginal effects. We have computed those effects for each data point and the results show the expected signs. The marginal product of management in (16) is positive for all observations. Evaluated at the mean estimated value of $m_i$* of 0.0634 and the geometric means of the inputs, this is roughly equal to 14500. While it is difficult to be specific about the margin involved – we could not state specifically what it would mean to increase management by one unit – this result does suggest that in the terms of the unobserved factor, $m_i^*$ is clearly important in the determination of output. At the same time, the second derivative of output with respect to $m_i^*$ in equation (17) is negative for all observations, indicating the existence of decreasing returns to management. Finally, an increase in $m_i^*$ increases the marginal products of all conventional inputs. The positive but decreasing marginal effect on output and positive effects on the marginal product of conventional (observed) inputs of $m_i^*$ can be seen as empirical evidence supporting the interpretation of this latent variable as an indicator of an unobserved input, namely managerial ability.

We have also calculated the set of output elasticities at the frontier for each of the inputs. The results are shown in Table 4 evaluated at the geometric mean of the inputs and estimated mean of $m_i^*$.

**Table 4. Output elasticities at the frontier**

| | Output elasticity at the frontier |
|---|---|
| Cows | 0.6344 |
| Land | 0.0408 |
| Labor | 0.0229 |
| Feed | 0.3095 |
| RTS | 1.0076 |

The sum of output elasticities for the conventional factors shown in Table 4 is a measure of returns to scale to conventional inputs. The measure depends on the sum of the parameters $\beta_{km}$, which by our estimates is extremely small since these parameters take on positive and negative values. Therefore, in terms of the observed inputs, management plays a very small role in explaining returns to scale. However, when management is considered directly as an input, it adds 0.1059, or about 11% to the total. This result suggests that when management is considered among the factors of production, at least for this application, there is a moderate degree of increasing returns to scale. We do note that the preceding results are in terms of management at the frontier, $m_i^*$, not actual management, $m_i$. It seems reasonable to assume that these would at least be highly positively correlated, so at least the qualitative result would persist.

Table 5 presents descriptive statistics for the inefficiency estimates produced from the basic stochastic frontier model and from the random coefficients stochastic frontier model. The distribution for the latter set of estimates has a much smaller mean and a much tighter distribution. Figure 1 below suggests the same pattern. This suggests that not accounting for the effect of management on production has somewhat inflated the estimated inefficiency. One might view this as a decomposition of the inefficiency into two parts, one explicitly accounted for by the management effect and the other apparently due to other unexplained factors.

**Table 5. Descriptive Statistics for Estimated Inefficiencies**

| | Mean | Standard Deviation | Minimum | Maximum |
|---|---|---|---|---|
| Basic Stochastic Frontier | 0.1352 | 0.0794 | 0.0107 | 0.4747 |
| Fixed Latent Management SF Model | 0.0581 | 0.0256 | 0.0116 | 0.2637 |

In summary, the results of the present paper show that management plays a complex role in production. As a result, the conventional production frontier with nonrandom coefficients (where management enters as a shifter) might not be a good instrument to analyze firm behavior when management is unobservable. This result is important in a number of settings. For example, unobserved management is a key factor in explaining firm size and growth (Jovanovic, 1982). Another example is farm policy, where the level of management is important for assessing the effects of increasing the size of farms (Alvarez and Arias, 2003). This is an important issue for policy purposes since in the agricultural sector it is common to implement farm policies oriented towards farm growth. These policies usually consist of low interest loans that allow farmers to buy more inputs and therefore disregard the important implications of holding management constant, as shown in previous sections.

## 5. Conclusions

This paper explores the relationship between managerial ability and technical efficiency. For that purpose, fixed managerial ability is introduced as an unobservable input in a translog production function. The interaction between the unobservable input and conventional inputs implies that technical efficiency depends not only on management but also on input use. As a result, fixed management can lead to time variant technical efficiency if input use changes over time. This is an important insight not considered in models with time invariant efficiency based in the assumption of fixed management.

The interaction between the unobservable input and conventional inputs creates a great deal of difficulty in estimating the resulting model. However, the model can be cast as a stochastic frontier with random coefficients. The unobserved variable is integrated out leading to a likelihood function with definite integrals that do not have a closed form. This is the reason why the model is estimated by maximum simulated likelihood.

We illustrate the feasibility of the proposed estimation procedure using a sample of dairy farms in Northern Spain. The mean of the random coefficients are of similar magnitude as the coefficients of a standard stochastic frontier, but managerial ability is found to affect the value of a number of random input coefficients. As a result, the random coefficients frontier provides a measure of technical efficiency that can be related with different levels of management depending on the circumstances of the farm (input use and output production). The feasibility of estimating the level of management that defines the production frontier is a clear advantage of our model.

The empirical model developed in the paper can be useful in analyzing firm policy issues since management is considered a key variable in assessing the effects of these policies. In fact, the model avoids treating management as a mere shifter of the production function, as in the conventional stochastic frontiers, and the econometric problems associated with the use of proxies.

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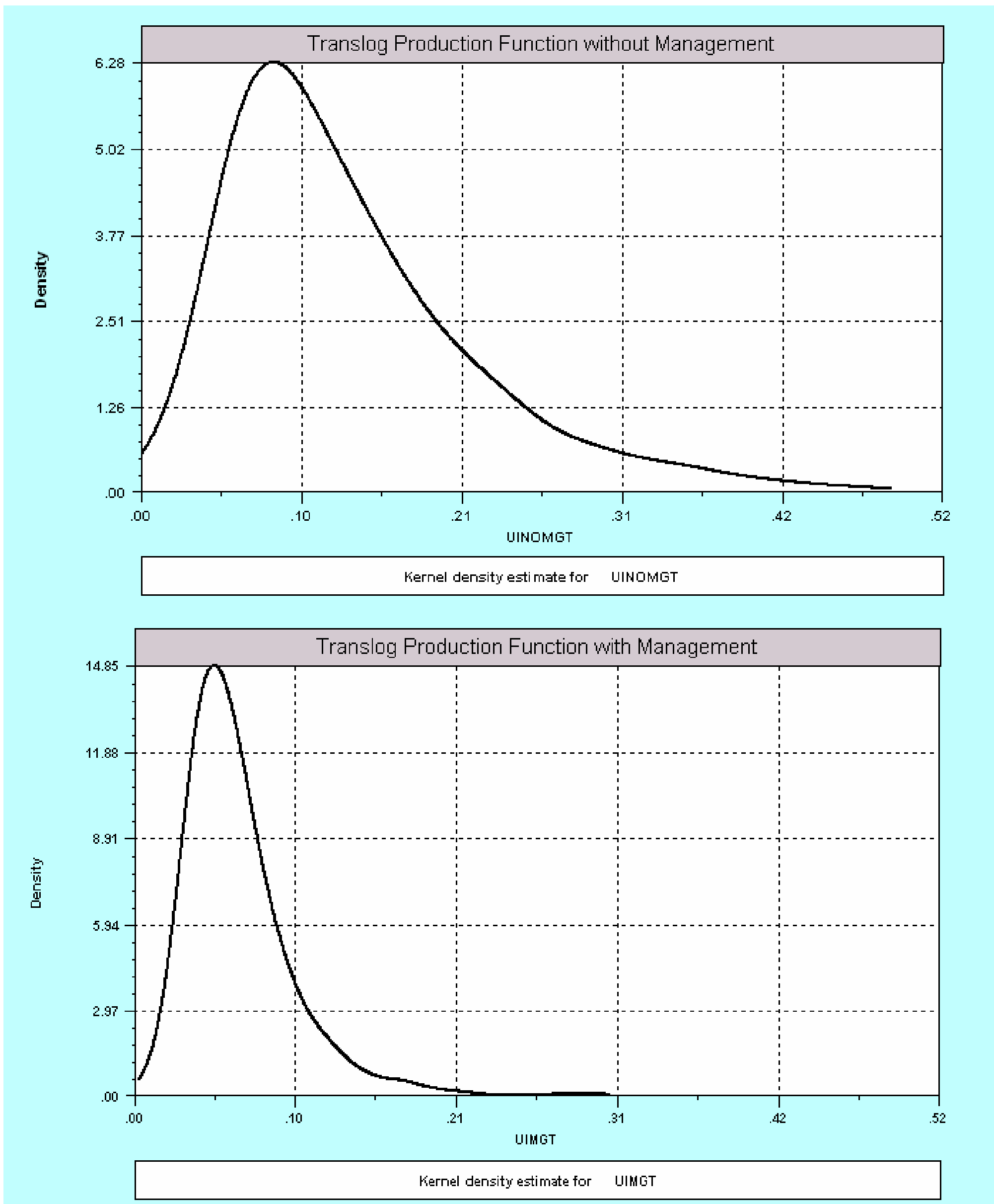


**Figure 1. Kernel density estimates for distribution of inefficiency across firms**

**APPENDIX.**

Let $m_{ir}$* denote the $r$-th random draw from the standard normal population of $m_i$* in a sample of $R$ such draws. Then, the contribution to the *simulated* likelihood function for firm $i$ is

$$L_i^S = \frac{1}{R}\sum_{r=1}^{R} \prod_{t=1}^{T} f(\varepsilon_{it} \mid m_{ir}^*) \qquad \text{(A.1)}$$

The simulated log likelihood that is maximized is

$$\log L^S(\boldsymbol{\delta}) = \sum_{i=1}^{N} \log\left[\frac{1}{R}\sum_{r=1}^{R}\prod_{t=1}^{T} f(\varepsilon_{it} \mid m_{ir}^*)\right] \qquad \text{(A.2)}$$

where $f(\varepsilon_{it}|m_i^*)$ appears in (12). This function is smooth and twice continuously differentiable in the parameters. (For conditions under which maximization of the simulated log likelihood produces an estimator with the same asymptotic properties as the true MLE, see Gourieroux and Monfort (1996), Train (2003) and Greene (2003)). The derivatives of the simulated log likelihood are obtained as follows:

$$\begin{aligned}\frac{\partial \log L_i^S}{\partial \boldsymbol{\delta}} &= \frac{\partial L_i^S / \partial \boldsymbol{\delta}}{L_i^S} \\ &= \frac{\tfrac{1}{R}\Sigma_{r=1}^{R}\left(\Pi_{t=1}^{T} f(\varepsilon_{it} \mid m_{ir}^*)\right)\Sigma_{t=1}^{T}\left[\partial \log f(\varepsilon_{it} \mid m_{ir}^*)/\partial \boldsymbol{\delta}\right]}{\tfrac{1}{R}\Sigma_{r=1}^{R}\left(\Pi_{t=1}^{T} f(\varepsilon_{it} \mid m_{ir}^*)\right)} \\ &= \sum_{r=1}^{R}\omega_{ir}\sum_{t=1}^{T}\mathbf{g}_{it}(m_{ir}^*) \\ &= \sum_{r=1}^{R}\omega_{ir}\mathbf{g}_{i}(m_{ir}^*)\end{aligned} \qquad \text{(A.3)}$$

where $\omega_{ir}$ is a set of nonnegative weights that sum over $r$ to one for each $i$ by construction and $\mathbf{g}_{it}(m_{ir}^*)$ is the vector of derivatives, $\partial \log f(\varepsilon_{it} \mid m_{ir}^*)/\partial \boldsymbol{\delta}$. Let

$\mathbf{x}_{itr}^* = [1,\ldots,\ln x_{itk},\ldots,\ldots, ½ \ln x_{itk}\ln x_{itl},\ldots,m_{ir}^*, ½m_{ir}^{*2},\ldots, ½\ln x_{itk}m_{ir}^*]$

so that $\varepsilon_{it}|m_{ir}^* = y_{it} - \boldsymbol{\beta}'\mathbf{x}_{it}(m_{ir}^*) = \varepsilon_{itr}^*$. For convenience, let $h_{itr}^* = \phi(-\lambda\varepsilon_{itr}^*/\sigma)/\Phi(-\lambda\varepsilon_{itr}^*/\sigma)$. The required first derivatives are

$$\partial \log f(\varepsilon_{itr}^*)/\partial\begin{pmatrix}\boldsymbol{\beta}\\ \sigma\\ \lambda\end{pmatrix} = \frac{1}{\sigma}\begin{bmatrix}\{(\varepsilon_{itr}^*/\sigma)+h_{itr}^*\lambda\}\mathbf{x}_{itr}^* \\ (\varepsilon_{itr}^*/\sigma)^2 - 1 + (\varepsilon_{itr}^*/\sigma)\lambda h_{itr}^* \\ -\varepsilon_{itr}^* h_{itr}^*\end{bmatrix} \qquad \text{(A.4)}$$

The integrals in (14) and its derivatives are approximated by obtaining a sufficient number of draws from the population generating $m_i$*. The law of large numbers is invoked to infer that the sample averages will converge to the underlying integral. Random draws from the population are sufficient for this process, but not necessary. What is essential is coverage of the range of variation of $m_i$*, not randomness of the draws. The method of Halton sequences [see Bhat (1999), for example] is used to provide much more efficient coverage of the range, and in turn, much faster estimation than the method of random simulation. [See, as well, Train (2003, pp. 224-238) for a discussion of Halton sequences]. Thus, $m_{ir}$* is the $r$th element of the Halton sequence for individual *i.* The elements of the Halton sequence, $H_{ir}$ are spread over the unit interval, (0,1). The draw of $m_{ir}$* is obtained by the inverse probability transform. Thus, $m_{ir}^* = \Phi^{-1}(H_{ir})$. The estimated standard errors of the parameter estimators are computed by using the BHHH estimator, as before, with simulation used for the derivative vectors. Since we are only integrating over a single dimension, the gain in efficiency, if this application is like others, is on the order of ten fold - that is, the same results are obtained with only about one tenth the number of draws needed. We have used 1,000 draws in our estimation, which would correspond to several thousand draws were they produced with a random number generator instead.

Computations were done using LIMDEP 9.0 (Econometric Software, Inc., www.limdep.com).